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| Georges Sadaka. Propagation of a Tsunamis wave. 2012. hal-00741584v1

HAL Id: hal-00741584

<https://hal.science/hal-00741584v1>

Submitted on 15 Oct 2012 (v1), last revised 20 Jan 2018 (v3)

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Propagation of a Tsunamis wave

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October 15, 2012

Abstract

`FreeFem++`¹ is an open source platform to solve partial differential equations numerically, based on finite element methods. The use of this software simplify also the construction of the domain in particular, one of the advantage of `FreeFem++` is that we can build a mesh through a photo and we can easily export bathymetric data in order to consider more realistic simulations.

We consider, in this paper, the BBM-BBM² Boussinesq system with a variable bottom in space (cf. [Sad11]) and try this system, first, using the mesh generated through a photo of the Mediterranean sea, then using the mesh generated through an xyz bathymetry for the East part of the Mediterranean sea.

Contents

1	Introduction	2
1.1	Boussinesq systems over variable bottom	2
1.2	Space discretization	3
1.3	Time discretization	3
2	Mesh generated through an imported a photo with FreeFem++	3
3	Mesh generated through an imported xyz bathymetry with FreeFem++	6
4	Numerical simulations	8
4.1	Propagation of a <i>Tsunami</i> in the Mediterranean sea with a flat bottom.	8
4.2	Propagation of a <i>Tsunami</i> in the Mediterranean sea with realistic bathymetry datas.	11
5	Conclusion and Outlook	13

1. <http://www.freefem.org/ff++/index.htm>
2. Benjamin, Bona and Mahony (BBM)

1 Introduction

We consider here to study the numerical simulation of the BBM-BBM Boussinesq system in 2D over a variable which has been derived in [Sad11] as approximations to the three-dimensional Euler equations by considering a domain $\Omega \subset \mathbb{R}^3$ that is occupied by a Newtonian, ideal, incompressible, homogeneous and irrotational fluid and which is limited from below by the bottom $-D(x, y)$ and from above by the free surface elevation $\eta(x, y, t)$ (cf. Figure 1).

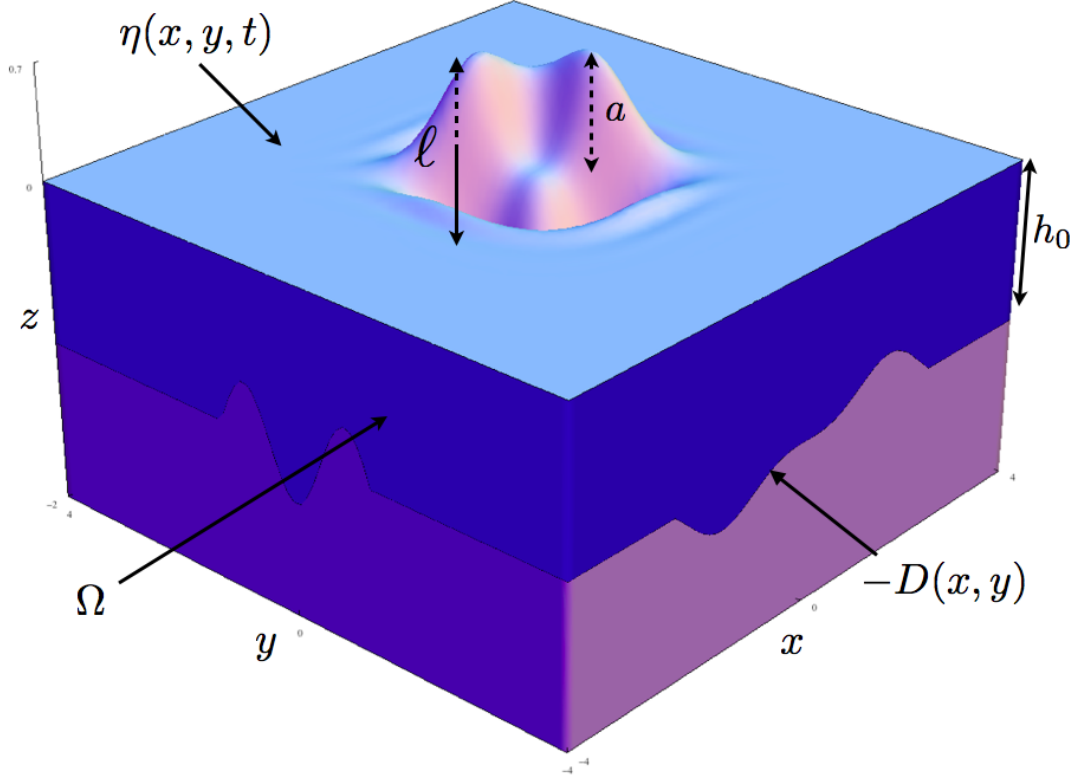


Figure 1: The domain Ω .

Remark. In our study, we suppose that $\eta = \mathcal{O}(a)$, where the amplitude a is the difference between the surface of water and the zero level, and the wave length $\lambda = \mathcal{O}(\ell)$, in addition we limit ourselves to the case where $\eta + D > 0$ (there is no dry zone) since we are in a big deep water wave regime.

1.1 Boussinesq systems over variable bottom

The BBM-BBM Boussinesq systems over a variable bottom $-D(x, y)$ in $\Omega \subset \mathbb{R}^2$ is given by:

$$\begin{aligned} \eta_t + \nabla \cdot ((D + \eta)V) - b\nabla \cdot (D^2 \nabla \eta_t) &= 0; \\ \mathbf{V}_t + \nabla \eta + \frac{1}{2} \nabla |\mathbf{V}|^2 - dD^2 \Delta \mathbf{V}_t &= 0, \end{aligned} \tag{1}$$

The variables in (1) are non-dimensional and unscaled : $\mathbf{X} = (x, y) \in \Omega$ and $t > 0$ are proportional to position along the channel and time, respectively, $\eta = \eta(\mathbf{X}, t)$ is proportional to the deviation of the free surface from its rest position, $\mathbf{V} = \mathbf{V}(\mathbf{X}, t) = \begin{pmatrix} u(\mathbf{X}, t) \\ v(\mathbf{X}, t) \end{pmatrix} = (u, v)^T = (u; v)$ is

proportional to the horizontal velocity of the fluid at some height, $\nabla \cdot = \begin{pmatrix} \partial_x \cdot \\ \partial_y \cdot \end{pmatrix}$ is the gradient, $\nabla \cdot (\star ; \cdot) = \partial_x \star + \partial_y \cdot$ is the divergence and $\Delta \cdot = \partial_{xx} \cdot + \partial_{yy} \cdot$ is the laplacian. The coefficients b and d are taken equal to $1/6$ in this paper.

