ICAR, a tool for Blind Source Separation using Fourth Order Statistics only

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***Abstract*— The problem of blind separation of overdetermined mixtures of sources, that is, with fewer sources than (or as many sources as) sensors, is addressed in this paper. A new method, named ICAR (Independent Component Analysis using Redundancies in the quadricovariance), is proposed in order to process complex data. This method, without any whitening operation, only exploits some redundancies of a particular quadricovariance matrix of the data. Computer simulations demonstrate that ICAR offers in general good results and even outperforms classical methods in several situations: ICAR (i) succeeds in separating sources with low signal to noise ratios, (ii) does not require sources with different SO or/and FO spectral densities, (iii) is asymptotically not affected by the presence of a Gaussian noise with unknown spatial correlation, (iv) is not sensitive to an over estimation of the number of sources.**

***Index Terms*— Independent Component Analysis, Blind Source Separation, Overdetermined Mixtures, Fourth Order Statistics.**

I. Introduction

INDEPENDENT Component Analysis (ICA) plays an important role in various application areas, including radiocommunications, radar, sonar, seismology, radio astronomy, data analysis, speech and medical diagnosis [4] [20]. In digital radiocommunications contexts for instance, if some sources are received by an array of sensors, and if the channel delay spread associated with the different sensors is significantly smaller than the symbol durations for each source, a *static* mixture of complex sources is observed on the sensors. On the other hand in Electrocardiography (ECG), it is possible to record the electrical activity of a fetal heart from ECG-recordings measured on the mother's skin. These *cutaneous* recordings can also be considered, in a first approximation, as instantaneous linear mixtures of potential signals generated by underlying bioelectric phenomena [20], hence again the *static* model considered.

The goal of Blind Source Separation (BSS) is to restore transmitted sources from the sole observation of sensor data. In some applications however, sources are not sought, and it is sufficient to identify the (static) mixture. Direction Of Arrival (DOA) estimation problems belong to this class [37], since the column vectors of the mixture contain all the information necessary to determine the location of transmitters. The column vectors of the mixture are the so-called source steering vectors. It is thus legitimate to distinguish between *blind identification* of source mixtures and *blind extraction* of sources; we shall go back to this in section II.

Some algorithms utilize Second Order statistics (SO), as classically Principal Component Analysis (PCA) in Factor Analysis. In contrast, ICA attempts to restore the independence of outputs using higher order statistics. The consequence is that the indeterminacy is reduced, so that ICA allows to blindly identify the static mixture, and transmitted sources can eventually be extracted. More precisely, the ICA concept relies on the core assumption that (i) sources should be independent in some way. Additionally, when a contrast functional is sought to be maximized, (ii) the mixture has to be *overdetermined*, which means that there should be at most as many sources as sensors [40]. In fact, there must exist a linear source separator [15] in the latter framework.

On the other hand, the more general case where there may be more sources than sensors is often referred to as Blind Identification of *underdetermined* mixtures, and is not considered in this paper but is addressed elsewhere; see [7] [19] [22] [35] [25] [3] [17] and references therein.

Since the first paper related to Higher Order (HO) BSS, published in 1985 [30], many concepts and algorithms have come out. For instance, the ICA concept was proposed a few years later, as well as the maximization of a Fourth Order (FO) contrast criterion (subsequently referred to as COM2) [15]. At the same time, a matrix approach was developed in [8] and gave rise to the joint diagonalization algorithm (JADE). A few years later, Hyvarinen et alterae developed the FastICA method, first for signals with values in the real field [31], and later for complex signals [6], using the fixed-point algorithm to maximize a FO contrast. This algorithm is of deflation type, as that of Delfosse et alterae [21], and must extract one source at a time, although some versions of FastICA extract all sources simultaneously. In addition, Comon proposed a simple solution [16], named COM1 in this paper, to the maximization of another FO contrast function previously published in [33] [18] [36]. Another algorithm of interest is SOBI, based only on SO statistics, developed independently by several authors in the nineties, and addressed in depth later in [5].

Each of these methods suffers from limitations. To start with, the SOBI algorithm is unable to restore components that have similar spectral densities. Moreover, the JADE method is very sensitive to an over estimation of the number of

sources as shown in the simulation section of this paper and in [2]. Note that in electronic war fare contexts, the number of sources needs to be estimated and may be over estimated, especially for low signal to noise ratios [38] [41]. On the other hand, though the previous methods [15] [16] [8] [31] [6] perform under some reasonable assumptions, they may be strongly affected by a Gaussian noise with unknown spatial correlation as shown in the simulation section of this paper. Such a noise appears for instance in some radiocommunications applications. It is in particular the case for ionospheric radiocommunications in the HF band where the external noise, composed of multiple emitting sources (industrial noise, atmospheric noise...), is much stronger than the thermal noise generated by the receivers. In order to deal with the correlated noise problem, Ferréol et alterae [26] and Abed-Meraim et alterae [1] have proposed a new family of BSS methods respectively exploiting the potential cyclostationarity of the received sources. In fact, the latter family of algorithms uses cyclic statistics of the data. Note that a cyclic covariance matrix associated with a stationary noise is null for non zero cyclic frequencies. Consequently, these cyclic methods allow the optimal separation of independent cyclostationary sources even in the presence of a stationary noise with unknown spatial correlation. However, the use of cyclic methods is more complex because of the estimation of cyclic frequencies and time delays. To overcome this drawback, Ferréol et alterae have recently introduced the FOBIUM algorithm [25], which, without SO whitening step, performs the blind source separation even in the presence of a Gaussian noise with unknown spatial correlation. Nevertheless, since FOBIUM is an extension of the SOBI method to FO statistics, it requires sources with different FO spectral densities. FOBIUM also allows to address the underdetermined case, but this is out of the scope of the present paper.

In order to overcome the limitations of the previous algorithms, the method named ICAR (Independent Component Analysis using Redundancies in the quadricovariance) shortly presented in [2] is proposed in this paper and addresses the case of complex mixture and sources, in the presence of additive (possibly spatially correlated) Gaussian noise. Only based on FO statistics, ICAR skips the SO whitening step in contrast to classical methods [5] [15] [16] [8] [31] [6], and consequently is asymptotically not affected by the presence of a Gaussian noise with unknown spatial correlation. Actually, ICAR exploits redundancies in a particular FO statistical matrix of the data, called *quadricovariance*. The latter algorithm assumes sources to have non zero FO marginal cumulants with the same sign, assumption that is verified in most radiocommunications contexts. Indeed, the kurtosis of most of radiocommunications signals is negative. For example M-PSK constellations have a kurtosis equal to -2 for $M=2$ and to -1 for $M\geq 4$. Continuous Phase Modulations (CPM), among which we find the GMSK modulation (GSM standard), are such that their kurtosis is smaller than or equal to -1, due to their constant modulus. Furthermore, the performance of ICAR is also analyzed in this paper, in different practical situations through computer simulations, and compared to those of classical algorithms, namely SOBI, COM1, COM2, JADE, FastICA and FOBIUM. It appears that ICAR exhibits good results in most cases even when classical methods fail.

The paper is organized as follows. Section II introduces the BSS problem, and assumptions needed in ICAR. Section III defines the SO and FO statistics considered in the paper, and section IV describes in detail the ICAR concept. Computer results are reported in section V. Section VI eventually concludes.

II. Assumptions and problem formulation

A noisy mixture of P statistically independent Narrow-Band (NB) sources $s_p(k)$ is assumed to be received by an array of N sensors. In accordance with the usual practice [34], only complex envelopes of NB signals are considered. The vector of complex envelopes of the signals at the sensor outputs, $\boldsymbol{x}(k)$, is thus given by

$$\boldsymbol{x}(k) = \boldsymbol{A}\,\boldsymbol{s}(k) + \boldsymbol{\nu}(k) \tag{1}$$

where $\boldsymbol{A}$, $\boldsymbol{s}(k)$, $\boldsymbol{\nu}(k)$ are the $N\times P$ constant mixing matrix, the $P\times 1$ source with components $s_p(k)$ and $N\times 1$ noise random vectors, respectively. In addition, for any fixed index k, $\boldsymbol{s}(k)$ and $\boldsymbol{\nu}(k)$ are statistically independent. We further assume the following hypotheses:

- **A1**) Vector $\boldsymbol{s}(k)$ is stationary, ergodic[1] with components a priori in the complex field and mutually uncorrelated at order 4;
- **A2**) Noise vector $\boldsymbol{\nu}(k)$ is stationary, ergodic and Gaussian with components a priori in the complex field too;
- **A3**) FO marginal source cumulants, called *kurtosis* (if normalized) and defined in section III-B, are not null and have all the same sign;
- **A4**) The mixture matrix $\boldsymbol{A}$ does not contain any null entry;
- **A5**) $\boldsymbol{A}$ is a full column rank matrix.

