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# Delay-Time and Thermopower Distributions at the Spectrum Edges of a Chaotic Scatterer

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We study chaotic scattering outside the wide band limit, as the Fermi energy  $E_F$  approaches the band edges  $E_B$  of a one-dimensional lattice embedding a scattering region of  $M$  sites. The Hamiltonian  $H_M$  of this region is taken from the Cauchy orthogonal ensemble. The scattering is chaotic at  $E_F$  if the average level density per site of  $H_M$  at  $E_F$  describes a semi-circle as  $E_F$  varies inside the conduction band. The edges of this semi-circle coincide with the band edges  $E_B$ . We show that the delay-time and thermopower distributions differ near the edges from the universal expressions valid in the bulk. To obtain the asymptotic universal forms of these edge distributions, one must keep constant the energy distance  $E_F - E_B$  measured in unit of the same energy scale  $\propto M^{-1/3}$  which is used for rescaling the energy level spacings at the spectrum edges of large Gaussian matrices. In particular the delay-time and the thermopower have the same universal edge distributions for arbitrary  $M$  as those for an  $M = 2$  scatterer, which we obtain analytically.

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Since nano-engineering makes it possible to fabricate devices which can be used for harvesting energy (Seebeck effect) or for cooling (Peltier effect) at the nanoscales, to study thermoelectric conversion beyond the semi-classical Boltzmann limit has become an important challenge. In the mesoscopic limit, quantum interferences can induce large fluctuations and the thermoelectric conversion can be mainly due to a large fluctuation around a zero average. This requires the knowledge of the whole distributions instead of simple averages. Such mesoscopic fluctuations of the thermopower have been observed [1] in quantum dots as one varies either the shape of the dot with an electrostatic gate or the magnetic flux threading the dot. Scattering theory, combined with random matrix theory (RMT), allows one to obtain [2] these distributions as long as the device behaves as an elastic scatterer (the low temperature limit).

Theoretical evaluation of the distribution of the thermopower  $S_k$  at a low temperature  $\mathcal{T}$  is complicated by the fact that one needs to know the distribution of not only the scattering matrix  $S$  but also its energy derivative at the Fermi energy  $E_F$ , which can be obtained from the Wigner-Smith time-delay matrix [3–5]  $Q \equiv -i\hbar S^{-1}\partial S/\partial E$ . The universal distributions of  $Q$ , and hence of  $S_k$ , have been obtained previously [2], but only in the wide band limit (WBL), restricted to the *bulk* of a wide conduction band. In that case, this is the bulk of the spectrum of the used chaotic scatterer which is probed at  $E_F$ . In the microscopic model which we study

exactly, one can consider not only the bulk of the conduction band, but also its *edges* where this is not bulk of the spectrum, but its edges which are probed at  $E_F$ . This is particularly interesting, since the universality of a distribution near the spectrum-edges can be quite different from that in the bulk, as is well-known from the Tracy-Widom vs Wigner nearest-neighbor spacing distributions [6–8]. In this letter we show that the distributions of  $Q$  and  $S_k$  near the edges are indeed different, and that the edge distributions give rise to a new asymptotic universality when the Tracy-Widom scaling is adopted. Furthermore, the results concerning  $Q$  which we obtain near the band edges could be relevant for other waves [9] (electromagnetic, acoustic, ...) than electron waves.

Let us consider a chaotic cavity opened to leads via single mode quantum point contacts. We assume time reversal and spin rotation symmetry, such that the scattering matrix  $S$  is a  $2 \times 2$  unitary symmetric matrix, diagonalizable by an orthogonal transformation. The generalization to other symmetries (unitary and symplectic) is straightforward, and will not be discussed here.  $S(E_F)$  can be assumed to be totally random at the Fermi energy  $E_F$ , the probability  $P(S)$  of finding  $S$  inside an infinitesimal volume element  $dS$  being constant. In other words,  $S(E_F)$  belongs to the Circular Orthogonal Ensemble (COE) [10]. Such an assumption can be justified if the corresponding classical trajectories are chaotic [9, 11]. The two eigenvalues  $e^{i\theta_j}$  of  $S(E_F)$  exhibit the COE level repulsion [10], and the distribution  $P(T)$  of the cavity transmission  $T$  at  $E_F$  reads [12–14]:

$$P(\theta_1, \theta_2) = \frac{1}{16\pi} |e^{i\theta_1} - e^{i\theta_2}|, \quad P(T) = \frac{1}{2\sqrt{T}}. \quad (1)$$

