Algebraic continued fractions : the contribution of R. de Montessus de Ballore (1870-1937)

Hervé Le Ferrand

Université de Bourgogne, Dijon, France

October 10-14, 2011

1 Who was Robert de Montessus de Ballore ?

- Photographies
- Biography

2 Algebraic continued fractions

- Articles by Robert de Montessus
- Context
- Genesis of his famous theorem (1902)
- Circulation of the theorem of 1902
- Other works

3 Conclusion

Photographies Biography

Photographies



Figure: Robert de Montessus in 1914



Figure: Robert de Montessus about 1930

同 ト イ ヨ ト イ ヨ ト

Photographies Biography

Photographies



Figure: Robert de Montessus and his friend the mathematician Robert d'Adhémar in front of the Lille Catholic University in 1905.

Robert de Montessus : biography

- Robert was born in Villeurbanne (Lyon) on May 20, 1870.
- Robert's mother was a grand-daughter of Philibert de Commerson, naturalist of the Bougainville's expedition and member of the french Academy of Sciences.
- Robert was the youngest among the four children of the couple. The oldest child was Fernand (1851-1923), graduated of the Ecole Polytechnique, the famous seismologist.
- He obtained his baccalaureate in 1886. He continued his education at Saint Etienne during two years. Unfortunately, his parents were on the road to ruin. He joined the french army, wich he left in 1893 and took a job at the *Compagnie des Chemins de fer de Paris à Lyon et à la Méditerranée*.
- During the years 1895-1902, he teached in different secondary schools (Evreux, Yzeure, Senlis).

Photographies Biography

Robert de Montessus : biography



Figure: Fernand de Montessus in 1871



Figure: Fernand de Montessus, Chili, first chief executive of the *Servicio Sismolgico de Chile* 1908-1923

- A - B - M

Robert de Montessus : biography

- During his academic year 1898-1899, he followed at the Sorbonne the lectures of Paul Appell, Gaston Darboux and Emile Picard. He obtained his *Licence es Sciences* the 24 October 1901.
- In 1902, his friend, the french mathematician Robert d'Adhémar helped him in joining the Lille Catholic University.
- Robert de Montessus defended his thesis on 8 May 1905 (the examiners were being his supervisor Paul Appell, together with Henri Poincaré and Edouard Goursat.) In the first part of the thesis, Robert de Montessus dealt with different problems of convergence of algebraic continued fractions. The second part was on probabilities (errors theory).
- Robert married Suzanne Montaudon (1884-1983) in 1906 (their daughter Simone born in 1907 is dead in March 2011).

Photographies Biography

Robert de Montessus : biography



Figure: subject of la seconde thèse, signature of Paul Appell

(日) (同) (日) (日) (日)

Robert de Montessus : biography

- In 1906, the subject proposed for the Grand Prix of the Paris Academy of 1906 was on the convergence of algebraic continued fractions. The Grand Prix was awarded to Robert de Montessus, Henri Padé and A. Auric.
- During World War I, he gave some lectures at the Sorbonne on elliptic functions, algebraic space curves particularly. In 1917, Robert de Montessus entered the team redaction of the *Journal de Mathématiques Pures et Appliquée* directed by Camille Jordan. The same year Robert de Montessus was employed in ballistics by the french government.
- After 1919, Robert devoted much attention to the theory of probabilities and applied statistics. In 1924, he was appointed searcher at the french National Office of Meteorology at Paris.
- In 1919, Robert de Montessus created the Index Generalis.

Photographies Biography

Robert de Montessus : biography

INSTITUT DE FRANCE ACADÉMIE DES SCIENCES Prais le 8 Diumber 1906 Les Secrétaires perpétuels de l'Académie/ à Monsieur R. de Montessus, à Selle Monsieur - Mensieur, Vous avens l'honneur de vous imfermer que l'Académie des Sciences vous a décerne une partie du Grand fin de Science mathim a tiques . Nous saisissons eithe occasion de vous offier nos filicitations personnelles et de vous témoigner l'intérêt que l'Académie prend à vos travaux et à vos succes Des billets, pour assister à la Léance publique de l'Académie, qui aura lieu le lundi 14 Oricembre, vous seront délivrés si vous voulez bien en faire la demande à Monsieur le Chef du Secrétariat. Veuillez agréer, Monsieur, l'assurance de notre consideration la plus distinguée. Darboux Berthelot

