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► **To cite this version:**

Houcem Gazzah, Jean-Pierre Delmas. Subspace blind MIMO channel equalization with quadratic complexity. ISWCS '12: The Ninth International Symposium on Wireless Communication Systems, Aug 2012, Paris, France. pp.236-240, 10.1109/ISWCS.2012.6328365 . hal-00746476

**HAL Id: hal-00746476**

**<https://hal.science/hal-00746476>**

Submitted on 16 Dec 2020

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# SUBSPACE BLIND MIMO CHANNEL EQUALIZATION WITH QUADRATIC COMPLEXITY

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## ABSTRACT

An SOS-based blind equalization algorithm for the SIMO channel has recently been proposed that has an unprecedented quadratic complexity in the channel memory, compared to all existing SOS-based techniques whose complexity is cubic in this regard. In this paper, we show that this technique can be adapted to the MIMO channel and we prove that its complexity is maintained independently from the number of input/output channels. Simulation tests are reported that sustain the feasibility of this technique in practical observation conditions.

## 1. INTRODUCTION

Recent developments in wireless communication systems include the introduction of Multiple-Input Multiple-Output (MIMO) channels that enable many useful functionalities: increasing channel capacity, signal enhancement through reception diversity, direction of arrival estimation and, relevant to this paper, blind equalization using Second-Order Statistics (SOS). These techniques (such that [2, 4, 3], among many others) are preferred to Higher-Order Statistics techniques, such that [6], which require large sample sizes, unaffordable in a time-varying environment. Blind equalization is sought after because it suppresses the need for training sequences. Hence, a larger bandwidth is made available to the system users. Real-time implementation of blind SOS-based techniques is hindered by a complexity that is cubic in both the number of receiving antennas and the channel memory. In practice, while the former is limited, the latter is typically large. In fact, such complexity is uncommon, compared to non-blind (supervised) equalization techniques which have a quadratic complexity. Hence, the introduction of the first SOS-based blind equalization technique with a quadratic complexity [7] is an important step towards rendering blind equalization attractive for practical implementation.

Originally proposed for Single-Input Multiple-Output (SIMO) channels, we extend the so-called compaction based algorithm [7] to MIMO channels. The principle of channel compaction is the processing of small-sized sub-blocks of the channel correlation matrix, instead of the whole channel

correlation matrix. Such sub-blocks are found that have an interesting kernel structure leading to a subspace technique with a negligible complexity. For instance, linear transformation are derived from these sub-blocks that shorten the channel response by one or more taps, hence, progressively reducing Inter-Symbol Interference (ISI) until complete complete Zero-Forcing (ZF) equalization is achieved. As a result, we obtain the first MIMO SOS-based blind channel equalization technique with a quadratic complexity. Equalizers computed in this manner are to be used in a non-trivial manner in order to generate an instantaneous mixture of data from the different users. In a last step, data streams relative to the different system users are differentiated using appropriate source separation techniques. Simulations examples are conducted that show the effectiveness of this unprecedented quadratic-complexity subspace technique in blindly equalizing arbitrary MIMO channels from a limited number of noise-corrupted snapshots.

We, first, present the MIMO model in Sec. 2. In Sec. 3, we explain the principle of blind channel compaction, a channel order reduction technique serving as the basis of the proposed blind equalization technique. A new ZF equalization technique is detailed in Sec. 4. Some practical issues are addressed in Sec. 5 before simulations are conducted and commented on in Sec. 6. A conclusion is given in Sec. 7.

## 2. DATA MODEL

The well-known MIMO channel model is adopted here, where the  $c$ -th receiving antenna,  $c = 1, \dots, C_2$ , delivers, at time index  $t$ , an observation  $y^{(c)}(t)$  that is a superposition of  $C_1$  data streams  $s^{(1)}(t), \dots, s^{(C_1)}(t)$  (corrupted by noise and ISI) that originate from different  $C_1$  transmitting antennas. Propagation from the  $i$ -th transmitting antenna to the  $j$ -th receiving antenna is modeled as an impulse response  $\mathbf{h}^{i,j} = [h_0^{i,j}, \dots, h_M^{i,j}]^T$  where  $M$  is the largest order among all impulses responses  $\mathbf{h}^{i,j}$ ,  $i = 1, \dots, C_1$  and  $j = 1, \dots, C_2$ . The  $C_2$ -dim MIMO channel output  $\mathbf{y}(t) \stackrel{\text{def}}{=} [y^{(1)}(t), \dots, y^{(C_2)}(t)]^T$  is expressed as function of

the  $C_1$ -dim input  $\mathbf{s}(t) \stackrel{\text{def}}{=} [s^{(1)}(t), \dots, s^{(C_1)}(t)]^T$  as follows

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{H} [\mathbf{s}^T(t), \dots, \mathbf{s}^T(t-M)]^T + \mathbf{n}(t) \\ &\stackrel{\text{def}}{=} \mathbf{x}(t) + \mathbf{n}(t) \end{aligned}$$

