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Vibration-based fault detection of sharp bearing faults in helicopters

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Abstract:

Many signal processing tools have been developed by the mechanical and signal processing community to find the characteristic symptoms of sharp bearing faults (like localized spalling) from vibratory analysis. However the context of helicopter imposes a limited sampling frequency regarding the observed phenomena, many noisy vibrations and flight regimes. The performances of the classical methods are limited in such an environment mainly in identifying fault frequencies. Local bearing faults induce temporal periodic and impulsive patterns that produce redundant harmonics in the spectral domain. In this article four methods are proposed to take advantage of that redundancy. These methods provide an estimator of the fault frequency and an indicator of the quality of that estimation. These indicators are used to assess the severity of the fault. The four methods are then tested on synthesized and flight data in order to illustrate and discuss their efficiency.

Keywords: vibration, helicopter, health monitoring, frequency estimation, bearing, HUMS

NOMENCLATURE

- $v(t)$: recorded vibrations
- $V(f)$: Fourier transform of $v(t)$
- f : frequency
- α : fault frequency to be found
- \bullet : pointwise product (also called Hadamard product)
- I : frequential band around the theoretical value of α
 $I = [\alpha_{th} - \Delta f; \alpha_{th} + \Delta f]$
- I^f : small frequential band around f : $[f - \delta f; f + \delta f] - \{f\}$
- $S(n, f)$: formatted spectrum of the harmonic n ($\in [1; N]$), for the fundamental frequency $f \in I$
- $S(., f)$: vector of the first N harmonics of the fundamental frequency f

1. INTRODUCTION

Early fault detection is a crucial problem in Helicopter maintenance strategy. Indeed, a small fault, if it is not detected, could increase and lead to a breakdown of the system and in some cases lead to an accident. Condition Monitoring Methods are effective ways to perform health monitoring of the system. Among Condition Monitoring Methods, vibration analysis using non-intrusive sensors matches well with helicopter requirements. Accelerometers are the most suitable sensors for non-intrusive health monitoring of helicopters in terms of space, cost and qualification Randall [2011].

Among helicopter elements, the health of rolling-element bearings plays a key role for the power transmission chain integrity. Its monitoring is therefore worth to be carried out with the most effective methods.

Efficient vibration analysis methods have been developed

for fault detection that takes advantages of signal processing for bearing's elements monitoring. A demodulation/envelope analysis is presented in Hochmann and Bechhoefer [2005] while spectrum/cepstrum indicators, are proposed in Keller and Grabill [2003]. However both of these works do not provide decision rule for automatic fault detection. Statistical and stochastic properties of bearings are exploited in Estupinan et al. [2008] with a second order cyclostationary analysis and in Sawalhi and Randall [2006] the authors describe a semi-automated way to detect bearing's faults.

However, helicopter environment is very noisy, do not permit to access high harmonics and vibrations induced by bearing faults are corrupted by neighbor elements: gears, shafts, local resonances... Performance of classical methods is limited under these conditions. In particular, statistical tests proposed in Antoni [2007a] 4.1 or 5.2 suppose that the sampling frequency is higher than fault frequencies, that the frequency band with highest signal-to-noise ratio is known and that the noise is Gaussian and does not contains other parasitical frequencies. Unfortunately, none of these assumptions are entirely met in helicopters.

In such an environment, a good understanding of the features of each sub-system of a bearing gives prior information that may improve the accuracy of the decision. Dynamical properties of bearings have been studied (Randall and Antoni [2011]) to characterize vibrations and deduce algorithms to differentiate vibrations of bearings from those of other sub-systems. Diagram 1 details the four elements of one bearing: inner race (attached to the shaft), rolling elements, cage (holding rolling elements together), and outer race (attached to the helicopter structure). These four sub-elements are prone to degradations

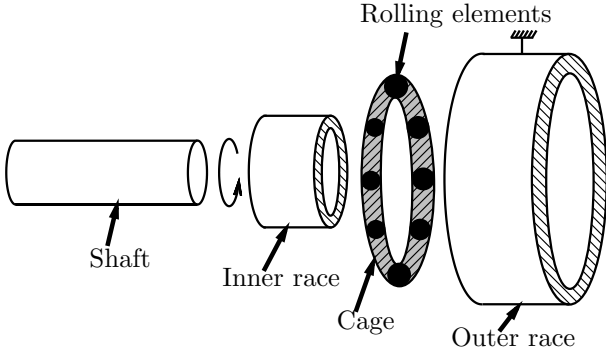


Fig. 1. Main components of a bearing.

at four different frequencies, (Su and Lin [1992]). It is then possible to distinguish faults from other components.

