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# Trajectory Planning and Re-planning Strategies

## Applied to a Quadrotor Unmanned Aerial Vehicle

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### I. Introduction

Path or trajectory planning is a complex problem. It involves meeting the physical constraints of the unmanned vehicles, constraints from the operating environment and other operational requirements [1]. The quadrotor helicopter is one of the unmanned aerial vehicles (UAVs) that has been considered for the trajectory planning problem. However, most of the existing works are optimization-based methods. A time-optimal motion planning approach is proposed in [2] for a non-linear model of a quadrotor helicopter. In [3], adaptive path planning algorithms are developed for an optimized trajectory. Several trajectory optimization algorithms are presented in [4] for a team of cooperating unmanned vehicles. Flatness has been also employed for the trajectory planning of the quadrotor helicopter: a method is presented in [5] to generate time-optimal trajectories for the quadrotor system whereas Cowling *et al.* [6] use the differential flatness to pose the trajectory planning as a constrained optimization problem in the output space.

Posing the trajectory planning as an optimization problem has one major limitation. The on-

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board implementation of the trajectory planning approach requires a microcontroller that is capable of solving a nonlinear optimization problem in real time. Moreover, most of the exciting works on trajectory planning are only illustrated in the simulation framework. The main contribution of this work with respect to existing works is in: a) proposing a simple and effective trajectory planning/re-planning method that can be solved in a straightforward manner without the need to solve a complex optimization problem in real time; and b) successfully flight-testing the approach through an experimental application to a cutting-edge quadrotor helicopter UAV.

The Note is organized as follows. Section II presents the mathematical model of the quadrotor helicopter. Section III presents the trajectory planning/re-planning method where two types of constraints are investigated: flight envelope and actuator limitations. The flight envelope is represented by the pitch and roll angles and the actuator limitations are the maximal thrusts that can be generated by the rotors. To initialize trajectory re-planning in the presence of actuator faults, fault detection and diagnosis problem is also considered. This is achieved by implementing on-board an unscented Kalman filter-based fault detection and diagnosis scheme. Experimental results are given in Section IV followed by conclusions in Section V.

## II. Dynamics of the Quadrotor UAV System

The quadrotor UAV available at the Networked Autonomous Vehicles (NAV) Lab in the Department of Mechanical and Industrial Engineering of Concordia University is the Qball-X4 testbed [7]. It is developed by Quanser Inc. under the financial support of an NSERC (Natural Sciences and Engineering Research Council of Canada) Strategic Project Grant lead by Concordia University with Quanser Inc. as one of three industrial partners. A detailed description of the Qball-X4 as well as its nonlinear model can be found in [8]. In this paper, the following simplified model will be employed for trajectory planning as well as for the design of the baseline LQR controller:

$$\begin{aligned}
\ddot{x} &= \theta g; & J_1 \ddot{\theta} &= u_\theta \\
\ddot{y} &= -\phi g; & J_2 \ddot{\phi} &= u_\phi \\
\ddot{z} &= u_z/m - g; & J_3 \ddot{\psi} &= u_\psi
\end{aligned} \tag{1}$$

This simplified model is obtained by assuming hovering conditions ( $u_z \approx mg$  in the  $x$  and  $y$

directions) with no yawing ( $\psi = 0$ ) and small roll and pitch angles.  $x$ ,  $y$  and  $z$  are the coordinates of the quadrotor UAV center of mass in the earth-fixed frame.  $\theta$ ,  $\phi$  and  $\psi$  are the pitch, roll and yaw Euler angles respectively.  $m$  is the mass,  $g$  is the gravitational acceleration and  $J_i$  ( $i = 1, 2, 3$ ) are the moments of inertia along  $y$ ,  $x$  and  $z$  directions respectively.  $u_z$  is the total lift generated by the four propellers and applied to the quadrotor UAV in the  $z$ -direction (body-fixed frame).  $u_\theta$ ,  $u_\phi$  and  $u_\psi$  are respectively the applied torques in  $\theta$ ,  $\phi$  and  $\psi$  directions. The Qball-X4 is equipped with four outrunner brushless motors driven by pulse-width modulated (PWM) inputs. By assuming that the motor dynamics is a simple gain  $K$ , it is possible to express the relation between the lift/torques and the PWM inputs  $u_i$  as follows:

$$\begin{aligned}
u_z &= K(u_1 + u_2 + u_3 + u_4) \\
u_\theta &= KL(u_1 - u_2) \\
u_\phi &= KL(u_3 - u_4) \\
u_\psi &= KK_\psi(u_1 + u_2 - u_3 - u_4)
\end{aligned} \tag{2}$$

where  $K$  and  $K_\psi$  are positive constants and  $L$  is the distance from the center of mass to each motor.