Above, in  $\mathbf{H} \stackrel{\text{def}}{=} [\mathbf{H}_{(0)}, \dots, \mathbf{H}_{(M)}]$ ,  $\mathbf{H}_{(k)}$  is the  $C_2 \times C_1$   $k$ -th tap of the MIMO channel whose  $l$ -th row is  $[h_k^{1,l}, \dots, h_k^{C_1,l}]$ . The noise vector  $\mathbf{n}(t)$  is defined analogously to output vector  $\mathbf{y}(t)$ . Successive outputs are stacked into the  $C_2 L$ -dim vector

$$\begin{aligned} \mathbf{y}_L(t) &\stackrel{\text{def}}{=} [\mathbf{y}^T(t) \dots \mathbf{y}^T(t-L+1)]^T \\ &= \mathbf{H}_L [s^{(1)}(t), \dots, s^{(C_1)}(t), \dots, \\ &\quad s^{(1)}(t-M-L+1), \dots, \\ &\quad s^{(C_1)}(t-M-L+1)]^T + \mathbf{n}_L(t) \end{aligned}$$

where  $\mathbf{H}_L \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{H} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{H} & \dots & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{H} \end{bmatrix}$  is the  $C_2 L \times C_1(M+L)$  channel filtering matrix,  $\mathbf{0}$  is  $C_2 \times C_1$  and  $\mathbf{n}_L(t)$  is defined similarly as  $\mathbf{y}_L(t)$ . We let  $\mathbf{D}_s \stackrel{\text{def}}{=} \text{Diag}(\sigma_1, \dots, \sigma_{C_1})$ , where  $\sigma_c^2$  stands for the power of symbols from user  $c$  and  $\sigma_n^2$  is the noise power. The channel correlation matrix is equal to

$$\begin{aligned} \mathbf{R}_L^y &\stackrel{\text{def}}{=} \mathbf{E} [\mathbf{y}_L(t) \mathbf{y}_L^H(t)] \\ &= \mathbf{H}_L (\mathbf{I} \otimes \mathbf{D}_s^2) \mathbf{H}_L^H + \sigma_n^2 \mathbf{I} \\ &= \mathbf{R}_L + \sigma_n^2 \mathbf{I} \end{aligned} \quad (1)$$

where  $\mathbf{R}_L \stackrel{\text{def}}{=} \mathbf{E} [\mathbf{x}_L(t) \mathbf{x}_L^H(t)]$  is the correlation matrix of the noise-free output. For (1) to hold, we assume (as commonly the case in SOS-based techniques such as [2, 4, 3]) that we have i.i.d. symbols uncorrelated from each other and from the white noise, all being zero-mean. To simplify notation, we drop index  $L$  from matrices  $\mathbf{R}_L$  and  $\mathbf{H}_L$ . The latter can be made left-invertible by deploying enough receiving antennas [5]. We also handle correlation coefficients (that are sub-blocks of the correlation matrices) defined as  $\mathbf{\Gamma}_k \stackrel{\text{def}}{=} \mathbf{E} [\mathbf{x}(t+k) \mathbf{x}^H(t)]$  and  $\mathbf{\Gamma}_k^y \stackrel{\text{def}}{=} \mathbf{E} [\mathbf{y}(t+k) \mathbf{y}^H(t)]$ , for  $k = 0, \dots, M$ . Outputs are processed by a linear equalizer

$$\mathbf{G} \stackrel{\text{def}}{=} [\mathbf{G}_{(0)}^T, \dots, \mathbf{G}_{(L-1)}^T]^T$$

where coefficients  $\mathbf{G}_{(k)}$  are  $C_2 \times C_2$ , resulting in an  $C_2$ -dim output  $\mathbf{G}^T \mathbf{y}_L(t)$ , or equivalently, in a channel-equalizer combined response  $\mathbf{G}^T \mathbf{H}_L$ . ZF equalization, with eventually a non-zero delay, is achieved when the latter has the form  $[\dots, \mathbf{0}, \mathbf{0}, \mathbf{W}, \mathbf{0}, \mathbf{0}, \dots]$ , where blocks are  $C_2 \times C_1$ . The so-generated output is, then, an instantaneous mixture of output symbols.