Figure: Grand prix de l'Académie des Sciences

A B M A B M

< 67 ▶

Photographies Biography

Robert de Montessus : biography

JOURNAL MATHÉMATIQUES

PURES ET APPLIQUÉES

FONDE EN 1816 ET PUBLIE JUSQU'EN 1874 Par Joseph LIQUVILLE

PUBLIE DE 1875 A 1814 Par H. RESAL

SEPTIÈNE SÉRIE

.....

PA CARLE JOINS

A REALE AND A MARKED AND A REALEST AND A REAL AND A

TOME QUATRIÈME - ANNÉE 1918.

dP Volume de la Collection)

PARIS

GAUTHIEN-VILLARS ET O', ÉDITEURS LIBRARS DE REGER DE LOSSITURS, DE L'ÉCOLE POLYTORINGE Qui des Grand-Augustins, 33

1918

Figure: JMPA 1918

SC2011 Hervé Le Ferrand Robert de Montessus de Ballore 1870-1937

э

Photographies Biography

Robert de Montessus : biography

About twenty years ago, de Montessus undertook the publication of the "Index Generalis", an annual reference work now well known throughout the scientific world and of inestimable value to every investigator. It is hard to conjecture the number of practical difficulties which de Montessus had to overcome in organizing this immense mass of data on the universities and learned societies of the world ; the scientific qualities of which he had given evidence elsewhere came to his aid here.

Henri Villat, in Nature, pp 226-227, n. 3536, vol 140, 1937

Mathematicals papers

Robert has written 57 papers about Mathematics -but also 13 books and numerous papers in journals like L'intermédiaire des mathématiciens (founded by C.A. Laisant), L'enseignement mathématique (founded by C.A Laisant and H. Fehr), La revue du mois (founded by E. Borel and his wife, the writter Camille Marbo, Appell's daughter.

Mathematicals papers

- Robert has written 57 papers about Mathematics -but also 13 books and numerous papers in journals like L'intermédiaire des mathématiciens (founded by C.A. Laisant), L'enseignement mathématique (founded by C.A Laisant and H. Fehr), La revue du mois (founded by E. Borel and his wife, the writter Camille Marbo, Appell's daughter.
- among those above, 11 papers deal with continued fractions. The period is quite short, 1897-1909.

Mathematicals papers

- Robert has written 57 papers about Mathematics -but also 13 books and numerous papers in journals like L'intermédiaire des mathématiciens (founded by C.A. Laisant), L'enseignement mathématique (founded by C.A Laisant and H. Fehr), La revue du mois (founded by E. Borel and his wife, the writter Camille Marbo, Appell's daughter.
- among those above, 11 papers deal with continued fractions. The period is quite short, 1897-1909.
- thesis : Sur les fractions continues algébriques. Palermo Rend., 1905.

・ 戸 ト ・ ヨ ト ・ ヨ

Mathematicals papers

- 1897 Développement en fractions continues périodiques d'ordre supérieur des racines des équations quelconques. Brux. S. Sc.
- 1902 Sur les fractions continues algébriques. C. R. 134, 1489-1491.
- 1902 Sur les fractions continues algébriques. Bull. Soc. Math. Fr.
- 1903 Sur la convergence de certaines fractions continues algébriques. Brux. S. Sc.
- 1904 Sur la représentation des fonctions par des suites de fractions rationnelles. C. R. 138, 471-474.
- 1905 Sur les fractions continues algébriques. C. R. 139, 846-848.
- 1905 Sur les fractions continues algébriques de Laguerre. C. R. 140.
- 1908 Sur les fractions continues algébriques. Ann. de l'Ec. Norm.
- 1909 Recherche effective des racines réelles des séries hypergéométriques. Bull. Soc. Math. Fr.
- 1909 Les fractions continues algébriques. Acta Math. 32, 257-281.

Articles by Robert de Montessus Context Genesis of his falous theorem (1902) Circulation Other works

Context

Why did Robert de Montessus work on the area of Algebraic Continued Fractions ?

