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Contribution of the added masses in the dynamic modelling of flexible airships

Selima Bennaceur, Naoufel Azouz

Abstract This paper presents an efficient modelling of autonomous flexible airships. These flying objects lighter than air (L.T.A.) are assumed to undergo large rigid-body motion and small elastic deformation. The formalism used is based on the Euler–Lagrange approach. The airship considered in this study is represented by a flexible ellipsoid of revolution. The coupling between the added masses issued from the overall body motion and those issued from the elasticity was determined by means of the velocity potential flow theory. We develop a fully analytical methodology with some assumptions. This feature distinguishes the current work from earlier treatments of the coupling, it allows one to minimise the number of degrees of freedom of the dynamical model, and renders the model suitable for use in the algorithms of stabilisation and trajectory generation. Numerical simulations are presented at the end of this paper. They underline the interest of the developed theoretical results.

Keywords Flexible airship · Dynamic modelling · Interaction fluid-structure · Ellipsoidal coordinates · Coupling added masses

1 Introduction

Capabilities of airships have expanded rapidly in the last decades, and the range of missions they were designed to allow is growing. We can mention climate research; surveillance. . . The human presence in those cases is highly undesirable. To elaborate efficient algorithms of stabilisation or navigation of these autonomous airships, it is important to develop precise dynamic models of these objects. In accordance with the increase of the volumes and the shapes of the current airships it is necessary to introduce the effect of the structural flexibility in the dynamic and aerodynamic model. We shall mention that several kinds of airships, usually called blimps, are mainly constituted of a balloon filled with gas. The only solid parts are the gondola and the tail fins (for more details see [1]). The integration of the structural flexibility in the dynamic analysis of the airships becomes useful then; however, it is now at an embryonic state and is only just emerging [2–4]. As opposed to other treatments [5], where it is merely assumed that the flexibility effects are sample perturbations, we introduce here the dynamic and aerodynamic effects of the flexibility in the global dynamic model of the airship, and we focus on the coupling between the rigid-body contribution and the flexibility. We shall note that in other flying objects, such as light aircraft, the introduction of the flexibility in the dynamic model becomes essential [6–9], and [10].

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The Newton–Euler description [11], used here, was extensively studied in the case of rigid flying objects. This choice is mainly motivated by the easiness to build control or stabilisation algorithms based on this model. However, the use of the Newton–Euler approach in the dynamic analysis of flexible complex structures is rare [12]. A method was proposed [13] to extend the classical rigid bodies’ model to the deformable bodies, without destroying the general formalism obtained. We adopt this method for our study. Other descriptions of flexible flying objects were proposed in the literature using the Updated Lagrangian Method [2, 14, 15], or the natural coordinates [10]. However, the Newton–Euler description seems the most promising method regarding the minimisation of the degrees of freedom.

On the other hand, airships are also governed by the aerodynamic forces that have to be modelled. Air-structure interaction is mainly represented by the added masses phenomena. We can mention two groups of researchers in this field: those studying the vibration of flexible bodies within a fluid around a reference configuration as in [16, 17] and those analysing the large motion of rigid bodies in a fluid as in [18]. The coupling between these two parts is seldom treated, and in that case numerical methods such as Finite or boundary Element Method are used [19–22]. A novel aspect of the approach advocated here is that the equations governing the interaction air-structure are resolved in an analytical setting. This approach departs clearly from many earlier treatments of the coupling, and permits to minimise the number of degrees of freedom of the dynamical model. Control laws can then be easily implemented in the dynamical model of the airship, taking into account the dynamic and aerodynamic coupling. This is important especially when the airship is manoeuvring above a target or over an unloading area.

2 Nomenclature

$\eta_1 = [x_0, y_0, z_0]^T$: vector position of the origin expressed in the fixed reference frame R_0
 $\eta_2 = [\varphi, \theta, \psi]^T$: vector orientation of the pointer R_m in regards to R_0 , given by the Euler angles
 $\eta = [\eta_1, \eta_2]^T$: vector attitude compared to R_0

$\dot{\eta}$: velocity vector in regards to R_0 expressed in R_0
 $\mathbf{v}_1 = [u, v, w]^T$: velocity vector expressed in R_m
 $\mathbf{v}_2 = [p, q, r]^T$: vector of angular velocities expressed in R_m
 $\mathbf{v}^T = [\mathbf{v}_1^T, \mathbf{v}_2^T]$: $\dot{\mathbf{v}}$ is the time derivative of \mathbf{v}
 $\bar{\mathbf{v}}^T = [\mathbf{v}_1^T, \mathbf{v}_2^T, \dot{\mathbf{Y}}_d]$: the generalised velocity vector of the flexible airship
 m : the mass of the airship
 I_3 : the identity matrix 3×3
 $\tilde{\mathbf{U}}$: skew-matrix of the vector \mathbf{U}
 ρ_f : the fluid (air) density
 ρ_A : the equivalent density of the airship
 \mathbf{F}_0 : column matrix of the whole forces applied on the airship as regards R_0
 \hat{M}_B : mass matrix of the body in R_m
 g : acceleration of gravity
 V : volume of the airship
 V_d : velocity of a point in the flexible airship
 $\nabla = [\partial/\partial x, \partial/\partial y, \partial/\partial z]^T$ is a differential operator
 \wedge : is the vector product
 Φ_{flex} : the flexible velocity potential
 Φ_s : spatial part of the flexible velocity potential
 l, m, k are the direction cosines
 a, b, c the polar and equatorial radii of an ellipsoid

3 Dynamic model

3.1 Dynamics

The analysis of the motion of the airship is made with respect to two reference frames, namely an earth-fixed frame $R_0 = (O, X_0, Y_0, Z_0)$, and a body-fixed one $R_m = (G, X_m, Y_m, Z_m)$ called also pointer.

The origin of the body-fixed frame R_m is fixed at the centre of gravity of the airship G ; its axes are selected as follows:

X_m : aligned with longitudinal axis, Y_m : the transverse axis, Z_m : the normal axis directed downward (see Fig. 1).

Commonly in aeronautics, a parameterisation in yaw, pitch and roll (ψ, θ, φ) is used to describe the orientation of the local frame R_m as regards the inertial reference frame.