### 3. BLIND CHANNEL COMPACTION

Blind channel compaction is a channel output processing technique introduced in [7] that builds a sequence of equivalent SIMO channels with a decreasing order, which ultimately suppresses ISI. We extend it here to MIMO channels. For this purpose, we adapt the notation  $\mathbf{H}_L$  to introduce the  $C_2 L \times C_1(M'+L)$  filtering matrix of an  $M'$ -order arbitrary channel with response  $\mathbf{H}'$  as follows

$$\mathbf{T}_L \{\mathbf{H}'\} \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{H}' \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}' & \dots & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{H}' \end{bmatrix}.$$

We also define a new, but equivalent, MIMO channel

$$\begin{aligned} \mathbf{F} &\stackrel{\text{def}}{=} [\mathbf{H}_{(0)} \mathbf{D}_s, \dots, \mathbf{H}_{(M)} \mathbf{D}_s] \\ &\stackrel{\text{def}}{=} [\mathbf{F}_{(0)}, \dots, \mathbf{F}_{(M)}] \end{aligned}$$

through which symbol sources can be seen as having unit power. In fact,  $\mathbf{R}_L^y = \mathbf{T}_L \{\mathbf{F}\} \mathbf{T}_L^H \{\mathbf{F}\} + \sigma_n^2 \mathbf{I}$  and

$$\mathbf{\Gamma}_k = [\mathbf{F}_{(k)}, \dots, \mathbf{F}_{(M)}] [\mathbf{F}_{(0)}, \dots, \mathbf{F}_{(M-k)}]^H$$

We can assume that any  $C_2$  columns of  $\mathbf{H}$  are linearly independent. Consequently, the rank of  $\mathbf{\Gamma}_k$  is equal to  $\min\{(M-k+1)C_1, C_2\}$  (in other words,  $\mathbf{\Gamma}_k$  is full column rank, unless it is large). We assume that  $C_2$  is a multiple of  $2C_1$  and that  $M+1 \geq C_2/(2C_1)$ . We define

$$K \stackrel{\text{def}}{=} M+1 - C_2/(2C_1)$$

so that  $M-K+1 = C_2/(2C_1)$ . If  $K$  is positive, then the rank of the  $C_2$ -square  $\mathbf{\Gamma}_K$  is given by  $(M-K+1)C_1 = C_2/2$ . We form a  $C_2 \times C_2/2$  matrix  $\mathbf{U}$  (resp.  $\mathbf{V}$ ) whose columns form an orthonormal set of vectors left (resp. right) orthogonal to  $\mathbf{\Gamma}_K$ , i.e. left orthogonal to  $[\mathbf{F}_{(K)}, \dots, \mathbf{F}_{(M)}]$  (resp. to  $[\mathbf{F}_{(0)}, \dots, \mathbf{F}_{(M-K)}]$ ). We can assume (with probability 1) that, if  $M-K < K$ , i.e. if  $K \geq (M+1)/2$ , then  $[\mathbf{F}_{(K)}, \dots, \mathbf{F}_{(M)}]$  is not related to  $[\mathbf{F}_{(0)}, \dots, \mathbf{F}_{(M-K)}]$ , and, so, a vector can not be both left and right orthogonal to  $\mathbf{\Gamma}_K$ . In such circumstances, the  $C_2$ -square  $[\mathbf{U}, \mathbf{V}]$  is full rank. It is, then, easy to verify that

$$[\mathbf{U}, \mathbf{V}]^H \mathbf{F} \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{F}^{\text{left}}, \mathbf{0}_{C_2/2, (M+1-K)C_1} \\ \mathbf{0}_{C_2/2, (M+1-K)C_1}, \mathbf{F}^{\text{right}} \end{bmatrix} \quad (2)$$

where  $\mathbf{F}^{\text{left}} \stackrel{\text{def}}{=} \mathbf{U}^H [\mathbf{F}_{(0)} \dots \mathbf{F}_{(K-1)}]$  and  $\mathbf{F}^{\text{right}} \stackrel{\text{def}}{=} \mathbf{V}^H [\mathbf{F}_{(M-K+1)} \dots \mathbf{F}_{(M)}]$  are  $C_2/2 \times KC_1$  matrices. Each row in (2) can be interpreted as an impulse response of order  $K-1$  only. However, the right hand side of (2) can not be considered as a MIMO channel of order  $K-1$  since none of the columns is strictly zero. One more step is needed

to achieve this goal. Thanks to the (block) Toeplitz structure of the channel filtering matrix, we can extend (2) as follows

$$\left( \mathbf{I} \otimes [\mathbf{U}, \mathbf{V}]^H \right) \mathbf{T}_{M+1} \{ \mathbf{F} \} = \begin{bmatrix} \mathbf{F}^{\text{left}}, \mathbf{0} \\ \mathbf{0}_{C_2 M, C_1}, \mathbf{T}_M \{ \mathbf{F}^{(1)} \}, \mathbf{0} \\ \mathbf{0}, \mathbf{F}^{\text{right}} \end{bmatrix} \quad (3)$$

where

$$\mathbf{F}^{(1)} = \begin{bmatrix} \mathbf{0}_{C_2/2, (M-K)C_1}, \mathbf{F}^{\text{right}} \\ \mathbf{F}^{\text{left}}, \mathbf{0}_{C_2/2, (M-K)C_1} \end{bmatrix}$$

is an  $(M-1)$ -order MIMO channel in the strict sense.