### III. Flatness-based Trajectory Planning/Re-planning

The objective of the trajectory planning/re-planning is to find the profile of the path that a quadrotor UAV will follow when moving from an initial to a final position while respecting system constraints in nominal and fault conditions. Differential flatness is the first step towards this objective as explained hereafter.

#### A. Differential Flatness of the Quadrotor Helicopter

Flatness can be defined as follows. A general nonlinear dynamic system  $\dot{x} = f(x, u)$ ,  $y = h(x)$  with  $x \in \mathbb{R}^n$  and  $u \in \mathbb{R}^m$ , is flat if and only if there exist variables  $F \in \mathbb{R}^m$  called the flat outputs such that:  $x = \Xi_1(F, \dot{F}, \dots, F^{(n-1)})$ ,  $y = \Xi_2(F, \dot{F}, \dots, F^{(n-1)})$  and  $u = \Xi_3(F, \dot{F}, \dots, F^{(n)})$  [11], [12].  $\Xi_1$ ,  $\Xi_2$  and  $\Xi_3$  are three smooth mappings and  $F^{(i)}$  is the  $i^{\text{th}}$  derivative of  $F$ . The parameterization of the control inputs  $u$  in function of the flat outputs  $F$  plays a key role in the trajectory planning problem: the nominal control inputs to be applied during a mission can be expressed in function

of the desired trajectories. This allows to tune the profile of the trajectories to keep the applied control inputs below the actuator limits. The same idea applies for the roll and pitch angles that the system will achieve during the mission. For the quadrotor UAV simplified model given in (1), the system is flat with flat outputs  $F_1 = z$ ,  $F_2 = x$ ,  $F_3 = y$  and  $F_4 = \psi$ . In addition to  $x$ ,  $y$ ,  $z$  and  $\psi$ , the parameterization of  $\theta$  and  $\phi$  in function of the flat outputs is:

$$\theta = \frac{\ddot{F}_2}{g}; \quad \phi = -\frac{\ddot{F}_3}{g} \quad (3)$$

The parameterization of the control inputs in function of the flat outputs is:

$$u_z = m(\ddot{F}_1 + g); \quad u_\theta = J_1 \frac{F_2^{(4)}}{g}; \quad u_\phi = -J_2 \frac{F_3^{(4)}}{g}; \quad u_\psi = J_3 \ddot{F}_4 \quad (4)$$

Let  $F_i^*$  be the reference trajectory for the flat output  $F_i$  with  $i = 1, \dots, 4$ . An important consequence of the differential parameterization (4) is that if the system is forced to follow the reference trajectories, then the nominal control inputs to be applied along the trajectories are:

$$u_z^* = m(\ddot{F}_1^* + g); \quad u_\theta^* = J_1 \frac{F_2^{(4)*}}{g}; \quad u_\phi^* = -J_2 \frac{F_3^{(4)*}}{g}; \quad u_\psi^* = J_3 \ddot{F}_4^* \quad (5)$$

## B. Trajectory Parametrization

Before considering the trajectory planning problem, the reference trajectory  $F_i^*$  will be first time-parameterized. The total time of the mission is then tuned so that the system constraints are respected. In [9], the authors use second-order systems for the reference trajectories in the framework of spacecrafts in formation flight. They also approximate the natural frequency as function of the rise time to define the transient specifications for the formation. However, they state that it is hard to choose a good rise time beforehand to achieve a good performance in the system. On one hand, if the rise time is small then the formation moves too fast and the formation members could fall far behind their desired positions. On the other hand, if the rise time is big then the formation moves too slowly and the mission cannot be achieved within short time. Inspired by that work, this paper proposes to solve the above-mentioned problem by using differential flatness. Let us first define the reference trajectories  $F_i^*$  ( $i = 1, \dots, 4$ ) as second-order systems:

$$\ddot{F}_i^*(t) = -2\xi\omega_n\dot{F}_i^*(t) - \omega_n^2 F_i^*(t) + \omega_n^2 r_i(t) \quad (6)$$

where  $\xi$  is the damping ratio and  $\omega_n$  is the natural frequency.  $r_i(t)$  ( $i = 1, \dots, 4$ ) are the references along  $z$ ,  $x$ ,  $y$  and  $\psi$  directions respectively and are defined as step functions of amplitude  $R_i$ . Consider a critically damped system by setting  $\xi = 1$  and assuming zero initial conditions, it turns out that the solutions of the reference models given in (6) are:

$$F_i^*(t) = [1 - (1 + \omega_n t) e^{-\omega_n t}] R_i \quad ; \quad i = 1, \dots, 4 \quad (7)$$