Note that sources with null kurtosis are tolerated but cannot be seen and processed by method ICAR. Such sources will be considered as noise. Moreover, the second part of (**A3**) will be discussed in section IV-C.1. Assumption (**A4**) is not a strong assumption, in particular in digital radiocommunications contexts, since it is more than just reasonable to assume the array of sensors in good repair. On the other hand, if the n-th sensor is defective, the n-th row of $\boldsymbol{A}$ will be null. It is then necessary to erase the contribution of this sensor and to assume that we have $N-1$ sensor outputs instead of N. As far as the masking phenomenon is concerned, it is more rare and may produce at most one null component in each column of $\boldsymbol{A}$ for arrays with space diversity. Forthcoming works will consist in studying the ICAR robustness with respect to this pathological phenomenon. As far as (**A5**) is concerned, it implies necessarily $P\leq N$. Under the previous assumptions, the problems addressed in the paper are both the blind identification and the blind extraction of the sources using solely the FO statistics of the data. The goal of Blind Mixture Identification (BMI) is to blindly identify the mixing matrix $\boldsymbol{A}$, to within a trivial matrix $\boldsymbol{\mathcal{T}}$; recall that a trivial matrix is of the form $\boldsymbol{\Lambda\Pi}$ where $\boldsymbol{\Lambda}$ is invertible diagonal

[1] the *cyclostationary* and *cycloergodic* case is addressed in section III-D

and $\mathbf{\Pi}$ a permutation. On the other hand, the goal of Blind Source Extraction (BSE) or Separation (BSS) is to blindly find a matrix $\boldsymbol{W}$, yielding a $P\times 1$ output vector $\boldsymbol{y}(k) = \boldsymbol{W}^{\mathsf{H}}\boldsymbol{x}(k)$ corresponding to the best estimate, $\hat{\boldsymbol{s}}(k)$, of the vector $\boldsymbol{s}(k)$, up to a multiplicative trivial matrix. Superscript ($^{\mathsf{H}}$) denotes the complex conjugate transpose of a matrix.

III. SO and FO data statistics

A. SO statistics

The SO statistics considered in the paper are given by

$$C^{i_2}_{i_1,\boldsymbol{x}}(k) = \mathsf{Cum}\{x_{i_1}(k), x_{i_2}(k)^*\} \tag{2}$$

Function (2) is well-known as the SO *cumulant* of $\boldsymbol{x}(k)$. Consequently, the SO *marginal cumulant* of source $s_p(k)$ is defined by

$$C^{p}_{p,\boldsymbol{s}}(k) = \mathsf{Cum}\{s_p(k), s_p(k)^*\} \tag{3}$$

B. FO statistics

The FO statistics considered in the paper are given by

$$C^{i_3,\,i_4}_{i_1,\,i_2,\,\boldsymbol{x}}(k) = \mathsf{Cum}\{x_{i_1}(k), x_{i_2}(k), x_{i_3}(k)^*, x_{i_4}(k)^*\} \tag{4}$$

where two terms $x_i(k)$ are not conjugate and two terms are conjugate. Function (4) is well-known as the FO *cumulant* of $\boldsymbol{x}(k)$. Consequently, the FO *marginal cumulant* of source $s_p(k)$ is defined by

$$C^{p,\,p}_{p,\,p,\,\boldsymbol{s}}(k) = \mathsf{Cum}\{s_p(k),\ s_p(k),\ s_p(k)^*,\ s_p(k)^*\} \tag{5}$$

Likewise, the *kurtosis* of source $s_p(k)$ is given by

$$\kappa^{p,p}_{p,p,\boldsymbol{s}}(k) = C^{p,\,p}_{p,\,p,\,\boldsymbol{s}}(k)/(\gamma_p(0))^2 \tag{6}$$

where $\gamma_p(0)$ is the variance of source $s_p(k)$. Note that in the presence of stationary sources, SO (2) and FO (4) statistics do not depend on time k, so that they can be denoted by $C^{i_2}_{i_1,\,\boldsymbol{x}}$ and $C^{i_3,\,i_4}_{i_1,\,i_2,\,\boldsymbol{x}}$, respectively.

C. Matrix arrangement

1) SO and FO statistical matrices: SO and FO statistics computed according to (2) and (4) may be arranged in two Hermitian statistical matrices, $\boldsymbol{R}_{\boldsymbol{x}} = \boldsymbol{\mathcal{C}}_{2,\,\boldsymbol{x}}$ and $\boldsymbol{Q}_{\boldsymbol{x}} = \boldsymbol{\mathcal{C}}_{4,\,\boldsymbol{x}}$, of size $N\times N$ and $N^2\times N^2$, respectively. These matrices are called the *covariance* and the *quadricovariance* of $\boldsymbol{x}(k)$, respectively. We limit ourselves to arrangements of SO and FO statistics that give different results, in terms of maximum number of processed sources at the output of the BSS methods. The impact of the chosen way to arrange statistics in a matrix is analyzed in [12]. It is shown in [12], through extensions of the Virtual Array concept initially introduced in [23] and [14] for the FO data statistics, that there exists an optimal arrangement of the FO cumulants in a quadricovariance matrix with respect to the maximal number of statistically independent sources to be processed by a method exploiting the algebraic structure of this quadricovariance. As far as SO statistics are concerned, there is a unique non redundant way to store them in a matrix $\boldsymbol{R}_{\boldsymbol{x}}$ under constraints of hermicity. Consider indeed the following arrangement

$$R_{\boldsymbol{x}}(i_1, i_2) = C^{i_2}_{i_1,\,\boldsymbol{x}} \tag{7}$$

where $R_{\boldsymbol{x}}(i_1, i_2)$ is the (i_1, i_2)-th entry of matrix $\boldsymbol{R}_{\boldsymbol{x}}$; the other possible arrangement $R'_{\boldsymbol{x}}(i_2, i_1) = C^{i_2}_{i_1,\,\boldsymbol{x}}$ just leads to $\boldsymbol{R}_{\boldsymbol{x}}^{\mathsf{H}}$ and hence to the same result in terms of maximum number of processed sources. On the other hand, there are two distinct non redundant ways associated with FO statistics under constraints of hermicity, which can be indexed by the integer ℓ ($\ell\in\{0,1\}$). Each way yields a statistical matrix $\boldsymbol{\mathcal{C}}^{\ell}_{4,\,\boldsymbol{x}}$ such that its $\left(I^{\ell}_1, I^{\ell}_2\right)$-th entry ($1\le I^{\ell}_1, I^{\ell}_2\le N^2$) is given by

$$\mathcal{C}^{\ell}_{4,\,\boldsymbol{x}}\left(I^{\ell}_1, I^{\ell}_2\right) = C^{i_3,\,i_4}_{i_1,\,i_2,\,\boldsymbol{x}} \tag{8}$$

where for any ℓ belonging to $\{0,1\}$ and for all i_1, i_2, i_3, i_4 ($1\le i_1, i_2, i_3, i_4\le N$),

$$I^{\ell}_1 = \begin{cases} i_2 + N\,(i_1 - 1) & \text{if } \ell = 0 \\ i_4 + N\,(i_1 - 1) & \text{if } \ell = 1 \end{cases} \tag{9}$$

and

$$I^{\ell}_2 = \begin{cases} i_4 + N\,(i_3 - 1) & \text{if } \ell = 0 \\ i_2 + N\,(i_3 - 1) & \text{if } \ell = 1 \end{cases} \tag{10}$$

Note that the optimal arrangement is shown in [12] to correspond to ℓ=1 and for this reason, we consider this arrangement in the following sections. So matrices $\boldsymbol{\mathcal{C}}^{1}_{4,\,\boldsymbol{s}}$, $\boldsymbol{\mathcal{C}}^{1}_{4,\,\boldsymbol{x}}$ will be denoted by $\boldsymbol{Q}_{\boldsymbol{s}}$ and $\boldsymbol{Q}_{\boldsymbol{x}}$ respectively.

