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SUBSPACE BLIND EQUALIZATION FOR CHANNELS WITH LOW SNR

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ABSTRACT

In the plethora of second-order statistics (SOS) based blind channel equalization techniques, only two algorithms are able to perform equalization with a pre-specified delay. Delay selection is a compelling feature in order to reduce noise enhancement of Zero-Forcing (ZF) equalizers. We show that channel output shifted correlation matrices with different time lags can be combined to obtain a rank-deficient SOS-based matrix whose kernel is made of ZF equalizers of a pre-determined delay. Contrarily to existing algorithms, such a ZF equalizer is obtained at a low complexity: It involves a single Eigen Vector Decomposition (EVD) and does not require prior knowledge nor estimation of the noise power. Such a straightforward estimation translates, also, into better equalization performance at low channel SNR, as confirmed by simulations.

1. INTRODUCTION

Blind equalization is a classic topic of statistical signal processing where the input of an unknown linear filter is to be restored from the noise-corrupted filter output. Among many engineering applications, blind equalization is of relevance, primarily, to communication systems, where it allows to save bandwidth, otherwise spent on channel training. An algorithm computes the equalizer taps using the statistics of the channel output. Naturally, SOS are preferred to the less accurately estimated higher-order statistics. However, the receiver, then, needs to display at least two receiver antennas and/or sample the channel output at a rate higher than the Baudrate. Very often, the so-called subspace algorithms identify a full subspace of ZF equalizers that restore the input, with the same delay but with different SNRs. At last, the optimal combination of these equalizers is found. In the process, many eigen/singular vector decompositions (EVD/SVD) are needed.

These are ZF equalizers that may result in significant noise enhancement if the channel SNR is low. Furthermore, the amount of noise enhancement depends significantly on the equalization delay. A recent result [8] is more specific as it proves that all delays larger than the channel order

are expected to lead to very comparable performance. This phenomenon is particularly severe at low SNR. Hence, it is important that the (blind) equalization algorithm detects all possible ZF equalizers with a delay that can be controlled. Linear prediction (LP) [1], probably the most referenced blind equalization algorithm, does not offer such a functionality: performance is not optimum and delay is fixed to zero. In [5], performance is made optimum but the zero delay, unchanged, leads to poor equalization performance [4]. Some other algorithms, for which the delay can be fixed, do not detect all the possible equalizers [3, 4], and hence, are not guaranteed to reach optimality. Up to our knowledge, the only existing algorithms that compute ZF equalizers with optimum performance and controllable delay are [2, 9]. They behave in a similar manner, in that they achieves the lowest Mean Square Error (MSE) among all ZF equalizers by: (i) first, detecting (the subspace of) all ZF equalizers with the specified length and delay, then, (ii) a criterion from [5, 6] is applied to select, within this subspace, the minimum MSE (MMSE) equalizer. The advantage of [9] is a complexity (typically) 4 times lower than for [2]. The two algorithms have the same asymptotic performance (for a large number of observations) but [2] performs better when the number of snapshots is limited.

The objective of this paper is to propose a new algorithm, with the same features as [2, 9], i.e. able to compute to the optimum (MMSE) linear equalizer and set the equalization delay to an arbitrarily chosen value. It involves less computation and shows performance better than both. This advantage can be explained by a number of SVD/EVD lower than for [2, 9]. We know that such decompositions are responsible for the majority of the computation load [14] and are sensitive to the channel condition (number of snapshots, noise level and near-singularity of the correlation matrix) [15].

The paper is organized as follows. First, we recall the data model in Sec. 2. In Sec. 3, we detail the proposed algorithm and compare it in Sec. 4 with the algorithms from [5, 2]. A conclusion is given in Sec. 5. Matrices (resp. vectors) are represented by bold or calligraphic upper case (resp. bold lower case) characters. Vectors are by default in column orientation, while The following notations are used. $\mathbf{0}_{a,b}$ is the $a \times b$ zero

matrix. \mathbf{I}_a is the $a \times a$ identity matrix. $\mathbf{J}_a \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I}_{a-1} & \mathbf{0} \end{bmatrix}$ is the $a \times a$ (down) shifting matrix. We denote $\mathbf{J}_a^{-k} \stackrel{\text{def}}{=} (\mathbf{J}_a^T)^k$. Dimensions are dropped when they can be inferred from the context. $\|\cdot\|$ denotes the Euclidean norm. $\mathbf{A} \otimes \mathbf{B}$ is the Kronecker product between matrices \mathbf{A} and \mathbf{B} defined such that its (i, j) block is $a_{i,j}\mathbf{B}$.