1.2 Space discretization

We let Ω be a convex, plane domain, let \mathcal{T}_h denote a regular, quasiuniform triangulation of Ω with triangles of maximum size $h < 1$, let $V_h = \{v_h \in C^0(\bar{\Omega}); v_h|_T \in \mathbb{P}_1(T), \forall T \in \mathcal{T}_h\}$ denote a finite-dimensional subspace of $H^1(\Omega) = \{u \in L^2(\Omega) \text{ s.t. } \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \in L^2(\Omega)\}$ where \mathbb{P}_1 is the set of polynomials of \mathbb{R} of degrees ≤ 1 and let $\langle \cdot ; \cdot \rangle$ denote the L^2 inner product on Ω . Consider the weak formulation of the system (1), find $\eta_h, u_h, v_h \in V_h$ such that $\forall \phi_h \in V_h$ we have :

$$\begin{aligned} \langle \eta_{ht}; \phi_h \rangle + b \langle D^2 \nabla \eta_{ht}; \nabla \phi_h \rangle + \langle \nabla \cdot (u_h; v_h) + \eta_{hx} u_h + \eta_h u_{hx} + \eta_{hy} u_h + \eta_h u_{hy}; \phi_h \rangle &= 0; \\ \langle (Id + d \nabla (D^2) \cdot \nabla) u_{ht}; \phi_h \rangle + d \langle D^2 \nabla u_{ht}; \nabla \phi_h \rangle + \langle \eta_{hx} + u_h u_{hx} + v_h v_{hx}; \phi_h \rangle &= 0; \\ \langle (Id + d \nabla (D^2) \cdot \nabla) v_{ht}; \phi_h \rangle + d \langle D^2 \nabla v_{ht}; \nabla \phi_h \rangle + \langle \eta_{hy} + u_h u_{hy} + v_h v_{hy}; \phi_h \rangle &= 0. \end{aligned} \tag{2}$$

We note also that when using a mesh adaptation method integrated in FreeFem++, a \mathbb{P}_2 -finite element space is used for better resolution. This method gives comparable results with \mathbb{P}_1 -finite element space without mesh adaptation technic however we have to more refine in the domain.

1.3 Time discretization

The time discretization of the problem is realized using an explicit second order Runge-Kutta where the study of it convergence in the flat bottom case is detailed in [Sad12a].

2 Mesh generated through an imported a photo with FreeFem++

We present here a method to build a mesh from a photo using Photoshop® and a script in FreeFem++ made by Frédéric Hecht³.

Through Google Earth®, we can download pictures of the earth for areas of interest. In this section, we choose the picture of the Mediterranean sea. The Mediterranean is being very large, in order to have better resolution, we takes pictures of many parts (cf. Figure 2) that subsequently assemble using Photoshop® so as to obtain a complete picture of the Mediterranean (cf. Figure 3).

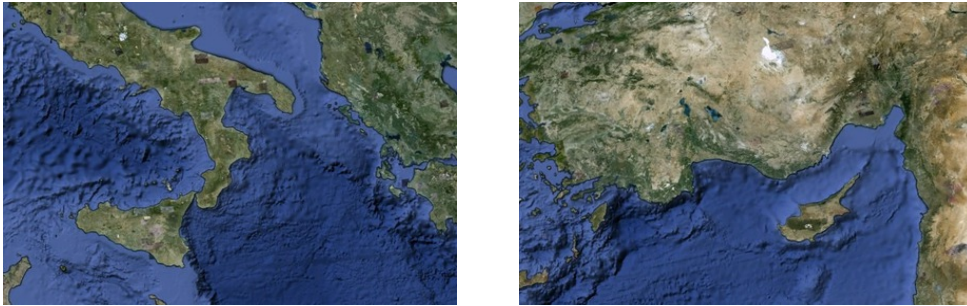


Figure 2: The pictures of two parts of the Mediterranean sea.

3. <http://www.ann.jussieu.fr/hecht/>



Figure 3: The Mediterranean sea after assembly with Photoshop®.

Using also Photoshop®, we can eliminate the dry areas that circumvent the Mediterranean (cf. Figure 4).

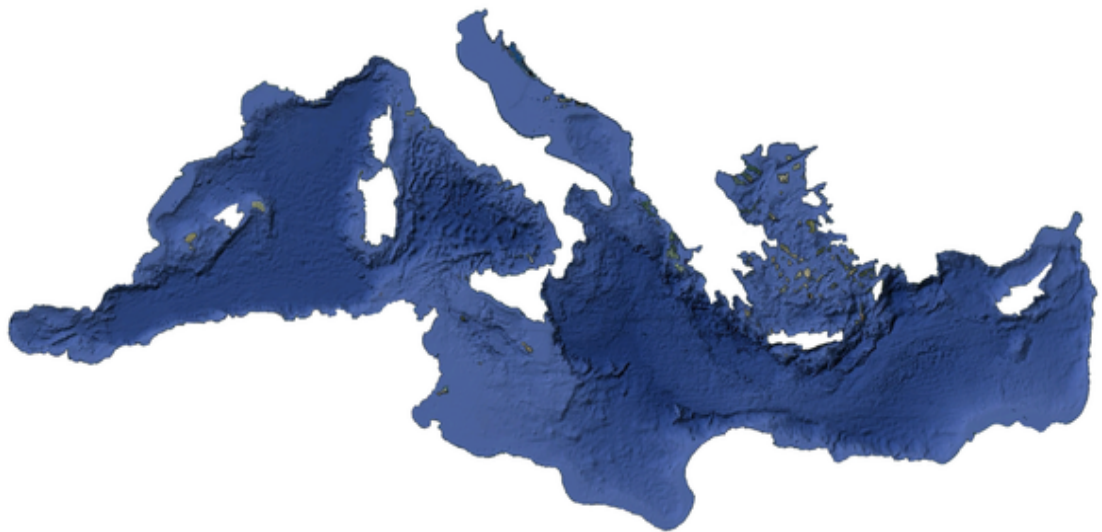


Figure 4: The Mediterranean sea.

We fill in one color the final domain and we use some filter in Photoshop® in order to smooth the boundary. Then we convert the jpg photo to a pgm photo which can be read by FreeFem++ using in a terminal window :

```
convert Medit_sea.jpg Medit_sea.pgm
```

In order to generate the mesh of the Mediterranean sea domain (cf. Figure 5), we use a FreeFem++ script made by Frédéric Hecht which can be downloaded from the following link <http://www.freefem.org/ff2a3/Stic-FF2A3-2010/edp/Chesapeake/Chesapeake-mesh.edp>.



