Variance/Covariance extension for time series discrimination

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Abstract. For time series discrimination,the main idea behind the proposed approach is to use a variance/covariance criterion to strengthen or weaken aligned observations according to their contribution to the variability within and between classes. To this end, the classical variance/covariance expression is extended to a set of time series, as well as to a partition of time series.
Keywords: time series alignment, classification, variance-covariance

1 Introduction

Time series originating from the same (or similar) sources are often noisy as the timing of salient events can be extremely variable. For example, in the context of electric networks, a particular peak associated with the same underlying event may appear at different times, depending on the use of the plugs monitored. To allow time series comparison while dealing with time delays, numerous alignments strategies have been proposed, as the ones based on Dynamic Time Warping (DTW) [1], which however proposes a too local view as the alignment depends only on the couple of time series under consideration; furthermore, the process of alignment is decoupled from the one of analysis (as clustering or classification), weakening the use of the alignment in real applications.

To partly overcome these problems, Gaffney et al. [2] propose a probabilistic framework to jointly handle the clustering and the alignment processes. However, the proposed alignments are limited to time series of a same class so that the discriminative power of the method is limited. In Listegarten et al. [3], a hierarchical Bayesian model is proposed to perform detection of rare differences between classes of time series. This model allows one to align time series simultaneously across all classes, while detecting and characterizing class-specific differences. Ramsay et al. [4] propose a time series clustering model where an alignment function is learned for each time series, parameterized with order one B-spline coefficients. The learned alignments account for a common shared structure within clusters.

In the context of discriminating complex time series, one thus needs to align time series with respect to the commonly shared features, pertaining to potentially many underlying global structures, within classes, and to identify the most differential features between classes. We propose to do so here through the use of a variance/covariance criterion to strengthen or weaken links according to their contributions to the variances within and between clusters. The variance/covariance measure is a classical criterion, used in many approaches, including discriminant analysis, dimensionality reduction, clustering and classification, and variants of it have already been proposed for graph-structured data (see for example [5–8]). Its use for learning alignments between time series has however never been investigated before, to our knowledge. We propose an extension of the classical variance/covariance expression to a set of time series, then to a partition based on classes of time series. Based on the learned alignments, a discriminative distance is defined for time series nearest neighbor classification.

2 The variance/covariance of time series data

We first recall here the definition of the conventional variance/covariance matrix, prior to extend it to a set of time series and then to a partitioned (according to classes) set of time series. Let X be the $(n \times p)$ data matrix providing the description of n observations by p numerical variables. The conventional $(p \times p)$ variance/covariance matrix expression is:

$$V = X^t(I - UP)^t P(I - UP)X \tag{1}$$

where, I is the diagonal identity matrix, U the unit matrix, and P a diagonal matrix of weights, generally set to $p_i = \frac{1}{n}$ for equally weighted observations.

In the case of a set of time series, let X be the $(nT \times p)$ matrix providing the description of n multivariate time series $S_1, ..., S_n$ by p numerical variables at T time stamps. The general term x_{ij}^l of X gives the value of the variable X_j $(j = 1, ..., p)$ taken by S_l $(l = 1, ..., n)$ at the ith time stamp $(i = 1, ..., T)$. Alignments between n time series can be encoded through a matrix M composed of n^2 block matrices $M^{ll'}$ $(l = 1, ..., n; l' = 1, ..., n)$. A block $M^{ll'}$ is a $(T \times T)$ matrix that specifies the alignment between S_l and $S_{l'}$, and its general term $m_{ii'}^{ll'} \in [0, 1]$ indicates the intensity of the linkage between the observation of S_l at time i and the observation of $S_{l'}$ at time i'.

3 The variance induced by a partition of time series

Let us now consider the set of time series $S_1, ..., S_n$ partitioned into K groups, with $y_i \in \{1, ..., K\}$ the class label of S_i and n_k the size of class k (i.e. the number of time series contained in class k). In such a case, the within variance/covariance matrix provides a measure of the dispersion of time series within classes, and the within variance a measure of how close time series are within classes. Following the definition for the variance of a set of time series, we define the within variance with an intra-class alignment matrix M as the sum, restricted over the time series of the same class, of the variance of each variable.

Similarly, the between variance (i.e. the variance between classes) can be defined as follows. The between variance with an inter-class alignment matrix M. The general form of the alignment matrix M is symmetric wrt to the preceding one, alignments between time series of the same class being forbidden this time, whereas alignments between time series of different classes are taken into account.

As one can note, alignments between time series play a crucial role (through the intra and inter class alignment matrices) in the definition of the within and between variances. To discriminate time series, the question which thus arises is how to learn such matrices so as to be able to minimize the within variance and maximize the between variance.

4 Conclusion

For time series discrimination, the main idea is based on strengthening or weakening links according to their contribution to the variability within and between classes. To this end, the classical variance/covariance expression is extended to a set of time series, and to a partition of time series. For time series classification, a new time series discriminative distance based on the learned alignments can be proposed.

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