2. DATA MODEL

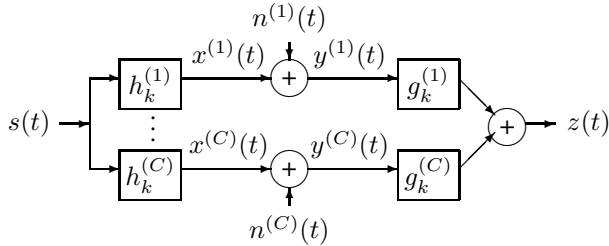


Fig. 1. Equalized SIMO channel.

When a transmitted signal is received by a fractionally spaced and/or multi-sensor receiver, the channel can be modeled as a Single Input Multiple Output (SIMO) channel. As depicted in Fig. 1, a set of C sub-channels are driven by a common scalar input $s(n)$. The impulse response of the c -th sub-channel is $[h_0^{(c)} \cdots h_M^{(c)}]$, M being the largest order among all sub-channels. It is stored in the c -th row of the $C \times (M + 1)$ matrix \mathbf{H} , that represents the impulse response of the SIMO channel. The $(k + 1)$ -th column $\mathbf{h}_k \stackrel{\text{def}}{=} [h_k^{(1)} \cdots h_k^{(C)}]^T$ of \mathbf{H} represents the C -dim k -th tap of the SIMO channel, $k \in \{0, \dots, M\}$. The C -dim SIMO output $\mathbf{y}(t) \stackrel{\text{def}}{=} [y^{(1)}(t) \cdots y^{(C)}(t)]^T$ is corrupted by additive white Gaussian noise (AWGN) $\mathbf{n}(t)$, defined similarly as $\mathbf{y}(t)$. The channel input-output relationship is given by

$$\mathbf{y}(t) = \mathbf{H}\mathbf{s}_{M+1}(t) + \mathbf{n}(t),$$

where $\mathbf{s}_l(t) \stackrel{\text{def}}{=} [s(t) \cdots s(t - l + 1)]^T$ for any l . To exploit time-invariance of the channel, we stack L successive outputs into the CL -dim vector $\mathbf{y}_L^T(t) \stackrel{\text{def}}{=} [\mathbf{y}^T(t) \cdots \mathbf{y}^T(t - L + 1)]^T$ which is related to the source symbols following

$$\mathbf{y}_L(t) = \mathbf{H}_L\mathbf{s}_{M+L}(t) + \mathbf{n}_L(t)$$

where

$$\mathbf{H}_L \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{H} & \mathbf{0} & \cdots & \mathbf{0} \\ & \cdot & & \\ & & \cdot & \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H} \end{bmatrix}$$

is the $CL \times (M + L)$ channel filtering matrix. There, $\mathbf{n}_L(t)$ is defined similarly as $\mathbf{y}_L(t)$ and $\mathbf{0}$ is the C -dim zero vector.

The matrix \mathbf{H}_L is full column rank if [7]

$$L \geq M. \quad (1)$$

It is also required that the transfer functions relative to the different sub-channels do not have any common zero. This condition is widely verified in practice [7].

