



Fitness landscapes and graphs: multimodularity, ruggedness and neutrality

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Fitness Landscapes and Graphs: Multimodularity, Ruggedness and Neutrality

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July, 8 2012

Fitness landscapes : Motivations

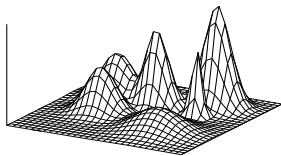
Why using fitness landscapes ?

- To analyse the structure of the search space
- To study problem (search) difficulty in combinatorial optimisation :
information on runtime for a given problem and a class of LS
- To design effective search algorithms

L. Barnett, U. Sussex, DPhil Diss. 2003

"the more we know of the statistical properties of a class of fitness landscapes, the better equipped we will be for the design of effective search algorithms for such landscapes"

Fitness landscapes in biology

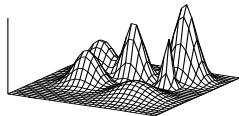
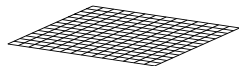
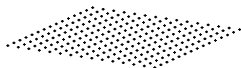


Biological science :
Wright 1930 [35]

Biological evolution :

- a metaphorical uphill struggle across a "fitness landscape"
- mountain peaks represent high "fitness", or ability to survive,
- valleys represent low fitness.
- evolution proceeds :
population of organisms
performs an "adaptive walk"

Fitness landscapes in biology



In biology :

- Modelisation of species evolution

Used to model dynamical systems :

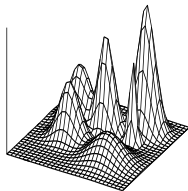
- statistical physic,
- molecular evolution,
- ecology, etc

Fitness landscapes in biology

2 sides for Fitness Landscapes :

- Powerful **metaphor** : most profound concept in evolutionary dynamics
 - give pictures of evolutionary process
 - be careful of misleading pictures : "smooth landscape without noise"
- **Quantitative** concept : predict the evolutionary paths
 - Quasispecies equation : mean field analysis with differential equations
 - Stochastic process : markov chain
 - Network analysis

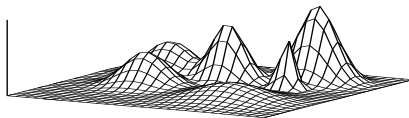
In combinatorial optimization



Fitness landscape $(\mathcal{S}, \mathcal{N}, f)$:

- \mathcal{S} : set of admissible solutions,
- $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$: neighborhood function,
- $f : \mathcal{S} \rightarrow \mathbb{R}$: fitness function.

Fitness landscapes for black-box optimisation



Tools for black-box optimisation

Blackbox scenario :

we have only $\{(x_0, f(x_0)), (x_1, f(x_1)), \dots\}$ given by an "oracle"

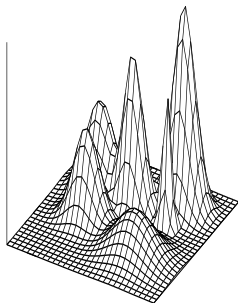
Search space analysis where "no" information is either not available or needed on the definition of fitness function.

Fitness landscapes in evolutionary computation

2 sides for Fitness Landscapes :

- Powerful **metaphor** : most profound concept
 - give pictures of the search dynamic :
"if the fitness landscapes have big valleys, I can use this algorithm"
 - be careful of misleading pictures : set of smooth mountains
- **Quantitative** concept : predict the evolutionary dynamic
 - Quasispecies equation : mean field analysis with differential equations
 - Stochastic process : markov chain
 - Network analysis

What is a neighborhood ?



Neighborhood function :

$$\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$$

Set of "neighbor" solutions associated to each solution

$$\mathcal{N}(x) = \{y \in \mathcal{S} \mid \mathbb{P}(y = op(x)) > 0\}$$

or

$$\mathcal{N}(x) = \{y \in \mathcal{S} \mid \mathbb{P}(y = op(x)) > \epsilon\}$$

or

$$\mathcal{N}(x) = \{y \in \mathcal{S} \mid d(y, x) \leq 1\}$$

Example of neighborhood : bit strings

Search space : $\mathcal{S} = \{0, 1\}^N$

Algorithm : simple GA,
hill-climbing, or simulated
annealing, etc.

$\mathcal{N}(01101) = \{$
 01101,
 01100,
 01111,
 01001,
 00101,
 11101,
 }

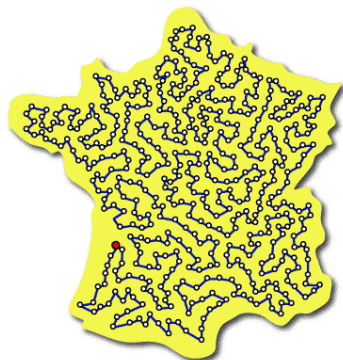
Important !

Definition of neighborhood must be
based on the local search operator
used in the algorithm

Neighborhood \Leftrightarrow Operator

$$\mathcal{N}(x) = \{y \in \mathcal{S} \mid d_{\text{Hamming}}(y, x) \leq 1\}$$

Example of neighborhood : permutations

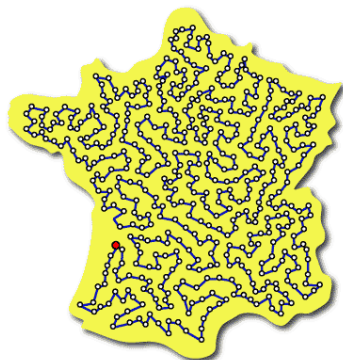


- *Search space* :
 $\mathcal{S} = \{ \sigma \mid \sigma \text{ permutations} \}$
- *Algorithm* : simple EA
operator : 2-opt

$$\mathcal{N}(x) = \{ y \in \mathcal{S} \mid \mathbb{P}(y = op_{2opt}(x)) > 0 \}$$

Traveling Salesman Problem :
find the shortest tour which
cross one time every town