Figure 5: The mesh generated by FreeFem++ for the Mediterranean sea.

Because of problem of non-smoothing boundary, we met some difficulties in the generation of the mesh in some regions as in the Figure 6 when using mesh adaptation technic. On the other side, without using the mesh adaptation technic, we no longer have this problem with the mesh generation since the mesh is generated at the beginning.

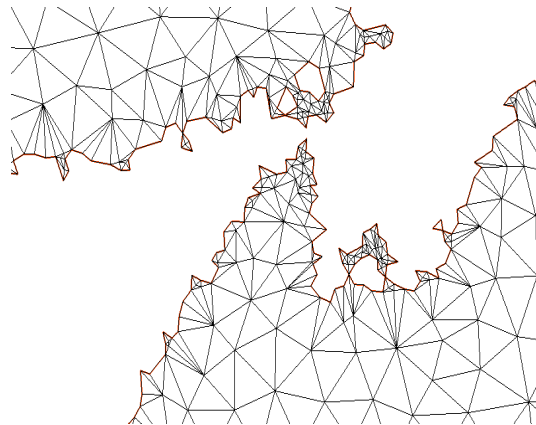


Figure 6: Mesh problem using the script “Chesapeake-mesh.edp”.

Recently, and in order to smooth the boundary, Frédéric Hecht has write another script and the result are better as shown in Figure 7. This script could be downloaded from the following link: http://lamfa.u-picardie.fr/sadaka/generate_a_mesh_using_a_photo.zip, and for more details see also [Sad12e].

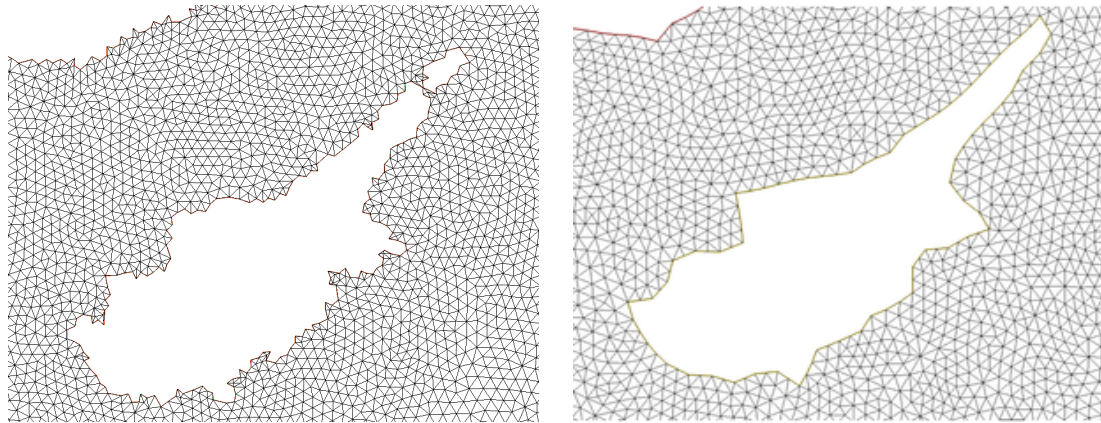


Figure 7: The mesh around Cyprus using the script of Frédéric Hecht (at left : the old one and at right : the new one).

3 Mesh generated through an imported xyz bathymetry with FreeFem++

In order to consider more realistic case *i.e.* the Mediterranean sea with a bathymetry, Olivier Pantz⁴ writes a script with FreeFem++ that allows, through the xyz bathymetry of a chosen area of the earth (downloaded from the NOAA website http://www.ngdc.noaa.gov/mgg/gdas/gd_designagrid.html, cf. Figure 8), to generate the mesh of the area where the amplitude is zero while using a level-set method to smooth the boundary (cf. Figure 10). In addition, we can through this script, on the one hand use the mesh adaptation method without having problem of generation of the mesh and in the other hand, we can have different label for each part of the boundary, which facilitates the use of different types of boundary condition.

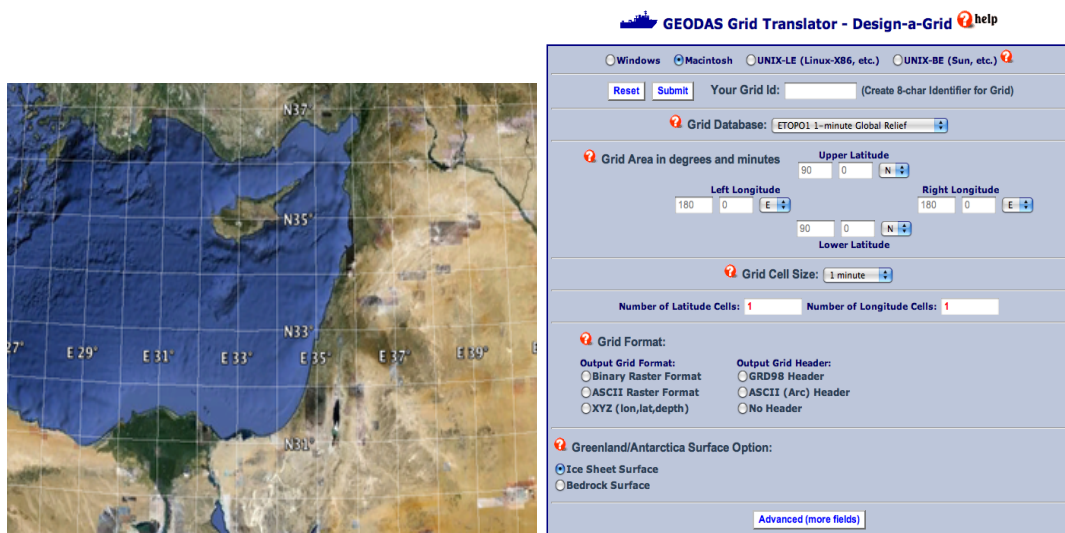


Figure 8: Importation of bathymetry datas through the NOAA website using Google Earth®.

