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# MULTIVARIATE TEXTURE RETRIEVAL USING THE KULLBACK-LEIBLER DIVERGENCE BETWEEN BIVARIATE GENERALIZED GAMMA TIMES AN UNIFORM DISTRIBUTION

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## ABSTRACT

This paper presents a new multivariate elliptical distribution, namely the multivariate generalized Gamma times an Uniform (MGFU) distribution. Because it generalizes the multivariate generalized Gaussian distribution (MGGD), the MGFU distribution is able to fit a wider range of signals. For the bivariate case, we provide a closed-form of the Kullback-Leibler divergence (KLD). We propose the MGFU distribution for modeling chrominance wavelet coefficients and exercise it in a texture retrieval experiment.

A comparative study between some multivariate models on the VisTex and Outex image database is conducted and reveals that the use of the MGFU distribution of chrominance wavelet coefficient allows an indexing gain compared to other classical approaches such as MGGD and Copula based model).

**Index Terms**— Texture, image retrieval, Kullback-Leibler divergence, Multivariate elliptical distribution.

## 1. INTRODUCTION

In the context of texture image recognition, the wavelet representation has been widely considered to characterize the texture. The multiscale recognition algorithm operates in two main steps. One is the feature extraction which consists in modeling each wavelet subband by a given probability density function (pdf). Its estimated parameters compose the signature of the texture. Then, a similarity measure based on a probabilistic metric is computed between the query image and all the other images of the database.

Many univariate models such as the generalized Gaussian [1] and the Weibull [2] distributions have been proposed for modeling the magnitudes of wavelet detail coefficients. Those works have further been extended by the use of the generalized Gamma (GF) distribution which generalizes these two models [3]. However, these approaches do not fully exploit the texture information in the image. Multivariate distributions have hence recently been introduced to model the spatial and/or color dependencies of the wavelet

coefficients such as the Gaussian [4] and Student-t [5] copulas. Other approaches based on the multivariate elliptical distributions have been proposed such as the multivariate generalized Gaussian distribution (MGGD) [6]. For this later one, the multivariate Laplace is a special case, and the multivariate  $\mathcal{G}^0$  distribution. For both models, a closed form of the geodesic distance (GD) [7] has been established for fixed shape parameters and an approximation of the GD has been given for the general case by assuming the geodesic coordinate functions as straight lines [8] [9]. Unfortunately, most of the time, only approximations of the GD can be derived for non trivial multivariate distributions (MGGD,  $\mathcal{G}^0$ , ...).

The main paper contribution is to propose a new multivariate model which generalizes the MGGD, namely the multivariate generalized Gamma times an Uniform (MGFU) distribution. For the special case of a bivariate generalized Gamma times an Uniform (BGFU) distribution, an analytical expression of the Kullback-Leibler divergence (KLD) [10] is derived. The paper is structured as follows. In section 2, we introduce the MGFU distribution and its parameters estimation. Section 3 provides the derivation of the analytical expression of the KLD for the BGFU distribution. The BGFU distribution is then proposed for modeling the chrominance wavelet coefficients. Some indexing results on the VisTex and Outex databases are also presented. Conclusions are finally reported in Section 4.

## 2. THE MGFU DISTRIBUTION

### 2.1. Definition

In this paper, we introduce a novel distribution, namely the MGFU distribution, which belongs to the family of elliptically contoured distributions. This model admits the following stochastic representation [11]:

$$\mathbf{k} = \tau \Sigma^{\frac{1}{2}} \mathbf{x}, \quad (1)$$

where  $\mathbf{x}$  is an uniformly distributed random vector on the unit sphere  $\mathbb{R}^p$ ,  $p$  being the dimension of vector  $\mathbf{k}$ .  $\Sigma$  is the normalized scatter matrix (i.e.  $\text{tr}(\Sigma) = p$ ).  $\tau$  is a positive

scalar random variable distributed according to a Generalized Gamma distribution, whose pdf  $p_\tau(\tau)$  is given by [12]:

$$p_\tau(\tau|\alpha, \beta, \gamma) = \text{GF}(\tau|\alpha, \beta, \gamma) = \frac{\alpha\tau^{\alpha\gamma-1}e^{-\left(\frac{\tau}{\beta}\right)^\alpha}}{\beta^\alpha\gamma\Gamma(\gamma)}, \quad (2)$$

where  $\alpha > 0$ ,  $\beta > 0$ ,  $\gamma > 0$  and  $\tau \in [0, +\infty[$ .  $\alpha$ ,  $\beta$  and  $\gamma$  are respectively the power, scale and shape parameters of the distribution.