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The thermopower  $S_k$  at a low temperature  $\mathcal{T}$  (in units of  $\pi^2 k_B^2 \mathcal{T}/3e$ ,  $k_B$  and  $e$  being the Boltzmann constant and the electron charge) is given by  $S_k = (dT/dE)/T$  where  $T$  and  $dT/dE$  are to be evaluated at  $E = E_F$ . These energy derivatives can be obtained from the matrix  $Q$ , the distribution of its eigenvalues (the delay-times) taking a universal form for chaotic scattering: Assuming the WBL limit and a chaotic scatterer of size  $M \rightarrow \infty$ , the distribution of the inverse delay-times turns out to be given by the Laguerre ensemble from RMT [3, 4]. Introducing the two rescaled delay-times  $\tilde{\tau}_j = \tau_j/\tau_H$  ( $\tau_H = 2\pi\hbar/\Delta_F$  being the Heisenberg time with  $\Delta_F$  the mean level spacing of the cavity at  $E_F$ ), the average eigenvalue density of the  $2 \times 2$  matrix  $Q$  can be calculated using Refs. [4, 15]:

$$P_B(\tilde{\tau}) = \frac{(4\tilde{\tau} + 1) \exp[-1/\tilde{\tau}]}{6\tilde{\tau}^4} - \frac{(4\tilde{\tau} - 1) \exp[-1/(2\tilde{\tau})]}{12\tilde{\tau}^4}, \quad (2)$$

while the distribution of the dimensionless thermopower  $\sigma_k \equiv (\Delta_F/2\pi)(dT/dE)/T$  can be found in Ref. [2] in the form of a multiple integral:

$$P_B(\sigma_k) = \int_{-1}^{+1} dc \int_0^\infty d\tilde{\tau}_1 \int_0^\infty d\tilde{\tau}_2 \int_0^{+1} dT f(c, \tilde{\tau}_1, \tilde{\tau}_2, T) \times \delta \left( \sigma_k - c(\tilde{\tau}_1 - \tilde{\tau}_2) \sqrt{\frac{1}{T} - 1} \right). \quad (3)$$

Here  $f(c, \tilde{\tau}_1, \tilde{\tau}_2, T) = \frac{1}{\pi\sqrt{1-c^2}} P(T) P(\tilde{\tau}_1, \tilde{\tau}_2) F(\tilde{\tau}_1, \tilde{\tau}_2)$  and  $P(\tilde{\tau}_1, \tilde{\tau}_2) = \frac{1}{48} |\tilde{\tau}_1 - \tilde{\tau}_2| (\tilde{\tau}_1 \tilde{\tau}_2)^{-4} \exp[-\sum_{j=1}^2 \frac{1}{2\tilde{\tau}_j}]$ . The charging effects inside the cavity are taken into account [2, 16] in the Hartree approximation by the function  $F(\tilde{\tau}_1, \tilde{\tau}_2)$ :  $C$  being the capacitance of the cavity,  $F(\tilde{\tau}_1, \tilde{\tau}_2) = \tilde{\tau}_1 + \tilde{\tau}_2$  if  $e^2/C \gg \Delta_F$  while  $F(\tilde{\tau}_1, \tilde{\tau}_2) = 1$  if  $e^2/C \ll \Delta_F$ .

The distributions (2-3) were obtained in the WBL limit, an approximation where the real part of the lead self energy  $\Sigma(E)$  and the energy dependence of its imaginary part around  $E_F$  are neglected. This is justified in the bulk of a wide conduction band, but not near the band edges. Moreover,  $H_M$  was taken from the Gaussian orthogonal ensemble (GOE), a distribution giving rise to chaotic scattering only when  $M \rightarrow \infty$  and  $\Delta_F \rightarrow 0$ . These assumptions, leading to distributions (2-3), cannot be made as  $E_F$  approaches the edges of the conduction band and the spectrum edges of the chaotic scatterer.