Remark 1 *Another way, perhaps more intuitive (especially for readers familiar with Matlab), to present the construction of $\boldsymbol{Q}_{\boldsymbol{x}}$ is the following: first, construct a 4-dimensional tensor $\boldsymbol{T}$, whose elements are given by*

$$\boldsymbol{T}(i_4, i_1, i_2, i_3) = C^{i_3,\,i_4}_{i_1,\,i_2,\,\boldsymbol{x}}$$

The matrix $\boldsymbol{Q}_{\boldsymbol{x}}$ is then given by a simple Matlab reshape operation as follows

$$\boldsymbol{Q}_{\boldsymbol{x}} = \mathit{reshape}(\boldsymbol{T}, N^2, N^2)$$

2) Multilinearity property: The SO and FO statistical matrices of the data, $\boldsymbol{R}_{\boldsymbol{x}}$ and $\boldsymbol{Q}_{\boldsymbol{x}}$, have a special structure, due to the multilinearity property under change of coordinate systems, which is enjoyed by all moments and cumulants. Since sources and noise are independent, this property can be expressed, for SO statistical matrices and according to (7), by

$$\boldsymbol{R}_{\boldsymbol{x}} = \boldsymbol{A}\,\boldsymbol{R}_{\boldsymbol{s}}\,\boldsymbol{A}^{\mathsf{H}} + \boldsymbol{R}_{\boldsymbol{\nu}} \tag{11}$$

Similarly, according to (8), (9) and (10), and since noise is Gaussian and independent of sources, the FO cumulant matrix can be expressed as follows, using the multilinearity property associated with $\ell = 1$:

$$\boldsymbol{Q}_{\boldsymbol{x}} = [\boldsymbol{A}\otimes\boldsymbol{A}^*]\,\boldsymbol{Q}_{\boldsymbol{s}}\,[\boldsymbol{A}\otimes\boldsymbol{A}^*]^{\mathsf{H}} \tag{12}$$

The $P\times P$ matrix $\boldsymbol{R}_{\boldsymbol{s}}$ and the $P^2\times P^2$ matrix $\boldsymbol{Q}_{\boldsymbol{s}}$ are the SO and FO statistical matrices of $\boldsymbol{s}(k)$ respectively. $\boldsymbol{R}_{\boldsymbol{\nu}}$ denotes the $N\times N$ SO statistical matrix of $\boldsymbol{\nu}(k)$.

D. Statistical estimation

In practical situations, SO and FO statistics have to be estimated from components of $\boldsymbol{x}(k)$. If components are stationary and ergodic, sample statistics may be used to estimate (2) and (4). Nevertheless, if sources are cyclostationary, cycloergodic,

potentially non zero-mean, SO and FO continuous time average statistics have to be used instead of (2) and (4), such as

$$C^{i_2}_{i_1,\boldsymbol{x}} = \left\langle C^{i_2}_{i_1,\boldsymbol{x}}(k)\right\rangle_{\rm c} \tag{13}$$

and

$$C^{i_3,\,i_4}_{i_1,\,i_2,\,\boldsymbol{x}} = \left\langle C^{i_3,\,i_4}_{i_1,\,i_2,\,\boldsymbol{x}}(k)\right\rangle_{\rm c} \tag{14}$$

where $\langle\cdot\rangle_{\rm c}$ is the continuous time average operation defined by

$$\forall\, f,\ f{:}\,t\longmapsto f(t),\quad \langle f(t)\rangle_{\rm c} = \lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{T/2} f(t)dt \tag{15}$$

These continuous-time temporal mean statistics need some knowledge on cyclic frequencies of the received signal and are thus computed using, for instance, the unbiased and consistent estimators described in [27], [29] and [28]. Moreover, ordering these continuous-time temporal mean statistics in matrices $\boldsymbol{R_x}$ and $\boldsymbol{Q_x}$ by means of (7), (8), (9) and (10) respectively, expressions (11) and (12) remain valid.

IV. The ICAR method

We present in this section a new method of BSS named ICAR, which exploits the algebraic structure of an alternative expression of matrix $\boldsymbol{Q_x}$.

A. Matrix notation

Define a columnwise Kronecker product, denoted $\oslash$ and referred to as the Khatri-Rao product [24] [39]. For any $N{\times}P$ rectangular matrices $\boldsymbol{G}$ and $\boldsymbol{H}$, the columns of the $N^2{\times}P$ matrix $\boldsymbol{G}\oslash\boldsymbol{H}$ are defined as $\boldsymbol{g}_j\otimes\boldsymbol{h}_j$, where $\otimes$ denotes the usual Kronecker product, if $\boldsymbol{g}_j$ and $\boldsymbol{h}_j$ denote the columns of $\boldsymbol{G}$ and $\boldsymbol{H}$ respectively.

B. The core equation

The ICAR method exploits several redundancies present in the quadricovariance matrix of the data, $\boldsymbol{Q_x}$. Although most BSS algorithms, such as JADE, exploit expression (12), the ICAR method uses an alternative form, described by

$$\boldsymbol{Q_x} = [\boldsymbol{A}\oslash\boldsymbol{A}^*]\,\boldsymbol{\zeta_s}\,[\boldsymbol{A}\oslash\boldsymbol{A}^*]^{\rm H} \tag{16}$$

where the $P{\times}P$ diagonal matrix $\boldsymbol{\zeta_s} = \mathrm{Diag}\Big[C^{1,1}_{1,1,\boldsymbol{s}}, C^{2,2}_{2,2,\boldsymbol{s}}, \cdots, C^{P,P}_{P,P,\boldsymbol{s}}\Big]$ (i.e. $\forall\ (p_1,p_2)$, $1\le p_1,p_2\le P$, $\boldsymbol{\zeta_s}(p_1,p_2){=}C^{p_1,p_1}_{p_1,p_1,\boldsymbol{s}}$ if $p_1{=}p_2$, 0 otherwise) is full rank, in contrast to $\boldsymbol{Q_s}$ in (12), and where the $N^2{\times}P$ matrix $\boldsymbol{A}\oslash\boldsymbol{A}^*$ is defined by

$$\boldsymbol{A}\oslash\boldsymbol{A}^* = [\,\boldsymbol{a}_1\otimes\boldsymbol{a}_1^*\ \ \boldsymbol{a}_2\otimes\boldsymbol{a}_2^*\ \ \cdots\ \ \boldsymbol{a}_P\otimes\boldsymbol{a}_P^*\,] \tag{17}$$

and can be written as

$$\boldsymbol{A}\oslash\boldsymbol{A}^* = [\ [\boldsymbol{A}^*\boldsymbol{\Phi}_1]^{\rm T}\,[\boldsymbol{A}^*\boldsymbol{\Phi}_2]^{\rm T}\ \cdots\ [\boldsymbol{A}^*\boldsymbol{\Phi}_N]^{\rm T}\ ]^{\rm T} \tag{18}$$

with

$$\boldsymbol{\Phi}_n = \mathrm{Diag}[\,A(n,1),\,A(n,2),\,\cdots,\,A(n,P)\,] \tag{19}$$

In other words, the non zero elements of the $P{\times}P$ diagonal matrix $\boldsymbol{\Phi}_n$ are the components of the n-th row of matrix $\boldsymbol{A}$. In addition, note that equation (16) can be easily derived from equation (12). Indeed, the latter equation straight implies, in view of the structure of the diagonal non inversible matrix $\boldsymbol{Q_s}$, that the only column vectors of matrix $\boldsymbol{A}\otimes\boldsymbol{A}^*$ which generate matrix $\boldsymbol{Q_x}$ are $\boldsymbol{a}_p\otimes\boldsymbol{a}_p^*$ $(1\le p\le P)$, hence result (16).

C. The ICAR concept

The algorithm proposed proceeds in three stages. Firstly, a unitary matrix $\boldsymbol{V}$ is estimated in the Least Square (LS) sense, and allows the estimation of $\boldsymbol{A}\oslash\boldsymbol{A}^*$ from $\boldsymbol{Q_x}$ (16). In a second stage, several algorithms may be thought of in order to compute an estimate of $\boldsymbol{A}$ from $\boldsymbol{A}\oslash\boldsymbol{A}^*$. Finally, an estimation of sources $\boldsymbol{s}(k)$ is computed using the estimate of $\boldsymbol{A}$.