The whole transformation between R_0 and R_m is then given by

$$J_1 = \begin{pmatrix} \cos \psi \cos \theta & -\sin \psi \cos \phi + \sin \phi \cos \psi \sin \theta & \sin \phi \sin \psi + \sin \theta \cos \psi \cos \phi \\ \cos \theta \sin \psi & \cos \psi \cos \phi + \sin \theta \sin \psi \sin \phi & -\cos \psi \sin \phi + \cos \phi \sin \theta \sin \psi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{pmatrix}$$

so that $J_1^T J_1 = J_1 J_1^T = I_3$.

Using the rotation matrix J_1 , the expression of the linear velocity in R_0 is given by

$$\dot{\eta}_1 = J_1 \mathbf{v}_1 \quad (1)$$

On the other hand, the angular velocity of the blimp \mathbf{v}_2 is the combination of the angular velocity around the three axes of yaw, pitch and roll.

We can then easily express the relation between \mathbf{v}_2 and $\dot{\eta}_2$ as

$$\mathbf{v}_2 = J_2 \dot{\eta}_2 \quad (2)$$

The transformation matrix J_2 is given by

$$J_2 = \begin{pmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \varphi & \sin \varphi \cos \theta \\ 0 & -\sin \varphi & \cos \varphi \cos \theta \end{pmatrix}$$

If we consider the flexibility of the airship, we can represent the position of a given point P such as

$$\mathbf{r} = OP' = OG + GP + PP' \quad (3)$$

P' represents the new position of the point P in the deformed configuration. We have

$$\mathbf{r} = \eta_1 + J_1(\mathbf{U}_0 + \mathbf{U}_d) = \eta_1 + J_1 \mathbf{U} \quad (4)$$

\mathbf{U}_0 is the local position of the point P .

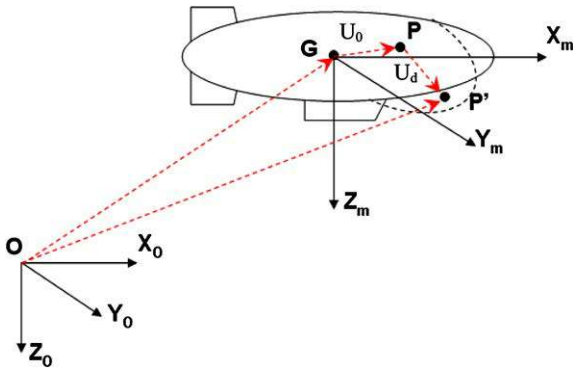


Fig. 1 Position vectors

The displacement due to the deformation $\mathbf{U}_d = \mathbf{U}_d(s, t)$ expressed in the local frame is a function of space and time. It can be broken up into a sum of two separate functions:

$$\mathbf{U}_d(s, t) = \sum_i^{nd} Y_d^i(t) S_i \quad (5)$$

where:

S_i : represents the i th shape function of the airship

$Y_d^i(t)$: the associated amplitude

This manner of writing can be condensed in the following way:

$$\mathbf{U}_d = S \bar{\mathbf{Y}}_d \quad (6)$$

S is a matrix representing the selected shape functions, and $\bar{\mathbf{Y}}_d$ is the column matrix composed by the various Y_d^i .

The body's flexibility can be represented by analytical approximate shape functions or by a limited number (nd) of lower frequencies deformation modes, resulting from the Raleigh–Ritz decomposition. The choice of the number of modes to be kept depends on the structure and the solicitations applied (see for example [23]).

We denote by $\bar{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \bar{\mathbf{Y}}_d \end{pmatrix}$ a generalised position vector of an arbitrary point in the flexible airship.

The kinetic energy of the deformable body can be expressed as follows:

$$E_{cdB} = \frac{1}{2} \iiint_V \rho_A \dot{\mathbf{r}}^T \dot{\mathbf{r}} dV = \frac{1}{2} \bar{\eta}^T \hat{M}_{dB} \bar{\eta} \quad (7)$$

\hat{M}_{dB} is known as the symmetric time varying mass matrix of the flexible body in regards of the reference frame R_0 .

By using Lagrange's equations [24], one can obtain the following dynamic relation:

$$\hat{M}_{dB} \ddot{\bar{\eta}} = \mathbf{F}_0 - K \bar{\eta} \quad (8)$$

\mathbf{F}_0 is the column matrix of the external forces and torques expressed in the frame R_0 , including the contribution of the actuators, the gyroscopic, aerodynamic, gravity and buoyancy forces.

K is the stiffness matrix that can be presented as follows:

$$K = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_{DD} \end{pmatrix} \quad (9)$$

The details of constructing such a matrix can be found in [25]; or [24].

For a better use of the data given by the embedded sensors, it is preferable to use the local parameters.

The velocity of a given point P in the deformable airship can then be expressed as follows:

$$\begin{aligned} V_d &= \mathbf{v}_1 + \mathbf{v}_2 \wedge \mathbf{U} + \dot{\mathbf{U}}_d \\ &= \mathbf{v}_1 + \mathbf{v}_2 \wedge \mathbf{U}_0 + \mathbf{v}_2 \wedge S\bar{\mathbf{Y}}_d + S\bar{\dot{\mathbf{Y}}}_d \end{aligned} \quad (10)$$

When using adequate transformations, the kinetic energy of the flexible body can be expressed using the generalised variables $\bar{\mathbf{v}}^T = [\mathbf{v}_1^T, \mathbf{v}_2^T, \bar{\dot{\mathbf{Y}}}_d]$ as:

$$E_{cdB} = \frac{1}{2} \bar{\mathbf{v}}^T M_{dB} \bar{\mathbf{v}} \quad (11)$$

M_{dB} is the mass matrix of the body in the local frame R_m . Let us designate by I_{RR} the inertia matrix of rotation of the flexible body in the local frame R_m such as: $I_{RR} = \iiint_V \rho_A \tilde{\mathbf{U}}^T \tilde{\mathbf{U}} dV$, I_{RD} defined by $I_{RD} = \int_V \rho_A \tilde{\mathbf{U}}^T S dV$ a term of inertial coupling rotation-deformation, I_{DD} such as $I_{DD} = \int_V \rho_A S^T S dV$ the constant matrix of deformation, and finally $I_{TT} = m I_3$ the translation term.