Blind subspace techniques make use of the channel SOS, represented by correlation matrices

$$\begin{aligned} \mathbf{R}_L^{(i)} &\stackrel{\text{def}}{=} \mathbf{E} [\mathbf{y}_L(i + i')\mathbf{y}_L^H(i')] \\ &= \begin{bmatrix} \Gamma_i & \cdots & \Gamma_{i+L-1} \\ \vdots & \ddots & \vdots \\ \Gamma_{i-L+1} & \cdots & \Gamma_i \end{bmatrix} \\ &= \sigma_s^2 \mathbf{H}_L \mathbf{J}_{M+L}^i \mathbf{H}_L^H + \sigma_n^2 (\mathbf{J}_L^i \otimes \mathbf{I}_C), \end{aligned} \quad (2)$$

where $\Gamma_k \stackrel{\text{def}}{=} \mathbf{E} [\mathbf{y}(t + k)\mathbf{y}^H(t)]$. A majority of techniques [11, 1, 10] use the standard correlation matrix $\mathbf{R}_L \stackrel{\text{def}}{=} \mathbf{R}_L^{(0)}$. However, shifted correlation matrices have been shown to be useful to directly detect ZF equalizers [9, 2]. We assume independent and identically distributed (i.i.d.) symbols that are also uncorrelated from the white noise components, in order for (2) to hold. We also assume the transmitted symbols and the noise samples to be zero-mean and denote by σ_s^2 and σ_n^2 their respective powers. These assumptions are common to all existing SOS-based blind channel identification, equalization and shortening techniques.

Channel outputs are fed to an $(L - 1)$ -order linear equalizer \mathbf{g} . The equalizer impulse response is denoted by $\mathbf{g} \stackrel{\text{def}}{=} [\mathbf{g}_0, \dots, \mathbf{g}_{L-1}]^T$, where $\mathbf{g}_k \stackrel{\text{def}}{=} [g_k^{(1)}, g_k^{(2)}, \dots, g_k^{(C)}]^T$ is the k -th equalizer tap, using similar notations as for the channel impulse response. An $(L - 1)$ -order equalizer \mathbf{g} achieves ZF equalization with delay d iff¹ $\mathbf{g}^H \mathbf{H} = [\mathbf{0}_{1,d} \ w \ \mathbf{0}_{1,M+L-1-d}]$ for some arbitrary scalar w . The equalizer output $z(t) = \mathbf{g}^H [\mathbf{y}^T(t) \mathbf{y}^T(t - 1) \cdots]^T$ restores the symbols with delay d and attenuation w . Signal and noise components have power $\sigma_s^2 |w|^2$ and $\sigma_n^2 \|\mathbf{g}\|^2$, respectively. The associated (normalized) MSE is defined as $\text{MSE} \stackrel{\text{def}}{=} \mathbf{E} [|s(t - d) - z(t)|^2]$. The ZF equalizer with the lowest MSE, referred to here as the MMSEZF equalizer, is [5] $(\mathbf{H}^H)^\dagger [\cdots 0 \ 1 \ 0 \ \cdots]^T$ where 1 is at position $d + 1$. It has been shown in [8] that for $d = M, \dots, L - 1$, all MMSEZF equalizer are expected to show comparable equalization MSE. However, for delays outside this range, performance degrade very significantly.

¹For notation simplicity, we adopt here an altered definition of the ZF equalizer, which, in reality, should verify $\mathbf{g}^T \mathbf{H} = [\cdots, 0, 1, 0, \cdots]$.

3. THEORETICAL DEVELOPMENT

The idea of this paper is to combine shifted correlation matrices in order to obtain an SOS-derived expression equal to

$$\mathbf{H}_L \mathbf{T}_{L+M} \mathbf{H}_L^H \quad (3)$$

where

$$\mathbf{T}_n \stackrel{\text{def}}{=} \begin{bmatrix} t_0 & \cdots & t_{-(n-1)} \\ \vdots & \ddots & \vdots \\ t_{n-1} & \cdots & t_0 \end{bmatrix}$$

is an arbitrarily chosen Toeplitz matrix. In a second step, we choose \mathbf{T}_{L+M} such that the matrix $\mathbf{H}_L \mathbf{T}_{L+M} \mathbf{H}_L^H$ is one whose (signal and/or noise) subspace has a special property, useful for identification and/or equalization purposes. For instance, using (2), one can prove that

$$\sum_{i=-(L+M-1)}^{L+M-1} t_i \mathbf{R}_L^{(i)} = \sigma_s^2 \mathbf{H}_L \mathbf{T}_{L+M} \mathbf{H}_L^H + \sigma_n^2 (\mathbf{T}_L \otimes \mathbf{I}_C)$$