э

Context

Why did Robert de Montessus work on the area of Algebraic Continued Fractions ?

• The supervisor of his thesis is Paul Appell (Robert met Appell thanks to C.A Laisant in 1900).

▲ □ ▶ ▲ □ ▶ ▲ □ ▶

Context

Why did Robert de Montessus work on the area of Algebraic Continued Fractions ?

- The supervisor of his thesis is Paul Appell (Robert met Appell thanks to C.A Laisant in 1900).
- Algebraic contitued fractions are an important subject of researchs at the period.

Context

Why did Robert de Montessus work on the area of Algebraic Continued Fractions ?

- The supervisor of his thesis is Paul Appell (Robert met Appell thanks to C.A Laisant in 1900).
- Algebraic contitued fractions are an important subject of researchs at the period.
- Algebraic continued fractions are connected to the problem of *Analytic continuation*. Particularly, Robert de Montessus knew the works of Hadamard and Fabry on that question.

・ 戸 ト ・ ヨ ト ・ ヨ

Articles by Robert de Montessus Context Genesis of his falous theorem (1902) Circulation Other works

Context



Source gallica.bnf.fr / Bibliothèque nationale de France

Figure: Paul Appell

SC2011 Hervé Le Ferrand Robert de Montessus de Ballore 1870-1937

э

<ロト <部ト < 注ト < 注ト

Articles by Robert de Montessus **Context** Genesis of his falous theorem (1902) Circulation Other works

Context

1" 5 the rest fin at the Star plus giveraure en fastant des hypothèse particulines sur la nature des ponits Mais una fonction sent avoir une infinite de poles sans prendre la infini sur la montérence de convergen Come précédente . Far exemple e (on infuriment gies), il mattemble $f(x) = 5 \frac{A_{v}}{(A_{v} - x)(A_{v} - x)} = 5 \frac{A_{v}}{(A_{v} - x)(A_{v} - x)}$ diffinile d'affirmer que le théorème est toryours exact, car il y a au moin ladri 5 (to) converge ; at it set un point singulier essentiel, limite des also, Casandant il va des cas ascos 2 ma indivite St polo Saigelos puise generaux où la demonstration 1'analique ancore . L' la fonction point se mettre, par evenyite, sous à priori cele ne parait pas impossible Li vous voulez avoi une ille B des travais publics en losponits singul (B-x)=+ = 0X-16 se & since so fatelor, vous sources le sirie E/A) étant convergentes trouver des renergements this complet IAI=9, 1×129 main terrent vero g dans un potit traite public par Quadors Haitemant au mois de mai dernier S = B = + + E Ar Large serie de lagtor at son prologans

(letter of January the 7th 1902.) [...] If you want to have an overview on the published research on Taylor series, you can find a lot of thorough informations on "Tuylor's series and its analytical continuation" in the small paper published the previous month of May by Mr. Hadamard in Scienta's collection.

< 日 > < 同 > < 三 > < 三 >

Figure: letter (extract) of Eugène Fabry, January 7th, 1902

Padé's approximants and continued fractions

 $f(z) = \sum_{i=0}^{+\infty} a_i z^i$, the Padé's approximant [L/M] de f is a rational fraction of type (L,M) such that $f(z) - [L/M](z) = O(z^{L+M+1})$.

Example

The Padé's approximant [3/4] of exp is
$$\frac{1+\frac{3}{7}z+\frac{1}{14}z^2+\frac{1}{210}z^3}{1-\frac{4}{7}z+\frac{1}{7}z^2-\frac{2}{105}z^3+\frac{1}{840}z^4}$$

伺 ト イ ヨ ト イ ヨ ト

Padé's approximants and continued fractions

 $f(z) = \sum_{i=0}^{+\infty} a_i z^i$, the Padé's approximant [L/M] de f is a rational fraction of type (L,M) such that $f(z) - [L/M](z) = O(z^{L+M+1})$.

Example

The Padé's approximant [3/4] of exp is
$$\frac{1+\frac{3}{7}z+\frac{1}{14}z^2+\frac{1}{210}z^3}{1-\frac{4}{7}z+\frac{1}{7}z^2-\frac{2}{105}z^3+\frac{1}{840}z^4}$$

Padé's table : a sequence of approximants "well" chosen gives an algebraic continued fractions whose convergents are rightly the approximants.