The theoretical frequencies are known for the four elements of a bearing from a kinematic prior analysis. However the actual dynamical conditions induce uncertainties on these frequencies. This uncertainty acts mainly for high speed bearings.

Therefore, the paper focuses on automatic bearings fault detection from vibration data in-flight conditions of helicopters. It proposes to take advantage of the redundancy induced by harmonics of fault frequencies. This redundancy allows estimating the actual frequency of fault even in a noisy and corrupted environment. Therefore, the monitoring of the fault is made easier and more accurate, furthermore some indicators are proposed from which decision rules are established.

The paper is organized as follows: section 2 deals with raw vibration pre-processing. Section 3 proposes four improved fault detection methods. Application of the proposed methods on synthesized and in-flight data is finally given in section 4 that allows a first comparison of their performances.

2. PREPROCESSING

Preprocessing aims at enhancing fault vibrations from bearings. Two features are utilized:

- *Randomness*: because of the random jitters and random contact surface of the balls inside the cage, random delays and random amplitude modulations are produced between two impacts of two consecutive balls Antoni and Randall [2002].
- *Impulsivity*: due to the size and depth of the defect at its early stages, it produces sharp impulses (Randall and Antoni [2011] section 2.2.3), and slow decrease of the harmonics of the fault.

In the presented approach, these two features are used to enhance the influence of the fault and the following preprocessing is applied:

- (1) *Whitening*: the vibration is decomposed into a random part and a predictable part. The implementation is made with an autoregressive filter to remove the linearly predictable part.
- (2) *Envelope analysis*: The random part of the signal extracted may also contain information about periodic patterns related to the fault. However as randomness turns harmonics into spread patterns in the spectrum,

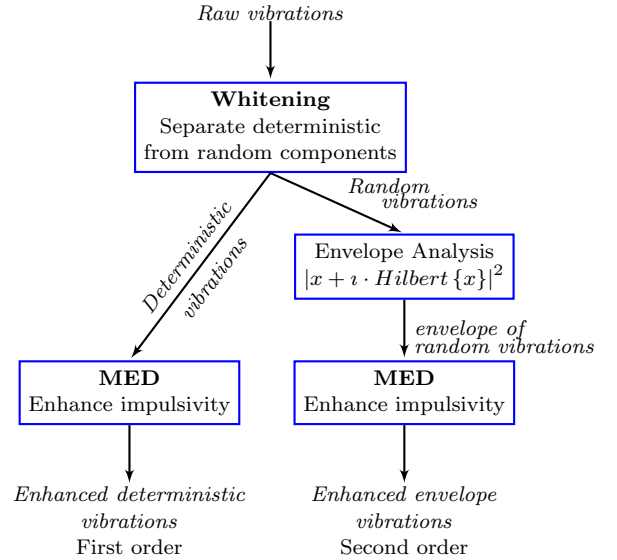


Fig. 2. Preprocessing steps. The data is preprocessed to enhance fault's features, and separates the deterministic and the random components. Then the envelope is extracted from the random part using the Hilbert transform to avoid aliasing.

a transformation is necessary to make the periodic component of the random part observable in the spectrum. The transformation that squares the norm of the analytical part of the vibrations achieves this objective, Antoni [2007b].

- (3) *MED, Minimum Entropy Deconvolution*: linear filter enhancing the impulsivity of vibrations. Such a preprocessing is particularly relevant for high speed systems with possible overlapping between two consecutive fault pulses. Further explanations are given Endo and Randall [2007].

The preprocessing produces two output signals, see diagram 2, which are the enhanced deterministic vibrations and the enhanced envelope vibrations. Both signals have to be analyzed in order to search the presence of a fault frequency.

3. REDUNDANCY-BASED ANALYSIS OF HARMONICS

3.1 Data formatting for redundancy analysis

The theoretical fault frequency associated to one of the four component of the bearing (figure 1) is noted α_{th} and is computed from kinematic properties, Su and Lin [1992].