Example of neighborhood



Traveling Salesman Problem :
find the shortest tour which
cross one time every town

- *Search space* :
 $\mathcal{S} = \{ \sigma \mid \sigma \text{ permutations} \}$
- *Algorithm* : simple EA
operators : 2-opt and 3-opt

$$\mathcal{N}(x) = \{ y \in \mathcal{S} \mid \mathbb{P}(y = op_{2opt}(x)) > 0 \text{ or } \mathbb{P}(y = op_{3opt}(x)) > 0 \}$$

Example of neighborhood : memetic algorithms

- *Algorithm* : memetic algorithm, EA + operator hill-climbing

$$\mathcal{N}(x) = \{y \in \mathcal{S} \mid y = op_{HC}(x)\}$$

- *Algorithm* : memetic algorithm, EA + operator hill-climbing and bit-flip mutation

2 possibilities :

- Study 2 landscapes :
one for *HC* operator, one for bit-flip mutation
- Study 1 landscape :
 $\mathcal{N}(x) = \{y \in \mathcal{S} \mid y = op_{HC}(x) \text{ or } \mathbb{P}(y = op_{bit-flip}(x)) > \epsilon\}$

It depends on what you want to know

Goal of the fitness landscapes study

- "Geometry" (features) of fitness landscape
⇒ dynamics of a local search algorithm
- Geometry is linked to the problem difficulty :
 - If there are a lot of local optima, the probability to find the global optimum is lower.
 - If the fitness landscape is flat, discovering better solutions is rare.
 - What is the best search direction in the landscape?

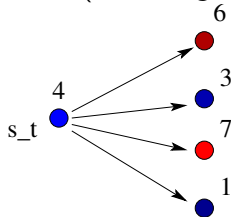
Study of the fitness landscape features
allows to study
the performance of search algorithms

Goal of the fitness landscapes study

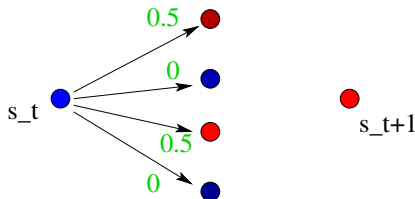
- ① To compare the difficulty of two search spaces :
 - One problem with 2 (or more) possible codings : $(S_1, \mathcal{N}_1, f_1)$ and $(S_2, \mathcal{N}_2, f_2)$
different coding, mutation operator, fitness function, etc.
Which one is easier to solve?
- ② To choose the algorithm :
 - analysis of global geometry of the landscape
Which algorithm can I use?
- ③ To tune the parameters :
 - *off-line* analysis of structure of fitness landscape
Which is the best mutation operator? the size of the population? etc.
- ④ To control the parameters during the run :
 - *on-line* analysis of structure of fitness landscape
Which is the optimal mutation rate according to the estimation of structure?

Point of view : Before putting a particular heuristic

FL = (Sol., Neighbors, Fitness)



Put prob. from your heuristic :



- Sample the neighborhood to have information on **local features** of the search space
- From local information : deduce some **global features** like general shape of search space, "difficulty", etc.

Goal of the fitness landscapes study

Study of the geometry of the landscape allows to study the difficulty, and design a good optimisation algorithm

Fitness landscape is a graph $(\mathcal{S}, \mathcal{N}, f)$ where the nodes have a value (fitness) : can be "pictured" as a "real" landscape

Two main geometries have been studied :

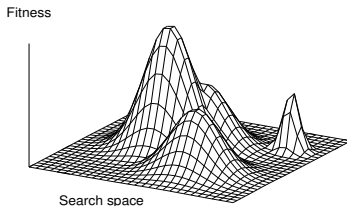
- multimodal and ruggedness
- neutral

Multimodal Fitness landscapes

Local optima s^* :

no neighbor solution with higher fitness value

$$\forall s \in \mathcal{N}(s^*), f(s) < f(s^*)$$



Multimodal Fitness landscapes

Adaptive walk : (s_0, s_1, \dots) where $s_{i+1} \in \mathcal{N}(s_i)$ and $f(s_i) < f(s_{i+1})$

Hill-Climbing (HC) algorithm

Choose initial solution $s \in S$

repeat

 choose $s' \in \mathcal{N}(s)$ such that $f(s') = \max_{x \in \mathcal{N}(s)} f(x)$

if $f(s) < f(s')$ **then**

$s \leftarrow s'$

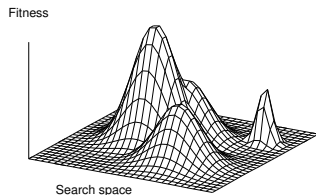
end if

until s is a Local optimum

Basin of attraction of s^* :

$$\{s \in S \mid \text{HillClimbing}(s) = s^*\}.$$

Multimodal Fitness landscapes



Optimisation difficulty :
number and size of attractive
basins (Garnier *et al* [10])

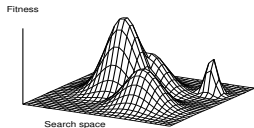
The idea :

- if the size of attractive basin of global optima is relatively "small"
- the problem is difficult to optimize

The measure :

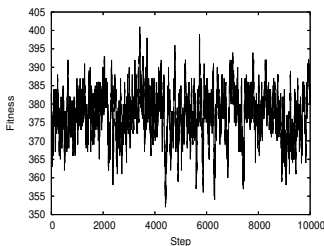
- Length of adaptive walks (distribution, avg, etc.)