If we neglect the coupling translation-deformation, and if we use the generalised Euler variables, the dynamic model of the flexible airship becomes [13]

$$\begin{pmatrix} I_{TT} & 0 & 0 \\ 0 & I_{RR} & I_{RD} \\ 0 & I_{RD}^T & I_{DD} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{v}}_1 \\ \dot{\mathbf{v}}_2 \\ \bar{\dot{\mathbf{Y}}}_d \end{pmatrix} = \boldsymbol{\tau} \quad (12)$$

or:

$$M_{dB} \bar{\mathbf{v}} = \boldsymbol{\tau} \quad (13)$$

$\bar{\mathbf{v}}$ is the generalised vector of acceleration expressed with Eulerian variables, and $\boldsymbol{\tau}$ is the vector involving all the forces and torques applied on the airship so that

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{Ac} + \boldsymbol{\tau}_{Gy} + \boldsymbol{\tau}_{Gb} + \boldsymbol{\tau}_{Grb} + \boldsymbol{\tau}_{Ac} \quad (14)$$

$\boldsymbol{\tau}_{Ac}$ is the column matrix of the forces and torques produced by the actuators (i.e. rotors and tail fins).

$\boldsymbol{\tau}_{Gy}$ is the column matrix of gyroscopic, Coriolis, and stiffness generalised forces and torques given by

$$\boldsymbol{\tau}_{Gy} = \begin{pmatrix} -I_{TT}(\mathbf{v}_2 \wedge \mathbf{v}_1) \\ -\mathbf{v}_2 \wedge (I_{RR}\mathbf{v}_2) - \mathbf{v}_2 \wedge (I_{RD}\bar{\dot{\mathbf{Y}}}_d) \\ -K_{DD}\bar{\dot{\mathbf{Y}}}_d \end{pmatrix} \quad (15)$$

$\boldsymbol{\tau}_{Gb}$ represents the gravity and the buoyancy. The gravity is applied on the centre of mass of the airship. The buoyancy is the main characteristic of the L.T.A. flying objects. It is applied in the centre of volume of the airship and has a vertical direction. We neglect here the influence of the small deformations on the buoyancy. We assume also that the distribution of masses is balanced between the top side and the bottom side of the airship, and thus the centre of volume coincides with the centre of gravity G .

$\boldsymbol{\tau}_{Grb}$ can then be computed in the local frame R_m according to this manner:

$$\boldsymbol{\tau}_{Gb} = (mg - \rho V) J_1^T \mathbf{Z}_0 \quad (16)$$

$\boldsymbol{\tau}_{Ac}$ is the column matrix of the aerodynamic forces F_A and torques M_A depending on several parameters such as the reference section of the airship, for example the medium transverse section, the Reynolds number, the angles of attack and of side-slip, the speed of the relative wind, and the aspect ratio. More details of the computation of these aerodynamic efforts for an ellipsoidal airship can be seen in [26].

3.2 Added masses

The lighter than air aerial vehicles undergo a particular aerodynamic phenomenon called added masses. If a voluminous and light body is moving in an inviscid and incompressible fluid, it will force a passage through this fluid, and will produce a displacement of the particles of the surrounding layers of fluid.

The kinetic energy of the fluid produces an effect equivalent to a significant growing of the mass and inertia of the body. As the airship displays a very large volume, its added masses and inertias become very significant.

Let us now present some assumptions for this study:

- The mass of the airship is constant. The use of electrical power makes this hypothesis acceptable.

- The added masses within the careen due to the motion of the helium are neglected.
- The air is supposed to be at rest. The only region perturbed by the motion of the airship is its close surroundings.

To take into account the interaction of the airship with the surrounding fluid, a model of the flow is needed. Here, we use the potential flow theory with the following assumptions.

- The air can be considered as an ideal fluid with irrotational flow, and uniform density ρ_f , i.e. an incompressible fluid.
- A velocity potential ϕ exists and satisfies the Laplace equation throughout the fluid domain $\nabla^2 \phi = 0$, and satisfies the non-linear free surface condition, body boundary condition, and initial conditions.

Finally, we suppose that the velocity of the air is null far from the airship ($\phi_\infty \rightarrow 0$).

- The velocity of the fluid obeys to the expression $V_f = \nabla \phi$.

The basis of the analysis of the motion of a rigid body in an unbounded incompressible fluid at rest at infinity was established more than one century ago by [27]. In his study, Lamb proves that the kinetic energy of the fluid surrounding a moving rigid body can be expressed as a quadratic function of the six components of the translation and rotation velocity $\mathbf{v} = (u, v, w, p, q, r)^T$, and can also be expressed as a function of the velocity potential of this fluid by the following relation:

$$E_{ca} = \frac{1}{2} \mathbf{v}^T M_a \mathbf{v} = -\frac{1}{2} \rho_f \iint_{\partial S} \phi \frac{\partial \phi}{\partial \mathbf{n}} dS \quad (17)$$

M_a is known as the added mass matrix of the body and can be presented as:

$$M_a = [Z_{ij}] \quad (18)$$

For a symmetric and fully immersed body in a fluid, the symmetry of this matrix appears as a reasonable assumption [18]. The experimental data have shown that the extra-diagonal terms are small compared to the diagonal terms.

To compute the kinetic energy of the fluid and to extract the terms of the added mass matrix Z_{ij} , we should determine the velocity potential of the moving fluid.

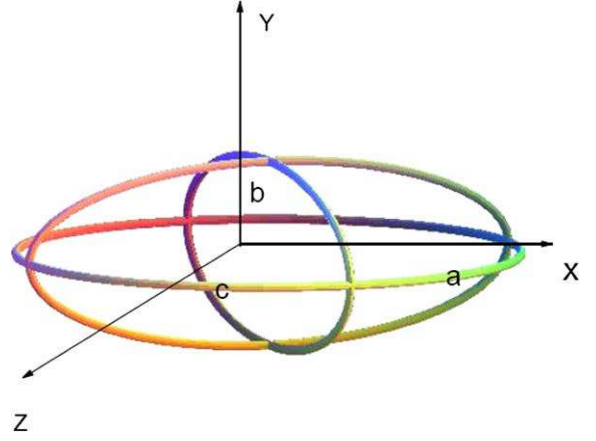


Fig. 2 A submerged ellipsoid

3.3 Computation of the velocity potential

The velocity potential of the fluid issued from the motion of a rigid immersed vehicle can be expressed as follows:

$$\phi_{rr} = u\phi_1 + v\phi_2 + w\phi_3 + p\phi_4 + q\phi_5 + r\phi_6 \quad (19)$$

where the ϕ_i are functions of x, y, z . They are determined exclusively from the geometry of the solid [27].