The actual fault frequency α is assumed to be close to the theoretical fault frequency α_{th} : $\alpha \in I = [\alpha_{th} - \Delta f; \alpha_{th} + \Delta f]$. For example $\alpha_{th} \approx 700Hz$, a 10% uncertainty on the speed of the shaft leads to a $2 \times 10\% \times 700 = 140Hz$ -long frequential band where the actual fault is. The objective is to identify fault severity at the most probable value of α .

The analysis is performed with classic Fourier transform rather than other methods. Indeed the power spectral density provides a trade-off between amplitude variance and frequential resolution, while our objective is to accurately estimate the fault frequency. Time-frequency or

time-scale methods have also been ruled out because the preprocessing detailed in section 2 reduces the problem to the detection of purely periodic patterns in vibrations. The cepstrum was also not selected as it produces very limited results when the spectrum contains parasitical components with a small signal-to-noise ratio. As the form and features of the fault pulse may change depending on the geometry of the bearing, on the fault, and on the helicopter or flight context and because of the low signal-to-noise ratio, the problem is made very difficult.

As helicopter is a highly vibrating structure, many remaining parasitical harmonics exist near the fault frequencies and can mislead the detection method. To overcome that difficulty, we proposed to take advantage of the redundancy of the fault harmonics among the amplitude of the Fourier coefficients.

As the fault frequency α is supposed to be contained in I , the Fourier transform of the vibrations is restrained to the first N multiples of that frequential band: $V(f)$ for $f \in n \times I; n = 1 \dots N$. Where $V(f)$ is the Fourier transform of the vibrations $v(t)$. In addition the area of each band n is scaled to 1 in order to balance comparisons between harmonics. Let us call $S(n, f)$ the formatted spectra:

$$S(n, f) = \frac{|V(n \times f)|}{\int_{\nu \in I} |V(n \times \nu)| d\nu} \in [0; 1]; f \in I \quad (1)$$

Then, $n \times I$ is the interval corresponding to the n^{th} harmonic. It implies that if α is the fault frequency, $S(n, \alpha)$ should be significantly different from 0 for all n .

3.2 Identification of redundant harmonics

In this part, four criteria are presented in order to find the most energetic and redundant harmonics using only $S(n, f)$.

Geometric mean Geometric mean penalizes frequencies with nearly null harmonics. It is then an indicator of frequential redundancy:

$$\forall f \in I : \Pi(f) = \prod_{n=1}^N S(n, f)^{1/N} \quad (2)$$

It requires most of the harmonics of the fault frequency to be different enough from 0. A simple decision rule is proposed to check the relevance of each frequency $f \in I$:

$$\Pi(f) \gg \frac{\int_{\nu \in I} \Pi(\nu) d\nu}{\text{length}(I)} \Rightarrow f \text{ is likely to be a fault frequency}$$

A frequency f with high harmonics should be different enough from the mean value of Π .

Minimum harmonic A similar approach is proposed to get rid of the parasitical frequencies, for each frequency the minimum harmonic is kept:

$$\forall f \in I : \text{MinH}(f) = \min_{n=1 \dots N} S(n, f) \quad (3)$$

That criterion is much more demanding than the geometric mean as it requires all of the harmonics of the

fault frequency to be different enough from 0. Like the geometrical mean, a simple decision rule is also proposed to check the relevance of each frequency:

$$\text{MinH}(f) \gg \frac{\int_{\nu \in I} \text{MinH}(\nu) d\nu}{\text{length}(I)} \Rightarrow f \text{ is likely to be a fault frequency}$$

Remark: Convex and Holder inequalities can help to find intermediate criteria between the geometric mean and the minimum, as $\forall (a_k)_{k=1 \dots K} \in [0; +\infty]^K, \forall M \geq 1$:

$$\prod_{k=1}^K a_k^{1/K} \geq \frac{K}{\sum_{k=1}^K 1/a_k} \geq \underbrace{\left[\frac{K}{\sum_{k=1}^K 1/a_k^M} \right]^{1/M}}_{\xrightarrow{M \rightarrow \infty} \min_{k=1 \dots K} a_k}$$

The different values of M provide then intermediary criteria.