As we are to develop a blind technique based on channel SOS, we seek a relationship between the correlation matrices relative to the original and shortened MIMO channels. The  $C_2 M$ -square (noise-free) correlation matrix of  $\mathbf{F}^{(1)}$  can be reconstructed from that of  $\mathbf{F}$  as follows. By left-multiplying (3) by  $[\mathbf{0}_{C_2 M, C_2/2}, \mathbf{I}_{C_2 M}, \mathbf{0}_{C_2 M, C_2/2}]$ , then calculating the outer product, we obtain the relationship

$$\begin{aligned} & \mathbf{T}_M \{ \mathbf{F}^{(1)} \} \mathbf{T}_M^H \{ \mathbf{F}^{(1)} \} \\ &= \begin{bmatrix} \mathbf{0}_{C_2/2, C_2 M} \\ \mathbf{I} \\ \mathbf{0}_{C_2/2, C_2 M} \end{bmatrix}^T \left( \mathbf{I} \otimes [\mathbf{U}, \mathbf{V}]^H \right) \mathbf{T}_{M+1} \{ \mathbf{F} \} \\ & \quad \times \mathbf{T}_{M+1}^H \{ \mathbf{F} \} \left( \mathbf{I} \otimes [\mathbf{U}, \mathbf{V}] \right) \begin{bmatrix} \mathbf{0}_{C_2/2, C_2 M} \\ \mathbf{I} \\ \mathbf{0}_{C_2/2, C_2 M} \end{bmatrix} \end{aligned}$$

Channel compaction relies on the correlation coefficients rather than on the correlation matrix. Hence, we transform the above relationship into one between the correlation coefficients  $\mathbf{\Gamma}_0^{(1)}, \dots, \mathbf{\Gamma}_{M-1}^{(1)}$  of the compacted impulse response  $\mathbf{F}^{(1)}$  and the original correlation coefficients  $\mathbf{\Gamma}_0, \dots, \mathbf{\Gamma}_M$ . After tedious manipulations, one can prove that

$$\begin{aligned} [\mathbf{\Gamma}_0^{(1)}, \dots, \mathbf{\Gamma}_{M-1}^{(1)}] &= \left[ \begin{bmatrix} \mathbf{V}^H \mathbf{\Gamma}_0 \\ \mathbf{U}^H \mathbf{\Gamma}_1^H \end{bmatrix} \mathbf{V}, \begin{bmatrix} \mathbf{V}^H \mathbf{\Gamma}_1 \\ \mathbf{U}^H \mathbf{\Gamma}_0 \end{bmatrix} [\mathbf{U}, \mathbf{V}], \right. \\ & \quad \left. \dots, \begin{bmatrix} \mathbf{V}^H \mathbf{\Gamma}_M \\ \mathbf{U}^H \mathbf{\Gamma}_{M-1} \end{bmatrix} \mathbf{U} \right] \end{aligned}$$

Naturally, this elementary step is to be repeated to ultimately reach an (equivalent) zero-order channel i.e., one with no ISI. We, hence, form a sequence  $\mathbf{F}^{(0)}, \dots, \mathbf{F}^{(M)}$  with  $C_2$ -rows and, respectively,  $(M+1)C_1, \dots, C_1$  columns. Actually, what we obtain are the associated SOS terms  $\mathbf{\Gamma}_0^{(i)}, \dots, \mathbf{\Gamma}_{M-i}^{(i)}$ ,  $i = 1, \dots, M$ .

At last, we obtain  $\mathbf{\Gamma}_0^{(M)} = \mathbf{F}^{(M)} (\mathbf{F}^{(M)})^H$  where  $\mathbf{F}^{(M)}$  is actually  $C_2 \times C_1$ . This has been the description of an  $M$ -steps blind procedure that reduces the MIMO channel order from  $M$  to zero. In the next section, we show how this principle can be used to achieve ZF equalization.

#### 4. COMPACTION-BASED EQUALIZATION

The purpose of this section is to show how ZF equalizers are built such that the combined channel-equalizer is ISI-free. We extend (3) to a filtering matrix with an arbitrary size  $L$ ,

$$\begin{aligned} & \left( \mathbf{I} \otimes [\mathbf{U}, \mathbf{V}]^H \right) \mathbf{T}_L \{ \mathbf{F} \} \\ &= \begin{bmatrix} \mathbf{F}^{\text{left}}, \mathbf{0} \\ \mathbf{0}_{C_2(L-1), C_1}, \mathbf{T}_{L-1} \{ \mathbf{F}^{(1)} \}, \mathbf{0}_{C_2(L-1), C_1} \\ \mathbf{0}, \mathbf{F}^{\text{right}} \end{bmatrix} \end{aligned}$$

In particular, rows  $C_2/2 + 1, \dots, C_2 L - C_2/2$  verify

$$\begin{aligned} & [\mathbf{0}_{C_2(L-1), C_2/2}, \mathbf{I}, \mathbf{0}] \left( \mathbf{I} \otimes [\mathbf{U}, \mathbf{V}]^H \right) \mathbf{T}_L \{ \mathbf{F} \} \\ &= [\mathbf{0}_{C_2(L-1), C_1}, \mathbf{T}_{L-1} \{ \mathbf{F}^{(1)} \}, \mathbf{0}_{C_2(L-1), C_1}] \end{aligned}$$