This work considers the actuator faults that may take place during the mission. When faults occur it is necessary to re-plan the nominal trajectories. This is because forcing the system to follow the same pre-fault trajectory may lead to saturation or even instability if the new limitations of the actuators are not taken into consideration. Eq. (7) is only valid for zero initial conditions. However, when faults occur and re-planning is needed, the initial conditions at the re-planning instant depend on the quadrotor status and they are not necessarily zero. With a critically damped system ( $\xi = 1$ ) and non-zero initial conditions, the solutions of the reference models given in (6) are:

$$F_i^*(t) = R_i - (1 + \omega_n t) e^{-\omega_n t} \Delta_i + \dot{F}_i^*(t_{rep}) t e^{-\omega_n t} \quad ; \quad i = 1, \dots, 4 \quad (8)$$

where  $\Delta_i = R_i - F_i^*(t_{rep})$  is the remaining distance to the desired position and  $F_i^*(t_{rep})$  and  $\dot{F}_i^*(t_{rep})$  are the position and the velocity of the system at the instant of re-planning  $t_{rep}$ . It can be shown that the presence of the term  $\dot{F}_i^*(t_{rep}) t e^{-\omega_n t}$  in the reference trajectory (due to the non-zero initial velocity) makes it impossible to derive a solution for the trajectory re-planning in a straightforward manner. Nevertheless, it is always possible to find the solution to the trajectory re-planning problem by posing it as an optimization problem where the function to minimize is the mission time and the constraints are those of the flight envelope or the actuator limits. However, the objective of this work is to minimize as much as possible the calculation requirements for the quadrotor on-board microcomputer. To this end, it is proposed to proceed as following. Once a fault is detected by a Fault Detection and Diagnosis (FDD) scheme, the trajectory is re-planned with a sufficiently large mission time. This big mission time will result in a very slow reference trajectory serving in two directions. First, it will give sufficient time to diagnose (isolate and identify) the fault before taking any action since the system will almost stay in hovering position. Second, the system velocity will drop to zero and therefore, the last term of (8) can be set to zero. This allows to easily re-plan the

trajectory as will be shown in the sequel.

### C. Fault Detection and Diagnosis: Unscented Kalman Filter for Parameter Estimation

Actuator faults can be modeled as  $u_i^f = (1 - w_i)u_i$  for  $i = 1, \dots, 4$ .  $w_i$  represents the loss of effectiveness in the  $i^{\text{th}}$  rotor.  $w_i = 0$  denotes a healthy rotor,  $w_i = 1$  denotes a complete loss of the  $i^{\text{th}}$  rotor and  $0 < w_i < 1$  represents a partial loss of control effectiveness. As will be shown in the subsequent sections, trajectory re-planning in fault cases requires the knowledge of the fault amplitude  $w_i$ . Thus, an FDD module is needed to detect, isolate and identify the fault. This is achieved by employing a parameter estimation-based FDD where the unscented Kalman filter [13] is used to estimate the parameters  $w_i$ . It should be noted that for FDD purpose, the nonlinear model [8] is employed. In the fault-free case, the estimates of  $w_i$  ( $i = 1, \dots, 4$ ) are close to 0 and a deviation from 0 will be an indication for a fault occurrence. In the subsequent sections, the trajectory planning/re-planning approach will be explained for two cases: flight envelope constraints and actuator constraints.