Linear optimization The problem is now written in terms of one linear optimization problem. For each frequency f , the set $S(\cdot, f) = [S(n, f)]_{n=1 \dots N}$ represents the coordinates of a vector in the \mathbb{R}^N Euclidian space. For each frequency, the projector P is calculated to maximize the projection $P \times S(\cdot, f)$ while keeping the mean projection on the other close frequencies I^f constant.

$$L(f) = \max_{P \in \mathbb{R}^N} |P \cdot S(\cdot, f)|^2 \text{ with } \frac{\|P \cdot S(\cdot, I^f)\|^2}{\text{length}(I^f)} = 1 \quad (4)$$

(1) The solution P of problem (4) is the eigenvector associated with the extremal eigenvalue of the pair of matrices:

$$S(\cdot, f)^T \cdot S(\cdot, f) \text{ and } \frac{[S(\cdot, I^f)]^T \cdot [S(\cdot, I^f)]}{\text{length}(I^f)} \quad (5)$$

(2) P is scaled by :

$$P^T \cdot \frac{[S(\cdot, I^f)]^T \cdot [S(\cdot, I^f)]}{\text{length}(I^f)} \cdot P = 1 \quad (6)$$

A decision rule for f to be a fault frequency is $L(f) \gg 1$ according to problem (4): the projection of a relevant frequency f should be much bigger than the averaged close frequencies.

Optimization with a penalized method It is also possible to construct a contrast function and to establish another optimization problem. The objective of the criterion is to find the frequencies presenting high amplitudes at harmonics where the close frequencies have a relative low value. This criterion tries to both maximize the projection of the harmonics of f and to minimize the correlation between that harmonics and the harmonics of the close frequencies in I^f . The criterion is:

$$\text{Pen}(f) = \max_{P \in \mathbb{R}^N} |P \cdot S(\cdot, f)|_1 - \underbrace{\frac{\int_{\nu \in I^f} |[P \cdot S(\cdot, \nu)] \bullet [P \cdot S(\cdot, f)]|_1 d\nu}{\text{length}(I^f)}}_{\text{Averaged correlation between } I^f \text{ and } f} \quad (7)$$

	Π	$MinH$	Linear	Penalization
$S(n_0, f) \ll 1$	\searrow	$\searrow \searrow$	\searrow	\searrow
$S(n_0, f) \sim 1$	\nearrow	\rightarrow	$\nearrow \nearrow$	\nearrow

Table 1. Impact on the criteria for isolated changes in one harmonic n_0 of a frequency f and starting from $S(n, f) \sim 1/\text{length}(I)$.

The absolute value (L_1 norm) is used as it is robust to parasitical and high amplitude peaks. The solution is explicit:

$$Pen(f) = \frac{1}{4} \cdot \sum_{n=1 \dots N} \frac{S(n, f)}{\frac{\int_{\nu \in I^f} S(n, \nu) d\nu}{\text{length}(I^f)}} \quad (8)$$

This means that the n^{th} harmonic of f is weighted by the inverse mean of the close frequencies at the same harmonic: the bigger the local noise, the smaller the criterion. That should avoid the parasitical frequency reaching high values. When no fault is present without any parasitical peaks: $S(n, f) \approx 1/\text{length}(I)$ and then $Pen(f) \approx N/4$. Then a rule for checking the relevance of a fault at frequency f :

$$Pen(f) \gg N/4 \quad (9)$$

3.3 Expected performances of the criteria

The four methods provide four estimates of the fault frequency α with decision rules.

$$\hat{\alpha} = \arg \max_{f \in I} Crit(f); Crit = \Pi, MinH, L, Pen \quad (10)$$

The associated rules for decision have been detailed before. Tabular 1 presents qualitative changes in the different criteria presented according to the method used. The geometric mean is expected to follow the global variations of the harmonics as it is a mean. The $MinH$ criterion should not be affected by parasitical peaks but requires all the harmonics to be high enough. One should expect huge drops in the performances of those two criteria when redundancy is not strictly observable among all the harmonics. As the linear approach exploits the Euclidian distance, high amplitude parasitical peaks are expected to boost excessively the criterion. Changes in the penalization criterion are hard to predict as it depends a lot on the other frequencies.

4. RESULTS

The four algorithms are now tested on a synthesized model of harmonics and on flight data.