The left hand side represents combined channel-equalizer responses that are zero-padded (i.e. shortened), as can be seen on the right hand side. Hence, we can progressively reduce ISI using the following sequence of matrices

$$\begin{aligned} \mathbf{G}^{(i)} &\stackrel{\text{def}}{=} \left( \mathbf{I} \otimes [\mathbf{U}^{(1)}, \mathbf{V}^{(1)}]^* \right) \begin{bmatrix} \mathbf{0}_{C_2/2, C_2(L-1)} \\ \mathbf{I} \\ \mathbf{0}_{C_2/2, C_2(L-1)} \end{bmatrix} \\ & \quad \dots \left( \mathbf{I} \otimes [\mathbf{U}^{(i)}, \mathbf{V}^{(i)}]^* \right) \begin{bmatrix} \mathbf{0}_{C_2/2, C_2(L-i)} \\ \mathbf{I} \\ \mathbf{0}_{C_2/2, C_2(L-i)} \end{bmatrix} \end{aligned}$$

$i = 1, \dots, \min\{M, L\}$ , where  $\mathbf{U}^{(i)}$  and  $\mathbf{V}^{(i)}$  are deduced from  $\mathbf{\Gamma}_0^{(i-1)}, \dots, \mathbf{\Gamma}_{M-i+1}^{(i-1)}$  the same way  $\mathbf{U}^{(1)} \stackrel{\text{def}}{=} \mathbf{U}$  and  $\mathbf{V}^{(1)} \stackrel{\text{def}}{=} \mathbf{V}$  are deduced from  $\mathbf{\Gamma}_0, \dots, \mathbf{\Gamma}_M$ . In fact, we have

$$\left( \mathbf{G}^{(i)} \right)^T \mathbf{T}_L \{ \mathbf{F} \} = [\mathbf{0}_{C_2(L-i), C_1}, \mathbf{T}_{L-i} \{ \mathbf{F}^{(i)} \}, \mathbf{0}]$$

where  $\mathbf{F}^{(i)}$  has  $M-i+1$  taps only. Each of the  $C_2(L-i)$  columns of  $\mathbf{G}^{(i)}$  can be seen as an  $L$ -taps equalizer for which the combined impulse response has a maximum of  $M-i+1$  non-zero taps. Using a similar proof in [7], we can show that matrices  $\mathbf{G}^{(i)}$  have the following interesting structure

$$\begin{aligned} \mathbf{G}^{(i)} &= \left[ \begin{bmatrix} \mathbf{G}_2^{(i)} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{0}_{C_2, C_2} \\ \mathbf{G}_1^{(i)}, \mathbf{G}_2^{(i)} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{0}_{2C_2, C_2} \\ \mathbf{G}_1^{(i)}, \mathbf{G}_2^{(i)} \\ \mathbf{0} \end{bmatrix}, \right. \\ & \quad \left. \dots, \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_1^{(i)}, \mathbf{G}_2^{(i)} \\ \mathbf{0}_{C_2, C_2} \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_1^{(i)} \end{bmatrix} \right] \end{aligned}$$

where  $iC_2 \times C_2/2$  matrices  $\mathbf{G}_1^{(i)}$  and  $\mathbf{G}_2^{(i)}$  are defined iteratively, as:  $\mathbf{G}_1^{(1)} \stackrel{\text{def}}{=} (\mathbf{U}^{(1)})^*$ ,  $\mathbf{G}_2^{(1)} \stackrel{\text{def}}{=} (\mathbf{V}^{(1)})^*$ , and for  $i \geq 2$

$$\mathbf{G}_1^{(i)} \stackrel{\text{def}}{=} \left[ \begin{bmatrix} \mathbf{G}_2^{(i-1)} \\ \mathbf{0}_{C_2, C_2/2} \end{bmatrix}, \begin{bmatrix} \mathbf{0}_{C_2, C_2/2} \\ \mathbf{G}_1^{(i-1)} \end{bmatrix} \right] (\mathbf{U}^{(i)})^* \quad (4)$$

$$\mathbf{G}_2^{(i)} \stackrel{\text{def}}{=} \left[ \begin{bmatrix} \mathbf{G}_2^{(i-1)} \\ \mathbf{0}_{C_2, C_2/2} \end{bmatrix}, \begin{bmatrix} \mathbf{0}_{C_2, C_2/2} \\ \mathbf{G}_1^{(i-1)} \end{bmatrix} \right] (\mathbf{V}^{(i)})^* \quad (5)$$