Walking on fitness landscapes



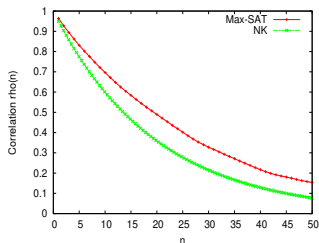
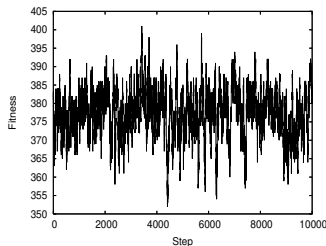
Random walk : (s_1, s_2, \dots)
such that $s_{i+1} \in \mathcal{N}(s_i)$ and
equiprobability on $\mathcal{N}(s_i)$

- Fitness seems to be very "chaotic"
- Analyse the fitness during the random walk as a signal



fitness vs. step of a random walk
(example of max-SAT problem)

Rugged/smooth fitness landscapes



Autocorrelation of time series of fitnesses $(f(s_1), f(s_2), \dots)$ along a random walk (s_1, s_2, \dots) [34] :

$$\rho(n) = \frac{E[(f(s_i) - \bar{f})(f(s_{i+n}) - \bar{f})]}{\text{var}(f(s_i))}$$

autocorrelation length $\tau = \frac{1}{\rho(1)}$

- small τ : **rugged landscape**
- long τ : **smooth landscape**

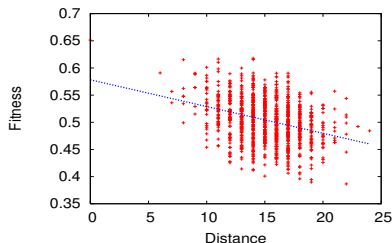
Results on rugged fitness landscapes (Stadler 96 [26])

Problem	parameter	$\rho(1)$
symmetric TSP	n number of towns	$1 - \frac{4}{n}$
anti-symmetric TSP	n number of towns	$1 - \frac{4}{n-1}$
Graph Coloring Problem	n number of nodes α number of colors	$1 - \frac{2\alpha}{(\alpha-1)n}$
NK landscapes	N number of proteins K number of epistasis links	$1 - \frac{K+1}{N}$

Ruggedness decreases with the size of those problems :
small variation has less effect on the fitness values

Fitness Distance Correlation (FDC) (Jones 95 [15])

Correlation between distance to global optimum and fitness



Classification based on experimental studies :

- $\rho < -0.15$, easy optimization
- $\rho > 0.15$, hard optimization
- $-0.15 < \rho < 0.15$, undecided zone

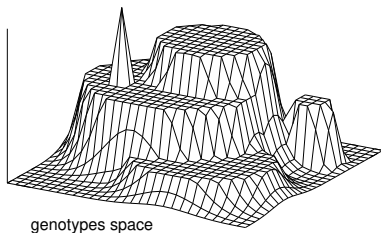
Neutral Fitness Landscapes

Neutral theory (Kimura \approx 1960 [17])

Theory of mutation and random drift

A considerable number of mutations have no effects on fitness values

Fitness



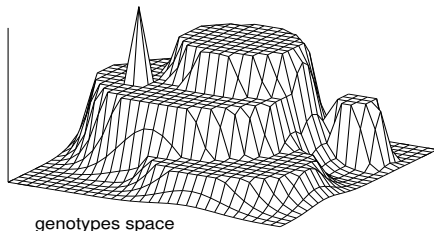
- plateaus
- neutral degree
- neutral networks
[Schuster 1994 [25],
RNA folding]

Neutral Fitness Landscapes

Combinatorial optimization

- Redundant problem (symetries, ...) (Goldberg 87 [12])
- Problem “not well” defined or dynamic environment (Torres 04 [14])

Fitness



Applicative problems :

- Robot controler
- Circuit design
- genetic programming
- Protein Folding
- learning problems

Neutrality and difficulty

- In our knowledge, there is no definitive answer about neutrality / problem hardness
- Certainly, it is dependent on the nature of neutrality of the fitness landscape

⇒ Sharp description of the geometry of neutral fitness landscapes is needed

Neutrality and difficulty

We know for certain that :

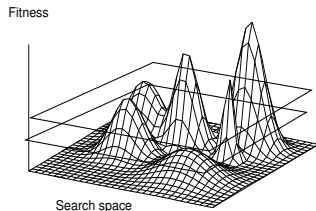
- **No information** is better than **Bad information** :
Hard trap functions are more difficult than
needle-in-a-haystack functions
 - **Good information** is better than **No information**
-
- When there is **No information** :
you should have a good method to find it !

In the following

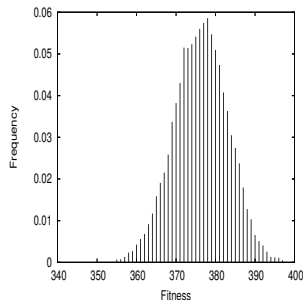
Description of neutral fitness landscapes :

- Neutral sets :
set of solutions with the same fitness
- Neutral networks :
add neighborhood information

Neutral sets : Density Of States



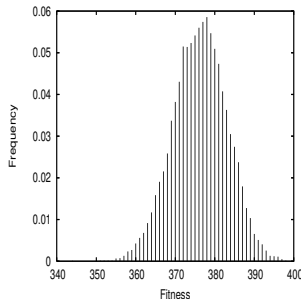
Set of solutions with fitness value



Density of states (D.O.S.)

- Introduce in physics (Rosé 1996 [24])
- Optimization (Belaidouni, Hao 00 [4])

Neutral sets : Density Of States

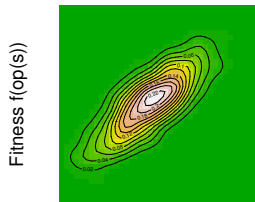
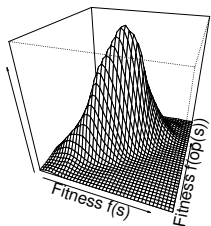


Density of states (D.O.S.)