4.1 Synthesized data

A simple model for $S(n, f)$ has been implemented to perform benchmarks and to help revealing qualitatively the performances of the different methods for some test cases. For each harmonic n a synthesized spectrum is synthesized:

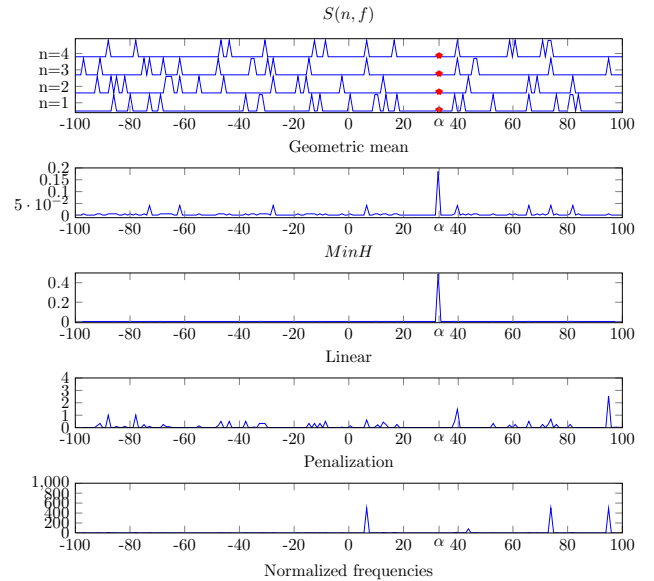


Fig. 3. Synthesized spectra $S(n, f)$ for $n = 1 \dots 4$ and $f = 1 \dots 200$, the four harmonic bands are plotted one above the other on the top graph and results for the four methods are plotted below. The parameter used to generate the model are $A = 10$ and $P_A = 0.01$ ($A/P_A \rightarrow \infty$) in equation 11 and frequencies are indexed as offsets of the theoretical fault frequency.

$$S(n, f) \propto \delta_\alpha(f) + A \times \underbrace{u(f)}_{\sim Ber(P_A)} \quad ; \text{ for } n=1 \dots N \quad (11)$$

Where the Dirac distribution $\delta_\alpha(f)$ stands for the harmonic amplitude at the redundant fault frequency, $u(f)$ follows an independent Bernoulli law of parameter P_A . A defines the amplitude of the parasitical frequencies. The symbol \propto in equation (11) means that synthesized data are scaled to reduce the area to 1 for each n :

$$\int_{\nu \in I} S(n, \nu) d\nu = 1 \quad (12)$$

The frequencies will be referred according to their relative position with the theoretical fault frequency: $f - \alpha^{\text{th}}$ rather than f .

High and rare amplitude peak $A = 10, P_A = 0.01$ The spectrum is corrupted with non-redundant, high amplitude peaks which should make detection more complex. A sample of that model for $N = 4$ harmonics has been generated in figure 3, where the “actual fault” frequency is highlighted at 33. The geometric mean Π and the $MinH$ criteria manage to reveal the fault frequency, whereas the other ones fail. The linear optimization overamplifies isolated peaks like the one at 40 in figure 3, since it does not include any penalization for non-redundancy. For the same reason penalization does not allow to find the actual frequency. Both linear and penalization method would have a false-alarm rate for that case.

Medium background noise $A = 1, P_A = 0.6$ The model has been changed in order to simulate numerous parasitical peaks with medium amplitude. The “actual fault”

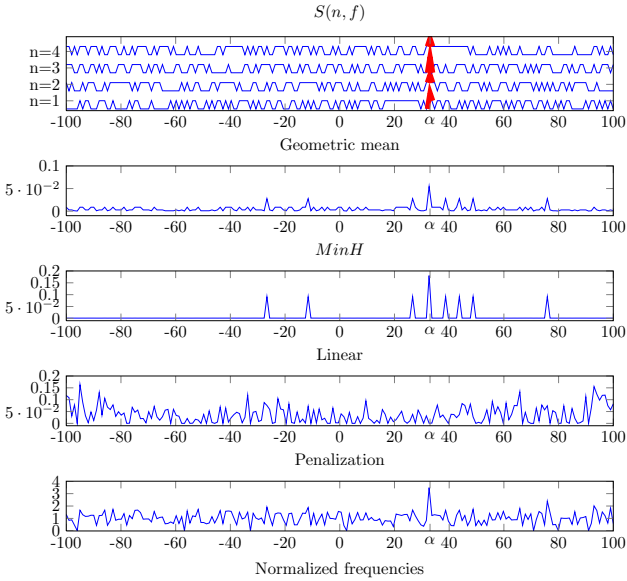


Fig. 4. Synthesized spectra $S(n, f)$ for $n = 1 \dots 4$ and $f = 1 \dots 200$, the four harmonic bands are plotted one above the other on the top graph and results for the four methods are plotted below. The parameter used to generate the model are $A = 1$ and $P_A = 0.6$ in equation 11 and frequencies are indexed as offsets of the theoretical fault frequency.

frequency is still highlighted at 33 on figure 4. All methods except the linear one reach their maxima at the fault frequency α and the rules can barely be applied.