Since we are completely free to choose \mathbf{T}_{L+M} , we will choose one with a particular property. To determine what would be the suitable choice of $\dots, t_{-1}, t_0, t_1, \dots$, we investigate the properties of the noise subspace of the matrix (3), assumed to be rank deficient. Let $\mathbf{f}^{(L)}$ and $\mathbf{f}^{(R)}$ be, respectively, left and right orthogonal to $\mathbf{H}_L \mathbf{T}_{L+M} \mathbf{H}_L^H$. Then $\mathbf{T}_{L+M} \mathbf{H}_L^H \mathbf{f}^{(R)} = \mathbf{0}$ and $\mathbf{T}_{L+M}^H \mathbf{H}_L \mathbf{f}^{(L)} = \mathbf{0}$, as a consequence of \mathbf{H}_L being left invertible. We choose the \mathbf{T}_{L+M} to have a rank $L + M - 1$ and \mathbf{e}_{d+1} to be its unique (up to a scaling and a rotation) right orthogonal vector, $d = 0, \dots, L + M - 1$. This implies that $t_{-d} = \dots = t_0 = \dots = t_{L+M-1-d} = 0$. This is also row $L + M - d$ of \mathbf{T}_{L+M} , so that we also have \mathbf{e}_{L+M-d} necessarily left orthogonal to the same \mathbf{T}_{L+M} . Hence, we can write

$$\mathbf{T}_{L+M} \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{U} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{L} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

where \mathbf{L} and \mathbf{U} are, respectively, d lower and $L + M - d - 1$ upper triangular matrices. For \mathbf{T}_{L+M} to be deficient by one, we need \mathbf{L} and \mathbf{U} to be both invertible, i.e., at least $t_{-(d+1)}$ and t_{L+M-d} to be non-zero. Consequently,

$$\begin{aligned} \mathbf{H}_L^H \mathbf{f}^{(R)} &= \alpha^{(R)} \mathbf{e}_{d+1} \\ \mathbf{H}_L^H \mathbf{f}^{(L)} &= \alpha^{(L)} \mathbf{e}_{L+M-d} \end{aligned}$$

The above is true for any \mathbf{T}_{L+M} verifying $t_{-(d+1)} \neq t_{-d} = \dots = t_0 = \dots = t_{L+M-1-d} = 0 \neq t_{L+M-d}$. For the remaining $t_{-(L+M-1)}, \dots, t_{-(d+2)}, t_{L+M-d+1}, t_{L+M-1}$, they are chosen such that \mathbf{T}_{L+M} is well conditioned. By checking $\mathbf{T}_{L+M} \mathbf{T}_{L+M}^H$, we realize that, apart from zero, the remaining singular values of \mathbf{T}_{L+M} have the same magnitude as those of \mathbf{L} and \mathbf{U} . To ensure the best condition number, we would like all singular values to have the same magnitude, say one.

A trivial way to do so is simply to choose \mathbf{L} and \mathbf{U} to be identity matrices. Hence, all t_i are zero, except $t_{-(d+1)}$ and t_{L+M-d} , both equal to one. We redefine \mathbf{T}_n as $\mathbf{T}_n^{(d)}$, the $n \times n$ Toeplitz matrix whose only non-zero diagonals are the $(d+1)$ upper diagonal and the $(L+M-d)$ lower diagonal.

Because \mathbf{H}_L is full column rank, $\mathbf{H}_L \mathbf{T}_{L+M}^{(d)} \mathbf{H}_L^H$ has rank $L+M-1$. Let $\mathbf{N}^{(R)}$ and $\mathbf{N}^{(L)}$ be its $CL \times [(C-1)L-M+1]$ right and left kernel respectively. For any $[(C-1)L-M+1]$ -dim \mathbf{f} , we have

$$\begin{aligned} \mathbf{H}_L^H \mathbf{N}^{(R)} \mathbf{f} &= \alpha^{(R)} \mathbf{e}_{d+1} \\ \mathbf{H}_L^H \mathbf{N}^{(L)} \mathbf{f} &= \alpha^{(L)} \mathbf{e}_{L+M-d} \end{aligned}$$

for some undetermined $\alpha^{(R)}$ and $\alpha^{(L)}$. Hence, $\mathbf{N}^{(R)} \mathbf{f}$ and $\mathbf{N}^{(L)} \mathbf{f}$ are ZF equalizers with a delay of, respectively, d and $L + M - d - 1$.