Informations given :

- Performance of random search
- Tail of the distribution is an indicator of difficulty :
 - the faster the decay, the harder the problem
- But do not care about the neighborhood relation

Neutral sets : Fitness Cloud



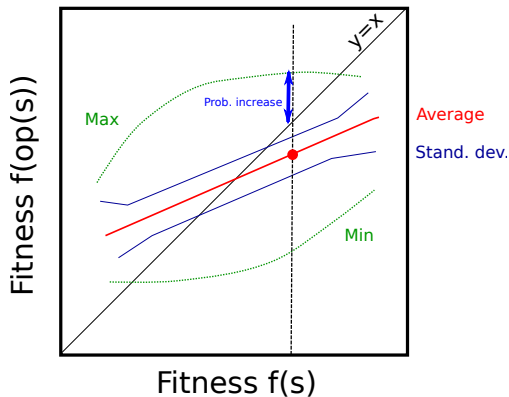
Fitness $f(s)$

- $(\mathcal{S}, \mathcal{F}, \mathbb{P})$: probability space
- $op : \mathcal{S} \rightarrow \mathcal{S}$ stochastic operator of the local search
- $X(s) = f(s)$
- $Y(s) = f(op(s))$

Fitness Cloud of op

Conditional probability density function of Y given X

Fitness cloud : Measure of evolvability

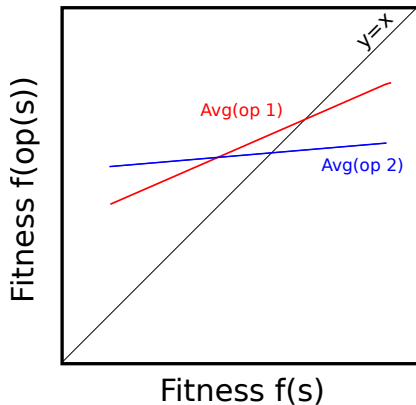


Evolvability

Ability to evolve : fitness in the neighborhood compared to the fitness of the solution

- Probability of finding better solutions
- Average fitness of better neighbor solutions
- Average and standard deviation of fitnesses

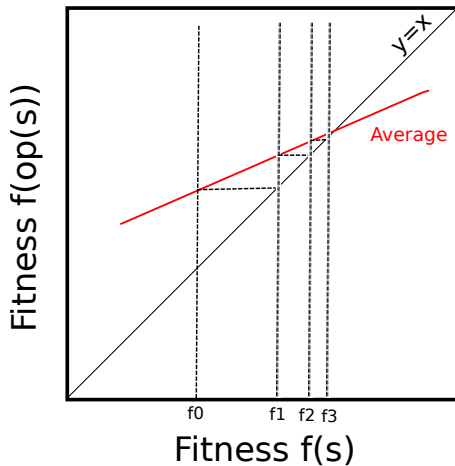
Fitness cloud : Comparaison of difficulty



- Operator 1 > Operator 2
- Because Average 1 more correlated to fitness
- Linked to autocorrelation
- Average is often a line :
 - See works on Elementary Landscapes (D. Whitley and others)
 - See Negative Slope Coefficient (NSC)

Fitness cloud

Prediction of fitness (CEC 2003)



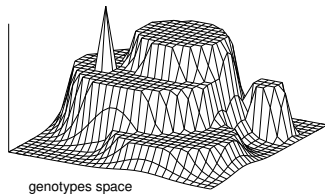
- Approximation (only approximation) of the fitness value after few steps of local operator
- Indication on the quality of the operator

Neutral fitness landscapes

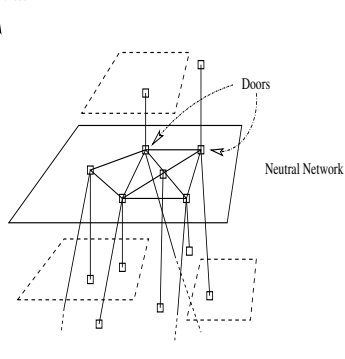
- Neutral sets (**done**) :
set of solutions with the same fitness
⇒ No structure
- Fitness cloud (**done**) :
Bivariate density ($f(s), f(op(s))$)
⇒ Neighborhood relation **between** neutral sets
- Neutral networks (**to be done**) :
⇒ Neighborhood structure **into** the neutral sets : Graph

Neutral networks (Schuster 1994 [25])

Fitness



Fitness



Definitions

Test of neutrality

$$isNeutral : S \times S \rightarrow \{true, false\}$$

For example, $isNeutral(s_1, s_2)$ is *true* if :

- $f(s_1) = f(s_2)$.
- $|f(s_1) - f(s_2)| \leq 1/M$ with M is the search population size.
- $|f(s_1) - f(s_2)|$ is under the evaluation error.

Neutral neighborhood

of s is the set of neighbors which have the same fitness $f(s)$

$$\mathcal{N}_{neut}(s) = \{s' \in \mathcal{N}(s) \mid isNeutral(s, s')\}$$

Neutral degree of s

Number of neutral neighbors : $nDeg(s) = \#(\mathcal{N}_{neut}(s) - \{s\})$.

Definitions

Neutral walk

$$W_{neut} = (s_0, s_1, \dots, s_m)$$

- for all $i \in [0, m-1]$, $s_{i+1} \in \mathcal{N}(s_i)$
- for all $(i, j) \in [0, m]^2$, $isNeutral(s_i, s_j)$ is true.

Neutral Network

graph $G = (N, E)$

- $N \subset \mathcal{S}$: for all s and s' from V , there is a neutral walk belonging to V from s to s' ,
- $(s_1, s_2) \in E$ if they are neutral neighbors : $s_2 \in \mathcal{N}_{neut}(s_1)$

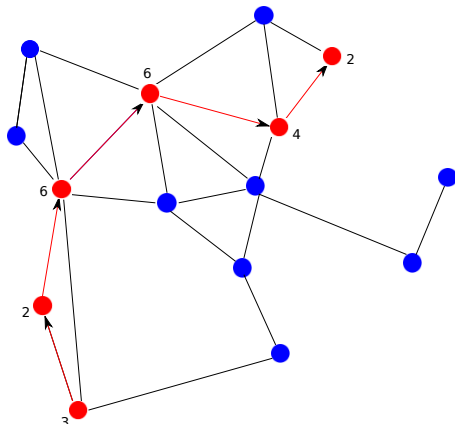
*A fitness landscape is neutral
if there are many solutions with high neutral degree.*