To derive the expression of the pdf of  $\mathbf{k} = \tau\Sigma^{\frac{1}{2}}\mathbf{x}$ , let first consider the scalar random variable  $u = \mathbf{k}^T\Sigma^{-1}\mathbf{k}$ . As  $\mathbf{x}$  is uniformly distributed on the unit sphere  $\mathbb{R}^p$ , it can be easily proved that  $u$  follows the same distribution as  $\tau^2$ . After the variable change  $u = \tau^2$  in (2), we obtain:

$$p_u(u|\alpha, \beta, \gamma) = \text{GF}\left(u\left|\frac{\alpha}{2}, \beta^2, \gamma\right.\right) = \frac{\alpha u^{\frac{\alpha\gamma}{2}-1}e^{-\frac{u}{\beta^2}}}{2\beta^\alpha\gamma\Gamma(\gamma)}. \quad (3)$$

By definition,  $\mathbf{k}$  follows a multivariate elliptical distribution, then:

$$p_{\mathbf{k}}(\mathbf{k}|\alpha, \beta, \gamma, \Sigma) = \frac{1}{|\Sigma|^{\frac{1}{2}}}h_p(\mathbf{k}^T\Sigma^{-1}\mathbf{k}), \quad (4)$$

where  $h_p(\cdot)$  is the density generator function. The change variable  $u = \mathbf{k}^T\Sigma^{-1}\mathbf{k}$  in (4) leads to:

$$p_u(u|\alpha, \beta, \gamma) = \frac{\pi^{\frac{p}{2}}}{\Gamma(p/2)}u^{\frac{p}{2}-1}h_p(u). \quad (5)$$

By identification between (3) and (5), we obtain the analytical expression of  $h_p(\cdot)$  for the MGFU distribution, it yields :

$$h_p(u) = \frac{\Gamma(p/2)}{\pi^{\frac{p}{2}}}\frac{\alpha}{2\beta^\alpha\gamma\Gamma(\gamma)}u^{\frac{\alpha\gamma-p}{2}}e^{-\frac{u}{\beta^\alpha}}. \quad (6)$$

Finally, the expression of the pdf for an MGFU distribution is obtained:

$$p_{\mathbf{k}}(\mathbf{k}|\alpha, \beta, \gamma, \Sigma) = \frac{1}{\pi^{\frac{p}{2}}|\Sigma|^{\frac{1}{2}}}\frac{\alpha\Gamma(p/2)}{2\beta^\alpha\gamma\Gamma(\gamma)}(\mathbf{k}^T\Sigma^{-1}\mathbf{k})^{\frac{\alpha\gamma-p}{2}} \times e^{-\frac{(\mathbf{k}^T\Sigma^{-1}\mathbf{k})^\alpha}{\beta^\alpha}}. \quad (7)$$

A particular case of the MGFU distribution is the MGGD which is obtained when  $\alpha = 2b$ ,  $\beta = 2^{\frac{1}{2b}}$  and  $\gamma = \frac{p}{2b}$  [13]. Note that the multivariate Gaussian distribution is obtained when  $\alpha = 2$ ,  $\beta = \sqrt{2}$  and  $\gamma = \frac{p}{2}$ , and the multivariate Laplace distribution is obtained when  $\alpha = 1$ ,  $\beta = 2$  and  $\gamma = p$ .