From the equalization point-of-view,  $\mathbf{G}^{(i)}$  is equivalent to  $\mathbf{G}_1^{(i)}$  and  $\mathbf{G}_2^{(i)}$  that verify

$$\begin{aligned} (\mathbf{G}_1^{(i)})^T \mathbf{T}_i \{\mathbf{F}\} &= \begin{bmatrix} \mathbf{0}_{C_2/2, i-1}, [\mathbf{0}, \mathbf{I}_{C_2/2}] \mathbf{F}^{(i)}, \mathbf{0}_{C_2/2, i} \end{bmatrix} \\ (\mathbf{G}_2^{(i)})^T \mathbf{T}_i \{\mathbf{F}\} &= \begin{bmatrix} \mathbf{0}_{C_2/2, i}, [\mathbf{I}_{C_2/2}, \mathbf{0}] \mathbf{F}^{(i)}, \mathbf{0}_{C_2/2, i-1} \end{bmatrix} \end{aligned}$$

They reduce the channel order from  $M$  to a desired  $M-i$ . We let  $\mathbf{F}_{\text{up}} \stackrel{\text{def}}{=} [\mathbf{I}_{C_2/2}, \mathbf{0}] \mathbf{F}^{(M)}$  and  $\mathbf{F}_{\text{down}} \stackrel{\text{def}}{=} [\mathbf{0}, \mathbf{I}_{C_2/2}] \mathbf{F}^{(M)}$  so that  $\mathbf{G}_1^{(M)}$  and  $\mathbf{G}_2^{(M)}$ , now denoted as  $\mathbf{G}_1$  and  $\mathbf{G}_2$ , respectively, verify

$$\begin{aligned} \mathbf{G}_1^T \mathbf{T}_M \{\mathbf{F}\} &= [\mathbf{0}_{C_2/2, C_1(M-1)}, \mathbf{F}_{\text{down}}, \mathbf{0}_{C_2/2, C_1 M}] \\ \mathbf{G}_2^T \mathbf{T}_M \{\mathbf{F}\} &= [\mathbf{0}_{C_2/2, C_1 M}, \mathbf{F}_{\text{up}}, \mathbf{0}_{C_2/2, C_1(M-1)}] \end{aligned}$$

Given the definition of  $\mathbf{F}$ , this is equivalent to have

$$\begin{aligned} \mathbf{G}_1^T \mathbf{H}_M &= [\mathbf{0}_{C_2/2, C_1(M-1)}, \mathbf{F}_{\text{down}} \mathbf{D}_s^{-1}, \mathbf{0}_{C_2/2, C_1 M}] \\ \mathbf{G}_2^T \mathbf{H}_M &= [\mathbf{0}_{C_2/2, C_1 M}, \mathbf{F}_{\text{up}} \mathbf{D}_s^{-1}, \mathbf{0}_{C_2/2, C_1(M-1)}] \end{aligned}$$

We recognize above combined channel-equalizer responses that are ISI-free, hence achieving ZF equalization. Let's have our equalizers  $\mathbf{G}_1^T$  and  $\mathbf{G}_2^T$  process a set of  $W$  channel outputs  $\mathbf{y}(t), \dots, \mathbf{y}(t-W+1)$ . Thanks to the above relationships, we obtain

$$\begin{aligned} \mathbf{G}_1^T \mathbf{y}_M(t) &= \mathbf{F}_{\text{down}} \mathbf{D}_s^{-1} \mathbf{s}(t-M+1) + \mathbf{G}_1^T \mathbf{n}_M(t) \\ \mathbf{G}_2^T \mathbf{y}_M(t) &= \mathbf{F}_{\text{up}} \mathbf{D}_s^{-1} \mathbf{s}(t-M) + \mathbf{G}_2^T \mathbf{n}_M(t) \end{aligned}$$

From the above, we can see that columns of  $\mathbf{G}_1$  and  $\mathbf{G}_2$  act as ZF equalizers in the sense that they deliver (combinations of) inputs that have been emitted by the channel some time earlier. However, the delay is different for each family of equalizers. In order to benefit from the total set of computed equalizers, we arrange their respective outputs in the following matrix

$$\begin{bmatrix} \mathbf{G}_1^T [\mathbf{y}_M(t-1), \dots, \mathbf{y}_M(t-W+1)] \\ \mathbf{G}_2^T [\mathbf{y}_M(t), \dots, \mathbf{y}_M(t-W+2)] \end{bmatrix} \quad (6)$$

which can be easily verified to be equal (up to an additive noise term) to

$$\begin{bmatrix} \mathbf{F}_{\text{down}} \\ \mathbf{F}_{\text{up}} \end{bmatrix} \mathbf{D}_s^{-1} \begin{bmatrix} \mathbf{s}^{(1)}(t-M) \dots, \mathbf{s}^{(1)}(t-M-W+2) \\ \vdots \\ \mathbf{s}^{(C_1)}(t-M), \dots, \mathbf{s}^{(C_1)}(t-M-W+2) \end{bmatrix}$$

Hence, if we (linearly) combine the channel outputs, using our equalizers following (6), we obtain an instantaneous mixture of source symbols. Individual symbol streams can be retrieved using appropriate source separation algorithms, a regular step in SOS-based blind equalization of MIMO channels [3].