Neutral Networks (NN) : Inside Metrics

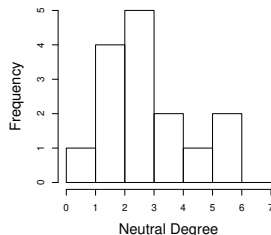
Classical graph metrics :

- Size of NN :
number of nodes of NN,
- Neutral degree distribution :
 - measure of the quantity of "neutrality"
- Autocorrelation of neutral degree (Bastolla 03 [3]) :
during neutral random walk
 - comparaison with random graph,
 - measure of the correlation structure of NN

Neutral Networks : Inside Metrics

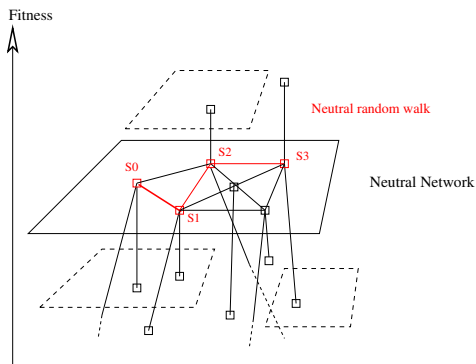


- Size : 15 solutions
Distribution of size
overall landscapes
- Neutral degree
distribution



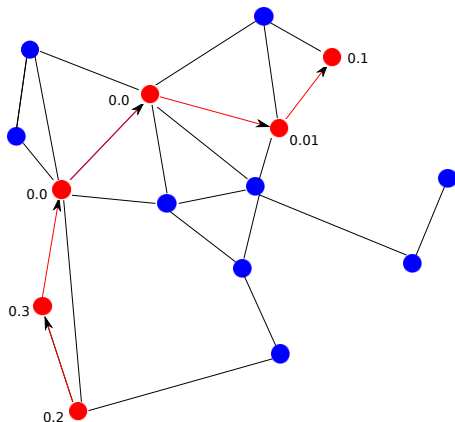
- Autocorrelation of

Neutral Networks : Outside Metrics



- 1 Rate of innovation (Huynen 96 [13]) :
The number of new accessible structures (fitness) per mutation
- 2 Autocorrelation of evolvability [32] :
autocorrelation of the sequence
($evol(s_0), evol(s_1), \dots$).

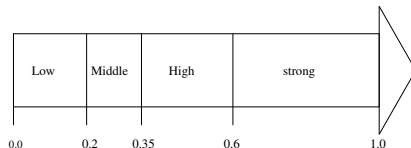
Neutral Networks : Outside Metrics



- Autocorrelation of evolvability :
 - Evolvability
 $evol = \text{avg fitness in the neighborhood}$
 - Autocorrelation of $(evol(s_0), evol(s_1), \dots)$.
- Informations :
 - if high correlation
 \Rightarrow "easy"
 (you can use this information)
 - if low correlation
 \Rightarrow "difficult"

Summary of metrics

- Neutral degrees distribution :
"How neutral is the fitness landscape ?"
- Autocorrelation of neutral degrees : network "structure"



- Rate of innovation :
low information for combinatorial optimization
- Autocorrelation of evolvability :
information on the links between NN

Basic Methodology of fitness landscapes analysis

- Density of States : pure random search, initialization ?
- Length of adaptive walks : multimodality ?
- Autocorrelation of fitness : ruggedness ?
- Neutral Degree Distribution : neutrality ?
- Fitness Cloud : Quality of the operator, evolvability ?
- Fitness Distance Correlation from best known
- Neutral walks and evolvability : neutral information ?
- ... be creative from your algorithm and problem point of view
- ... be careful on the computed measures : one measure is not enough, and must be very well understand

Software to perform fitness landscape analysis

Framework ParadisEO 1.3

<http://paradiseo.gforge.inria.fr/newWebsite/index.php?n=Doc.Tutorials> and tutorials : <http://paradiseo.gforge.inria.fr/newWebsite/index.php?n=Doc.Tutorials>

```
moAutocorrelationSampling<Neighbor> sampling(randomInit,  
                                              neighborhood,  
                                              fullEval,  
                                              incrementalEval,  
                                              nbStep);  
  
sampling();  
  
sampling.fileExport(str_out);
```

Motivation and general idea : Levels of description



- **Fitness landscapes** : based on an huge number of solutions
- **One metric** : based on one real number, or curve to catch all the complexity
- **Local optima Network** : based on local optima

Overview and Motivation

- Bring the tools of *complex networks* analysis to the study the structure of combinatorial fitness landscapes
- **Goals** : Understand problem difficulty, design effective heuristic search algorithms
- **Methodology** : Extract a network that represents the landscape (Inspiration from energy landscapes (Doye, 2002)¹)
 - **Vertices** : local optima
 - **Edges** : a notion of adjacency between basins
- Conduct a network analysis
- Relate (exploit ?) network features to search algorithm design

1. J. P. K. Doye, The network topology of a potential energy landscape : a static scale-free network., *Phys. Rev. Lett.*, 88 :238701, 2002.