Example

$$exp(z) = 1 + \frac{z}{1 + \frac{z}{-2 + \frac{z}{-3 + \frac{z}{2 + \dots}}}}$$
. The convergents are the Padé's approximants $[n/n]$ or $[n + 1/n]$ of exp

The theorem of 1902

• Thanks to Appell, Robert corresponded with Padé since 1901.

・ 同 ト ・ ヨ ト ・ ヨ ト

э

The theorem of 1902

- Thanks to Appell, Robert corresponded with Padé since 1901.
- the result of Robert de Montessus in *Sur les fractions continues algébriques*, Bulletin de la SMF, tome 30 (1902), pp 28-36.

▲ □ ▶ ▲ □ ▶ ▲ □ ▶

The theorem of 1902

- Thanks to Appell, Robert corresponded with Padé since 1901.
- the result of Robert de Montessus in *Sur les fractions continues algébriques*, Bulletin de la SMF, tome 30 (1902), pp 28-36.

•
$$f(x) = \sum_{0}^{+\infty} s_n x^n$$
 analytic at zero with poles :
 $|\alpha_1| \le |\alpha_2| \le \cdots \le |\alpha_p| < |\alpha_{p+1}| \le |\alpha_{p+2}| \le \cdots$.

▲ □ ▶ ▲ □ ▶ ▲ □ ▶

,,

- 同 ト - ヨ ト - - ヨ ト

The theorem of 1902

- Thanks to Appell, Robert corresponded with Padé since 1901.
- the result of Robert de Montessus in *Sur les fractions continues algébriques*, Bulletin de la SMF, tome 30 (1902), pp 28-36.

(Padé's table : row of rank p).

. . .

Articles by Robert de Montessus Context Genesis of his falous theorem (1902) Circulation Other works

The theorem of 1902

- Thanks to Appell, Robert corresponded with Padé since 1901.
- the result of Robert de Montessus in *Sur les fractions continues algébriques*, Bulletin de la SMF, tome 30 (1902), pp 28-36.

•
$$f(x) = \sum_{n=0}^{+\infty} s_n x^n$$
 analytic at zero with poles :

 $|\alpha_1| \le |\alpha_2| \le \dots \le |\alpha_p| < |\alpha_{p+1}| \le |\alpha_{p+2}| \le \dots$

- "ligne horizontale d'un Tableau de fractions normales " (Padé's table : row of rank p).
- "la fraction continue déduite (...) représente la fonction f(x) dans un cercle de rayon $|\alpha_{p+1}|$..." (uniform convergence of the convergents on compact sets of domain obtained from $|z| < |\alpha_{p+1}|$ by deleting the poles.)

イロト イポト イラト イラト

The article at the Bulletin de la SMF

• a short text, very dense.

伺 ト く ヨ ト く ヨ ト

э

The article at the Bulletin de la SMF

- a short text, very dense.
- Robert de Montessus went to the gist.

同 ト イ ヨ ト イ ヨ ト

The article at the Bulletin de la SMF

- a short text, very dense.
- Robert de Montessus went to the gist.
- The proof is based on some Hadamard's results on the location of polar singularities of a function represented by a Taylor series. There is no integral expression of the error between the function and his approximants.

The article at the Bulletin de la SMF

- a short text, very dense.
- Robert de Montessus went to the gist.
- The proof is based on some Hadamard's results on the location of polar singularities of a function represented by a Taylor series. There is no integral expression of the error between the function and his approximants.
- In his other papers on algebraic continued fractions, Robert de Montessus proved his great abilities in "computational mathematics" and in using algorithms. Those abilities allowed Robert to work later in differents domains like Elliptic functions and Algebraic space curvesy, Ballistics (during World War I) and especially in Applied Statistics (as an example of his virtuosity, let us mention his paper with Duarte : *Table à* 12 décimales de In n! pour toutes les valeurs de n de 1 1000.)