1) Identification of $\boldsymbol{A}\oslash\boldsymbol{A}^*$: Matrix $\boldsymbol{A}\oslash\boldsymbol{A}^*$ is an unobservable square root of $\boldsymbol{Q_x}$ to within a diagonal matrix, as shown by (16). In this context, the idea is to built an observable square root, $\boldsymbol{Q_x}^{1/2}$, of $\boldsymbol{Q_x}$, differing from $[\boldsymbol{A}\oslash\boldsymbol{A}^*]\,\boldsymbol{\zeta_s}^{1/2}$ only by a unitary matrix $\boldsymbol{V}$ and then to identify the latter from the exploitation of the algebraic structure of $\boldsymbol{Q_x}$. So consider the following proposition:

Proposition 1 *If $\boldsymbol{A}$ is of full column rank (**A5**) and contains no null entries (**A4**), then the $N^2{\times}P$ matrix $\boldsymbol{A}\oslash\boldsymbol{A}^*$ is full column rank.*

The proof is given in appendix I. So proposition 1 and assumption (**A3**) allow together to prove that matrix $\boldsymbol{Q_x}$, given by (16), is of rank P. Moreover, assumption (**A3**) and equation (16) imply that $\boldsymbol{Q_x}$ is positive if the FO marginal source cumulants are positive, which we assume in the following. Thus, a square root of $\boldsymbol{Q_x}$, denoted $\boldsymbol{Q_x}^{1/2}$, and defined such that $\boldsymbol{Q_x}{=}\boldsymbol{Q_x}^{1/2}[\boldsymbol{Q_x}^{1/2}]^{\rm H}$, may be computed. If the FO marginal source cumulants are negative, matrix $-\boldsymbol{Q_x}$ can be considered instead for computing the square root. In the case where there are terms with a different sign, our derivation can be reformulated in terms of an unknown $\boldsymbol{J}$-unitary matrix[2] $\boldsymbol{V}$, instead of unitary. Then we deduce from (16) that matrix $[\boldsymbol{A}\oslash\boldsymbol{A}^*]\,\boldsymbol{\zeta_s}^{1/2}$ is a natural square root of $\boldsymbol{Q_x}$. Yet, another possibility is to compute this square root via the Eigen Value Decomposition (EVD) of $\boldsymbol{Q_x}$ given by

$$\boldsymbol{Q_x} = \boldsymbol{E_s}\,\boldsymbol{L_s}\,\boldsymbol{E_s}^{\rm H} \tag{20}$$

where $\boldsymbol{L_s}$ is the real-valued diagonal matrix of the non zero eigenvalues of $\boldsymbol{Q_x}$. Since matrix $\boldsymbol{Q_x}$ is of rank P, $\boldsymbol{L_s}$ is of size $P{\times}P$. Besides, $\boldsymbol{E_s}$ is the $N^2{\times}P$ matrix of the associated orthonormalized eigenvectors. Consequently, a square root of $\boldsymbol{Q_x}$ can be computed as

$$\boldsymbol{Q_x}^{1/2} = \boldsymbol{E_s}\,\boldsymbol{L_s}^{1/2} \tag{21}$$

where $\boldsymbol{L_s}^{1/2}$ denotes a square root of $\boldsymbol{L_s}$.

Proposition 2 *For a full rank matrix $\boldsymbol{A}\oslash\boldsymbol{A}^*$, source kurtoses are not null and have all the same sign (**A3**) if and only if the diagonal elements of $\boldsymbol{L_s}$ are not null and have also the same sign, corresponding to that of the FO marginal source cumulants.*

The proof is given in appendix II. In addition, equation (34) can be rewritten as

$$\boldsymbol{Q_x}^{1/2} = \boldsymbol{E_s}\,\boldsymbol{L_s}^{1/2} = [\boldsymbol{A}\oslash\boldsymbol{A}^*]\,\boldsymbol{\zeta_s}^{1/2}\,\boldsymbol{V}^{\rm H}, \tag{22}$$

[2]a $\boldsymbol{J}$-unitary matrix $\boldsymbol{V}$ is such that $\boldsymbol{V}\,\boldsymbol{J}\,\boldsymbol{V}^{\rm H} = \boldsymbol{J}$ where $\boldsymbol{J}$ is a sign diagonal matrix

showing the link between $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}$ and $\boldsymbol{A}\odot\boldsymbol{A}^*$. Plugging (18) into (22), matrix $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}$ can be eventually rewritten as

$$\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}=\left[[\boldsymbol{A}^*\boldsymbol{\Phi}_1\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}\boldsymbol{V}^{\mathsf{H}}]^{\mathsf{T}}\ldots[\boldsymbol{A}^*\boldsymbol{\Phi}_N\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}\boldsymbol{V}^{\mathsf{H}}]^{\mathsf{T}}\right]^{\mathsf{T}}=[\boldsymbol{\Gamma}_1{}^{\mathsf{T}}\ldots\boldsymbol{\Gamma}_N{}^{\mathsf{T}}]^{\mathsf{T}} \quad (23)$$

where the N matrix blocks $\boldsymbol{\Gamma}_n$ of size $N\times P$ are given by

$$\forall\, n,\ 1\le n\le N,\qquad \boldsymbol{\Gamma}_n=\boldsymbol{A}^*\boldsymbol{\Phi}_n\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}\boldsymbol{V}^{\mathsf{H}} \quad (24)$$

Proposition 3 *For any n ($1\le n\le N$), matrix $\boldsymbol{\Gamma}_n$ is full column rank.*

The proof is given in appendix III. Using proposition 3, the pseudo-inverse $\boldsymbol{\Gamma}_n^\sharp$ of the $N\times P$ matrix $\boldsymbol{\Gamma}_n$ is defined by

$$\forall\, n,\ 1\le n\le N,\qquad \boldsymbol{\Gamma}_n^\sharp=(\boldsymbol{\Gamma}_n{}^{\mathsf{H}}\boldsymbol{\Gamma}_n)^{-1}\boldsymbol{\Gamma}_n^{\mathsf{H}} \quad (25)$$

Then, consider the $N(N-1)$ matrices $\boldsymbol{\Theta}_{n_1,n_2}$ below

$$\forall\ (n_1,n_2),\ 1\le n_1\ne n_2\le N,\qquad \boldsymbol{\Theta}_{n_1,n_2}=\boldsymbol{\Gamma}_{n_1}^\sharp\boldsymbol{\Gamma}_{n_2} \quad (26)$$

which can be rewritten, from (24) and (25), as

$$\begin{aligned}\boldsymbol{\Theta}_{n_1,n_2} &= \boldsymbol{V}\,\boldsymbol{\zeta}_{\boldsymbol{s}}^{-1/2}\,\boldsymbol{\Phi}_{n_1}^{-1}\,\boldsymbol{\Phi}_{n_2}\,\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}\,\boldsymbol{V}^{\mathsf{H}} \quad (27)\\ &= \boldsymbol{V}\,\boldsymbol{\Phi}_{n_1}^{-1}\,\boldsymbol{\zeta}_{\boldsymbol{s}}^{-1/2}\,\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}\,\boldsymbol{\Phi}_{n_2}\,\boldsymbol{V}^{\mathsf{H}}=\boldsymbol{V}\,\boldsymbol{\Phi}_{n_1}^{-1}\,\boldsymbol{\Phi}_{n_2}\,\boldsymbol{V}^{\mathsf{H}}\end{aligned}$$

where $\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}$, $\boldsymbol{\Phi}_{n_1}$, $\boldsymbol{\Phi}_{n_2}$ and $\boldsymbol{D}_{n_1,n_2}=\boldsymbol{\Phi}_{n_1}^{-1}\boldsymbol{\Phi}_{n_2}$, are $P\times P$ diagonal full rank matrices (the full rank character of matrices $\boldsymbol{\Phi}_{n_1}$, $\boldsymbol{\Phi}_{n_2}$ and $\boldsymbol{D}_{n_1,n_2}$ is due to assumption (**A4**)). It appears from (27) that matrix $\boldsymbol{V}$ jointly diagonalizes the $N(N-1)$ matrices $\boldsymbol{\Theta}_{n_1,n_2}$.