To focus on d -delay equalizers, they are located in the right kernel of $\mathbf{H}_L \mathbf{T}_{L+M}^{(d)} \mathbf{H}_L^H$ and in the left kernel of $\mathbf{H}_L \mathbf{T}_{L+M}^{(L+M-d+1)} \mathbf{H}_L^H \mathbf{S}$. Blind equalization is made possible because we can calculate $\mathbf{H}_L \mathbf{T}_{L+M}^{(d)} \mathbf{H}_L^H \mathbf{S}$ as

$$\mathbf{R} = \mathbf{R}_L^{-(d+1)} + \mathbf{R}_L^{(L+M-d)} - \sigma_n^2 (\mathbf{T}_L^{(d)} \otimes \mathbf{I}) \quad (4)$$

Using the above development means that we have to perform SVD of two matrices: $\mathbf{H}_L \mathbf{T}_{L+M}^{(d)} \mathbf{H}_L^H \mathbf{S}$ and

$\mathbf{H}_L \mathbf{T}_{L+M}^{(L+M-d+1)} \mathbf{H}_L^H \mathbf{S}$. Since computation burden is an issue, we will focus on a particular parametrization of the equalizer, for instance judicious choices of d and L that increases performance and reduce computation.

First, we notice that $\mathbf{T}_L^{(d)}$ is zero when both $d+1$ and $L+M-d$ are larger or equal to L ; i.e.

$$L-1 \leq d \leq M \quad (5)$$

Then, the computation (estimation) or σ_n^2 is not needed in (4), to the point that it can be assumed to be equal to zero (or any arbitrary value). Taken the condition (1) into consideration, condition (5) leads to the following possible choices

$$(L, d) \in \{(M, M-1), (M, M), (M+1, M)\}$$

For the above selected equalizer length and delay, the noise power is not needed (to be estimated), and SVD of \mathbf{R} is less motivated. As shown in [8], near-optimal equalization performance is met at any delay larger than the channel order, and, of course, the larger the equalizer, the better the performance. Consequently, we choose

$$(L, d) = (M+1, M)$$

In this case, we also have $d+1 = L+M-d$, i.e. the matrix $\mathbf{H}_L \mathbf{T}_{L+M}^{(d)} \mathbf{H}_L^H$ is Hermitian. Left and right kernels are identical, and can be computed using EVD, instead of SVD.

Because $\mathbf{H}_L \mathbf{T}_{L+M}^{(d)} \mathbf{H}_L^H$ is now Hermitian, its left and right kernels $\mathbf{N}^{(R)}$ and $\mathbf{N}^{(L)}$ are commonly referred to as \mathbf{N} . It is now obtained by EVD, rather than by SVD of $\mathbf{H}_L \mathbf{T}_{L+M}^{(d)} \mathbf{H}_L^H$. There, lie all possible ZF equalizers, including undesirable ones (restoring no signal because are left orthogonal to \mathbf{H}). Instead of excluding these ones by projecting \mathbf{N} on the signal subspace, which would require an EVD of the CL -square matrix \mathbf{R} , we propose to conduct this elimination indirectly by retaining, among all vectors (equalizers) in the column span of \mathbf{N} , the one that restores the signal with the largest SNR. A ZF equalizer can be written as $\mathbf{N}\mathbf{f}$, where \mathbf{f} is $[(C-1)L - M + 1]$ -dim. It has a combined (with the channel) response equal to $\mathbf{H}^H \mathbf{N}\mathbf{f} = \alpha \mathbf{e}_{d+1}$. It generates a signal components with power $\sigma_s^2 |\alpha|^2$ and a noise components with a power $\sigma_s^2 \|\mathbf{N}\mathbf{f}\|^2 = \sigma_s^2 \|\mathbf{f}\|^2$, assuming \mathbf{N} is made by orthonormal columns (which is the case when it is obtained as a result of an EVD, as the case here). Maximizing the equalizer output SNR implies maximizing the ratio $|\alpha|^2 / \|\mathbf{f}\|^2$, or equivalently $\mathbf{f}^H \mathbf{N}^H \mathbf{H} \mathbf{H}^H \mathbf{N} \mathbf{f} / \|\mathbf{f}\|^2$. Hence, the optimal equalizer is given by $\mathbf{N}\mathbf{f}$ where \mathbf{f} is the eigen vector associated to the largest eigenvalue of $\mathbf{N}^H (\mathbf{R} - \sigma_n^2 \mathbf{I}) \mathbf{N}$, or equivalently, the largest eigenvalue of $\mathbf{N}^H \mathbf{R} \mathbf{N}$. Hence we can describe the procedure as follows