It implies that the frequency would not be detected with enough confidence to take a decision, making uncertain future maintenance actions. The linear method fails in identifying a fault frequency but the criterion $L(f)$ remains low and so no alarm is triggered.

4.2 In-flight data

The four criteria are now tested on flight data during a stabilized flight regime, where a gradual bearing degradation has been observed. The shaft of interest is turning at 90 Hz, the sampling frequency is 20 kHz, the recording time is 2.5 second and 97 data sets have been recorded. All the other information are unknown, in particular nothing is known neither about the time elapsed between each recording, nor about the maintenance actions on the whole helicopter apart from the fact that that nothing has been done to repair the fault bearing. A growing inner bearing race wear has been identified by the operators on the studied data set. The frequencies will be referred according to their relative position with the theoretical fault frequency: $f - \alpha^{th}$ instead of f .

The performances of the four methods are presented in figure 5 and 7 for $N = 4$ harmonics. As the fault was not present in the enhanced envelope vibrations, the data utilized in the following are only the enhanced deterministic vibrations. An off-line version of the Generalized Likelihood Ratio (GLR) algorithm has been applied with a minimum expected change of 10 times the lower thresholds established in subsection 3.2 in order to identify the most probable change. The GLR algorithm is a classic algo-

rithm for detection of abrupt changes, see Basseville and Nikiforov [1993] section 2.4.3 for further details. The most probable fault frequency are strongly fluctuating before the 33rd recording for the four methods, whereas the associated rules of relevance are not fulfilled. The geometric mean and penalization methods start changing around the 30th recording, that is confirmed by the recorded spectra for the 33rd recording in figure 6. One can also check that the most relevant frequency is always the same (around -197) from 31st to the last recording in figure 7 for the first and last criteria.

There is a slight increase in the $MinH$ criterion starting at the 60th recording, however its performances are low because of the low level of the fault frequency ($\alpha - \alpha^{th} = -197$) at the second harmonic ($n = 2$). The linear method performances are unsteady as the criterion fluctuates a lot (figure 5) and the most relevant frequency oscillates from one recording to another (figure 7), that may be due to the high amplitude parasitical peaks.

The rules established in 3.2 allow changes in the criteria to be found without trend analysis: the geometric mean and the penalization methods start being more than 10 times bigger than their threshold ($\int_{\nu \in I} \Pi(\nu) d\nu / length(I) \approx 14 \cdot 10^{-3}$ and $N/4 = 1$) after the 33rd recording and that holds for the $MinH$ ($\int_{\nu \in I} MinH(\nu) d\nu / length(I) \approx 7 \cdot 10^{-4}$) but with a delay of 28 recordings. The linear optimization also achieves values significantly different from 1 (> 10) after the 30th recording.

Regarding these results geometric mean and penalization method have similar performances; however penalization should be rejected when high amplitude parasitical harmonics are present in the spectrum near the theoretical fault frequency. The two other ones are rejected because $MinH$ is actually too harsh and the linear method is inefficient as the estimated fault frequency is too erratic even when the fault is well-detectable in figure 7.

5. CONCLUSION

The problem of vibratory detection of sharp bearing faults in helicopters has been addressed in this article. Even if most of the faults can be localized according to the theoretical frequency of the produced vibrations, many obstacles hide it. The difficulties of this problem arise from the low signal-to-noise ratio, the numerous parasitical peaks and the wide types bearing uses. Classical preprocessings have been introduced to enhance fault pattern in the vibrations. These preprocessings are based upon stationary and random separation, impulsivity enhancement and envelope analysis.