Articles by Robert de Montessus Context Genesis of his falous theorem (1902) Circulation Other works

Padé

Chiking le 26 horande, 1901 It was converger to mine larger gen atte lettre. Van y Knowery I'kutrickin I'm minin In Mr. Hereite, in & him suget. Dan ce minune, M. Hermite Arin o la unthile de hornen polyenteres doranis for de Counded d'am lana [...] Bo the way, I have met Mr. Appell of the be-Se ougestin hep wentremer in not compile printies a caled intripal , it scrat, sam ginning of this month. We talked about you and he did communicate nour letter to me. Se reporte à vote lettre countil que jel danly am, bie intermand I apprefaction The theory about the continuous fractions opens a Auran Leine. 1. Q.) Aillen the M. Spull source tage as rappets cake le calel intiged wide field of researches, but in which it is not so evident at he loss be ricorround de la Missue de an commencement is a hors ; il m'a pedi to be aware at the beginning of the obstacles that will forditions Cathines be encountered later on. Thus, non must take with a It was at in a sit fait prevention write pinch of salt the detailed study on the generalization of letter. I similal gon views in decoming doir continuous fractions that I will indicate non as most inas there as for the antimes offer mind olten me pareit de plu remarguables, mes teresting. I wrote in a paper published some years ago Change P.h. Vate & redenter, man a' Just it som Juke sommis a de energhing in the journal of Mr. Jordan an overview of the subject and it will be my pleasure to send it with this lefter. it " 'at par Kongons, faile de so rent any wontrames. Je loba dan plasin' vok You will find in it a reference to another paper by Mr. Compte à l'ansen de deffentites que l'a Himmentowthin your Van l' Aury Hermite on the same topic. In the latest paper, Mr. Herpolice mite is lead to the method of the associated polynoms by Verintfeed. Cal Jon an Balle seck te Verilly depin, hornors, l'aprecion to considering integral calculus : It would be for sure as in-Reames que à Viens rendiquerdi , Occume teresting to further investigate the relationship between Want public in grand inkarol, une un rurrante la plu Bittingers integral calculus and the laws of recurrence of the theory these appropriate so be generalisething see of continuous fractions. The result that you told me you obtained seems to for them Contrines. It is a fait you fleve be most impressive, but it must be submitted to a lot of & suget stam sur minusic gon a pars, exceptions. I will be happy to read your demonstration I y a gonlynes amile, then to proved be when it will be published. Mr. Jordan , al gen & an fai un plais

Figure: letter of Padé, November 26, 1901

Articles by Robert de Montessus Context Genesis of his falous theorem (1902) Circulation Other works

Appell





Figure: letter of Appell, December 3,1901 Figure: letter of Appell, December 3, 1901 According to me, your results are interesting and I think you should write it down in a note and send it to the Société Mathématique. [...]

< 日 > < 同 > < 三 > < 三 >

Articles by Robert de Montessus Context Genesis of his falous theorem (1902) Circulation Other works

Drafts

Lunde le reprisentation de function qui de prisent le prencie à l'épart de I dideparent in she to present. They went to apain station for an presen retirements, question to bary dais to parmerer. To can prestion retremally U = 2, et reter ... reprised on franking f(s), be practiced Schronaller Vill To Perturn Stor. In prostores on mines on draph folg bound , we activity restriction reprinter encer to produce flat. I as with for prestion rationale format of some sure with two which at it on them to providen I go flow. Our but see endente per freching red sound to place an and to some determine of a providence of the second men it with or it is the set of the set of the set of a set of the An array array array ar a presidence of the contrain a lon spirit le president to president and a contrain a for the presidence of the presidenc The Scaleport on and to fine reason the start of the post of the start Settime - 2 stan To be fraction for), the l'on as stage in mate an date 1/2 , 1/2 , the , ... profi < profi < profi < profi < profi < m de freshing rationally and shine, down to practice to be with as I histopray S deener be hacking and agreed materity for the term common and to similar recargon very at this there, he with tiping drive lifer topsent deneamplijist Vacuum with La forskin generation to be deri (5)reinischution from its while for these from at for whattat going by por undersite publicit all a custo to Hallows a moto to aquintation the marries & rate for institute or monthing ou the restlectuation in application por fraction continues Cover times in April case with be franking appendix $\overline{U}_{i}^{i} + \overline{U}_{i+1}^{i} + \overline{U}_{i+1}^{i}$ he inter without the to the of a requester, on sang alow the apply to