In the case of an ellipsoid totally submerged in the fluid the expression of ϕ_{rig} is

$$\phi_{rr} = -ux - vy - wz - \frac{c^2 - a^2}{c^2 + a^2} qzx - \frac{a^2 - b^2}{a^2 + b^2} rxy \quad (20)$$

Here a, b and c are the semi-axes of the ellipsoid (see Fig. 2). We consider here an extended ellipsoid of revolution ($b = c$).

The ellipsoidal coordinates are suitable to describe the motion and the behaviour of such an ellipsoid.

The expressions of the Cartesian coordinates x, y and z in regard to the ellipsoidal coordinates (ξ, μ, Θ) are [28]:

$$\begin{cases} x = k_o \mu \xi & 1 \leq \xi \leq \infty \\ y = k_o (1 - \mu^2)^{1/2} (\xi^2 - 1)^{1/2} \cos \Theta & -1 \leq \mu \leq 1 \\ z = k_o (1 - \mu^2)^{1/2} (\xi^2 - 1)^{1/2} \sin \Theta & 0 \leq \Theta \leq 2\pi \end{cases} \quad (21)$$

The surface of the ellipsoid corresponds to $\xi = \xi_o$, and the constant k_o is defined by $k_o = \frac{a}{\xi_o}$.

The outward normal vector to the ellipsoid is

$$\mathbf{n} = \left(\frac{\partial x}{\partial \xi}, \frac{\partial y}{\partial \xi}, \frac{\partial z}{\partial \xi} \right) \quad (22a)$$

$$\text{with } \begin{cases} \frac{\partial x}{\partial \xi} = k_o \mu \\ \frac{\partial y}{\partial \xi} = k_o (1 - \mu^2)^{1/2} \xi (\xi^2 - 1)^{-1/2} \cos \Theta \\ \frac{\partial z}{\partial \xi} = k_o (1 - \mu^2)^{1/2} \xi (\xi^2 - 1)^{-1/2} \sin \Theta \end{cases} \quad (22b)$$

Hence we can write the velocity potential of the fluid due to a perturbation by the motion of a rigid body as

$$\begin{aligned} \phi_{rr} = & -uk_o \mu \xi - vk_o (1 - \mu^2)^{1/2} (\xi^2 - 1)^{1/2} \cos \Theta \\ & - w(1 - \mu^2)^{1/2} (\xi^2 - 1)^{1/2} \sin \Theta \\ & - \frac{c^2 - a^2}{c^2 + a^2} q k_o^2 \mu \xi (1 - \mu^2)^{1/2} (\xi^2 - 1)^{1/2} \sin \Theta \\ & - \frac{a^2 - b^2}{a^2 + b^2} r k_o^2 \mu \xi (1 - \mu^2)^{1/2} (\xi^2 - 1)^{1/2} \cos \Theta \end{aligned} \quad (23)$$

The boundary conditions on the surface of interaction ∂S are

$$\begin{aligned} \frac{\partial \phi_{rr}}{\partial \mathbf{n}} = & l(u + qz - ry) + m(v + rx - pz) \\ & + k(w + py - qx) \end{aligned} \quad (24)$$

Beginning with the development of the kinetic energy in quadratic form,

$$\begin{aligned} 2E_{ca} = & Z_{11}u^2 + Z_{22}v^2 + Z_{33}w^2 + Z_{44}p^2 + Z_{55}q^2 \\ & + Z_{66}r^2 + 2Z_{23}vw + 2Z_{13}wu + 2Z_{12}uv \\ & + 2Z_{56}qr + 2Z_{46}rp + 2Z_{45}pq \\ & + 2p(Z_{14}u + Z_{24}v + Z_{34}w) \\ & + 2q(Z_{15}u + Z_{25}v + Z_{35}w) \\ & + 2r(Z_{16}u + Z_{26}v + Z_{36}w) \end{aligned} \quad (25)$$

we refer to the fact that here $\phi = \phi_{rr}$, we identify the terms of (17) according to (18–20), and we can then find the expression of the added mass matrix compo-

nents:

$$Z_{ij} = -\rho_f \iint_S \phi_i \frac{\partial \phi_j}{\partial \mathbf{n}} dS \quad (26)$$

3.4 Aerodynamic effect of the flexibility

Let us now consider the case of a flexible airship. This has an overall rigid-body motion and a flexible displacement. In addition to the previous hypothesis concerning the fluid, we assume that the careen is impermeable to the surrounding fluid.

The kinetic energy of the air surrounding the flexible body in motion can be expressed as follows:

$$E_{cad} = \frac{1}{2} \iiint_V \rho_f \dot{\mathbf{r}}^T \dot{\mathbf{r}} dV = \frac{1}{2} \bar{\mathbf{\eta}}^T \hat{M}_{ad} \bar{\mathbf{\eta}} \quad (27)$$

Considering the assumption of incompressibility of the air, and when using the Lagrange equations for the system combining the deformable airship and the surrounding fluid, one can obtain the following equation:

$$(\hat{M}_{dB} + \hat{M}_{ad}) \ddot{\bar{\mathbf{\eta}}} = \mathbf{F}_0 - K \bar{\mathbf{\eta}} \quad (28)$$

The dynamic model of the whole system is similar to that obtained in (8). The fluid effect appears as an increase of the inertia of the flexible body.

If we use the Euler variables, the dynamic system becomes

$$(\mathbf{M}_{dB} + \mathbf{M}_{ad}) \ddot{\bar{\mathbf{v}}} = \boldsymbol{\tau} + \boldsymbol{\tau}_a \quad (29)$$

\mathbf{M}_{ad} is the added mass matrix for the flexible airship. Compared to that presented in (18), this $(6 + nd, 6 + nd)$ matrix includes the added inertial effects issued from the rigid-body motion, the deformation, and the coupling. It will be presented in the following section.