Proposition 4 *If $\boldsymbol{A}$ is of full column rank (**A5**) and contains no null entries (**A4**), then, for all pairs (p_1,p_2), $1\le p_1\ne p_2\le P$, at least one pair (n_1,n_2), $1\le n_1\ne n_2\le N$ exists such that $D_{n_1,n_2}(p_1,p_1)\ne D_{n_1,n_2}(p_2,p_2)$.*

The proof is given in appendix IV. Under proposition 4, paper [5] allows to assert that if $\boldsymbol{V}_{sol}$ jointly diagonalizes matrices $\boldsymbol{\Theta}_{n_1,n_2}$, then $\boldsymbol{V}_{sol}$ and $\boldsymbol{V}$ are related through $\boldsymbol{V}_{sol}=\boldsymbol{V}\boldsymbol{\mathcal{T}}$ where $\boldsymbol{\mathcal{T}}$ is a trivial unitary matrix. So matrix $\boldsymbol{V}_{sol}$ allows, in accordance with (22), to recover $\boldsymbol{A}\odot\boldsymbol{A}^*$ up to a multiplicative trivial matrix:

$$\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}\,\boldsymbol{V}_{sol}=[\boldsymbol{A}\odot\boldsymbol{A}^*]\,\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}\,\boldsymbol{\mathcal{T}} \quad (28)$$

2) Identification of mixture $\boldsymbol{A}$: Three algorithms are proposed in this section to identify $\boldsymbol{A}$ from the estimate, $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}\boldsymbol{V}_{sol}$, of $\boldsymbol{A}\odot\boldsymbol{A}^*$. These algorithms optimize differently the compromise between performance and complexity.

Note that equation (28) can be rewritten from (18) in the form of N matrix blocks $\boldsymbol{\Sigma}_n=\boldsymbol{A}^*\boldsymbol{\Phi}_n\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}\boldsymbol{\mathcal{T}}$ of size $N\times P$ as

$$\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}\,\boldsymbol{V}_{sol}=[\boldsymbol{\Sigma}_1{}^{\mathsf{T}}\ \boldsymbol{\Sigma}_2{}^{\mathsf{T}}\cdots\boldsymbol{\Sigma}_N{}^{\mathsf{T}}]^{\mathsf{T}} \quad (29)$$

So a first approach to estimate $\boldsymbol{A}$ up to a trivial matrix, called ICAR1 in the sequel, consists of merely keeping the matrix block $\boldsymbol{\Sigma}_1^*$ made up of the N first rows of $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}\boldsymbol{V}_{sol}$ such that

$$\boldsymbol{\Sigma}_1=\boldsymbol{A}^*\,\boldsymbol{\Phi}_1\,\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}\,\boldsymbol{\mathcal{T}} \quad (30)$$

where $\boldsymbol{\Phi}_1$ and $\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}$ are diagonal matrices, and where $\boldsymbol{\mathcal{T}}$ is a unitary trivial matrix.

It is also possible to take into account all the matrix blocks $\boldsymbol{\Sigma}_n^*$ and to compute their average. This yields a second algorithm, named ICAR2, of higher complexity.

A third algorithm, called ICAR3, is now described, and yields a more accurate solution to the BMI problem: since matrix $\boldsymbol{A}\odot\boldsymbol{A}^*$, given by (17), has been identified from the previous section by $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}\boldsymbol{V}_{sol}$ to within a trivial matrix, ICAR3 consists first of mapping each $N^2\times 1$ column vector $\boldsymbol{b}_p$ of $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}\boldsymbol{V}_{sol}$ into a $N\times N$ matrix $\boldsymbol{B}_p$ (the n-th column of $\boldsymbol{B}_p$ is made up from the N consecutive entries of vector $\boldsymbol{b}_p$, between $[N(n-1)+1]$ and Nn), and secondly of diagonalizing each matrix $\boldsymbol{B}_p^*$.

Proposition 5 *For any matrix $\boldsymbol{B}_p$ ($1\le p\le P$) built from $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}\boldsymbol{V}_{sol}$, there exists a unique column vector $\boldsymbol{a}_q$ ($1\le q\le P$) of $\boldsymbol{A}$ such that the eigenvector of $\boldsymbol{B}_p^*$ associated with the largest eigenvalue corresponds, up to a scale factor, to $\boldsymbol{a}_q$.*

The proof is given in appendix V. In addition, the indeterminacy of the norms of columns of $\boldsymbol{A}$ is related to matrices $\boldsymbol{\Phi}_p$, $\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}$, a unitary diagonal matrix (whose product by a permutation matrix gives $\boldsymbol{\mathcal{T}}$) and the way to identify $\boldsymbol{A}$ from matrix $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}\,\boldsymbol{V}_{sol}$. As far as the permutation indeterminacy is concerned, it is related to matrix $\boldsymbol{\mathcal{T}}$.

3) Extraction of the P independent components: Finally, to estimate the signal vector $\boldsymbol{s}(k)$ for any value k, it is sufficient, under (**A5**), to apply a linear filter built from the identified matrix $\boldsymbol{A}$: such a filter may be the Spatial Matched Filter (SMF) given by [11] $\boldsymbol{W}=\boldsymbol{R}_{\boldsymbol{x}}^{-1}\boldsymbol{A}$, which is optimal in the presence of decorrelated signals. In practical situations, since matrix $\boldsymbol{A}$ is estimated up to a trivial matrix according to section (IV-C.2), neither the order of sources $\boldsymbol{s}(k)$ nor their amplitude can be identified.

D. Implementation of the ICAR methods

The different steps of the ICAR method are summarized hereafter when K samples of the observations, $\boldsymbol{x}(k)$ ($1\le k\le K$), are available.

Step1 Compute an estimate of FO statistics $C_{i_1,i_2,\boldsymbol{x}}^{i_3,i_4}$ from the K samples $\boldsymbol{x}(k)$ and store them, using the $(\ell=1)$-arrangement, into matrix $\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}$, which is an estimate of $\boldsymbol{Q}_{\boldsymbol{x}}$.

Step2 Compute the EVD of the Hermitian matrix $\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}$, estimate $\widehat{P}$, the number of sources, from this EVD. Restrict $\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}$ to the $\widehat{P}$ principal components : $\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}=\widehat{\boldsymbol{E}}_{\boldsymbol{s}}\,\widehat{\boldsymbol{L}}_{\boldsymbol{s}}\,\widehat{\boldsymbol{E}}_{\boldsymbol{s}}^{\mathsf{H}}$, where $\widehat{\boldsymbol{L}}_{\boldsymbol{s}}$ is the diagonal matrix of the $\widehat{P}$ eigenvalues of largest modulus and $\widehat{\boldsymbol{E}}_{\boldsymbol{s}}$ is the matrix of the associated eigenvectors.

Step3 Estimate the sign, ϵ, of the diagonal elements of $\widehat{\boldsymbol{L}}_{\boldsymbol{s}}$.

Step4 Compute a square root matrix $[\epsilon\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}]^{1/2}$ of $\epsilon\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}$: $[\epsilon\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}]^{1/2}=\widehat{\boldsymbol{E}}_{\boldsymbol{s}}\,|\widehat{\boldsymbol{L}}_{\boldsymbol{s}}|^{1/2}$, where $|\cdot|$ denotes the absolute value operator.

Step5 Compute from $[\epsilon\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}]^{1/2}$ the N matrices $\widehat{\boldsymbol{\Gamma}}_n$, construct matrices $\widehat{\boldsymbol{\Theta}}_{n_1,n_2}=[\widehat{\boldsymbol{\Gamma}}_{n_1}^{\ \sharp}\,\widehat{\boldsymbol{\Gamma}}_{n_2}]$ for all (n_1,n_2), $1\le n_1\ne n_2\le N$, and compute $\widehat{\boldsymbol{V}}_{sol}$, an estimate of $\boldsymbol{V}_{sol}$, from the joint diagonalization of the $N(N-1)$ matrices $\widehat{\boldsymbol{\Theta}}_{n_1,n_2}$; one possible joint diagonalization algorithm may be found in [9].