After that step, the fault should be noticeable in the spectra by its numerous harmonics. For that reason the spectra of the resulting signals are formatted in order to help the search for redundant frequencies among the spectrum around a theoretical fault frequency. Then four methods are presented to identify the most redundant frequency with relevance indicators. The first two methods are based on indicators and the last two take advantage of optimization formulation. The linear method fails for the real and synthetic data. The $MinH$ method works with a long delay for detection on real data. The penalization

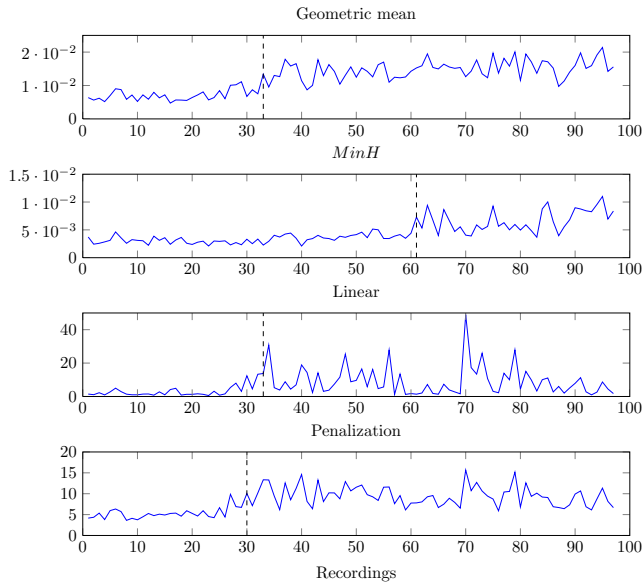


Fig. 5. Values of the four criteria for 97 consecutive recordings. Off-line GLR algorithm has been applied to detect an abrupt change, the minimum changes have been specified as 10 times the lower thresholds established in subsection 3.3 and the abrupt changes found are plotted as a vertical dashed line.

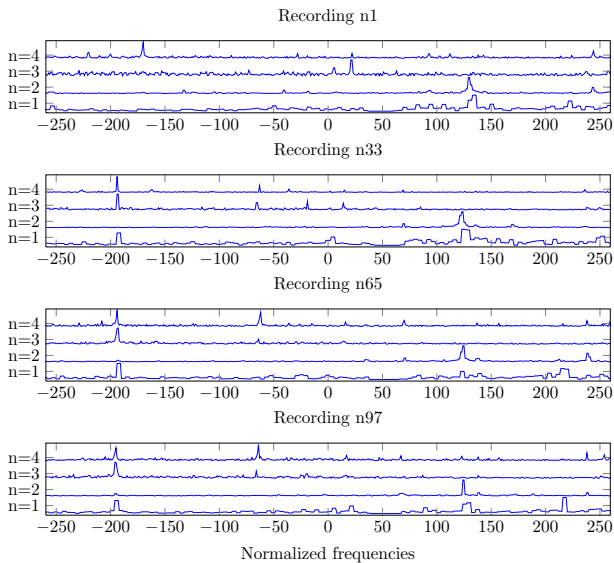


Fig. 6. Formatted spectra for four recordings at $n = 1, 33, 65, 97$. The frequencies are referred according to their relative position with the theoretical fault frequency. The actual fault frequency is located at $\alpha - \alpha^{th} = -197$.

has good performances only for real data. The geometric mean gives good results for both synthetic and real data, and should be preferred for further developments.

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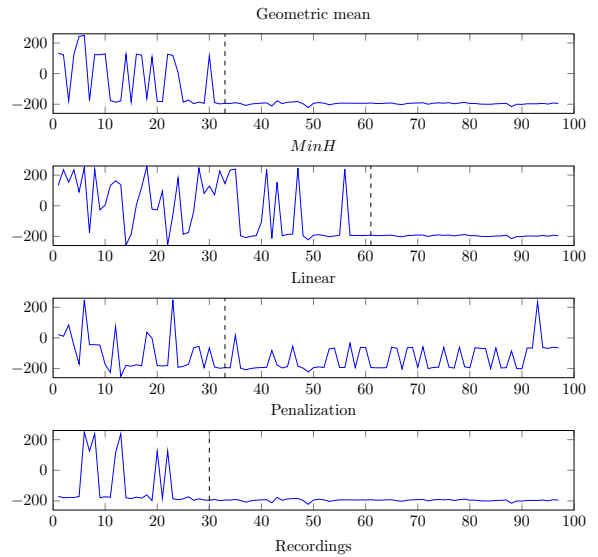


Fig. 7. Position of the estimated fault frequency $\hat{\alpha} - \alpha^{th}$ for the four methods. Position of the abrupt changes is drawn as in figure 5.

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