$\boldsymbol{\tau}_a$ is the added masses vector, due to the use of the non-Galilean frame. Since its calculation is quite lengthy, we merely state the result, which is

$$\boldsymbol{\tau}_a = \begin{bmatrix} -\mathbf{M}_{ad}^{TT} (\mathbf{v}_2 \wedge \mathbf{v}_1) \\ -\mathbf{v}_2 \wedge (\mathbf{M}_{ad}^{RR} \mathbf{v}_2) - \mathbf{v}_2 \wedge (\mathbf{M}_{ad}^{Rd} \bar{\mathbf{Y}}_d) \\ 0 \end{bmatrix} \quad (30)$$

\mathbf{M}_{ad}^{TT} , \mathbf{M}_{ad}^{RR} , and \mathbf{M}_{ad}^{Rd} are sub-matrices corresponding, respectively, to the translation motion, rotation motion and the coupling rotation-deformation.

The kinetic energy of the surrounding fluid can also be expressed in terms of Euler variables as an extension of (17) so that

$$E_{cad} = \frac{1}{2} \bar{\mathbf{v}}^T M_{ad} \bar{\mathbf{v}} = -\frac{1}{2} \rho_f \iint_{\partial S} \phi \frac{\partial \phi}{\partial \mathbf{n}} dS \quad (31)$$

ϕ designates the velocity potential of the fluid displaced by the motion of the flexible airship. We will define its expression later.

4 Analytic computation of the potential

4.1 Kinetic energy of the air

By using the boundary conditions of the airship in contact with the air:

$$V_d \cdot \mathbf{n} = V_f \cdot \mathbf{n} \quad (32)$$

Hence by replacing each velocity by its expression, we obtain:

$$\underbrace{(\mathbf{v}_1 + \mathbf{v}_2 \wedge \mathbf{U}_0)}_{V_r} + \underbrace{\mathbf{v}_2 \wedge S \bar{\mathbf{Y}}_d}_{V_{rd}} + \underbrace{S \bar{\dot{\mathbf{Y}}}_d}_{V_{dd}} \cdot \mathbf{n} = \nabla \phi \cdot \mathbf{n} \quad (33)$$

According to this presentation, we can split up the velocity potential into three terms:

$$\phi = \phi_{rr}(\mathbf{v}_1, \mathbf{v}_2) + \phi_{rd}(\bar{\mathbf{Y}}_d, \mathbf{v}_2) + \phi_{dd}(\bar{\dot{\mathbf{Y}}}_d) \quad (34)$$

ϕ_{rr} is the velocity potential corresponding to the rigid motion and presented in (19), while ϕ_{rd} appears as a velocity potential corresponding to the coupling between the rigid motion and the deformation.

It could be expressed as a function of a spatial component ϕ_s and time varying component $g_1(t)$

$$\phi_{rd} = \phi_s \cdot g_1(t) \quad (35)$$

The spatial component ϕ_s is issued from the shape matrix S in (33).

ϕ_{dd} is the velocity potential issued from the effect of the flexibility. It could be expressed similarly to ϕ_{rd} as

$$\phi_{dd} = \phi_s g_2(t) \quad (36)$$

To make the computation of $g_1(t)$ in (35) easier we use a canonical decomposition that we can summarise as follows:

The shape function matrix S can be broken up into

$$S = \sum_{i=u}^w \sum_{j=1}^{n_d} S_{ij} B_{ij} = S_{u1} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{B_{u1}} + S_{u2} \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{B_{u2}} \dots$$

Hence the term $\mathbf{v}_2 \wedge S \bar{\mathbf{Y}}_d$ in (33) is written as

$$\mathbf{v}_2 \wedge S \bar{\mathbf{Y}}_d = \sum_i \sum_j S_{ij} \underbrace{\mathbf{v}_2 \wedge B_{ij} \bar{\mathbf{Y}}_d}_{g_1(t)} \quad (37)$$

According to (31) the computation of the kinetic energy of the fluid shows new terms, which indicate the effect of the flexibility of the hull on the air flow. We present them in the global mass matrix of the added masses.

The added mass matrix defined in (31) can be presented as

$$M_{ad} = \begin{bmatrix} M_a & \vdots & 0 \\ \dots & \vdots & \dots \\ 0 & \vdots & 0 \end{bmatrix} + M_{ad}^{rd} + \begin{bmatrix} 0 & \vdots & 0 \\ \dots & \vdots & \dots \\ 0 & \vdots & M_{ad}^{dd} \end{bmatrix} \quad (38)$$

with

$$M_{ad}^{dd} = \begin{bmatrix} m_{11}^{dd} & \dots & m_{1n_d}^{dd} \\ \vdots & \ddots & \vdots \\ m_{n_d 1}^{dd} & \dots & m_{n_d n_d}^{dd} \end{bmatrix}$$

is a (nd, nd) square matrix, and can be seen as the effect of the flexibility of the careen on the air potential flow, while the matrix M_{ad}^{rd} represents the effects of the coupling between the rigid-body motion and the flexibility. It is a time varying matrix.

$$M_{ad}^{rd} = \begin{pmatrix} 0 & 0 & 0 & T_{14} & T_{15} & T_{16} & W_{17} & \cdot & \cdot & W_{1(n_d+6)} \\ 0 & 0 & 0 & T_{24} & T_{25} & T_{26} & W_{27} & \cdot & \cdot & W_{2(n_d+6)} \\ 0 & 0 & 0 & T_{34} & T_{35} & T_{36} & W_{37} & \cdot & \cdot & W_{3(n_d+6)} \\ T_{41} & \cdot & \cdot & K_{44} & K_{45} & K_{46} & H_{47} + W_{47} & \cdot & \cdot & H_{4(n_d+6)} + W_{4(n_d+6)} \\ T_{51} & \cdot & \cdot & \cdot & K_{55} & K_{56} & H_{57} + W_{57} & \cdot & \cdot & H_{5(n_d+6)} + W_{5(n_d+6)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & K_{66} & H_{67} + W_{67} & \cdot & \cdot & H_{6(n_d+6)} + W_{6(n_d+6)} \\ & & & H_{74} + W_{74} & \cdot & \cdot & 0 & & & 0 \\ & & & \cdot & \cdot & \cdot & \cdot & \ddots & & \\ & & & \cdot & \cdot & \cdot & \cdot & & & \\ & & & \cdot & \cdot & \cdot & 0 & & & 0 \end{pmatrix} \quad (39)$$