To study the delay-time and the thermopower distributions at the edges, we use a 1d tight-binding lattice (hopping term  $t$ ) embedding a scatterer of  $M$  sites, as shown in Fig. 1.  $S(E_F)$  is calculated from the scatterer Hamiltonian  $H_M$  and from the *exact* expressions of the lead self-energies  $\Sigma(E)$ . The COE distribution for  $S$  at  $E_F$  can be obtained by taking for  $H_M$  a Cauchy (Lorentzian) distribution of center  $E_F/2$  and of width  $\Gamma_F = t\sqrt{1 - (E_F/2t)^2}$ . The advantage of taking such a

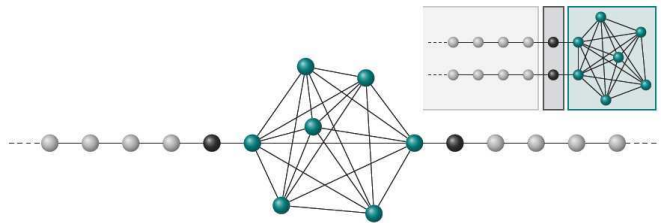


FIG. 1: (Color online) Chaotic scatterer of  $M = 7$  sites (indicated in green) embedded in a 1d tight-binding lattice (grey sites with nearest-neighbor hopping  $t = 1$ ). The two end sites of the 1d leads are shown in a darker grey. Upper right: Partition of the same infinite system used for deriving Eqs. (5-6).

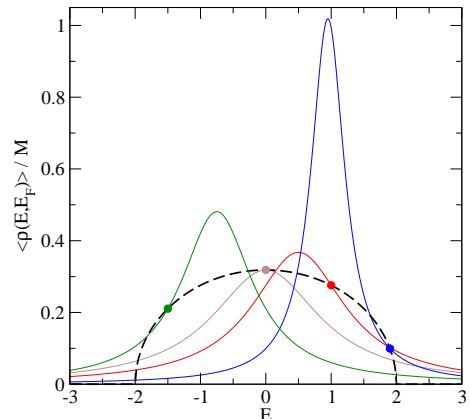


FIG. 2: (Color online) Average density of states per site (in unit of  $t^{-1}$ ),  $\langle \rho(E, E_F) \rangle / M$ , as a function of the energy  $E$  (in unit of  $t$ ), for 4 Cauchy ensembles ( $H_M \in \mathcal{C}(M, E_F/2, \Gamma_F)$ ), each of them giving rise to chaotic scattering at  $E_F = 0$  (brown line),  $E_F = 1$  (red line),  $E_F = -1.5$  (green line) and  $E_F = 1.9$  (blue line). The circles indicate for each curve the value of the density of states at  $E = E_F$ . The black dashed line is the semi-circle law  $(\Delta_F M)^{-1}$  (see Eq. (12)).

distribution for  $H_M$  is that it gives rise [17] to chaotic scattering at  $E_F$  when  $M \geq 2$ , while one needs to take the limit  $M \rightarrow \infty$  if  $H_M \in GOE$ . A remarkable property of these Cauchy distributions is shown in Fig. 2: the average density of states per site at  $E_F$  varies on a semi-circle of center and diameter given by the center and the width of the conduction band, as one varies  $E_F$ . As  $E_F$  approaches the edges  $\pm 2t$  of the conduction band,  $E_F$  approaches the spectrum tails of the Cauchy scatterer. We will show that this gives rise to edge distributions for the time delay and for the thermopower which differ from the bulk distributions (2-3). The edge distributions turn out to be universal after an energy rescaling similar to the one used by Tracy and Widom for the energy levels [6-8].