Step6 Compute an estimate $\widehat{\boldsymbol{A}}$ of the mixture $\boldsymbol{A}$ from the $N^2\times P$ matrix $[[\epsilon\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}]^{1/2}\,\widehat{\boldsymbol{V}}_{sol}]$ by either one of the following:

1) (ICAR1) taking the matrix block made up of the N first rows of $[[\epsilon\widehat{\boldsymbol{Q}}_{\boldsymbol{x}}]^{1/2}\,\widehat{\boldsymbol{V}}_{sol}]^*$;

2) (ICAR2) taking the average of the N matrix blocks, of size $N \times P$, made up of the successive rows of $[[\epsilon \widehat{\boldsymbol{Q}}_{\boldsymbol{x}}]^{1/2}\, \widehat{\boldsymbol{V}}_{sol}]^*$;
3) (ICAR3) taking each column vector $\widehat{\boldsymbol{b}}_p$ of $[[\epsilon \widehat{\boldsymbol{Q}}_{\boldsymbol{x}}]^{1/2}\, \widehat{\boldsymbol{V}}_{sol}]$ remodeling them into $N \times N$ matrices $\widehat{\boldsymbol{B}}_p$, and building the matrix whose p-th column vector is the eigenvector of matrix $\widehat{\boldsymbol{B}}_p^*$ associated with the largest eigenvalue.

Step7 Estimate the signal vector $\boldsymbol{s}(k)$ for any value k by applying to $\boldsymbol{x}(k)$ a linear filter built from $\widehat{\boldsymbol{A}}$, like for example the SMF defined by $\widehat{\boldsymbol{W}} = \widehat{\boldsymbol{R}}_{\boldsymbol{x}}^{-1} \widehat{\boldsymbol{A}}$.

V. Computer Results

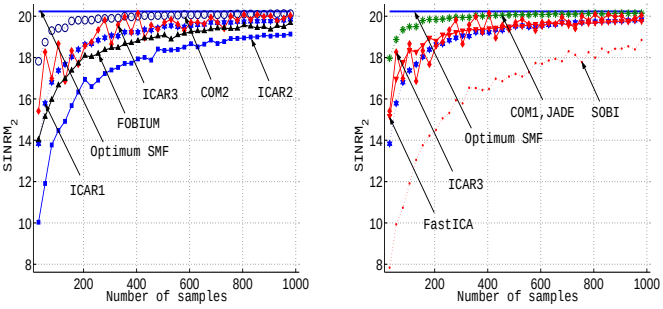
In this section, a comparative performance analysis of seven BSS methods (SOBI, COM1, COM2, JADE, FastICA, FOBIUM and ICAR) in various scenarios is presented. For this purpose, we consider a Uniform Linear Array (ULA) of $N = 4$ sensors, except for figure 2 where $N = 2$, equispaced half a wavelength apart [32]. $P = 2$ QPSK sources are linearly modulated with a pulse shape filter corresponding to a $1/2$-Nyquist filters with a roll off equal to 0.3 [34]. In addition, the $P = 2$ sources have the same symbol period T and the same Signal to Noise ratio (SNR) equal to 15 dB, except for figures 4(a) and 4(b). The sources are assumed to be well angularly separated except for figure 6, where the other cases are also considered. The source carrier residuals are such that $f_{c1}\, T_e = 0$, $f_{c2}\, T_e = 0.65$, except for figure 2 where $f_{c2}\, T_e = 0$. The sample period T_e corresponds to the symbol period T. As a result, the used SO and FO statistics are time invariant, so that classical sample estimators may be employed. As far as the background noise is concerned, it is temporally and spatially white except for section V-.1. Eventually, the simulation results are averaged over 200 realizations. Note that we resample the sources and the noise between these 200 experiments. On the other hand, the mixing matrix does not change except for figure 6 where its influence on the BSS methods performance is pointed out.

Moreover, the criterion used in this paper, in order to evaluate performance of BSS algorithms, is the well-known SINRM (Signal to Interference plus Noise Ratio Maximum) criterion defined in [11, section 3]. In other words, for each source $s_p(k)$ $(1 \le p \le P)$, the Signal to Interference plus Noise Ratio for the source p at the output of a spatial filter $\boldsymbol{w}_i$ is defined by

$$SINR_p\,[\boldsymbol{w}_i] = \gamma_p(0) \frac{|\boldsymbol{w}_i{}^{\mathsf{H}}\, \boldsymbol{a}_p|^2}{\boldsymbol{w}_i{}^{\mathsf{H}}\, \boldsymbol{R}_{\boldsymbol{\nu}_p}\, \boldsymbol{w}_i} \tag{31}$$

where $\gamma_p(0)$ is the variance of the p-th source. Moreover, $\boldsymbol{R}_{\boldsymbol{\nu}_p}$ is the total noise covariance matrix for source p, corresponding to matrix $\boldsymbol{R}_{\boldsymbol{x}}$ in the absence of source p. In these conditions, the restitution quality of source p at the output of separator $\boldsymbol{W}$, whose columns are the $\boldsymbol{w}_i$, can be evaluated by the maximum value of $SINR_p\,[\boldsymbol{w}_i]$ when i varies from 1 to P, and may be denoted $SINRM_p$.

1) The white noise case: The performance of ICAR at the output of the considered source separator is firstly illustrated in the presence of a Gaussian noise, spatially and temporally white, and compared with some well-known BSS algorithms. Figures 1(a), 1(b) and 2 show the variations of $SINRM_2$



(a) Case I (b) Case II

Fig. 1. Behavior of BSS methods in the presence of a white noise

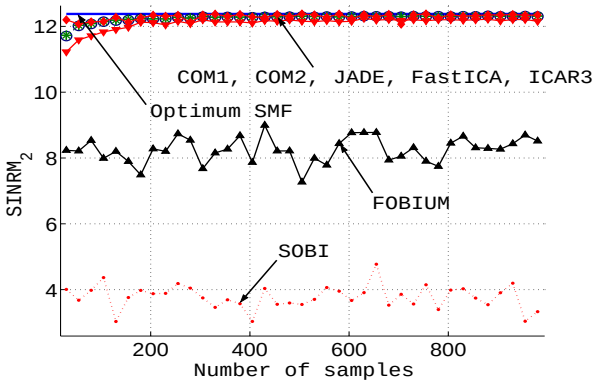


Fig. 2. Behavior of BSS methods for sources with identical trispectra

(source 2 performance) at the output of the previous methods as a function of the number of samples. Figures 1(a) and 1(b) show the good performance of the ICAR algorithm, especially ICAR3 (the third method given in section IV-C.2), facing the well-known SOBI, COM1, COM2, JADE, FastICA and FOBIUM methods. As for the SOBI method, it requires about 450 snapshots to obtain good results, due to a mild difference between the spectral densities of the sources. Note that similar results have been obtained for the other source. In addition, since the best results between the three ICAR methods are obtained in particular for ICAR3, we report in the following sections the comparison results only for this third method. Contrary to the other figures, figure 2 shows performance results when the two QPSK are chosen *in baseband*, i.e. taking $f_{c1}\, T_e = f_{c2}\, T_e = 0$, which implies that the two source signals have identical trispectra. Consequently, the SOBI and FOBIUM algorithms are unable to separate them correctly. However, we note that the FOBIUM method seems to be more robust than SOBI with respect to a spectrum difference of the sources. Moreover, other simulations have shown that the FOBIUM results are better as quotient $\frac{N}{P}$ increases, even if they remain suboptimal.

Figure 3 shows, for a number of 400 samples, the variations of $SINRM_2$ at the output of the previous methods as a function of the input SNR, identical for the two sources. All the BSS methods have approximately the same behavior. First, when the SNR is very small, they do not succeed perfectly in extracting the third source. On the contrary, for signal to noise ratios beteen -4 and 20 dB, the source separation is optimal.