Using the `triangulate` function in FreeFem++, we start by reading the xyz downloaded data (cf. Figure 9), then we separate the positive part from the negative one of the bathymetry by a curve which will be the boundary of our domain, we use the level-set method in order to smooth

4. <http://www.cmap.polytechnique.fr/spip.php?article105>

the boundary (cf. Figure 10). The details of this script could be downloaded from the following link: http://lamfa.u-picardie.fr/sadaka/generate_a_mesh_using_xyz_bathymetry.zip

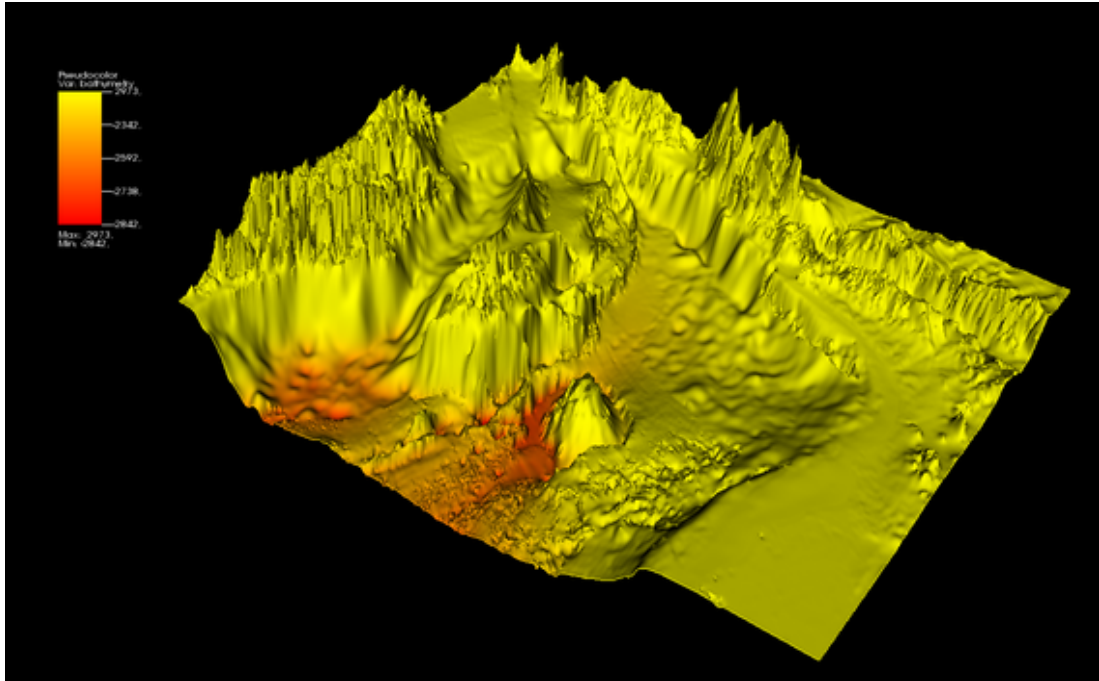


Figure 9: Bathymetry downloaded from the NOAA website, (min = $-2.842km$ and max = $2.973km$).

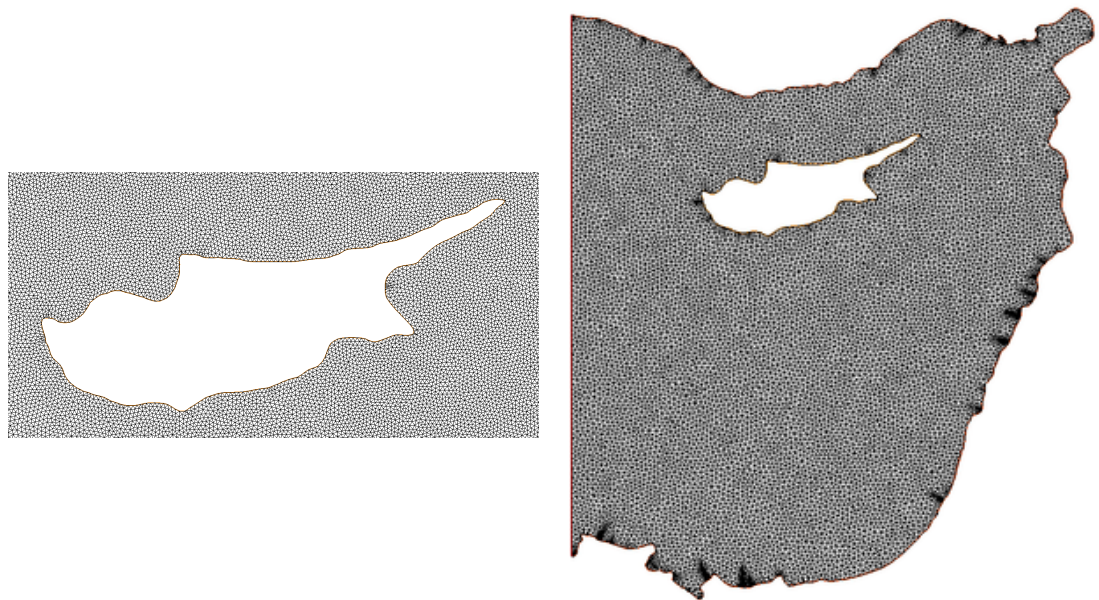


Figure 10: The mesh around Cyprus (left) and the mesh generated (right) using the script of Olivier Pantz.

4 Numerical simulations

In this section, we will use FreeFem++ in order to simulate the propagation of a wave, that looks like a *Tsunami* generated by an earthquake, in the Mediterranean sea over the BBM-BBM systems (1) with a flat bottom $-D(x, y) = -1$ and with a variable one in space using the mesh generated from the photo of the Mediterranean sea (5) and using from the xyz bathymetry of a part of the Mediterranean sea (7).

4.1 Propagation of a *Tsunami* in the Mediterranean sea with a flat bottom.

We simulate here, the propagation of a wave, that looks like a *Tsunami* generated by an earthquake, in the Mediterranean sea over the BBM-BBM system with a flat bottom $-D(x, y) = -1$, where Dirichlet boundary conditions are taken on the hole boundary of our domain for η, u and v and as initial datas :

$$\eta_{h0} = 0.1 \cdot \left\{ 1 - 1 / (1 + 10^3 \cdot \exp \left[- \left(\frac{-70 + (y - PY) - 0.2 \cdot (x - PX)}{10} \right)^2 - \left(\frac{-13 \cdot 10^4 + 0.2 \cdot (y - PY) + (x - PX) \cdot 10^3}{10^{(7/2)}} \right)^2 \right] \right\},$$

$$u_{h0}(x, y) = v_{h0}(x, y) = 0.$$

Here $PX = 2270$, $PY = 500$, the step time is $\Delta t = 0.1$.

In the Figure 11, we represent the maximum of the propagation of the solution at $t = 1000 \text{sec}$, for the same datas above using \mathbb{P}_1 -finite element space and without mesh adaptation technic.