## 2.2. Parameter estimation

Let  $(\mathbf{k}_1, \dots, \mathbf{k}_N)$  be  $N$  vectors distributed according to an independent and identically distributed random MGFU distribution. The expression of the maximum likelihood estimators (MLE) for the parameters of an MGFU distribution can

be easily derived by differentiating the joint distribution of  $(\mathbf{k}_1, \dots, \mathbf{k}_N)$  with respect to  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\Sigma$ . An iterative algorithm can be easily derived to solve the ML equations as in [14]. Unfortunately, this algorithm is very complex since the MLEs of the MGFU distribution parameters are all dependent of each other. To circumvent this difficulty, we propose a simpler estimation algorithm based on a mixed moment and ML estimation procedure. Given the  $N$  vectors  $(\mathbf{k}_1, \dots, \mathbf{k}_N)$ , the normalized scatter matrix is estimated by using a moment-based approach, it yields:

$$\hat{\Sigma} = \frac{1}{N}\sum_{i=1}^N\mathbf{k}_i\mathbf{k}_i^T \text{ with } \text{tr}(\hat{\Sigma}) = p. \quad (8)$$

Then, the  $N$  scalar variables  $u_i = \mathbf{k}_i^T\hat{\Sigma}^{-1}\mathbf{k}_i$  are computed. According to (3), the scalar random variable  $u$  follows a GF distribution of parameters  $\alpha/2$ ,  $\beta^2$  and  $\gamma$ . Those parameters can hence be estimated by using the ML method.

## 3. TEXTURE INDEXING

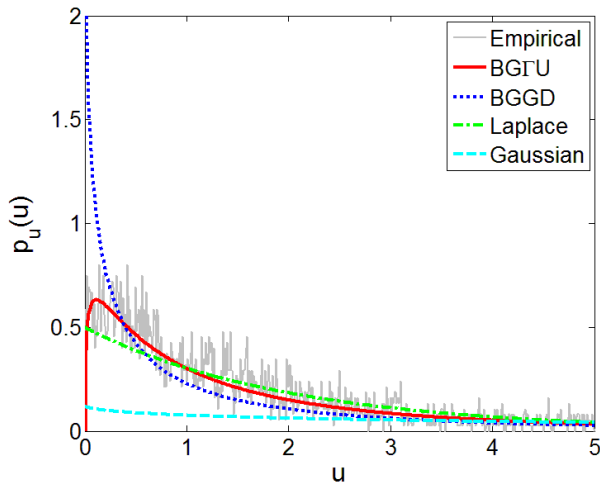
In the following, only the bivariate case will be considered (*i.e.*  $p = 2$ ). In this context, we can prove that the Kullback-Leibler divergence (KLD) can be analytically derived for a BGFU distribution. We then propose this distribution as a suitable model for the chrominance wavelet coefficients in a texture retrieval experiment.

### 3.1. Kullback-Leibler divergence

After some cumbersome computations, for the bivariate case, one can derive the KLD between two populations distributed according to a BGFU distributions of parameters  $(\alpha_1, \beta_1, \gamma_1, \Sigma_1)$  and  $(\alpha_2, \beta_2, \gamma_2, \Sigma_2)$ , it yields:

$$\begin{aligned} \text{KLD}(\alpha_1, \beta_1, \gamma_1, \Sigma_1 || \alpha_2, \beta_2, \gamma_2, \Sigma_2) &= \ln\left(\frac{\alpha_1}{\alpha_2}\right) + \alpha_2\gamma_2 \ln \beta_2 \\ &- \alpha_1\gamma_1 \ln \beta_1 + \ln\left(\frac{\gamma_2}{\gamma_1}\right) + \frac{1}{2}\ln\left(\frac{|\Sigma_2|}{|\Sigma_1|}\right) - \gamma_1 \\ &+ \left(\frac{\alpha_1\gamma_1 - \alpha_2\gamma_2}{2}\right)\left[2\ln \beta_1 + \frac{2}{\alpha_1}\Psi(\gamma_1)\right] \\ &- \left(\frac{\alpha_2\gamma_2}{2} - 1\right)\left[\frac{1}{2}\ln(g_1g_2) + \frac{1}{3}\frac{(g_2 - g_1)}{g_2} {}_2F_1\left(1, \frac{3}{2}; \frac{5}{2}; \frac{g_2 - g_1}{g_2}\right)\right. \\ &\left. + \frac{1}{3}\frac{(g_1 - g_2)}{g_1} {}_2F_1\left(1, \frac{3}{2}; \frac{5}{2}; \frac{g_1 - g_2}{g_1}\right)\right] \\ &+ \left(\frac{\beta_1}{\beta_2}\right)^{\alpha_2}\frac{\Gamma\left(\gamma_1 + \frac{\alpha_2}{\alpha_1}\right)}{2\Gamma(\gamma_1)}\left[g_2^{\alpha_2/2} {}_2F_1\left(-\frac{\alpha_2}{2}, \frac{1}{2}; \frac{3}{2}; \frac{g_2 - g_1}{g_2}\right)\right. \\ &\left. + g_1^{\alpha_2/2} {}_2F_1\left(-\frac{\alpha_2}{2}, \frac{1}{2}; \frac{3}{2}; \frac{g_1 - g_2}{g_1}\right)\right], \quad (9) \end{aligned}$$