Figure: draft of Robert de Montessus

a to plan in the stranger that - but a factor and water of the the A Thomas & Strangerthe Span at A to Bath and the as Aller + hereby - all hiller at in the two and a form and a long angent to all all and deal and to form on a far and the loss have been a some and and any to form and a some of the loss have been a some approximation and a some and a some a some a considering and Some film at I are the an some as considering and Alfren . R. Jan . als hilfren al 16 + (V1 - V0)+...+ (Un - Un)+ ...the R = TT(M VA Van per the You Phylas M/W/ALY =0 M. Ru= Votal Vn / V Theil Var / KV The still First Va Vais 1 Par 1/2 + 120 $\begin{array}{c} \frac{\sqrt{\pi}(m)}{\sqrt{\pi}(m)} \left| \sqrt{\pi}(m) \frac{1}{\sqrt{\pi}} \right| \sqrt{\sqrt{\pi}(m)} \frac{1}{\sqrt{\pi}(m)} \left| \sqrt{\pi}(m) \frac{1}{\sqrt{\pi}(m)} \frac{1}{\sqrt{\pi}(m)} \right| \frac{1}{\sqrt{\pi}(m)} \frac{1}{\sqrt{\pi$ [2, lain Bbr. L +1 =0 ALV TON I car the find room hints bit exclused became Bhi - n's anon hante -

Figure: draft of Robert de Montessus

э

SC2011 Hervé Le Ferrand

Robert de Montessus de Ballore 1870-1937

Circulation

A fast circulation

Van Vleck, Boston 1903 :

In investigating the convergence of the horizontal lines the first case to be considered is naturally that of a function having a number of poles and no other singularities within a prescribed distance of the origin. It is just this case that Montessus [33, a] has examined very recently. Some of you may recall that four years ago in the Cambridge colloquium Professor Osgood took Hadamard's thesis as the basis of one of his lectures. This notable thesis is devoted chiefly to series defining functions with polar singularities. Montessus builds upon this thesis and applies it to a table possessing a normal character. Although his proof is subject to this limitation, his conclusion is nevertheless valid when the table is not normal. as I shall show in some subsequent paper. 4 3 5 4

A fast circulation

æ

<ロト <部ト < 注ト < 注ト

A fast circulation

• 1901-1902, of course, correspondance with Padé

SC2011 Hervé Le Ferrand Robert de Montessus de Ballore 1870-1937

・ 同 ト ・ ヨ ト ・ ヨ ト

э

A fast circulation

- 1901-1902, of course, correspondance with Padé
- 1905, Montessus wrote to Mittag-Leffler.

▲ □ ▶ ▲ □ ▶ ▲ □ ▶

э

A fast circulation

- 1901-1902, of course, correspondance with Padé
- 1905, Montessus wrote to Mittag-Leffler.
- 1910, Nörlund took an interest in Montessus works.



Figure: Nörlund

□ > < = > <

Articles by Robert de Montessus Context Genesis of his falous theorem (1902) **Circulation** Other works

Mittag-Leffler



Dear Sir, Please welcome in the Acta. It will be my pleasure to publish your work. Please, make sure that it is possible to read you without the necessity to resort to other published works. Best regards,

< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

Figure: letter of Mittag Leffler, December 30, 1905

Articles by Robert de Montessus Context Genesis of his falous theorem (1902) **Circulation** Other works

Nörlund



Figure: letter of Nörlund, March 3, 1910

The "Rendiconti di Palermo" cannot be found in any public library in Copenhague, however I have received your Ph D thesis today.

Now I am happy to be able to cite your dissertation with your name. The dissertation that I had the honour to send you a particular copy will be published only in <u>1911</u> in Acta Mathematic number 34. [...]