The terms T_{ij} , K_{ij} , H_{ij} and W_{ij} are coupling terms. Using the development of the expression of the kinetic energy:

$$\begin{aligned} E_{cad} &= -\frac{1}{2}\rho_f \iint_S \phi \frac{\partial \phi}{\partial \mathbf{n}} dS \\ &= -\frac{1}{2}\rho_f \iint_S (\phi_{rr}(\mathbf{v}_1, \mathbf{v}_2) + \phi_{rd}(\bar{\mathbf{Y}}_d, \mathbf{v}_2) \\ &\quad + \phi_{dd}(\dot{\bar{\mathbf{Y}}}_d)) \left(\frac{\partial \phi_{rr}(\mathbf{v}_1, \mathbf{v}_2)}{\partial \mathbf{n}} \right. \\ &\quad \left. + \frac{\partial \phi_{rd}(\bar{\mathbf{Y}}_d, \mathbf{v}_2)}{\partial \mathbf{n}} + \frac{\partial \phi_{dd}(\dot{\bar{\mathbf{Y}}}_d)}{\partial \mathbf{n}} \right) dS \end{aligned} \quad (40)$$

the terms T_{ij} appear as the coupling between Φ_{rr} and Φ_{rd} , H_{ij} the coupling between Φ_{dd} and Φ_{rd} , W_{ij} the coupling between Φ_{rr} and Φ_{dd} , and K_{ij} the coupling between Φ_{rd} and Φ_{rd} .

For example:

$$\begin{aligned} K_{56} &= \left(\phi_5 \frac{\partial \phi_s}{\partial \xi} + \phi_6 \frac{\partial \phi_s}{\partial \xi} + \phi_s \frac{\partial \phi_6}{\partial \xi} \phi_s \frac{\partial \phi_5}{\partial \xi} + 2 \frac{\partial \phi_s}{\partial \xi} \phi_s \right) \\ &\quad \times \left(\sum_{i=1}^{n_d} B_{wi} Y_{d_i} - \sum_{i=1}^{n_d} B_{ui} Y_{d_i} \right) \\ &\quad \times \left(\sum_{i=1}^{n_d} B_{ui} Y_{d_i} - \sum_{i=1}^{n_d} B_{vi} Y_{d_i} \right) \end{aligned} \quad (41)$$

and:

$$W_{ij} = -\rho_f \iint_S \phi_i \frac{\partial \phi_{sj}}{\partial \xi} dS \quad (42)$$

The components of the matrix M_{ad}^{dd} can be presented as follows:

$$m_{ij}^{dd} = -\rho_f \iint_S \phi_{si} \frac{\partial \phi_{sj}}{\partial \xi} dS \quad (43)$$

4.2 Computation of ϕ_s

In the following part, we compute analytically the flexible velocity potential flow. We show here the influence of the surrounding air on the boundary surface of the ellipsoid, i.e. the hull ($\xi = \xi_0$).

According to [28], the superposition of two or more distributions of velocities on the ellipsoid is allowable. This is so in the sense that for example, for the deformable contribution V_{dd} seen in (33), the boundary condition could also be written as

$$S_{uj} \frac{\partial x}{\partial \xi} + S_{vj} \frac{\partial y}{\partial \xi} + S_{wj} \frac{\partial z}{\partial \xi} = k_o F_j(\mu, \theta) = \frac{\partial \phi_{sj}}{\partial \xi} \quad (44)$$

with

$$\begin{aligned} F_j(\mu, \theta) &= \mu S_{uj} + (1 - \mu^2)^{\frac{1}{2}} \xi_0 (\xi_0^2 - 1)^{-\frac{1}{2}} \\ &\quad \times (S_{vj} \cos \Theta + S_{wj} \sin \Theta) \end{aligned} \quad (45)$$

In order to define the expression of ϕ_s we have to resolve the Laplace equation:

$$\Delta \phi_s = 0 \quad (46)$$

Using the ellipsoidal coordinates (ξ, μ, Θ) , the previous equation could be written as

$$\begin{aligned} \frac{\partial}{\partial \mu} \left\{ (1 - \mu^2) \frac{\partial \phi_s}{\partial \mu} \right\} + \frac{\partial}{\partial \xi} \left\{ (\xi^2 - 1) \frac{\partial \phi_s}{\partial \xi} \right\} \\ + \frac{\xi^2 - \mu^2}{(1 - \mu^2)(\xi^2 - 1)} \frac{\partial^2 \phi_s}{\partial \Theta^2} = 0 \end{aligned} \quad (47)$$

The shape functions $S_j : (S_{uj}, S_{vj}, S_{wj})^T$ are defined here analytically as

$$\begin{aligned} S_{v1} &= K_1 \sin 2\Theta (1 - \mu^2) \\ S_{w2} &= K_2 \cos 2\Theta (1 - \mu^2) + K_3 \mu^2 \cos 2\Theta \end{aligned} \quad (48)$$

All the other components are null. K_i are constants.

These functions correspond to the bending of the airship in the plane $G X_m Y_m$, and in the plane $G X_m Z_m$.

We used this analytical form of the bending of an ellipsoid, given first by [29], because it represents an acceptable approximation of the vibration of the airship and allows us to conduct a fully analytical study. However, a vibrating study by the Finite Element Method including the effects of the solid parts, such as the gondola and the tail fins should be able to give us more realistic velocity distributions.

Using the Legendre function $P_n^m(\mu)$, $Q_n^m(\xi)$, the potential function of vibration for the motion of the air surrounding the flexible airship is given by (the reader can see [27] for more details):

$$\begin{aligned} \phi_s &= \sum_{n=0}^{\infty} \sum_{m=0}^n (\chi_n^m \sin m\theta + \varpi_n^m \cos m\theta) \\ &\quad \times P_n^m(\mu) Q_n^m(\xi) \end{aligned} \quad (49)$$

χ_n^m and ϖ_n^m are coefficients to be determined in such a way that the boundary conditions are fulfilled.

The coefficients are found by applying the integral properties of the circular and the Legendre functions.

We study the solutions of (49) for different values of m and for each function F related in (45).

The case $m = 0$ does not provide useful information, and the only non-zero values that we get are for $m = 1$, with the coefficient $n = 1$.