*Scattering Matrix:* Usually, the infinite system is divided into a scatterer and two attached leads, and the scattering matrix  $S$  is given [18] in terms of the scatterer Hamiltonian  $H_M$  and of the lead self-energies  $\Sigma(E)$ . For

the 1d-model sketched in Fig. 1, it is more convenient when  $M > 2$  to divide the infinite system as indicated in the upper right corner of Fig. 1: a “system” made of the two sites (dark grey) located at the lead ends (Hamiltonian  $H_0 = V_0 \mathbf{1}_2$ ), with the scatterer (Hamiltonian  $H_M$ , green) at its right side and the two leads (light grey) without their end sites at its left side. The scatterer and the leads are described by their self-energies [18]. Using this partition, one obtains the matrix  $S$  at an energy  $E$  in terms on an effective  $2 \times 2$  energy-dependent “Hamiltonian” matrix  $\tilde{H}_2(E)$ :

$$S(E) = -\mathbf{1}_2 + 2i\Gamma(E)A_2(E) \quad (4)$$

$$A_2(E) = \frac{1}{E\mathbf{1}_2 - H_0 - \tilde{H}_2(E) - \Sigma(E)} \quad (5)$$

$$\tilde{H}_2(E) = W^\dagger \frac{1}{E\mathbf{1}_M - H_M} W. \quad (6)$$

Here  $\mathbf{1}_M$  is the  $M \times M$  identity matrix and  $W$  is an  $M \times 2$  matrix with  $W_{i \neq j} = 0$ ,  $W_{11} = W_{22} = t$ .  $E = -2t \cos k$ ,  $\Sigma(E) = -te^{ik} \mathbf{1}_2$  is the lead self-energy and  $\Gamma(E) = t \sin k = t\sqrt{1 - (E/2t)^2}$ . Since  $V_0 = 0$ ,  $H_0$  disappears from Eq. (6). Using this expression for  $S$ ,  $\partial S/\partial E = 2i\partial(\Gamma(E)A_2(E))/\partial E$  and it becomes obvious that the distribution of  $\partial S/\partial E$  at  $E_F$  depends only on the distribution of the  $2 \times 2$  effective Hamiltonian  $\tilde{H}_2(E_F)$  as  $E_F$  approaches the band edges  $\pm 2t$  ( $\Gamma_F \rightarrow 0$  in this limit). We draw the conclusion that for having the distribution of the delay-time and of the thermopower as  $E_F \rightarrow \pm 2t$ , one needs only to have the distribution of  $\tilde{H}_2(E_F)$ . Let us notice that when  $M = 2$ , it is simpler not to use the previous partition, and to put the  $2 \times 2$  Hamiltonian matrix  $H_2$  instead of  $\tilde{H}_2(E)$  in Eq. (6). The matrices  $S$  given by the two methods are identical up to a phase factor  $e^{2ik}$ .

*Chaotic Scattering and Cauchy Ensembles:* An  $M \times M$  Hamiltonian  $H_M$  has a Cauchy (or Lorentzian) distribution  $\mathcal{C}(M, \epsilon, \Gamma)$  of center  $\epsilon$  and width  $\Gamma$  if the probability of finding  $H_M$  inside the infinitesimal volume element  $\mu(dH_M) = \prod_{i < j}^M dH_{M,ij}$  is given by

$$P(H_M) \propto \det((H_M - \epsilon \mathbf{1}_M)^2 + \Gamma^2 \mathbf{1}_M)^{-\frac{M+1}{2}}. \quad (7)$$

For  $M = 2$ , it is straightforward to show that

$$S(E_F) \in COE \Leftrightarrow H_2 \in \mathcal{C}(2, E_F/2, \Gamma_F). \quad (8)$$

Indeed,  $H_2$  and  $S$  are diagonalizable for  $M = 2$  by the same rotation  $R_\theta$  and the eigenvalues of  $S$  are simply related to the eigenenergies  $E_j$  of  $H_2$ :

$$e^{i\theta_j} = -1 + \frac{2i\Gamma_F}{E_F/2 - E_j + i\Gamma_F}; \quad j = 1, 2. \quad (9)$$

Using Eq. (1) and Eq. (9), one obtains the condition (8).