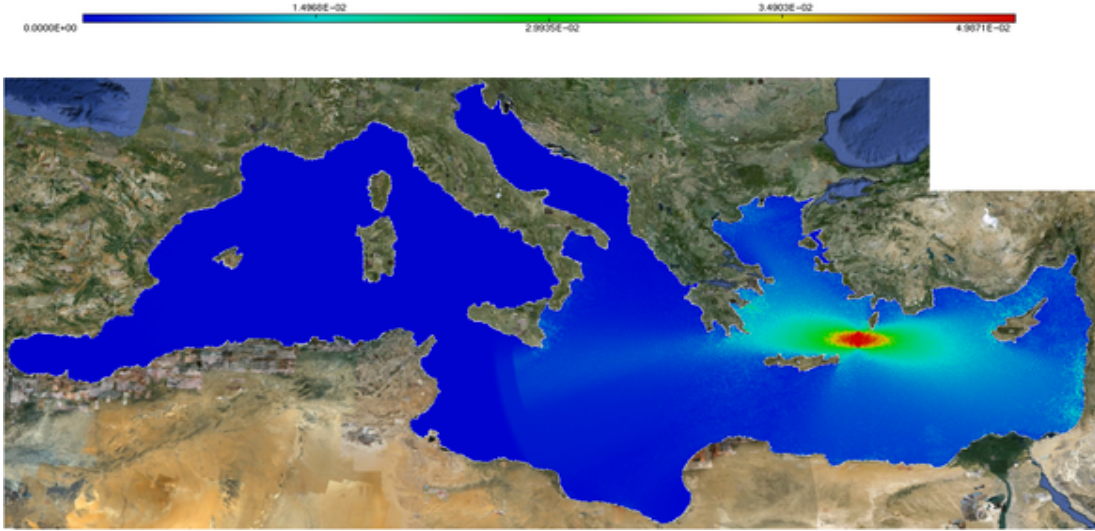


Figure 11: The maximum of the propagation of the solution of a *Tsunami* in the Mediterranean sea for $t = 1000 \text{sec}$. Video available on :

http://lamfa.u-picardie.fr/sadaka/tsunami_mediterranee_MAX_P1.avi.

On the other side, we present in the Figures 12 and 13 the propagation of the same wave, using a mesh adaptation method and a \mathbb{P}_2 -finite element space for better resolution. This method gives comparable results with \mathbb{P}_1 -finite element space without mesh adaptation technic, where in the last case, we no longer have a problem with the mesh generation, however we have to more refine, this makes the computation very heavy.

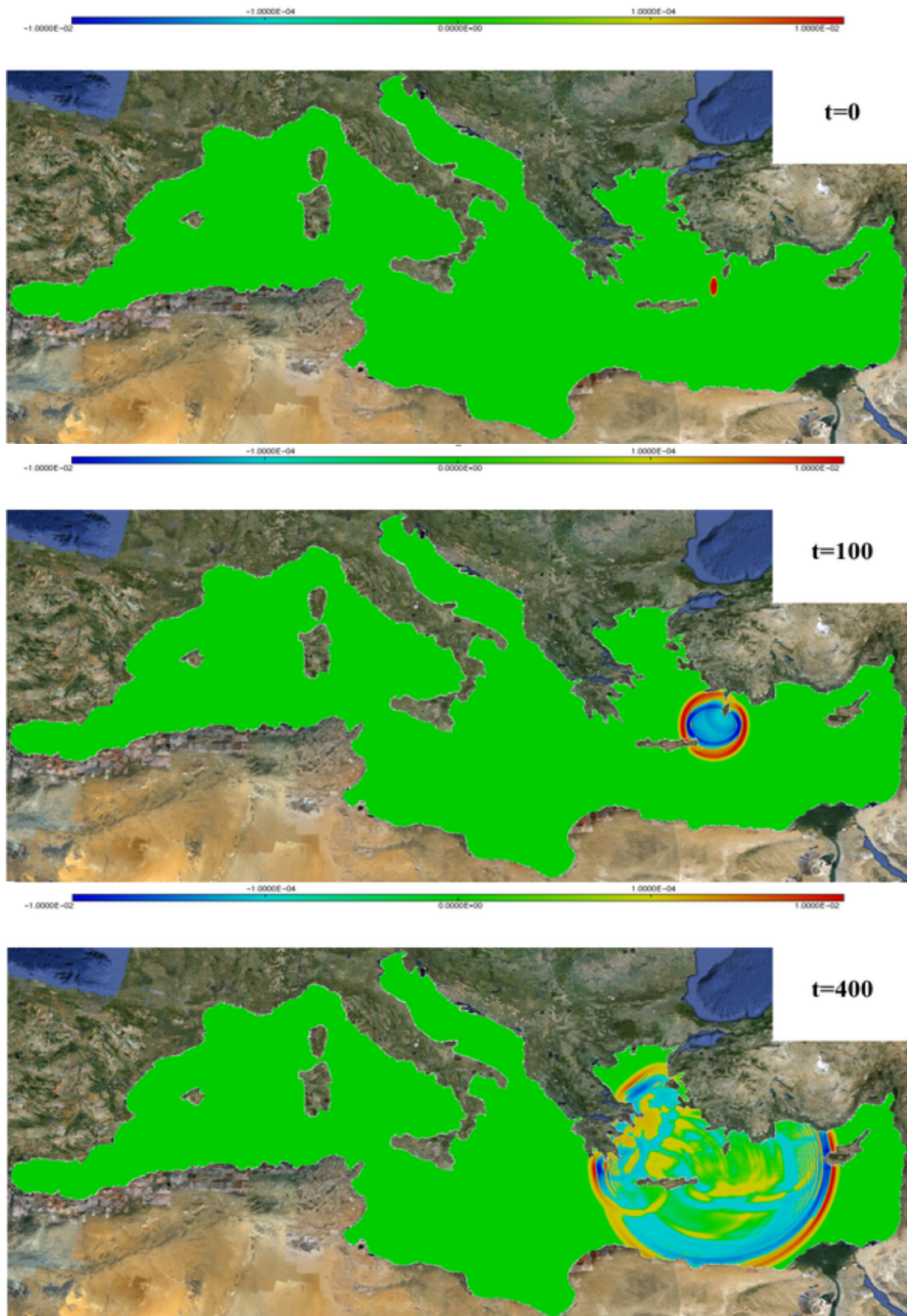


Figure 12: Propagation of the solution of a *Tsunami* in the Mediterranean sea for $t = \{0, 100, 400\}sec$. The *Tsunami* is generated near Crete.
 Video available on : http://lamfa.u-picardie.fr/sadaka/tsunami_mediterranee.avi.