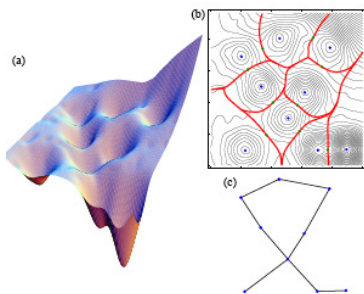
Small – world networks (Watts and Strogatz, 1998)

- Neither ordered nor completely random
- Nodes highly clustered yet path length is small
- Network topological measures :
 - C : clustering coefficient, measure of local density
 - l : shortest path length global measure of separation

Scale – free networks (Barabasi and Albert, 1999)

- The distribution of the number of neighbours (the degree distribution) is *right – skewed* with a heavy tail
- Most of the nodes have less-than-average degree, whilst a small fraction of hubs have a large number of connections
- Described mathematically by a power-law

Energy surface and inherent networks (Doye, 2002)



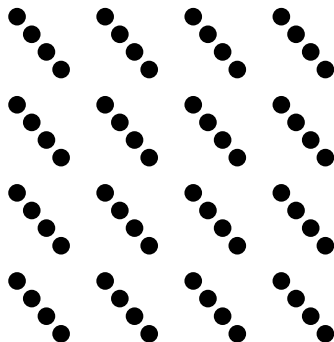
- a Model of 2D energy surface
- b Contour plot, partition of the configuration space into basins of attraction surrounding minima
- c landscape as a network

Inherent network :

- **Nodes** : energy minima
- **Edges** : two nodes are connected if the energy barrier separating them is sufficiently low (transition state)

Basins of attraction in combinatorial optimisation

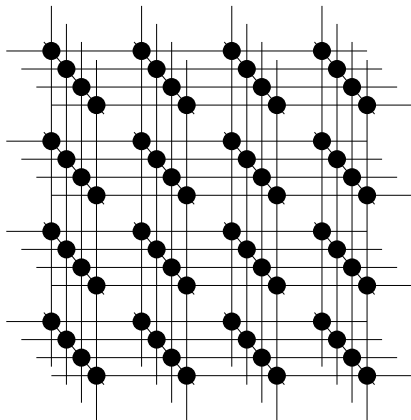
Example of small NK landscape with $N = 6$ and $K = 2$



- Bit strings of length $N = 6$
- $2^6 = 64$ solutions
- one point = one solution

Basins of attraction in combinatorial optimisation

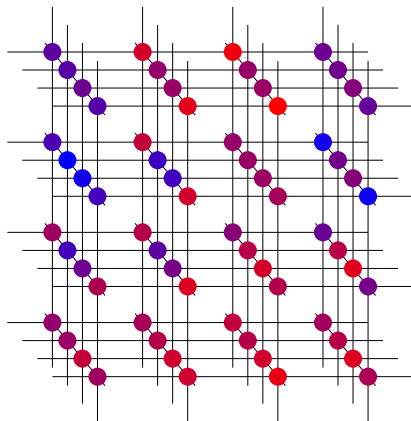
Example of small NK landscape with $N = 6$ and $K = 2$



- Bit strings of length $N = 6$
- Neighborhood size = 6
- Line between points = solutions are neighbors
- Hamming distances between solutions are preserved (except for at the border of the cube)

Basins of attraction in combinatorial optimisation

Example of small NK landscape with $N = 6$ and $K = 2$

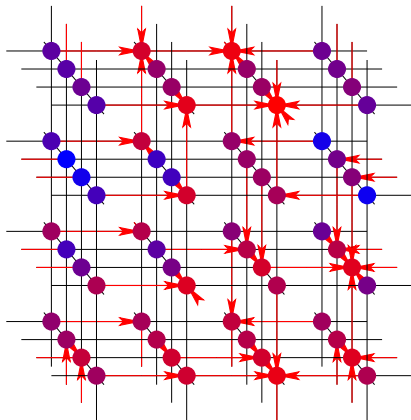


Color represent fitness value

- high fitness
- low fitness

Basins of attraction in combinatorial optimisation

Example of small NK landscape with $N = 6$ and $K = 2$



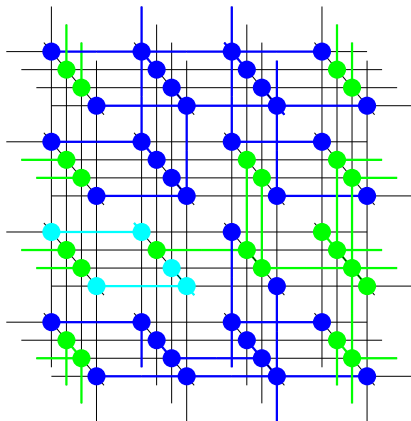
- Color represent fitness value
 - high fitness
 - low fitness
- → point towards the solution with highest fitness in the neighborhood

Exercise :

Why not make a Hill-Climbing walk on it ?

Basins of attraction in combinatorial optimisation

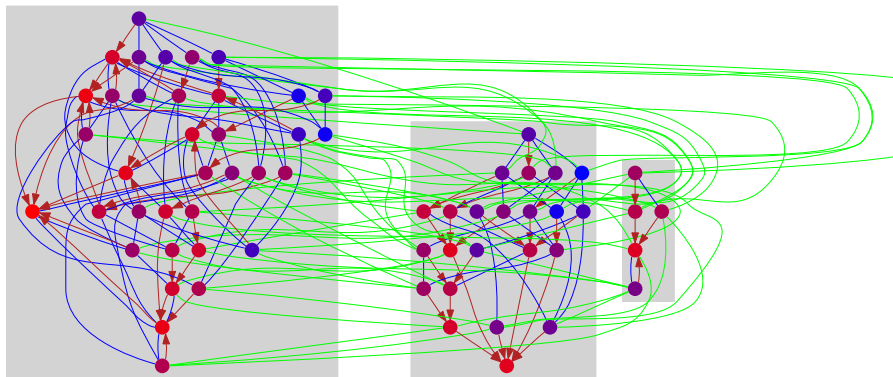
Example of small NK landscape with $N = 6$ and $K = 2$



- Each color corresponds to one basin of attraction
- Basins of attraction are interlinked and overlapped
- Basins have no "interior"

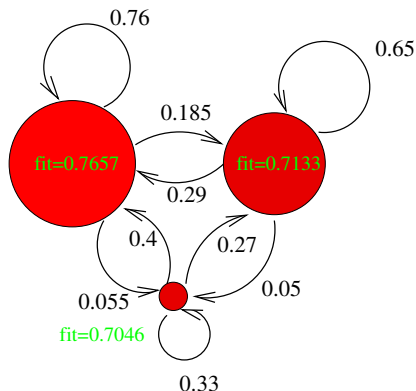
Basins of attraction in combinatorial optimisation