For  $M > 2$ , the condition (8) for the effective Hamiltonian  $\tilde{H}_2(E_F)$  instead of  $H_2$  is sufficient and necessary

for having chaotic scattering. Moreover, one can use two properties of the Cauchy distributions [17],

$$H_M \in \mathcal{C}(M, \epsilon, \Gamma) \Rightarrow H_M^{-1} \in \mathcal{C}(M, \frac{\epsilon}{D}, \frac{\Gamma}{D})$$

$$H_M \in \mathcal{C}(M, \epsilon/D, \Gamma/D) \Rightarrow W^\dagger H_M W \in \mathcal{C}(2, \frac{t^2\epsilon}{D}, \frac{t^2\Gamma}{D})$$

where  $D = \epsilon^2 + \Gamma^2$ , to obtain the following result:

$$H_M \in \mathcal{C}(M, E_F/2, \Gamma_F) \Rightarrow S(E_F) \in COE \quad (10)$$

since  $t^2/D = 1$  when  $D = E_F^2/4 + \Gamma_F^2$ .

*Chaotic Scattering and Semi-Circle Law:* Hereafter, we study the distributions  $P(\tilde{\tau})$  of the delay-time and  $P(\sigma_k)$  of the thermopower when  $H_M \in \mathcal{C}(M, E_F/2, \Gamma_F)$ . In this case,  $S(E_F) \in COE$  and the average energy level density of  $H_M$  at an energy  $E$  and for a Fermi energy  $E_F$  reads [17]

$$\langle \rho(E, E_F) \rangle = \frac{1}{\delta} \frac{\Gamma_F}{(E - E_F/2)^2 + \Gamma_F^2} \quad (11)$$

where  $\delta = \frac{\pi t}{M}$  is the level spacing at  $E = 0$ . This gives for  $E = E_F$

$$\Delta_F^{-1} = \langle \rho(E = E_F, E_F) \rangle = \frac{1}{\delta} \sqrt{1 - (E_F/2t)^2}. \quad (12)$$

As shown in Fig. 2, the average level density per site at  $E_F$  of the scattering region must vary on a semi-circle as  $E_F$  varies inside the conduction band in order to give rise to chaotic scattering. This semi-circle is also the limit when  $M \rightarrow \infty$  of the average level density of a unique asymptotic Gaussian Ensemble where  $\text{tr} H_M^2$  have a zero average and a variance  $V^2 = 2\delta^2 M/\pi^2$ . Thus, chaotic scattering with  $\langle \rho(E = E_F, E_F) \rangle \rightarrow 0$  becomes possible when  $H_M \in \mathcal{C}(M, E_F/2, \Gamma_F)$  only if one of the energy distances  $\epsilon_F^\mp \equiv E_F \pm 2t \rightarrow 0$ .

*Energy Rescaling:* In our quest for universal asymptotic distributions inside the conduction band (edges included), let us introduce the dimensionless parameter

$$\alpha \equiv \frac{\Gamma_F^2}{\Delta_F t} = \frac{1}{8\pi} \left| \frac{\epsilon_F^-}{tM^{-1/3}} \right|^{3/2} \left| \frac{\epsilon_F^+}{tM^{-1/3}} \right|^{3/2}. \quad (13)$$

The reasons for introducing  $\alpha$  (i. e. for measuring the energy distance from the edges in unit of  $tM^{-1/3}$ ) are twofold.

Firstly,  $\alpha$  appears as the relevant parameter in the study of the case  $M = 2$ , where the delay-time and thermopower distributions read [19]:

$$P_{M=2}(\tilde{\tau}, \alpha) = \frac{4\alpha}{\sqrt{1 - (4\pi\alpha\tilde{\tau})^2}} \quad (14)$$

$$P_{M=2}(\sigma_k, \alpha) = 2\alpha \ln \frac{1 + \sqrt{1 - (2\pi\alpha\sigma_k)^2}}{2\pi\alpha|\sigma_k|}, \quad (15)$$

where  $\tilde{\tau} = \tilde{\tau} - \hbar/(2\Gamma_F\tau_H)$  is a shifted delay-time. These distributions are valid in the whole conduction band

for  $M = 2$ . Moreover, as pointed out after Eq. (6), they give the edge distributions for any Cauchy scatterer  $H_M \in \mathcal{C}(M, E_F/2, \Gamma_F)$  of finite size  $M$ , in the limit  $\alpha \rightarrow 0$ .