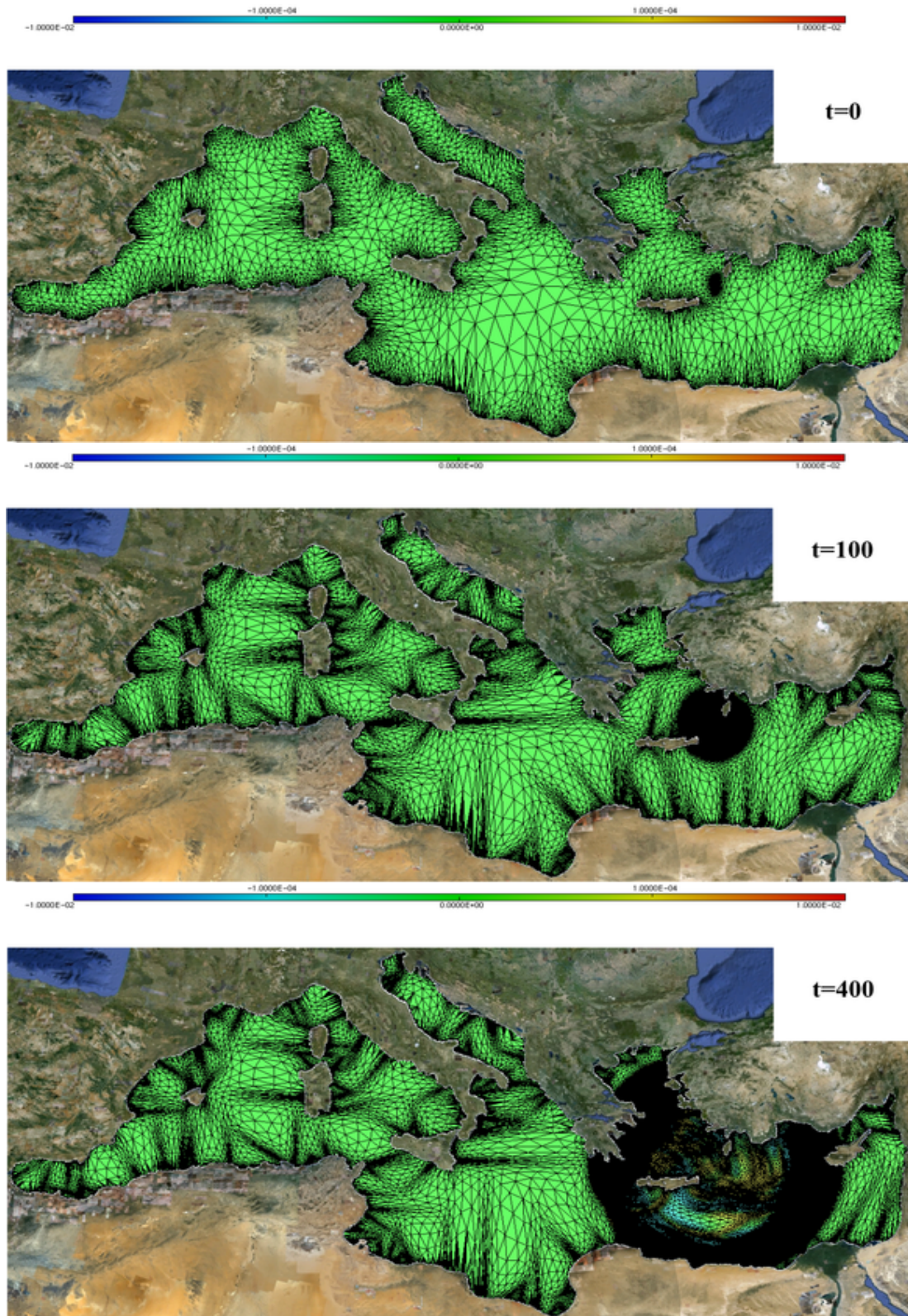


Figure 13: Propagation of the solution and the mesh of a *Tsunami* in the Mediterranean sea for $t = \{0, 100, 400\}sec$.

Video available on : http://lamfa.u-picardie.fr/sadaka/tsunami_mediterranee_mesh.avi.

4.2 Propagation of a *Tsunami* in the Mediterranean sea with realistic bathymetry datas.

In order to consider more realistic case *i.e.* the Mediterranean sea with a bathymetry, the bottom $-D(x, y)$ of our study domain will be the same of the one downloaded (9), but since we are in a big deep water wave regime *i.e.* $\eta(x, y, t) + D(x, y) > 0$, all the points of the bathymetry that are greater than or equal to $-10m$ will be equal to $-10m$ (cf. Figure 14).

We remark also that to download the bathymetry data of the NOAA website, we must know the degree of Latitude and of Longitude of our domain. The spherical shape of the earth was taken into account, even if it does not play significant role because of the small spatial scale of the experiments. So, since the radius of the earth near the equator is $R = 6378,137km$, we take into account that :

$$1 \text{ degree of Latitude} = \pi \cdot R/180 = 111,263km,$$

$$\text{and } 1 \text{ degree of Longitude} = \cos(1 \text{ degree of Latitude} \cdot \pi/180) \cdot \pi \cdot R/180.$$

We limit our study for this example by making a translation of our domain considering that 1 degree of Latitude is equal to 1 degree of Longitude is equal to $100km$. We use an homogeneous Dirichlet boundary conditions for η_h , u_h and v_h for the shoreline boundary, and an homogeneous Neumann boundary conditions η_h , u_h and v_h at the open sea. We use also \mathbb{P}_1 -finite element space and we start from the following initial datas :

$$\eta_{h0} = 0.01 \cdot \exp\left(-\left(\frac{x - 33.5 \cdot 100}{3}\right)^2 - \left(\frac{y - 33.8 \cdot 100}{10}\right)^2\right),$$