1. Set $d = M$ and $L = M + 1$
2. Calculate $\mathcal{R} = \mathbf{R}_L^{d+1} + (\mathbf{R}_L^{d+1})^H$ which is nothing but

$$\begin{bmatrix} \mathbf{0} & \Gamma_M^H & \cdots & \Gamma_1^H \\ \Gamma_M & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \Gamma_M^H \\ \Gamma_1 & \cdots & \Gamma_M & \mathbf{0} \end{bmatrix}$$
3. Perform EVD of \mathcal{R} and form \mathbf{N} whose columns are eigen vectors associated to the $(C-1)L - M + 1$ eigenvalue with the lowest magnitude².
4. Determine the unit-norm right eigen vector \mathbf{f} associated to the largest eigenvalue of $\mathbf{N}^H \mathbf{R} \mathbf{N}$
5. Calculate the equalizer $\mathbf{g} \stackrel{\text{def}}{=} (\mathbf{N}\mathbf{f})^*$

The procedure does not require estimation of the noise power, a feature shared with the SS algorithm [11] only. In addition to the EVD of the CL -square \mathcal{R} , its involves, in step (4), the EVD of a matrix of dimension $(C-1)L - M + 1 = (C-2)M + C$. If $C = 2$, this EVD has a complexity that does not depend on the channel order so that the whole procedure require only one EVD. Among all blind (equalization and identification) algorithms, only the LP algorithm [1, 5] exhibits such a feature.

²Matrix \mathcal{R} is Hermitian, but not necessarily positive definite.

4. SIMULATIONS

The proposed algorithm is tested and compared to the two major techniques: the LP algorithm for zero-delay equalization (we implement the optimized version in [5]) and the MD algorithm [2] for a delay-controllable equalization. As a benchmark, we use the well-referenced SIMO channel from [11] that has $C = 4$ sub-channels and an order $M = 4$. The channel is driven by unit-variance i.i.d. QPSK symbols. The equalizer length is set to $L = M + 1$ and the delay is set to $d = M$, as explained in Sec. 3. The channel SNR is defined as

$$\mathbf{E} [\|\mathbf{x}(t)\|^2] / \mathbf{E} [\|\mathbf{n}(t)\|^2] = \sigma_s^2 \|\mathbf{h}\|^2 / (C\sigma_n^2)$$

An estimated ZF equalizer is tested with a sequence $s(0), \dots, s(N-1)$ of randomly generated source symbols ($N = 200$). To account for scale and rotation ambiguities systematically encountered in blind techniques, performance is evaluated in terms of the MSE on the equalized symbols sequence, defined [13] as

$\mathbf{E} [\min_{\beta} \|\mathbf{s}_N(N-1-d) - \kappa \mathbf{z}_N(N-1)\|^2] / (\sigma_s^2 N)$ where κ is complex-valued. W.r.t. this equalization MSE, equalizers that are equal up to a multiplicative factor are equivalent. This MSE can be proved to be equal to [5]

$$\frac{1}{\sigma_s^2 N} \left[\|\mathbf{s}_N(N-1)\|^2 - \left(\frac{\mathbf{s}_N^H(N-1) \mathbf{z}_N(N-1)}{\|\mathbf{z}_N(N-1)\|} \right)^2 \right].$$