Secondly,  $\alpha$  is simply related near the band edges to the scale  $x$  leading to asymptotic universal level distributions at the spectrum edges of large Gaussian matrices. In particular, for an Hamiltonian  $H_M \in GOE$  having the semi-circle density shown in Fig. 2 when  $M \rightarrow \infty$ , the tail of its density  $\langle \rho^{GOE}(E) \rangle / M$  takes the Tracy-Widom form outside the conduction band [20]

$$\frac{\langle \rho^{GOE}(x) \rangle}{M} = \frac{1}{4\sqrt{\pi}x^{1/4}} \exp\left[-\frac{2}{3}x^{3/2}\right], \quad (16)$$

in terms of the scaling variable  $x = \epsilon/(tM^{-2/3})$ . Since  $\epsilon \equiv |E + 2t|$  ( $\epsilon \equiv |E - 2t|$ ) if  $E \approx -2t$  ( $E \approx 2t$ ) respectively, one gets that  $x \rightarrow (\pi\alpha)^{2/3}$  near the edges. It is clear from Eq. (16) that the distributions (2-3) correspond to the limit where  $x$  and hence  $\alpha \rightarrow \infty$ , since they were obtained in the asymptotic limit where  $\rho^{GOE}(x) = 0$  outside the band. This limit is out of reach for  $M = 2$  ( $\alpha < 2/\pi$ ). For finding the distributions (2-3) using our 1d lattice model, we must consider larger scattering regions. This leads us to numerically check that our 1d model does exhibit the bulk distributions (2 - 3) when  $\alpha \rightarrow \infty$  and the edge distributions (14-15) when  $\alpha \rightarrow 0$ . In addition, our results do confirm that the scaling leading to Eq. (16) leads also to asymptotic universal distributions for the delay-time and for the thermopower at intermediate values of  $\alpha$ .

*Universal  $\alpha$ -dependent asymptotic distributions:* We study how the thermopower distribution  $P(\sigma_k, M, \alpha)$  varies as one increases the size  $M$  of an Hamiltonian  $H_M \in \mathcal{C}(M, E_F/2, \Gamma_F)$ , taking  $E_F$  closer to the nearest edge for keeping  $\alpha$  constant. To measure the size dependence of  $P(\sigma_k, M, \alpha)$ , we introduce the parameter

$$\eta(\alpha, M) = \frac{\int d\sigma_k |P(\sigma_k, M, \alpha) - P_B(\sigma_k)|}{\int d\sigma_k |P_{M=2}(\sigma_k, \alpha) - P_B(\sigma_k)|}, \quad (17)$$

where  $P_B(\sigma_k)$  [ $P_{M=2}(\sigma_k, \alpha)$ ] is given by Eq. (3) [Eq. (15)] respectively. As shown in the inset of Fig. 3,  $P(\sigma_k, M, \alpha)$  does reach an asymptotic limit when  $M \rightarrow \infty$  if  $\alpha$  keeps a constant value. If this value is very small,  $\eta \approx 1$  independently of  $M$ , confirming that  $P_{M=2}(\sigma_k, \alpha)$  gives the asymptotic edge distribution when  $\alpha \rightarrow 0$ . For intermediate values of  $\alpha$ , the finite size effects on  $P(\sigma_k, M, \alpha)$  are noticeable when  $M$  is small, but become rapidly negligible at larger  $M$ . We show also in Fig 3 how  $\eta$  depends on  $\alpha$  as  $M$  increases. The curve obtained taking  $M = 100$  gives an excellent approximation of the asymptotic limit, and exhibits a crossover around  $\alpha \approx 0.25$  between the edge limit  $\alpha \rightarrow 0$  and the bulk limit  $\alpha \rightarrow \infty$ . The same conclusion can be drawn from the study of  $P(\tilde{\tau}, M, \alpha)$ . In Fig. 4, these asymptotic distributions (calculated taking