Example of small NK landscape with $N = 6$ and $K = 2$



- Basins of attraction are interlinked and overlapped !
- Most neighbours of a given solution are outside its basin

Local optima network



- **Nodes** : local optima
- **Edges** : transition probabilities

Basin of attraction

Hill-Climbing (HC) algorithm

Choose initial solution $s \in S$

repeat

 choose $s' \in \mathcal{N}(s)$ such that $f(s') = \max_{x \in \mathcal{N}(s)} f(x)$

if $f(s) < f(s')$ **then**

$s \leftarrow s'$

end if

until s is a Local optimum

Basin of attraction of s^* :

$$\{s \in S \mid \text{HillClimbing}(s) = s^*\}.$$

local optima network

Local optima network

- Nodes : set of local optima \mathcal{S}^*
- Edges : notion of connectivity between basins of attraction
 - e_{ij} between i and j if there is at least a pair of neighbours s_i and $s_j \in \mathcal{N}(s_i)$ such that $s_i \in b_i$ and $s_j \in b_j$ (GECCO 2008 [21])
 - weights w_{ij} is attached to the edges, account for transition probabilities between basins (ALIFE 2008 [33], Phys. Rev. E 2008 [30], CEC 2010)

Weights of edges

- From each s and s' , $p(s \rightarrow s') = \mathbb{P}(s' = op(s))$

For example, $S = \{0, 1\}^N$ and bit-flip operator

- if $s' \in \mathcal{N}(s)$, $p(s \rightarrow s') = \frac{1}{N}$
- if $s' \notin \mathcal{N}(s)$, $p(s \rightarrow s') = 0$
- Probability that a configuration $s \in S$ has a neighbor in a basin b_j

$$p(s \rightarrow b_j) = \sum_{s' \in b_j} p(s \rightarrow s')$$

- w_{ij} : Total probability of going from basin b_i to basin b_j is the average over all $s \in b_i$ of the transition prob. to $s' \in b_j$:

$$p(b_i \rightarrow b_j) = \frac{1}{\#b_i} \sum_{s \in b_i} p(s \rightarrow b_j)$$

\Rightarrow local optima network : weighted oriented graph

NK fitness landscapes : ruggedness and epistasis

NK-landscapes : Model of problems

N size of the bit-strings

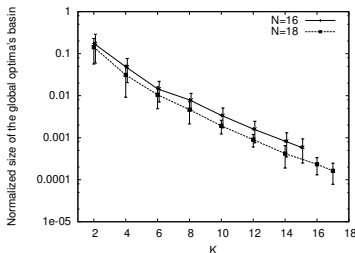
K from 0 to $N - 1$, NK landscapes can be **tuned** from smooth to rugged (easy to difficult respectively) :

- $K = 0$ no correlations, f is an additive function, and there is a **single maximum**
- $K = N - 1$ landscape **completely random**, the expected number of local optima is $\frac{2^N}{N+1}$
- Intermediate values of K interpolate between these two extreme cases and have a variable degree of **epistasis** (i.e. gene interaction)

Methods

- Extracted and analysed networks
 - $N \in \{14, 16, 18\}$,
 - $K \in \{2, 4, \dots, N - 2, N - 1\}$
 - 30 random instances for each case
- Measures :
 - Statistics on **basins** sizes and fitness of optima
 - **Network features** : clustering coefficient, shortest path to the global optimum, weight distribution, disparity, boundary of basins

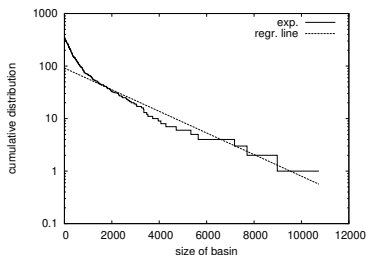
Global optimum basin size versus K



Size of the basin corresponding to the global maximum for each K

- Trend : the basin shrinks very quickly with increasing K.
- for higher K, more difficult for a search algorithm to locate the basin of attraction of the global optimum

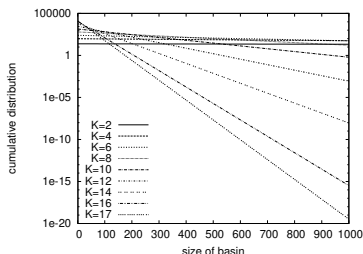
Analysis of basins : basin size



Cumulative distribution of basins sizes for $N = 18$ and $K = 4$

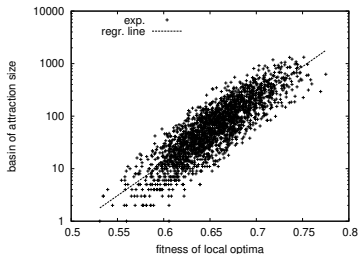
- Trend : small number of large basin, large number of small basin
- Log-normal cumulative distribution : not uniform !
- Slope of correlation increases with K
- When K large : basin sizes are nearly equals the distribution becomes more uniform

Analysis of basins : basin size



- Trend : small number of large basin, large number of small basin
- log-normal cumulative distribution
- slope of correlation increases with K
- when K large : basin sizes are nearly equals

Analysis of basins : fitness vs. basin size



Correlation fitness of local optima
vs. their corresponding basins
sizes

- Trend : clear positive correlation between the fitness values of maxima and their basins' sizes

The highest, the largest

- On average, the global optimum easier to find than one other local optimum
- But more difficult to find, as the number of local optima increases exponentially with increasing K

General network statistics

Weighted clustering coefficient

local density of the network

$$c^w(i) = \frac{1}{s_i(k_i - 1)} \sum_{j,h} \frac{w_{ij} + w_{ih}}{2} a_{ij} a_{jh} a_{hi}$$

where $s_i = \sum_{j \neq i} w_{ij}$, $a_{nm} = 1$ if $w_{nm} > 0$, $a_{nm} = 0$ if $w_{nm} = 0$ and $k_i = \sum_{j \neq i} a_{ij}$.