$$u_{h0}(x, y) = v_{h0}(x, y) = 0.$$

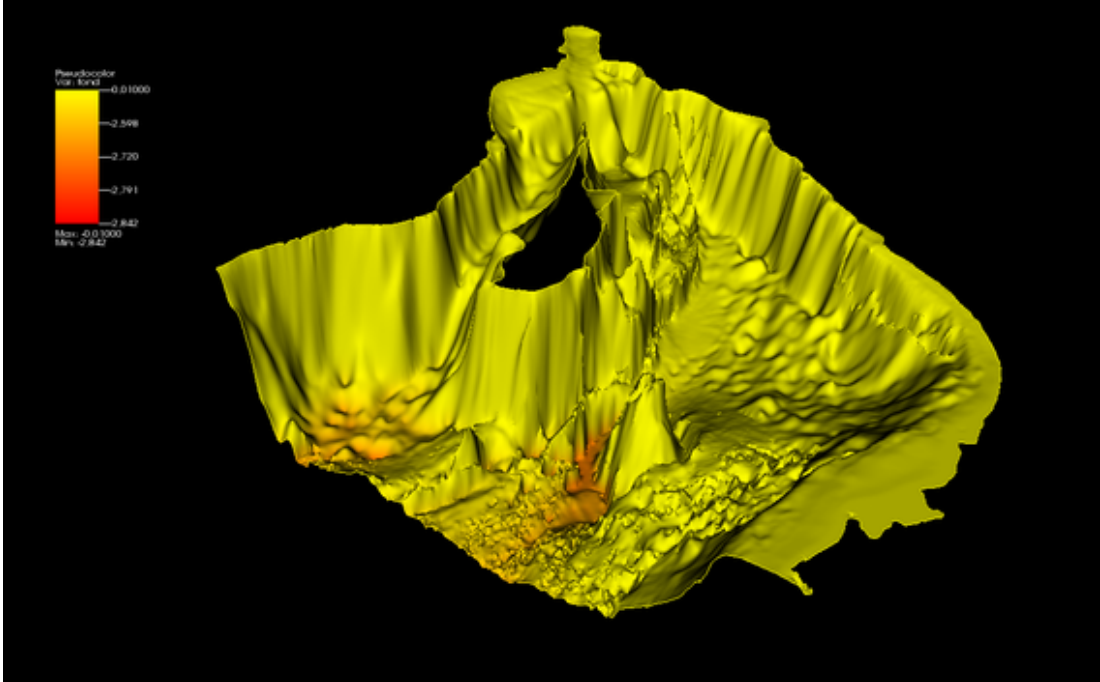


Figure 14: The bottom of the bathymetry downloaded from the NOAA website, (min = $-2.842km$ and max = $-0.01km$).

We present in Figure 15 the propagation of a wave generated near Cyprus in the Mediterranean sea that looks like the one generated through an earthquake. We present in the Figure 16 the maximum of the propagation of the solution for $t = 230sec$. As we can see also in the same figure that a mesh adaptation technic is used too without having any difficulties in mesh generation.

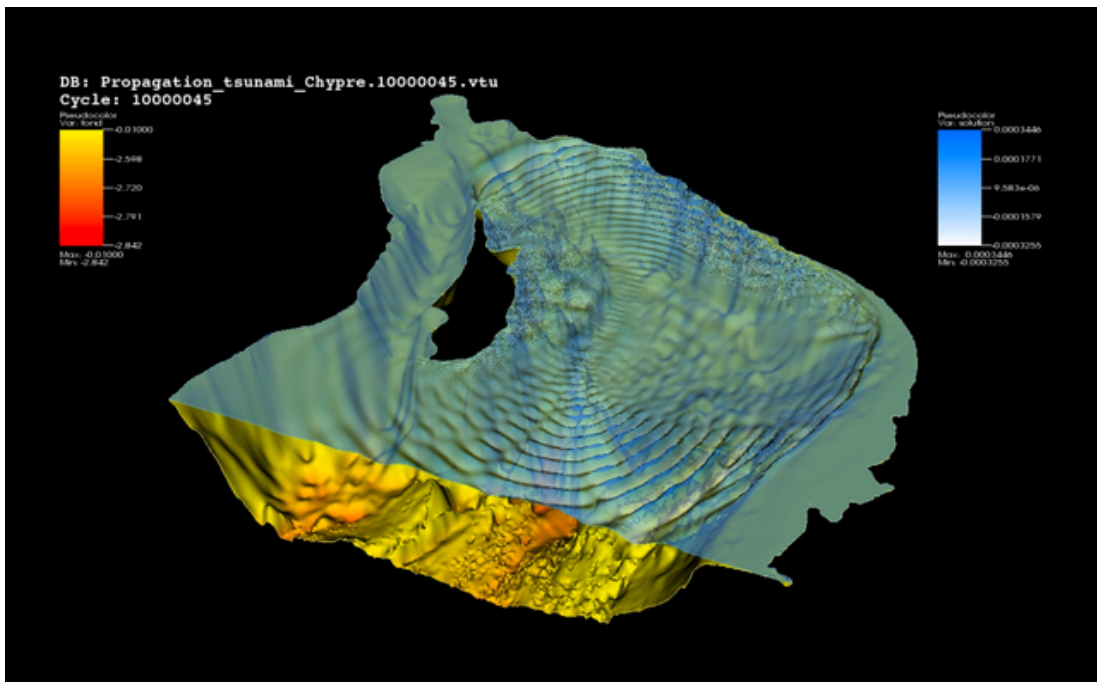
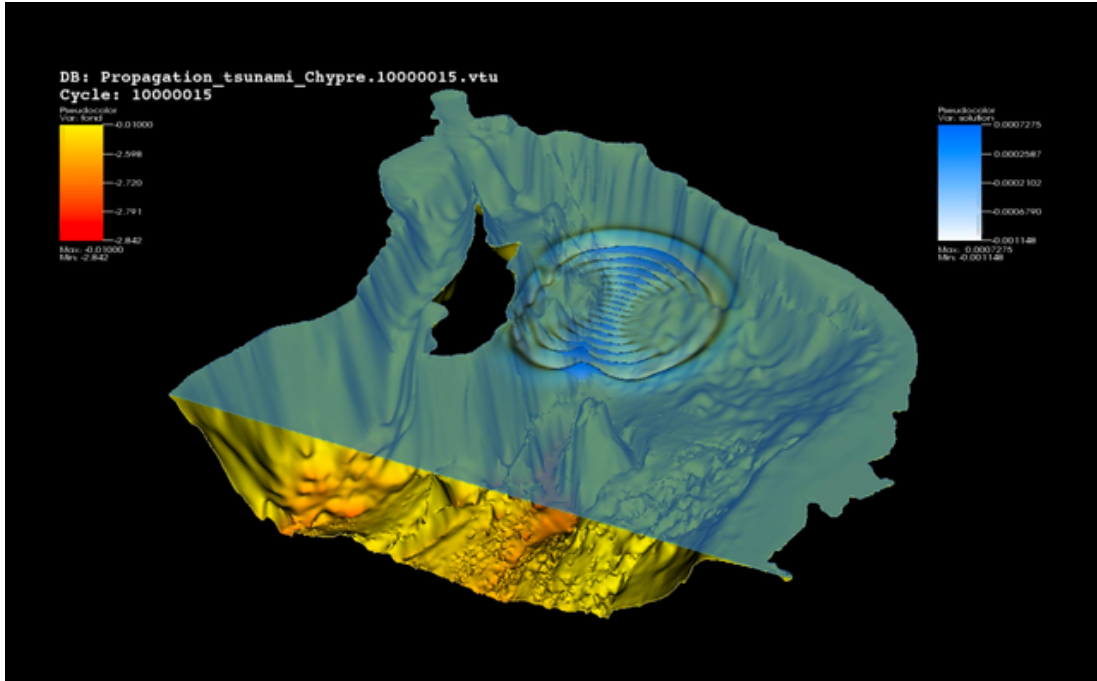


Figure 15: Propagation of a *Tsunami* in the Mediterranean sea, generated near Cyprus for $t = 75$ and 225sec .