### D. Trajectory Planning/Re-planning Under Flight Envelope Constraints

A trajectory with a small travel time involves aggressive maneuvers with big roll and pitch angles. A controller based on the nonlinear model would be able to guarantee the system stability. However, a controller based on a linearized model (such as (1)) may not be able to keep system's stability when the quadrotor goes outside the linear operating zone. To avoid this latter situation, it is possible to impose bounds on the maximal pitch and roll angles that are attained during the system's travel from the initial to the final position. According to (3), the pitch angle of the system when it is forced to follow the reference trajectory is given by:

$$\theta^* = \frac{\ddot{F}_2^*}{g} \quad (9)$$

Using (9) and the double time derivative of (7), one can find the nominal pitch angle  $\theta^*$  in function of the time  $t$ , the natural frequency  $\omega_n$ , and the amplitude  $R_2$  as following:

$$\theta^* = -\frac{\omega_n^2}{g} (t\omega_n - 1) e^{-\omega_n t} R_2 \quad (10)$$

For a critically damped second-order system ( $\xi = 1$ ) and for a settling time specification of 2% of the steady state value, it is possible to approximate the natural frequency as:

$$\omega_n \approx \frac{5.83}{t_s} \quad (11)$$

where  $t_s$  is the settling time of the reference model. Eq. (10) reads then:

$$\theta^* = -\frac{5.83^2}{gt_s^2} \left( \frac{5.83t}{t_s} - 1 \right) R_2 e^{-\frac{5.83t}{t_s}} \quad (12)$$

It is necessary at this stage to find a relation between the settling time  $t_s$  and the total time of the mission  $t_t$ . For a critically damped system ( $\xi = 1$ ), it is reasonable to assume that the settling time approximately equals the total time of the mission ( $t_s \approx t_t$ ). The objective turns out to determine  $t_s$  such that  $-\rho_\theta \theta_{max} \leq \theta^* \leq \rho_\theta \theta_{max}$  where  $\theta_{max}$  is the maximal pitch angle that the system is allowed to make during its travel. Since (9) is derived using the simplified model (1), the parameter  $0 < \rho_\theta < 1$  is introduced to create a safety margin to deal with model uncertainties. One way to calculate  $t_s$  so that  $-\rho_\theta \theta_{max} \leq \theta^* \leq \rho_\theta \theta_{max}$  is to determine where the maximal pitch is taking place and then tune  $t_s$  such that the angle constraints are respected. To determine where the maximal pitch is taking place, let us calculate first the time derivative of the pitch angle with respect to time:

$$\frac{d\theta^*}{dt} = \frac{5.83^3}{gt_s^3} \left( \frac{5.83t}{t_s} - 2 \right) R_2 e^{-\frac{5.83t}{t_s}} \quad (13)$$

By setting (13) to zero, one finds that the maximal pitch angle is taking place at  $t = 2t_s/5.83$ .

Plugging this value of  $t$  in (12) gives the extrema:

$$\theta_{Ext}^* = -\frac{5.83^2}{gt_s^2 e^2} R_2 \quad (14)$$

where the extrema collectively denote the minima and the maxima of a function. It is also necessary to check the value of the pitch angle at the beginning and at the end of the mission, i.e. for  $t = 0$  and  $t = t_s$ . These are given by:

$$\theta^*(0) = \frac{5.83^2}{gt_s^2} R_2 \quad \text{and} \quad \theta^*(t_s) = -\frac{4.83 \times 5.83^2}{gt_s^2 e^{5.83}} R_2 \quad (15)$$

Finally, to ensure that  $|\theta^*| \leq \rho_\theta \theta_{max}$  three solutions are obtained from (14) and (15):

$$t_s \geq \sqrt{\frac{5.83^2 |R_2|}{g\rho_\theta \theta_{max}}} \quad ; \quad t_s \geq \sqrt{\frac{5.83^2 |R_2|}{ge^2 \rho_\theta \theta_{max}}} \quad \text{and} \quad t_s \geq \sqrt{\frac{4.83 \times 5.83^2 |R_2|}{ge^{5.83} \rho_\theta \theta_{max}}} \quad (16)$$



which respectively corresponds to  $t = 0$ ,  $t = 2t_s/5.83$ , and  $t = t_s$ . The solution to consider is the maximal one among the three possible solutions. It is easy to see however that the solution corresponding to  $t = 0$  is the maximal one and hence it is the one to be considered. Similarly to the study carried out for the pitch angle  $\theta$ , it is possible to derive the solution for the roll angle  $\phi$  which is associated to the reference  $r_3(t)$  along the  $y$ -direction as following:

$$t_s \geq \sqrt{\frac{5.83^2 |R_3|}{g\rho_\phi\phi_{max}}} \quad (17)$$

In the fault case, the trajectory re-planning can be achieved by replacing  $R_2$  and  $R_3$  in (16) and (17) with the remaining distances to travel  $\Delta_2 = R_2 - F_2(t_{rep})$  and  $\Delta_3 = R_3 - F_3(t_{rep})$  respectively.