where  ${}_2F_1$  is the Gauss hypergeometric function and  $\Psi$  the digamma function.  $g_i = \frac{1}{\lambda_i}$  where  $\lambda_i$  are the eigenvalues of  $\Sigma_1^{-1}\Sigma_2$ , and  $\lambda_1 \geq \lambda_2$ . This expression of the KLD for the BGFU distribution generalizes the KLD found for the



**Fig. 1.** Modeling of the scalar random variable  $u$  for the first scale and orientation of the grayscale *Bark6* image.

BGGD [6], bivariate Laplace and bivariate Gaussian distributions when we replace the value of  $\alpha$ ,  $\beta$  and  $\gamma$  by their corresponding expressions given in Section 2.

### 3.2. Indexing results on the VisTex and Outex databases

As an analytical expression of the KLD exists for the BGFU distribution, this new model is of potential interest for modeling the chrominance wavelet coefficients. An indexing experiment is hence conducted on the VisTex and Outex databases. For the VisTex database, 40 texture classes from the MIT Vision Texture database have been selected. From each of these texture images of size  $512 \times 512$  pixels, 16 subimages ( $128 \times 128$ ) are created. Hence, the VisTex dataset contains 640 texture images [1]. For the Outex database, the 319 color images in BMP format with 600dpi under inca lightning conditions from the Outex website have been used [15]. Those images were then cropped to  $128 \times 128$  pixels starting from the top-left hand corner, hence creating a dataset of 5109 images. In this experiment, the orthogonal wavelet decomposition (with 1 scale) with Daubechies' filter db4 is used for the wavelet decomposition.

To evaluate the potential of the BGFU distribution for modeling the chrominance dependency of the wavelet coefficients, the empirical histogram of the scalar random variable  $u = \mathbf{k}^T \hat{\Sigma}^{-1} \mathbf{k}$  has been computed from the first scale and orientation of the grayscale *Bark6* image of the VisTex database. This histogram is then modeled by the corresponding distributions of  $u$  (see (3)) for the BGFU, the BGGD, the Laplace and the Gaussian distributions as shown in Fig. 1. As observed in Fig. 1, the BGFU distribution allows a better fit of the scalar random variable  $u$ . As the BGFU contains one more parameter compared to the BGGD, this new model is more flexible and seems promising for the modeling of the

	Gaussian	Laplace	BGGD	BGFU	Gaussian copula Weibull	GG
L	-	-	-	-	-	69.54
Ch	63.86	63.96	69.35	<b>70.77</b>	66.28	-
Ch + L	77.73	79.46	80.48	<b>81.46</b>	78.48	-
RGB	79.49	-	-	-	79.88	-
L	-	-	-	-	-	21.79
Ch	22.06	21.82	22.19	<b>25.21</b>	23.39	-
Ch + L	30.70	30.76	29.80	<b>32.39</b>	31.33	-
RGB	29.84	-	-	-	<b>33.15</b>	-

**Table 1.** Average retrieval rate (in %) on the VisTex (top) and Outex (bottom) databases.

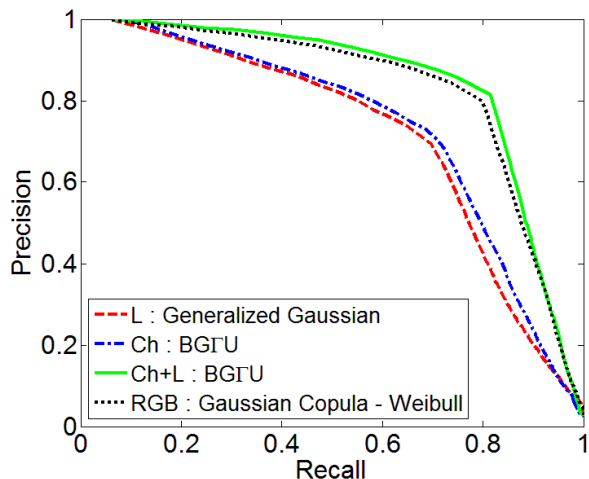
chrominance wavelet coefficients.