Video available on : http://lamfa.u-picardie.fr/sadaka/propagation_tsunami_Chypre_P1.mpeg.

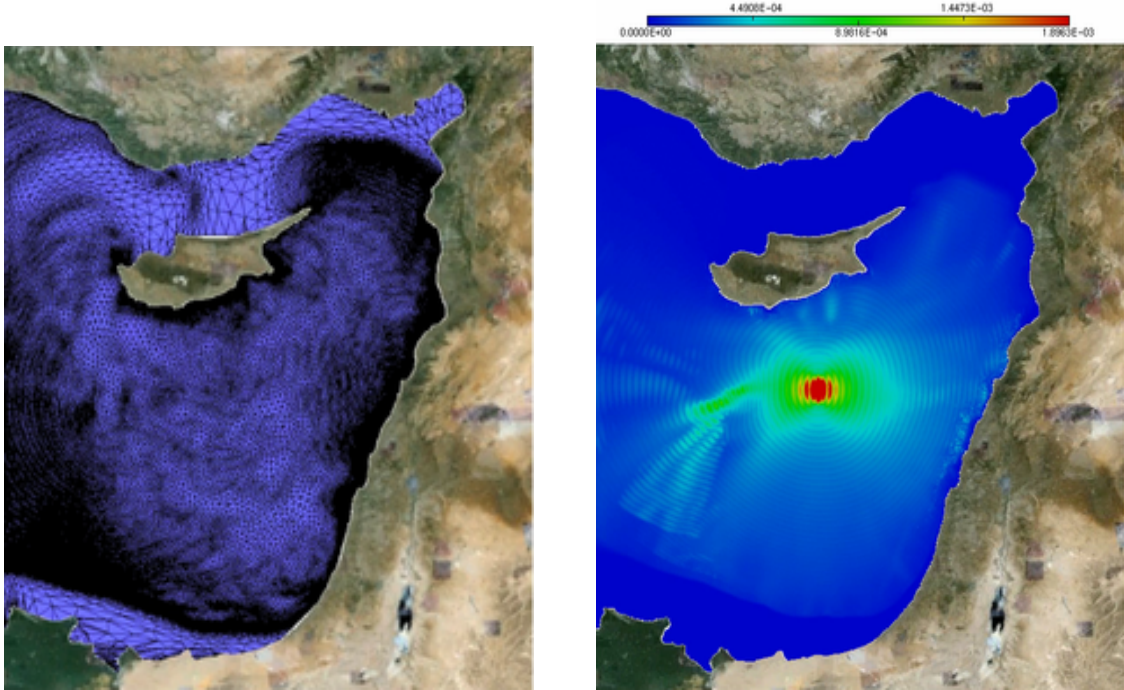


Figure 16: Left : mesh adaptation around Cyprus after the script of Olivier Pantz.
 Video available on : http://lamfa.u-picardie.fr/sadaka/chypre_adaptmesh.avi
 Right : the maximum of the propagation of the solution of a *Tsunami* in the Mediterranean sea for $t = 230sec$.
 Video available on : http://lamfa.u-picardie.fr/sadaka/max_solution_tsunami_Chypre_P1.avi.

5 Conclusion and Outlook

We show in this paper, the utilities of FreeFem++ for the Boussinesq system by building the domain, on the one hand through a photo taken from Google Earth® and on the other hand through an xyz bathymetry downloaded from the NOAA website. Concerning the simulation of a tsunami in the Mediterranean sea, the digital computing environment that we developed allows the integration of realistic data (bathymetry and geography) in a relatively simple framework.

As we see, the mesh generation through a picture presents some difficulties, other than smoothing the boundary, we are limited to take only homogeneous Dirichlet boundary condition on the hole boundary of our domain for η , u and v . In the other hand, for the simulation of a *Tsunami* near Cyprus, we remark that an artificial reflection come from the open sea after $t = 230sec$ which is due to the Neumann boundary conditions. In this case we must take some absorbent boundary condition which is an open problem. We see also, near the shoreline some oscillation because here, we are not in the big deep water wave regime and we must solve another equations such as Shallow Water equations [Sad12b]. Another approach is to use the sponge layer conditions as in [Lop*11].

More realistic cases for the Boussinesq system over a variable bottom are consider in [Sad12c] and the influence of the bottom on the wave energy are consider in [Sad12d].

Acknowledgements : This work is supported by the regional program "Appui à l'émergence" of the Region Picardie. I would like to thank my PhD advisor Jean-Paul Chehab (LAMFA, Amiens), Denys Dutykh (LAMA, Savoie), Frédéric Hecht (LJLL, Paris) and Olivier Pantz (CMAP, Paris) for fruitful discussions and remarks.

References

- [Lop*11] NUNO DAVID DE JESUS LOPES, PEDRO JORGE SILVA PEREIRA ET LUÍS MANUEL TRABUCHO DE CAMPOS. A numerical analysis of a class of generalized Boussinesq-type equations using continuous/discontinuous FEM. *Int. J. Numer. Meth. Fluids*, DOI: 10.1002/flid.2631, 2011. [PDF](#). 13
- [Sad11] GEORGES SADAKA. *Etude mathématique et numérique d'équations d'ondes aquatiques amorties. Thèse de l'Université de Picardie Jules Verne - Amiens*, 2011. [PDF](#). 1, 2
- [Sad12a] GEORGES SADAKA. *Solution of 2D Boussinesq systems with FreeFem++ : the flat bottom case. submitted to Journal of Numerical Mathematics*, 2012. [PDF](#). 3
- [Sad12b] GEORGES SADAKA. *Solving Shallow Water flows in 2D with FreeFem++ on structured mesh. In preparation*, 2012. 13
- [Sad12c] GEORGES SADAKA. *Generation, propagation and inundation of a Tsunami wave with FreeFem++*. *In preparation*, 2012. 13
- [Sad12d] GEORGES SADAKA. *Influence of the bottom on the wave energy for the Boussinesq system. In preparation*, 2012. 13
- [Sad12e] GEORGES SADAKA. *FreeFem++, a tool to solve PDEs numerically. This work was done during the CIMPA School - Caracas 16-27 of April 2012, hal-00694787*, 2012. [PDF](#). 5