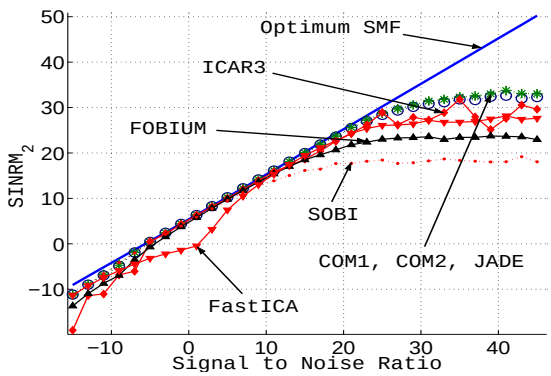


Fig. 3. Behavior of BSS methods for different signal to noise ratios

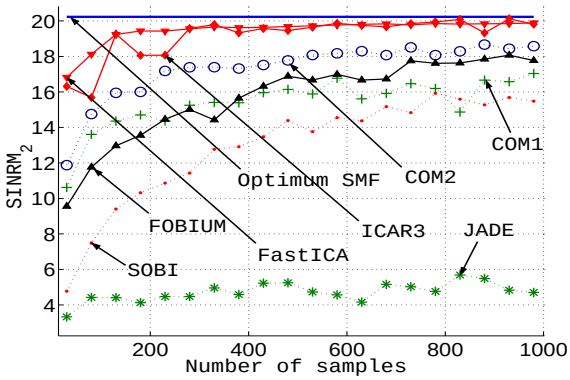
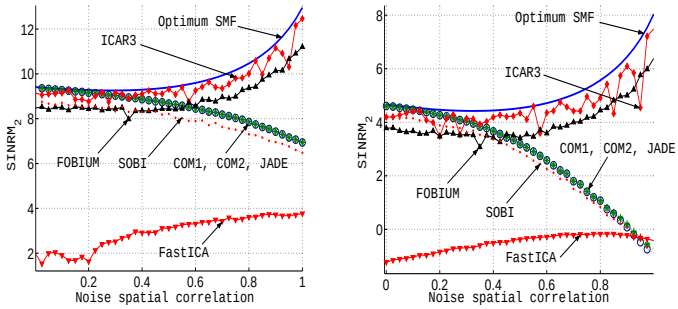


Fig. 5. Behavior of BSS methods for an over estimated number of sources



(a) Case I : a SNR of 5 dB (b) Case II : a SNR of 0 dB

Fig. 4. Behavior of BSS methods for a colored noise

Finally, although the variations of $SINRM_2$ for signal to noise ratios greater than 20 dB are somewhat surprising, this result has already been observed by Monzingo and Miller in [32] for optimal separators when mixture $\boldsymbol{A}$ is known. Note that similar results have been obtained for the other source.

2) The colored noise case: Then, the ICAR3 method is compared with the other algorithms in the presence of a Gaussian noise with unknown spatial correlation. Figure 4(a) and 4(b) show the variations of $SINRM_2$ at the output of the previous methods as a function of the noise spatial correlation factor ρ. SNR of the two sources is taken equal to 5 dB (figure 4(a)), next 0 dB (figure 4(b)). In addition, 400 samples are used to extract the two sources. Note that the Gaussian noise model employed in this simulation is the sum of an internal noise $\boldsymbol{\nu_{in}}(k)$ and an external noise $\boldsymbol{\nu_{out}}(k)$, of covariance matrices $\boldsymbol{R}_{\boldsymbol{\nu}}^{in}$ and $\boldsymbol{R}_{\boldsymbol{\nu}}^{out}$ respectively such that

$$\boldsymbol{R}_{\boldsymbol{\nu}}^{in}(r,q) \stackrel{\text{def}}{=} \sigma^2\delta(r-q)/2 \quad \boldsymbol{R}_{\boldsymbol{\nu}}^{out}(r,q) \stackrel{\text{def}}{=} \sigma^2\rho^{|r-q|}/2 \quad (32)$$

where σ^2, ρ are the total noise variance per sensor and the noise spatial correlation factor respectively. Note that $\boldsymbol{R}_{\boldsymbol{\nu}}(r,q) \stackrel{\text{def}}{=} \boldsymbol{R}_{\boldsymbol{\nu}}^{in}(r,q) + \boldsymbol{R}_{\boldsymbol{\nu}}^{out}(r,q)$ is the (r,q)-th component of the total noise covariance matrix. It appears in figure 4(a) that FOBIUM and ICAR3 are insensitive to a Gaussian noise with unknown spatial correlation, whereas ICAR3 seems to be a bit more robust than FOBIUM. On the other hand, the well-known COM1, COM2, JADE and SOBI methods are strongly affected as soon as the noise spatial correlation increases beyond 0.5. In fact, the classical BSS methods require a prior spatial whitening based on second order moments. This stage theoretically needs the perfect knowledge of the noise covariance. If this is not the case, a whitening of the observed data is performed instead, which is biased. ICAR does not suffer from this drawback, since it uses only FO cumulants, which are (asymptotically) insensitive to Gaussian noise, regardless of its space/time color. Note the poor performance of FastICA due to the presence of weak sources. Besides, similar results have been observed for source 1. As far as figure 4(b) is concerned, it confirms the fact that the performance differences between ICAR3 and the classical BSS methods increases as the source SNR decreases.

3) Over estimation of the number of sources: On the other hand, in operational contexts, the number of sources may be over estimated. It is then interesting to compare the ICAR method with other algorithms in such situations. To this aim, we assume that the estimated number of sources is equal to $\widehat{P} = 3$. Figure 5 shows the variations of $SINRM_2$ (source 2 performance) at the output of the previous methods as a function of the number of samples while the input SNR of the two sources is assumed to be equal to 15 dB. Similar results have been observed for source 1. More particularly, it appears that the FastICA and ICAR3 methods are robust with respect to an over estimation of the number of sources whereas, in this simulation configuration, the JADE algorithm looses 15 dB, for less than 1000 samples, with respect to the case where $\widehat{P} = 2$. As for the other methods, such as the FOBIUM algorithm, they are also affected by this over estimation, but less than the JADE algorithm since they lose on average 3 dB. The explanation of this surprising phenomenon is not easy and is beyond the scope of this paper. However, a similar behavior had been observed in [13] [10] when comparing JADE and COM algorithms. The lack of robustness of JADE stems from the fact that only a subset of cross-cumulants are minimized, which means that some cross-cumulants are implicitly maximized along with marginal ones.

4) The mixing matrix influence: Finally, the performance of the seven BSS methods (SOBI, COM1, COM2, JADE, FastICA, FOBIUM and ICAR3) are compared for different mixing matrices. Indeed, figure 6 shows the variations of $SINRM_2$ at the output of the previous methods as a function of the source spatial correlation $c_{1,2}$, which is defined as the normalized modulus of the scalar product between the two

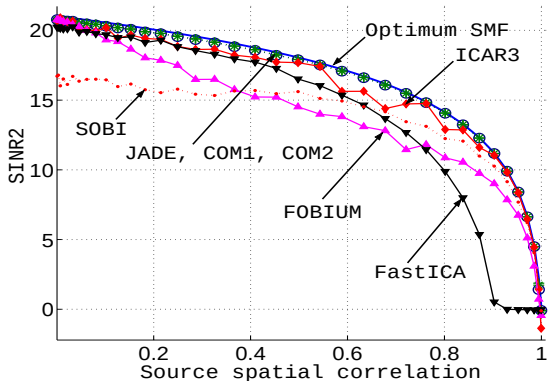


Fig. 6. Behavior of BSS methods for different source spatial correlations

steering vectors, i.e. the two column vectors of matrix $\boldsymbol{A}$:

$$c_{1,2} = \frac{1}{N} \left| \boldsymbol{a}_1^{\mathsf{H}} \, \boldsymbol{a}_2 \right| \tag{33}$$

The input SNR of the two sources is assumed to be equal to 15 dB. Similar results have been observed for source 1. More particularly, it appears that ICAR3 presents results generally close to the optimum SMF, except for some isolated values. In addition, FastICA seems to be more sensitive to sources that are not enough angularly separated. On the other hand, this simulation section allows to evaluate the robustness of the previous methods with respect to assumption **A5**, which is a basic but needed assumption in blind source separation, as shown in figure 6.

VI. Conclusion

The ICAR algorithm, exploiting the information contained in the data statistics at fourth order only, has been proposed in this paper. This algorithm allows to process overdetermined (including square) mixtures of sources, provided the latter have marginal FO cumulants with the same sign, which is generally the case in radio communications contexts. Three conclusions can be drawn: first, in the presence of a Gaussian noise spatially and temporally white, the proposed method yields satisfactory results. Second, contrary to most BSS algorithms, the ICAR method is not sensitive to a Gaussian colored noise whose spatial coherence is unknown. Last, the ICAR algorithm is robust with respect to an over estimation of the number of sources, which is not the case for some methods such as JADE. Forthcoming works include the search for a contrast criterion associated with ICAR in order to analyse accurately its performance.