Thus for $m = 1$ the constants χ_n^m and ϖ_n^m could be expressed as follows:

$$\begin{aligned} \chi_{nj}^1 &= \frac{2n+1}{2\pi Q_{\xi_n}^1(\xi_0)} \frac{(n-1)!}{(n+1)!} k_o \\ &\quad \times \int_0^{2\pi} \int_{-1}^1 F_j(\mu, \theta) P_n^1 \sin \Theta d\mu d\Theta \\ \varpi_{nj}^1 &= \frac{2n+1}{2\pi Q_{\xi_n}^1(\xi_0)} \frac{(n-1)!}{(n+1)!} k_o \\ &\quad \times \int_0^{2\pi} \int_{-1}^1 F_j(\mu, \theta) P_n^1 \cos \Theta d\mu d\Theta \end{aligned} \quad (50)$$

By using the shape function S_1 we obtain for each n

$$\begin{aligned} k_o \int_0^{2\pi} \int_{-1}^1 (1 - \mu^2)^{\frac{3}{2}} \xi_0 (\xi_0^2 - 1)^{-\frac{1}{2}} \\ \times (K_1 \sin 2\theta \cos^2 \theta) P_n^1 d\mu d\theta = 0 \end{aligned} \quad (51)$$

For the shape function S_2 we obtain

$$\begin{aligned} k_o \int_0^{2\pi} \int_{-1}^1 (1 - \mu^2)^{\frac{1}{2}} \xi_0 (\xi_0^2 - 1)^{-\frac{1}{2}} \\ \times [K_2 \cos 2\Theta (1 - \mu^2) + K_3 \mu^2 \cos 2\Theta] \\ \times \sin \Theta P_n^1 d\mu d\theta = 0 \end{aligned} \quad (52)$$

Hence all the coefficients ϖ_{nj}^1 are zero as we can see in (51) and (52).

Thus the potential fluid is reduced to

$$\phi_{sj} = \chi_{1j}^1 \sin \Theta P_1^1(\mu) Q_1^1(\xi) \quad (53)$$

with

$$\chi_{1j}^1 = C \int_0^{2\pi} \int_{-1}^1 F_j(\mu, \theta) P_1^1(\mu) \sin \Theta d\mu d\Theta \quad (54)$$

and

$$C = \frac{3k_o}{4\pi \underbrace{\frac{\partial}{\partial \xi} Q_1^1(\xi_0)}_{\text{constant}}} \quad (55)$$

Consequently for the first function, the constant is as follows:

$$\chi_{11}^1 = \frac{8\pi K_1 C \xi_0 (\xi_0^2 - 1)^{-\frac{1}{2}}}{15} \quad (56)$$

And for the second shape function, the constant is as follows:

$$\chi_{12}^1 = \frac{-2\pi (K_3 + 4K_2) C \xi_0 (\xi_0^2 - 1)^{-\frac{1}{2}}}{15} \quad (57)$$

Therefore we obtain the final expression of the flexible potential ϕ_s as follows:

$$\text{For } S_1: \quad \phi_{s1} = \chi_{11}^1 \sin \Theta P_1^1(\mu) Q_1^1(\xi) \quad (58a)$$

$$\text{For } S_2: \quad \phi_{s2} = \chi_{12}^1 \sin \Theta P_1^1(\mu) Q_1^1(\xi) \quad (58b)$$

In the matrix M_{ad}^{rd} the constant coupling terms corresponding to the two shape functions of deformation

chosen are equal to zero, except the terms W_{37} , W_{38} , W_{73} and W_{83} given by (42).

This is due to the fact that when computing the terms W_{ij} , some generic functions arise. We distinguish three functions of μ as

$$f_1(\mu) = \mu$$

$$f_2(\mu) = (1 - \mu^2)^{\frac{1}{2}}$$

$$f_3(\mu) = \mu(1 - \mu^2)^{\frac{1}{2}}$$

and three functions of Θ as $h_1(\Theta) = 1$; $h_2(\Theta) = \cos \Theta$; $h_3(\Theta) = \sin \Theta$.

For example the expression of W_{37} is as follows:

$$\begin{aligned} W_{37} &= -\rho_f \iint_S \underbrace{-f_2(\mu)h_3(\Theta)(\xi^2 - 1)^{\frac{1}{2}}}_{\phi_3} \\ &\quad \times \chi_{11}^1 P_1^1(\mu) \frac{\partial Q_1^1}{\partial \xi} dS|_{\xi=\xi_0} \\ &= \rho_f \underbrace{\int_0^{2\pi} \int_{-1}^1 f_2(\mu) P_1^1(\mu) d\mu h_3(\Theta) \sin \Theta d\Theta}_I \\ &\quad \times \chi_{11}^1 (\xi_0^2 - 1)^{\frac{1}{2}} dS \\ &= \frac{4\pi\rho_f}{3} \chi_{11}^1 (\xi_0^2 - 1)^{\frac{1}{2}} \end{aligned} \quad (59)$$

By using the same way, we compute the other coupling terms. The integral I in (59) leads to the nullity of the terms W_{ij} other than W_{37} , W_{38} , W_{73} and W_{83} .

5 Numerical simulations

In this section we present some numerical examples demonstrating the utility of the proposed formulation. As an illustration, we use the characteristics of the blimp AS-200 belonging to the LSC-IBISC laboratory and featuring the following characteristics.

The envelope:

Axis: $a = 3.125$ m; $b = c = 0.76$ m; $e = \frac{1}{\xi_0} = 0.94$;
 $\xi_0 = 1.06$; $k_o = 2.93$

Fabric: kevlar

Young modulus $E = 1.4 \times 10^5$ MPa; density $d = 1.4$;

Volume: 7.56 m^3

Mass of the airship: 5.8 kg

Payload: 1.58 kg

The blimp is powered by four contrarotating propellers.

By using the two shape functions presented before corresponding to the bending, we obtain some numerical results for the added masses as M_{ad}^{dd} for the flexible motion:

$$M_{ad}^{dd} = \begin{pmatrix} 0.4473 & -0.11 \\ -0.11 & 0.6989 \end{pmatrix}$$

And the rigid part in M_{ad} in (38) is

$$M_a = \begin{bmatrix} 0.377 & & & & \\ & 3.28 & & & \\ & & 3.28 & & \\ & & & 0 & \\ & & & & 2.91 \\ & & & & & 2.91 \end{bmatrix}$$

The calculations were performed using the numerical code Matlab. The problem discussed is solved within Newmark equation-solving strategy.

The test represents a typical manoeuvre of a blimp over an unloading area, and concerns a change of heading.

This yaw rotation of 90° is controlled through a simple proportional-derived law.

We will compare in this manoeuvre the behaviour of the blimp assumed as rigid, and that of the blimp with flexible characteristics.