**Remark 1** *The solution obtained under flight envelope constraints does not explicitly consider the actuator limits. Therefore, it is necessary to calculate  $t_s$  by considering actuator constraints as explained in the subsequent section. If the solution obtained under actuator constraints is bigger than the one obtained under flight envelope constraints then the former must be considered since the actuator constraints are more restrictive than those of the flight envelope.*

### E. Trajectory Planning/Re-planning Under Actuator Constraints

If aggressive maneuvers are allowed, the constraints on the pitch and roll angles can be ignored. However, it is necessary to consider actuator constraints. When actuators hit their limits and cannot deliver the actuation inputs desired by the controller this results in degraded performance or can even lead the closed-loop system to an unstable behavior [10]. The control inputs to be applied along the reference trajectories are given in (5). In this work, the system is assumed to not changing altitude during the mission and thus  $F_1^*$  is constant and  $\ddot{F}_1^* = 0$ . It is also assumed that the system is not yawing, thus  $F_4^* = 0$  and  $\ddot{F}_4^* = 0$ . By using (5), the control inputs are then:

$$u_z^* = mg; \quad u_\theta^* = J_1 \frac{F_2^{(4)*}}{g}; \quad u_\phi^* = -J_2 \frac{F_3^{(4)*}}{g}; \quad u_\psi^* = 0 \quad (18)$$

The relation between the lift/moments and the PWM inputs is given by (2). Taking the inverse of (2) and using (18) gives the PWM inputs to be applied along the reference trajectories. For instance, the first PWM input  $u_1^*(t)$  is:

$$u_1^*(t) = \frac{1}{4K}mg + \frac{J_1}{2gKL}F_2^{(4)*}(t) \quad (19)$$

Since  $F_2^*(t)$  is given in (7) it is possible to calculate the fourth derivative of  $F_2^*(t)$  with respect to time. Then, approximating the natural frequency as  $\omega_n \approx 5.83/t_s$  gives the nominal PWM inputs in function of  $t$  and  $t_s$  as:

$$u_1^*(t) = \frac{1}{4K}mg - \frac{J_1}{2gKL} \frac{5.83^4}{t_s^4} \left( \frac{5.83t}{t_s} - 3 \right) R_2 e^{-\frac{5.83t}{t_s}} \quad (20)$$

The objective is to determine the settling time  $t_s$  so that the nominal PWM inputs are less than the maximal allowable input  $u_{max}$ , i.e.  $u_i^*(t) \leq u_{max} \forall t \in [0, t_s]$  for  $i = 1, \dots, 4$ . Similarly as before, the extrema of  $u_i^*(t)$  must be determined as well as its terminal values at  $t = 0$  and  $t = t_s$ . For  $u_1^*(t)$ , the extrema is taking place at  $t = 4t_s/5.83$  and its value is:

$$u_{1Ext}^* = \frac{1}{4K}mg - \frac{J_1}{2gKL} \frac{5.83^4}{e^4 t_s^4} R_2 \quad (21)$$

The terminal values at  $t = 0$  and  $t = t_s$  are:

$$u_1^*(0) = \frac{1}{4K}mg + \frac{3J_1}{2gKL} \frac{5.83^4}{t_s^4} R_2 \quad \text{and} \quad u_1^*(t_s) = \frac{1}{4K}mg - \frac{J_1}{2gKL} \frac{2.83 \times 5.83^4}{t_s^4 e^{5.83}} R_2 \quad (22)$$

From (21) and (22), one can determine  $t_s$  for which  $u_1^*(t) \leq \rho_u u_{max}$ . That is:

$$t_s \geq \max \left\{ \left( -\frac{5.83^4 J_1 R_2}{2gKL e^4 \bar{u}_{max}} \right)^{\frac{1}{4}}; \left( \frac{3 \times 5.83^4 J_1 R_2}{2gKL \bar{u}_{max}} \right)^{\frac{1}{4}}; \left( -\frac{2.83 \times 5.83^4 J_1 R_2}{2gKL e^{5.83} \bar{u}_{max}} \right)^{\frac{1}{4}} \right\} \quad (23)$$