Results are averaged over 100 Monte Carlo runs and are summarized in Fig. 2. Clearly, the proposed algorithm exhibits the same asymptotic performance as the MD algorithm. Both significantly outperform the optimum (but zero-delay) LP algorithm [5] at low-to-moderate SNR, a fact confirmed in the low sample size case.

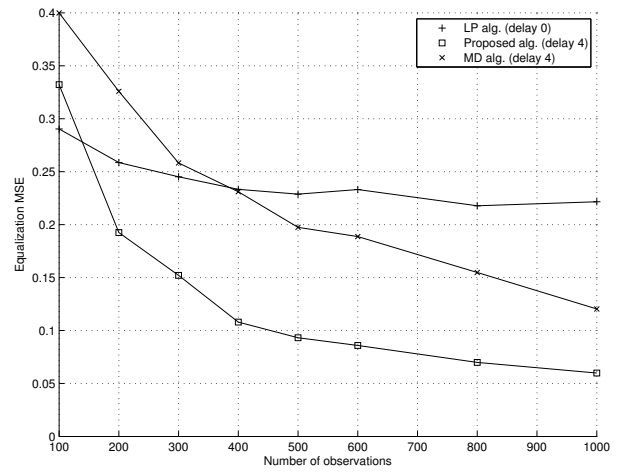
5. CONCLUSION

A two-step procedure is defined to, first, locate the subspace of all ZF equalizers with a specific length and (more importantly) delay, then, second, find the best (MMSE) equalizer within this subspace. The latter step is not needed in the practical case of two sub-channels. The short procedure means limited error propagation and, consequently, better performance, at low SNR for instance.

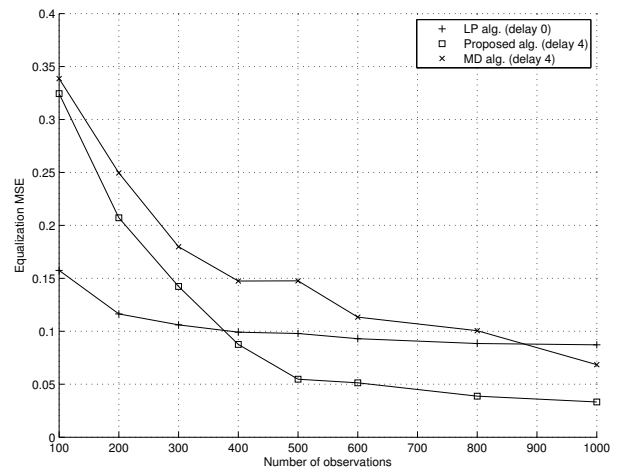
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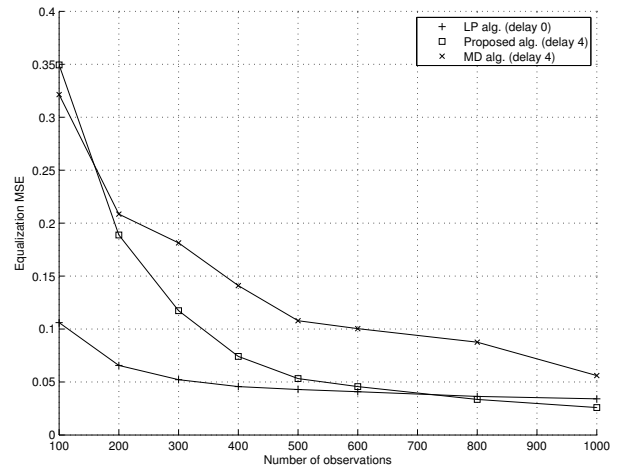
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(a) SNR=15 dB



(b) SNR=20 dB



(c) SNR=25 dB

Fig. 2. MSE on equalized symbols.