In Fig. 3 we superimpose the yaw angle and the desired one for both rigid (dashed) and flexible assumptions. Although the choice of the shape functions is not optimal (particularly with regards to capturing bending behaviour), it nonetheless demonstrates the capability of the proposed model to display the effect of the flexibility of the airship on the dynamic response to the current solicitations.

This effect is also seen on the response of the actuators. Figure 4 depicts the situation considered.

We see that the flexibility effects are best illustrated in the response of the actuators.

In Fig. 5 we superimpose the angular velocity around the z -axis for the rigid case and the flexible device.

The impact of flexibility on the velocity is palpable.

In Fig. 6 we visualised the bow of the airship, and we see its oscillation compared to the ‘‘rigid’’ behaviour. The deformations are about 0.28 m, which is significant considering the size of the airship.

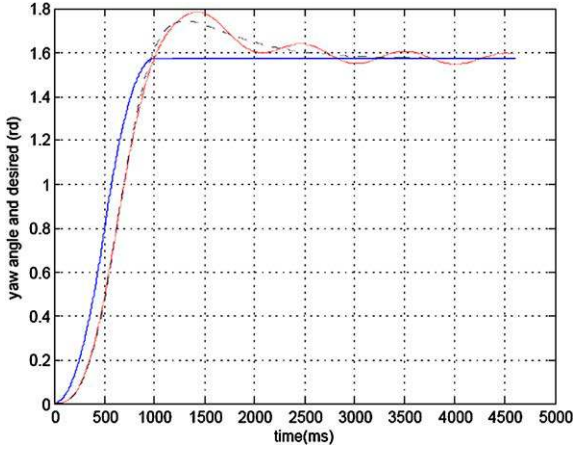


Fig. 3 Superposition of yaw angle and desired one

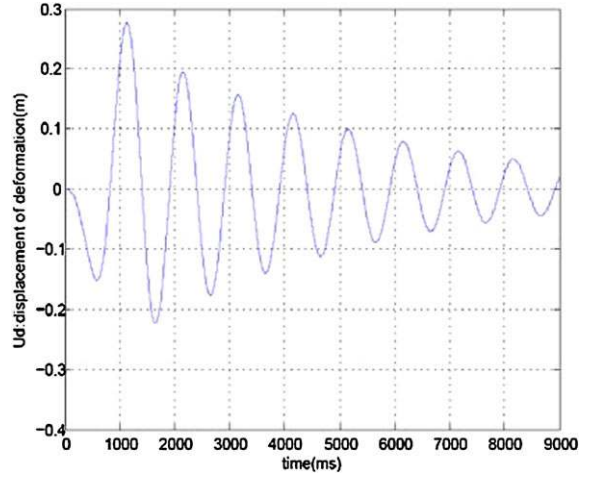


Fig. 6 Displacement due to the deformation

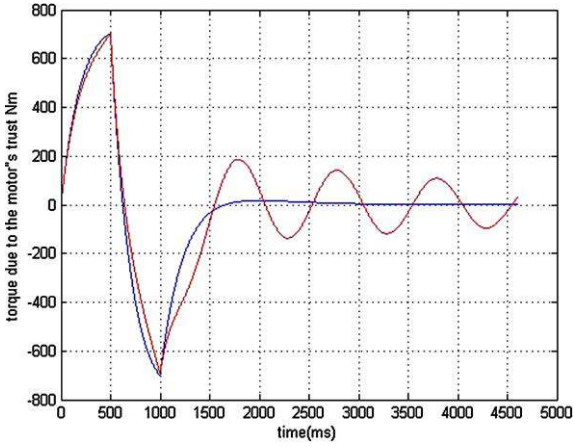


Fig. 4 Superposition of torques

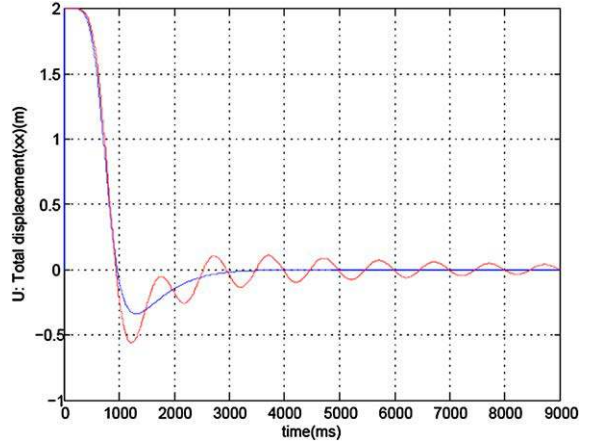


Fig. 7 Superposition of hole displacements

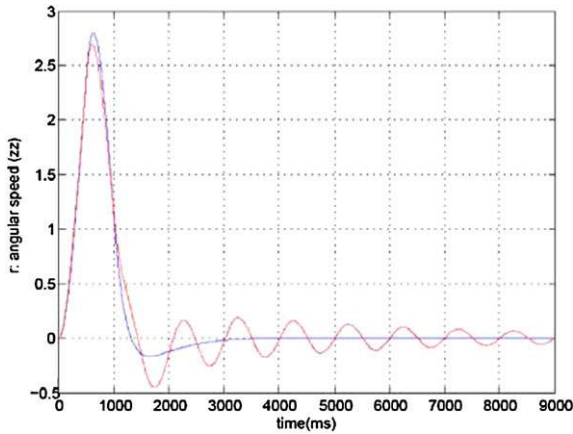


Fig. 5 Superposition of angular velocity of rigid and flexible airships

In Fig. 7, the impact of this displacement of deformation is highlighted.

6 Conclusion

In this paper a method has been presented for the aeroelastic analysis of the flexible airships. The strength of the different shape function for flexible careen, and the lightness of the Newton–Euler description of motion are combined for this purpose. A special focus is put on the computation of the added masses terms and the coupling between those issued from the whole rigid-body motion and those issued from the deformation of the careen. We develop for this purpose a fully analytical method with

some assumptions. This feature distinguishes the current work from earlier treatments of the coupling, and permits to minimise the number of the degrees of freedom, with the aim of optimising the ratio precision/computational time. This result will enable an easy implementation of control and stabilisation algorithms for the flexible flying objects. Simulation results are given for the case of an ellipsoidal airship manoeuvring over an unloading area. The results prove that the integration of the flexibility in the dynamic system of the airship is important and should not be neglected.

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