Appendix I
Proof of Proposition 1

The proof of proposition 1 follows immediately from equations (18), (19) and assumption (**A4**), i.e. matrix $\boldsymbol{A}$ does not contain any null entry. In fact, suppose that $\boldsymbol{A} \oslash \boldsymbol{A}^*$ is not full column rank. Then there exists some $P \times 1$ vector $\boldsymbol{\beta} \neq 0$ such that $[\boldsymbol{A} \oslash \boldsymbol{A}^*] \, \boldsymbol{\beta} = 0$, which, due to the structure of $\boldsymbol{A} \oslash \boldsymbol{A}^*$ (18) implies that for all $1 \leq n \leq N$, $\boldsymbol{A}^* \boldsymbol{\Phi}_n \, \boldsymbol{\beta} = 0$. So it implies that $\boldsymbol{A}$ cannot be full column rank (since matrices $\boldsymbol{\Phi}_n$ are $P \times P$ diagonal with nonzero entries, due to equation (19) and assumption (**A4**)), which contradicts the fact that $\boldsymbol{A}$ is of full column rank (**A5**).

Appendix II
Proof of Proposition 2

The proof is straightforward. In fact, two square roots of a matrix (here $\boldsymbol{Q}_{\boldsymbol{x}}$) are always equal to within a unitary matrix, which yields

$$[\boldsymbol{A} \oslash \boldsymbol{A}^*] \, \boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2} = \boldsymbol{E}_{\boldsymbol{s}} \, \boldsymbol{L}_{\boldsymbol{s}}^{1/2} \, \boldsymbol{V} \tag{34}$$

for some $P \times P$ unitary matrix $\boldsymbol{V}$. Equation (34) shows that the right-hand side is the SVD of the left-hand side, hence the proposition 2 result, since $\boldsymbol{E}_{\boldsymbol{s}}^{\mathsf{H}} [\boldsymbol{A} \oslash \boldsymbol{A}^*] \boldsymbol{\zeta}_{\boldsymbol{s}} [\boldsymbol{A} \oslash \boldsymbol{A}^*]^{\mathsf{H}} \boldsymbol{E}_{\boldsymbol{s}} = \boldsymbol{L}_{\boldsymbol{s}}$ is a real positive matrix.

Appendix III
Proof of Proposition 3

$\boldsymbol{A}^*$ is a full column rank matrix according to (**A5**). The diagonal matrices $\boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2}$ and $\boldsymbol{\Phi}_n$ (note that the diagonal elements of the latter are components of $\boldsymbol{A}$), are invertible according to (**A3**) and (**A4**) respectively, in other words, because source kurtoses are not null and because matrix $\boldsymbol{A}$ does not contain any null entry. As far as the square matrix $\boldsymbol{V}$ is concerned, it is invertible because of its unitary structure. So matrices $\boldsymbol{\Gamma}_n$ are the product of a full column rank matrix, $\boldsymbol{A}^*$, and an invertible matrix $\boldsymbol{F}_n = \boldsymbol{\Phi}_n \boldsymbol{\zeta}_{\boldsymbol{s}}^{1/2} \boldsymbol{V}^{\mathsf{H}}$. The fact that this particular product is of full column rank remains to be proved. In fact, suppose that $\boldsymbol{A}^* \boldsymbol{F}_n$ is not of full column rank. Then there exists some $P \times 1$ vector $\boldsymbol{\beta} \neq 0$ such that $\boldsymbol{A}^* \boldsymbol{F}_n \boldsymbol{\beta} = 0$. So it implies that $\boldsymbol{A}^*$ cannot be full column rank (since matrices $\boldsymbol{F}_n$ are $P \times P$ invertible), which contradicts the first sentence of this section.

Appendix IV
Proof of Proposition 4

If assumptions (**A4**) and (**A5**) are equivalent to assume $\boldsymbol{A}$ with no null entries and of full column rank respectivelly then, proposition 4 may be rewritten as

$$\begin{aligned} (\mathbf{A4}) + (\mathbf{A5}) \; \Rightarrow \{ \forall \; (p_1, p_2) \, , \, 1 \leq p_1 \neq p_2 \leq P, \, \exists \, (n_1, n_2) \, , \\ 1 \leq n_1 \neq n_2 \leq N \text{ s.t. } D_{n_1,n_2}(p_1, p_1) \neq D_{n_1,n_2}(p_2, p_2) \, \} \end{aligned} \tag{35}$$

where *s.t.* means *such that*. To prove assertion (35), assume the contrary:

$$\begin{aligned} (\mathbf{A4}) + (\mathbf{A5}) \; \Rightarrow \; \{ \exists \; (p_1, p_2) \, , \, 1 \leq p_1 \neq p_2 \leq P, \, \forall \; (n_1, n_2) \, , \\ 1 \leq n_1 \neq n_2 \leq N \text{ s.t. } D_{n_1,n_2}(p_1, p_1) = D_{n_1,n_2}(p_2, p_2) \, \} \end{aligned} \tag{36}$$

This implies, since $\boldsymbol{D}_{n_1,n_2} = \boldsymbol{\Phi}_{n_1}^{-1} \boldsymbol{\Phi}_{n_2}$ are $P \times P$ diagonal full rank matrices, that $\exists \; (p_1, p_2)$, $1 \leq p_1 \neq p_2 \leq P$, $\forall \; (n_1, n_2)$, $1 \leq n_1 \neq n_2 \leq N$ s.t.

$$\frac{\Phi_{n_2}(p_1, p_1)}{\Phi_{n_1}(p_1, p_1)} = \frac{\Phi_{n_2}(p_2, p_2)}{\Phi_{n_1}(p_2, p_2)} \tag{37}$$

which is equivalent, according to (19), to

$$\frac{A(n_2, p_1)}{A(n_1, p_1)} = \frac{A(n_2, p_2)}{A(n_1, p_2)} \tag{38}$$

This means

$$\exists\ (p_1, p_2)\ ,\ 1 \leq p_1 \neq p_2 \leq P \quad \text{s.t.} \quad \boldsymbol{a}_{p_1} \propto \boldsymbol{a}_{p_2} \tag{39}$$

In other words, assuming (36) under (**A4**) and (**A5**) implies that at least two column vectors of $\boldsymbol{A}$ are collinear, which contradicts (**A5**). Consequently, assertion (35) and hence proposition 4 are true.

APPENDIX V
PROOF OF PROPOSITION 5

Each column $\boldsymbol{b}_p$ of $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}\boldsymbol{V}_{sol}$ is defined, according to (28), by

$$\forall\, p,\ 1 \leq p \leq P, \qquad \boldsymbol{b}_p = \lambda_{\xi(p)}\, \boldsymbol{a}_{\xi(p)} \otimes \boldsymbol{a}^*_{\xi(p)} \tag{40}$$

where $\xi(\cdot)$ is a bijective function of $\{1, 2, \ldots, P\}$ into itself (i.e. a permutation) and where $|\lambda_p| = [\,|\, C^{p_1\, p}_{p_1\, p_1\, \boldsymbol{s}}\,|\,]^{1/2}$, $|\cdot\,|$ denoting the complex modulus operator. So we transform the P vectors $\boldsymbol{b}_p$ of size $N^2 \times 1$ into $N \times N$ matrices $\boldsymbol{B}_p$ where the (n_1, n_2)-th component of $\boldsymbol{B}_p$ corresponds to the $[N(n_2 - 1) + n_1]$-th component of $\boldsymbol{b}_p$ such that

$$\boldsymbol{B}_p = \lambda_{\xi(p)}\, \left[\, \boldsymbol{a}_{\xi(p)}\, \boldsymbol{a}_{\xi(p)}{}^{\mathsf{H}} \,\right]^* \tag{41}$$

Note that $\boldsymbol{B}_p$ is a rank one matrix. Consequently, a simple diagonalization of each matrix $\boldsymbol{B}_p^*$ indeed allows to extract, in a unique way up to a scale and permutation factor, each column vector of $\boldsymbol{A}$.

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