## 5. ALGORITHM DESCRIPTION

Developments have so far assumed noise-free SOS. Noise power  $\sigma_n^2$  needs to be estimated. Its contribution is removed from channel SOS (from  $\mathbf{\Gamma}_0^y$  actually). This needs to be achieved without compromising the main advantage of the proposed blind equalization technique, i.e. the quadratic complexity. Such a goal is reached thanks to the relationship  $\mathbf{\Gamma}_0^y + \sum_{k=1}^M [\mathbf{\Gamma}_k^y + (\mathbf{\Gamma}_k^y)^H]$  extended from a similar result in [7]. Hence,  $\sigma_n^2$  is the lowest eigenvalue of the positive-definite left hand side above, with multiplicity  $C_2 - C_1$ , or the average of the  $C_2 - C_1$  lowest eigenvalues if SOS are estimated. Finally, the equalization algorithm can be described as follows:

1. Estimate  $\mathbf{\Gamma}_j^{(0)} = \mathbf{E} [\mathbf{x}(n)\mathbf{x}^H(n-j)]$ ,  $j = 0 \dots, M$
2. Initialize  $i = 0$ . While  $i \leq M$ , repeat
  - (a) Set  $m = M - i$  and  $k = \max \left\{ m + 1 - \frac{C_2}{2C_1}, 1 + \lfloor \frac{m}{2} \rfloor \right\}$
  - (b) Let  $\mathbf{u}_j$  (resp.  $\mathbf{v}_j$ ) be the left (resp. right) unit-norm singular vector associated to the  $j$ -th lowest singular value of  $\mathbf{\Gamma}_k^{(i)}$ . Built  $\mathbf{U}^{(i+1)} \stackrel{\text{def}}{=} [\mathbf{u}_1, \dots, \mathbf{u}_{C_2/2}]$  and  $\mathbf{V}^{(i+1)} \stackrel{\text{def}}{=} [\mathbf{v}_1, \dots, \mathbf{v}_{C_2/2}]^H$
  - (c) Calculate  $\mathbf{G}_1^{(i)}$  and  $\mathbf{G}_2^{(i)}$  using (4)-(5)
  - (d) For  $j = 0, \dots, m-1$ , calculate  $\mathbf{\Gamma}_j^{(i+1)} = \begin{bmatrix} (\mathbf{V}^{(i+1)})^H \mathbf{\Gamma}_j^{(i)} \\ (\mathbf{U}^{(i+1)})^H \mathbf{\Gamma}_{j-1}^{(i)} \end{bmatrix} \mathbf{V}^{(i)}$ ,  $\begin{bmatrix} (\mathbf{V}^{(i+1)})^H \mathbf{\Gamma}_{j+1}^{(i)} \\ (\mathbf{U}^{(i+1)})^H \mathbf{\Gamma}_j^{(i)} \end{bmatrix} \mathbf{U}^{(i)}$  and increment  $i$  by 1
  - (e) Break if  $i > M$
  - (f) Repeat  $m - k$  times
    - i. Set  $\mathbf{U}^{(i)} = [\mathbf{0}, \mathbf{I}_{C_2/2}]^T$  and  $\mathbf{V}^{(i)} = [\mathbf{I}_{C_2/2}, \mathbf{0}]^T$
    - ii. Repeat steps (2c), (2d) and (2e)
  - (g) Set  $\mathbf{G}_1 \stackrel{\text{def}}{=} \mathbf{G}_1^{(M)}$  and  $\mathbf{G}_2 \stackrel{\text{def}}{=} \mathbf{G}_2^{(M)}$  and generate new ISI-free outputs using (6).

Since the blind algorithm processes channel SOS in the form of  $C_2 \times C_2$  matrices  $\mathbf{\Gamma}_k$ , it is independent from  $C_1$ , except for rank computation. Hence, the overall computation for large  $M$  is the same as that of the algorithm [7] originally developed for SIMO channels, i.e. of order  $O((6 + C_2)C_2^2 M^2)$ . This complexity will not be affected by the source separation step (next section) since this one does not depend on  $M$ , but on  $C_1$  and  $C_2$  instead, which both take limited values in practice.