A texture retrieval experiment is now conducted on the VisTex database. The symmetric Kullback-Leibler divergence (SKLD) has been considered as similarity measure, *i.e.* half of the double-sided KLD. Moreover, as the subbands are assumed to be independent, the SKLD between two images  $I_1$  and  $I_2$  is obtained by computing the sum of the SKLDs evaluated for each subbands. Table 1 shows the average retrieval rate on the VisTex (top) and Outex (bottom) databases. The rows L and Ch means that an univariate generalized Gaussian (GG) distribution and a bivariate model have respectively been considered for the luminance and chrominance wavelets coefficients. The five proposed bivariate models are the Gaussian, Laplace, BGGD, BGFU distributions and the Gaussian Copula with Weibull distributed margins. The row Ch + L shows the indexing results when combining the chrominance and luminance models. As the chrominance and luminance images can be considered as independent, the SKLD computed between two images  $I_1$  and  $I_2$  can be decomposed as follows:

$$\text{SKLD}(I_1||I_2) = \text{SKLD}(\text{Ch}_1||\text{Ch}_2) + \text{SKLD}(\text{L}_1||\text{L}_2), \quad (10)$$

where  $\text{Ch}_i$  and  $\text{L}_i$  are the chrominance and luminance images of  $I_i$  ( $i = 1, 2$ ). The line RGB corresponds to indexing results with a trivariate model for modeling the RGB wavelet coefficients dependencies. Note that for the trivariate case, only the Gaussian distribution and the Gaussian copula with Weibull distributed margins have been considered since their SKLD admits an analytical expression [16].

Note that here, indexing results on the Outex database seems to be very low, but this dataset is composed of very close texture classes (*e.g.* 11 classes of barleyrice). As shown in Table 1, the proposed BGFU distribution seems promising for modeling the chrominance wavelet coefficients. A gain of 1% and 3% are respectively observed on the VisTex and Outex databases compared to the BGGD. Moreover, for both databases, combining the chrominance and luminance models (Ch+L) yields to a raise of the retrieval rate of about 10% compared to a separate indexing (Ch or L). For the VisTex database, the combination of the BGFU model for the chrominance and the univariate generalized Gaussian model for the luminance channel yields to the best indexing results.



**Fig. 2.** Average retrieval rate as a function of the number of retrieved images for the VisTex database.

A gain of 2% is observed compared to a trivariate model applied directly on the RGB channels. Concerning the Outex, this gain is confirmed with respect to the trivariate Gaussian. If compared with the trivariate Gaussian copula, the indexing performance are approximately equivalent (32.39 vs. 33.15). Note also that the combination of chrominance and luminance models (Ch+L) allows parallelization of the code since the chrominance and luminance channels are treated independently.

Fig. 2 draws the recall/precision computed on the VisTex database for the four best models in each category (L, Ch, Ch+L and RGB). In a retrieval context, the best model is the one which has the highest recall and precision values. This graph confirms that the best indexing results are obtained with a combination of the proposed BGFU distribution (solid green line) for modeling chrominance wavelet coefficients and the univariate generalized Gamma distribution for luminance wavelet coefficients.

#### 4. CONCLUSION

In this paper, a new stochastic model has been introduced *i.e.* the multivariate generalized Gamma times an Uniform (MGFU) distribution. The multivariate Gaussian, Laplace and generalized Gaussian distribution (MGGD) can be considered as a special case. For bivariate data, a closed-form of the Kullback-Leibler divergence (KLD) has been established. The MGFU distribution has hence be proposed for modeling chrominance wavelet coefficients, and implemented in a texture retrieval context. Some experiments on the VisTex and Outex databases have been conducted and revealed that the proposed MGFU model allows a better retrieval rate compared to other classical approaches (MGGD, Copula based models).

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