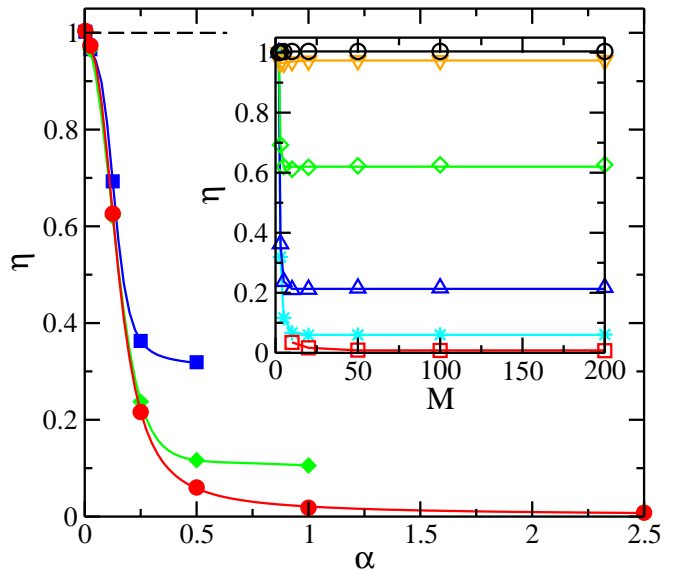


FIG. 3: (Color online)  $\eta$  parameter as a function of  $\alpha$  for  $M = 2$  (dashed line),  $M = 3$  (squares),  $M = 5$  (diamonds) and  $M = 100$  (circles). Inset :  $\eta$  as a function of the size  $M$  of the cavity for  $\alpha = 0.0025$  (circles),  $\alpha = 0.025$  (down triangles),  $\alpha = 0.125$  (diamonds),  $\alpha = 0.25$  (up triangles),  $\alpha = 0.5$  (star) and  $\alpha = 2.5$  (squares). In both panels, full lines are guides to the eye.

$M = 200$ ) are given for the delay-time and the thermopower for different values of  $\alpha$ . The distributions  $P(\sigma_k, \alpha)$ , which are symmetrical around the origin, are shown for  $\sigma_k > 0$  only. The curves valid at the band edges and corresponding to the case  $M = 2$  (Eq. (14) when  $\alpha/\Gamma_F \rightarrow 0$  and Eq. (15)) are also shown in Fig. 4, together with the bulk behaviors, given by Eq. (2) for the delay-time and by the numerical integration of Eq. (3) for the thermopower, taking  $F(\tilde{\tau}_1, \tilde{\tau}_2) = 1$ . To show the convergence towards the edge limit  $\alpha \rightarrow 0$ , we have multiplied the abscisses by  $\alpha$  in the insets. One can see that the bulk distributions (2,3) given in Refs. [2, 3] for chaotic cavities opened to leads via single mode quantum point contacts characterize also an infinite 1d lattice embedding a Cauchy scatterer as  $\alpha \rightarrow \infty$ , while Eqs. (14) and (15) give the edge distributions as  $\alpha \rightarrow 0$ . The change of the delay-time distribution as a function of  $\alpha$  is striking. When  $\alpha \rightarrow \infty$ , the average delay-time  $\langle \tau \rangle \rightarrow \pi\hbar/\Delta_F$  and the fluctuations do not give rise to negative times. The scattering region is always attractive. When  $\alpha \rightarrow 0$ ,  $\langle \tau \rangle \rightarrow \hbar/(2\Gamma_F)$  (see Eq. (14)). This change of average delay time induced by a decrease of  $\alpha$  is similar [21] to the one induced in compound nuclei by a decrease of the density of resonance levels. Around this average value, there are huge symmetrical fluctuations which can yield positive or negative values for  $\tau$ , reflecting either an attractive or a repulsive character [5] of the scattering region at the edges.