## 6. SIMULATIONS

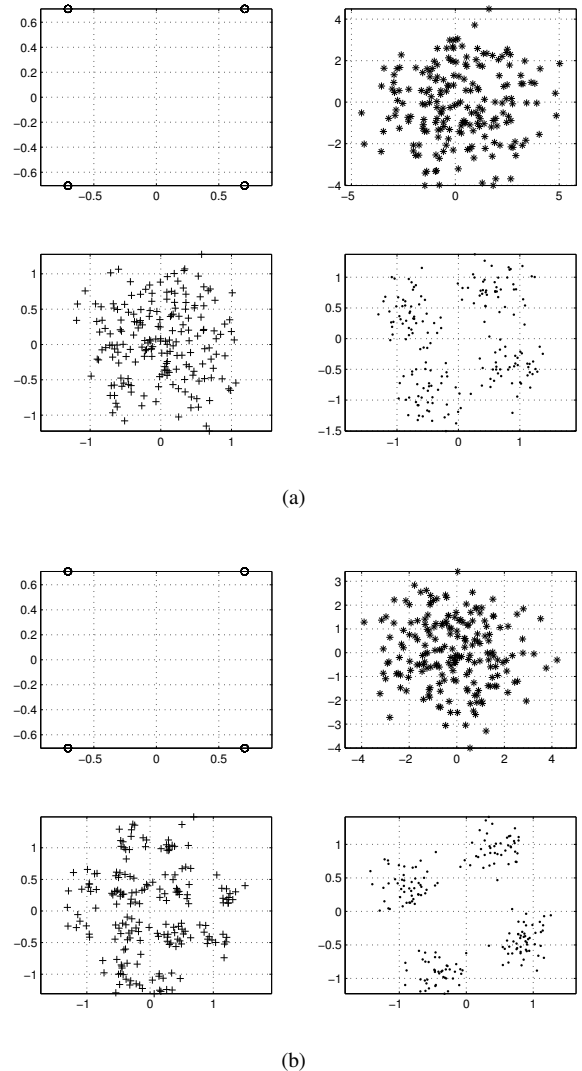
To demonstrate the feasibility of blind MIMO channel equalization using the proposed quadratic-complexity subspace algorithm, we test randomly generated ISI channels driven by i.i.d. unit-variance QAM symbols. After application of the proposed equalizers, the instantaneous source mixture is separated using the JADE algorithm [1]. The number of transmitting and receiving antennas have been fixed to 2 and 4, respectively. Channel coefficients of the MIMO channel have been generated as unit-norm i.i.d. random variables. Equalizers are computed from the channel correlation matrix estimated from a limited number of source symbols (500 in Fig. 1.a and 1000 in Fig. 1.b). The so-obtained equalizer is later tested with 200 QAM source symbols from each user. Detected symbols (i.e. 200) relative to source 1 are represented in Fig. 1 for a noise variance equal to  $\sigma_n^2 = 0.1$ . Source symbols are, hence, shown to be recovered with an undetermined rotation ambiguity, inherent to blind equalization, and that is usually combatted using differential encoding.

## 7. CONCLUSION

Channel compaction is based on the proved rank deficiency of some small-sized sub-blocks of the MIMO channel correlation matrix. This property is exploited to construct a subspace technique that forces to zero some leading/trailing terms of the MIMO channel impulse response. Because this procedure is to be repeated a number of times not larger than the channel memory  $M$ , the overall complexity is in  $M^2$ , unlike all existing comparable techniques whose complexity is at least in  $M^3$ . The feasibility of this blind equalization technique in realistic conditions is proved by simulations.

## 8. REFERENCES

- [1] J. F. Cardoso and A. Souloumiac, "Blind beamforming for non gaussian signals," *IEE Proceedings F*, vol. 140, no. 6, pp. 362-370, Dec. 1993.
- [2] J. Shen and Z. Ding, "Direct blind MMSE channel equalization based on second-orders statistics," *IEEE Trans. Signal Processing*, vol. 48, no. 4, pp. 1015-1022, April 2000.
- [3] V. Zarzoso, A. K. Nandi, J. I. Garcia and L. V. Dominguez, "Blind identification and equalization of MIMO FIR channels based on second-order statistics and blind source separation," in *Proc. Int. Conf. on Digital Signal Processing*, 2002, vol. I, pp. 135-138.
- [4] S. An, J. H. Manton and Y. Hua, "A Sequential Subspace Method for Blind Identification of General FIR MIMO Channels," *IEEE Trans. Signal Processing*, vol. 53, no. 10, pp. 3906-3909, Oct. 2005.



**Fig. 1.** Equalization results for a 4-th order  $2 \times 4$  MIMO channel. From left to right and from top to bottom: QAM symbols from source 1, output collected at receiving antenna 1, equalized symbols at receiving antenna 1, reconstructed symbols of source 1.

- [5] J. Fang, A. R. Leyman, Y. H. Chew and H. Duan, "Some further results on blind identification of MIMO FIR channels via second-order statistics", *Signal Processing*, Elsevier, vol. 87(6), pp. 1434-1447, June. 2007.
- [6] J. Via, I. Santamaria and J. Perez, "Deterministic CCA-based algorithms for blind equalization of FIR-MIMO channels," *IEEE Trans. Signal Processing*, vol. 55, no. 7, pp. 3867-3878, July 2007.
- [7] H. Gazzah, "SOS-based blind channel equalization with quadratic complexity," *IEEE Trans. Signal Processing*, vol. 59, no. 2, pp. 837-841, Feb. 2011.