< 日 > < 同 > < 三 > < 三 >

A fast circulation

- 1910 Watson G. N., *The solution of a certain transcendental equation.*, [J] Lond. M. S. Proc. (2) 8, 162-177.
- 1913 Perron O. *Die Lehre von den Kettenbruchen*, Leipzig und Berlin, Druck und Verlag von B.G. Teubner 1913.
- 1924 Nörlund N. E., *Vorlesungen uber Differenzenrechnung.* Berlin Verlag von Julius Springer, 1924.
- 1927 Wilson R., Divergent continued fractions and polar singularities, Proceedings L. M. S. (2) 26, 159-168, 1927.
- 1929 Perron O. *Die Lehre von den Kettenbruchen*, Second edition, revised. Leipzig and Berlin, Teubner, 1929.
- 1935 Walsh J.L., Interpolation and approximation in the complex domain, New York, American Mathematical Society (Amer. Math. Soc. ColloquiumPubl. Vol. XX), 1935.

Articles by Robert de Montessus Context Genesis of his falous theorem (1902) **Circulation** Other works

Wilson

UNIVERSITY COLLEGE OF SWANSEA INGLETON PARK SWANSEA. 19 .10.23,)en Sie, am interested in the development of M. Pade's work on the representation of a function by means of a continued fraction ; and find that you have contributed much recent work to this subject. should esteemit a great avour if you could let we have copies of your fublished fators onthis subject, remaining ; Day to minerely ; T. Wilson.

Figure: letter of Wilson, October 19, 1923

UNIVERSITY COLLEGE OF SWANSEA SINGLETON PARK SWANSEA. 31-10-23 lear In. I for the phrasing of my premisorate was somewhat infortunate . Your infortant work on the continued fraction, and the singularities of this function, is of especial interest An attempt to defferentiate between this fort of the work , and these corlier portions which have now into common usage , led to an error in phraseday Wilson negates, I by to Iman , Town Linning, Pil Con with Kindert regards,

Figure: letter of Wilson, October 31, 1923

イロト イ団ト イヨト イヨト

Other works

Robert de Montessus extends works of Laguerre :

< 日 > < 同 > < 三 > < 三 >

3

Other works

Robert de Montessus extends works of Laguerre :

• Convergence of the continued fraction associated to $\left(\frac{x+1}{x-1}\right)^{\omega}$.

・ 同 ト ・ ヨ ト ・ ヨ ト

Other works

Robert de Montessus extends works of Laguerre :

- Convergence of the continued fraction associated to $\left(\frac{x+1}{x-1}\right)^{\omega}$.
- Development (and convergence) in continued fractions of the function Z which satisfies the EDO

$$(az+b)(cz+d)\frac{dZ(z)}{dz}=(pz+q)Z+s.$$

伺 ト イ ヨ ト イ ヨ ト

Other works

Robert de Montessus extends works of Laguerre :

- Convergence of the continued fraction associated to $\left(\frac{x+1}{x-1}\right)^{\omega}$.
- Development (and convergence) in continued fractions of the function Z which satisfies the EDO

$$(az+b)(cz+d)\frac{dZ(z)}{dz}=(pz+q)Z+s.$$

Convergence of continued fractions which represent functions like $\frac{F(\alpha,\beta+1,\gamma+1,z)}{F(\alpha,\beta,\gamma,z)}$.

▲ □ ▶ ▲ □ ▶ ▲ □ ▶

Conclusion and questions

 Nowadays the Robert de Montessus theorem (1902) remains of a great interest. Many generalizations of the theorem exist.

Conclusion and questions

- Nowadays the Robert de Montessus theorem (1902) remains of a great interest. Many generalizations of the theorem exist.
- J. L. Walsh seems to play an important role in the diffusion of Montessus theorem. More informations ?

Conclusion and questions

- Nowadays the Robert de Montessus theorem (1902) remains of a great interest. Many generalizations of the theorem exist.
- J. L. Walsh seems to play an important role in the diffusion of Montessus theorem. More informations ?
- Robert de Montessus and Probabilities. His second thesis
 (1905) was on Probabilites and Robert is also known for being
 one of the first mathematicien to make reference to
 Bachelier's work (in his book published in 1908). Robert was
 also in contact with E. Borel. Thus it is surprising to notice
 that Robert de Montessus never dealt with the metric theory
 of continued fractions. Why ?

.

...July 2011, the cottage of Robert de Montessus...



Figure: Lavaud, Sainte Feyre, near Guéret, Creuse