In summary, we have obtained asymptotic edge dis-

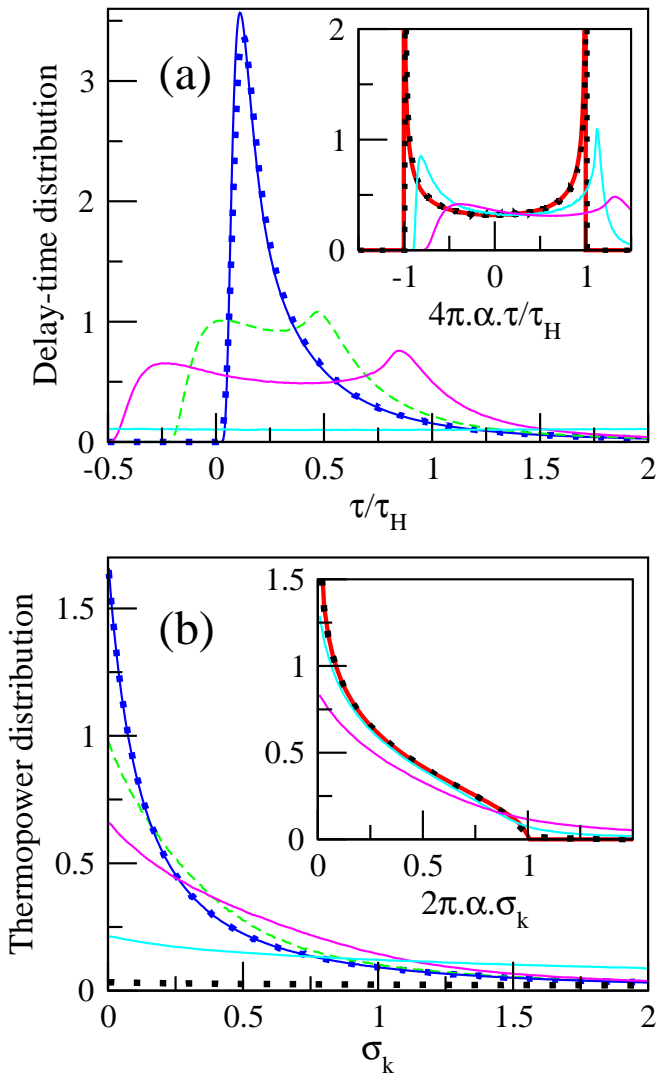


FIG. 4: (Color online) Asymptotic delay-time (a) and thermopower (b) distributions for different values of  $\alpha$ . Data are plotted as a function of  $\tilde{\tau} = \tau/\tau_H$  (a) and  $\sigma_k$  (b) in the main panels and as a function of a rescaled variable in the insets. Data for  $\alpha = 2.5 \times 10^{-5}$  (black dotted line in (a)),  $\alpha = 2.5 \times 10^{-3}$  (black dotted line in (b)),  $\alpha = 2.5 \times 10^{-2}$  (cyan line),  $\alpha = 0.125$  (magenta line),  $\alpha = 0.25$  (green dashed line) and  $\alpha = 2.5$  (blue dotted line) are shown. In the main panels, the data for  $\alpha = 2.5$  (blue dotted line) agree with the distributions (continuum blue line) expected in the limit  $\alpha \rightarrow \infty$  [Eqs. (2-3)]. In the insets, the data for  $\alpha = 2.5 \times 10^{-5}$  (black dotted line in (a)) and  $\alpha = 2.5 \times 10^{-3}$  (black dotted line in (b)) agree with the distributions (continuum red line) expected in the limit  $\alpha \rightarrow 0$  [Eqs. (14-15)].

tributions for the delay-time and for the low  $\mathcal{T}$  limit of the thermopower using a 1d model embedding a chaotic scatterer. Taking a Cauchy distribution for the scatterer Hamiltonian and the exact expressions for the lead self-energies, we have shown that chaotic scattering is associated to a semi-circle law for the average level density per site at  $E_F$  of the scattering region. This intriguing

coincidence between the asymptotic limit of a unique Gaussian ensemble and a continuum family of Cauchy ensembles has led us to use the same energy rescaling near the edges as the one leading to the Tracy-Widom expressions near the GOE spectrum edges. Doing so, we have shown that the delay-time and thermopower distributions converge towards universal distributions which depend on the scale  $\alpha$  used for Tracy-Widom distributions. In particular  $\alpha \rightarrow \infty$  corresponds to the bulk and  $\alpha \rightarrow 0$  to the edges where we obtain analytic distributions. We have numerically studied the universal crossover from the edges towards the bulk using the parameter  $\eta(\alpha)$ .

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