Disparity

dishomogeneity of nodes with a given degree

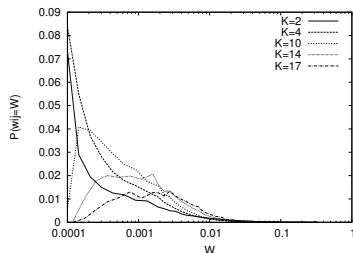
$$Y_2(i) = \sum_{j \neq i} \left(\frac{w_{ij}}{s_i} \right)^2$$

General network statistics $N = 16$

K	# nodes	# edges	\bar{C}^w	\bar{Y}	\bar{d}
2	33 ₁₅	516 ₃₅₈	0.96 _{0.0245}	0.326 _{0.0579}	56 ₁₄
4	178 ₃₃	9129 ₂₉₃₀	0.92 _{0.0171}	0.137 _{0.0111}	126 ₈
6	460 ₂₉	41791 ₄₆₉₀	0.79 _{0.0154}	0.084 _{0.0028}	170 ₃
8	890 ₃₃	93384 ₄₃₉₄	0.65 _{0.0102}	0.062 _{0.0011}	194 ₂
10	1,470 ₃₄	162139 ₄₅₉₂	0.53 _{0.0070}	0.050 _{0.0006}	206 ₁
12	2,254 ₃₂	227912 ₂₆₇₀	0.44 _{0.0031}	0.043 _{0.0003}	207 ₁
14	3,264 ₂₉	290732 ₂₀₅₆	0.38 _{0.0022}	0.040 _{0.0003}	203 ₁
15	3,868 ₃₃	321203 ₂₀₆₁	0.35 _{0.0022}	0.039 _{0.0004}	200 ₁

- **Clustering Coefficient** : For high K , transition between a given pair of neighboring basins is less likely to occur
- **Disparity** : For high K the transitions to other basins tend to become equally likely, an indication of the randomness of the landscape

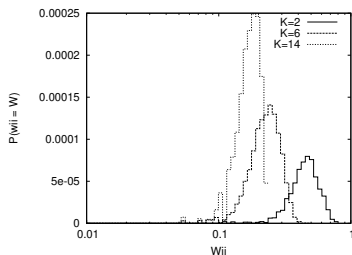
Weights distribution : transition probability between basins



distribution of the network weights w_{ij} for outgoing edges with $j \neq i$ in log-x scale, $N = 18$

- Weights are small
- For high K the decay is faster
- Low K has longer tails
- On average, the transition probabilities are higher for low K (less local optima)

Weight distribution remain in the same basin



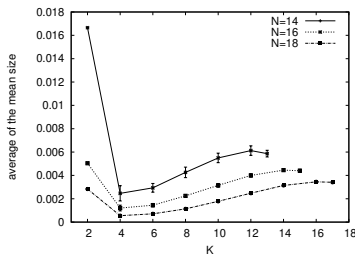
Average weight w_{ii} according to the parameter N and K

Question :

Is it easy to escape a basin ?

- Weights to remains in the same are large compare to w_{ij} with $i \neq j$
- w_{ii} are higher for low K
- Easier to leave the basin for high K : high "natural" exploration
- But : number of local optima increases fast with K

Interior and border size



Average of the mean size of
basins interiors

Question :

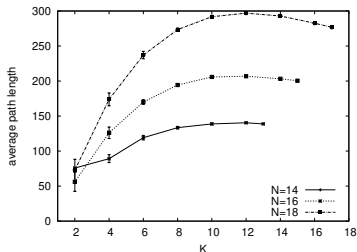
Do basins look like a "mountain"
with interior and border ?

solution is in the interior if all
neighbors are in the same basin

Answer

- Interior is very small
- Nearly all solution are in the border

Shortest path length between local optima



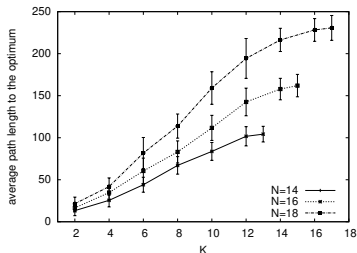
Average distance (shortest path)
between nodes

Question :

Are the basins "far" from each other?

- Increase with N (# of nodes increases exponentially)
- For a given N , increase with K up to $K = 10$, then stagnates

Shortest path length to global optima



Average path length to the global optimum from all the other basins

Question :

Is the global optimum basin is far ?

- More relevant for optimisation
- Increase steadily with increasing K

Summary on local optima network

- Medium level of description : proposed characterization of combinatorial landscapes as networks
- a new model for landscape analysis
- New findings about basin's structure : sizes, fitness vs. size, etc.
- Related some *network features* to *search difficulty*

Future on local optima network

- Design a method for sampling large search space (under construction)
- Compare the properties of Loc. Opt. Network and the optimal tradeoff between exploration and exploitation
- Study the LON like a fitness landscape
- Deduce some approximation of the runtime from the properties of LON

Summary on fitness landscapes

Fitness landscape is a representation of

- search space
- notion of neighborhood
- fitness of solutions

Goal :

- **local description** : fitness between neighbor solutions
Ruggedness, local optima, fitness cloud, neutral networks, local optima networks...
- and to deduce **global features** :
 - Difficulty !
 - To decide (and control) a good choice of the representation, operator and fitness function

Open questions

- How to control the parameters and/or operators of the algorithm with the local description of fitness landscape?
- Can fitness landscape describe the dynamics of a population of solutions?
- Links between neutrality and fitness difficulty?
- Which intermediate description shows relevant properties of the optimization problem according to the local search heuristic?
- What is the fitness landscapes for a *multiobjective problem*?

Integration of the FL tools into the open framework *paradisEO*

<http://paradisEO.gforge.inria.fr>



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