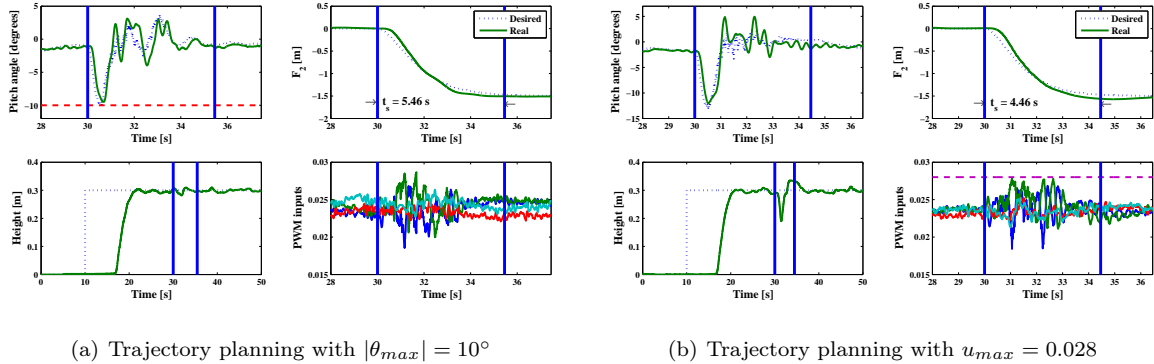
where  $\bar{u}_{max} = \rho_u u_{max} - mg/4K$  and  $\rho_u$  works similarly to  $\rho_\theta$ . Similar development can be carried out for the three remaining PWM inputs  $u_2^*(t)$ ,  $u_3^*(t)$  and  $u_4^*(t)$ . After ignoring the complex values of  $t_s$ , four solutions are obtained and the maximal one is the solution to be considered.

Trajectory re-planning consists in solving (23) by replacing  $R_2$  with the remaining distance to travel  $\Delta_2 = R_2 - F_2(t_{rep})$  and  $\bar{u}_{max}$  with the new actuator constraint  $\bar{u}_{max}^1 = (1 - \hat{w}_1)\rho_u u_{max} - mg/4K$ .  $\hat{w}_1$  is the estimate of effectiveness loss in the first actuator provided by the unscented Kalman filter of Section III C. Based on the obtained solution  $t_s$ , the damaged system must decide whether to continue the mission, return to the base or land safely while taking into consideration the remaining fuel or battery power and/or the attainable speed.

#### IV. Experimental Results

The trajectory planning/re-planning approach is applied to the quadrotor UAV testbed. The baseline controller for the Qball-X4 is an LQR controller which is designed based on the simplified

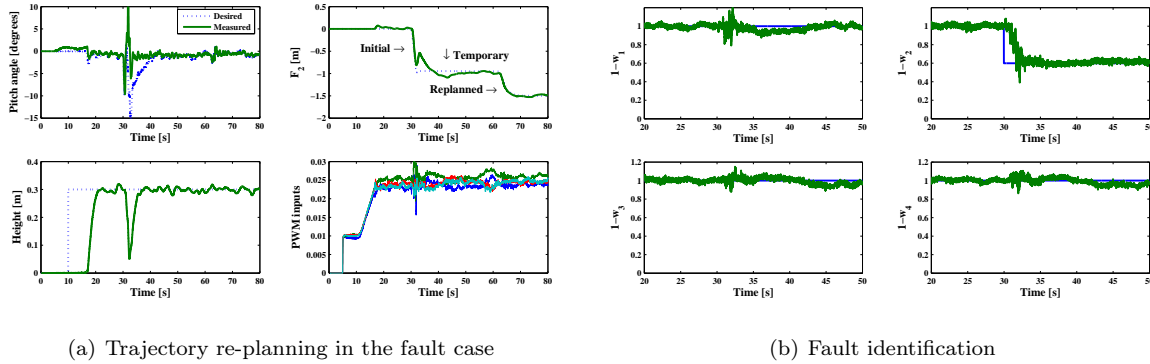
model (1). The fault-free trajectory planning is illustrated in Figures 1(a) and 1(b) with  $R_2 = -1.5 \text{ m}$  and  $R_3 = 0$ . The figures show the pitch angle and position tracking as well as the system's height which is set to  $0.3 \text{ m}$  and the PWM inputs to the four rotors. The reference trajectory along the  $x$ -direction is given by (7) whereas the desired pitch angle is generated by the LQR controller and its evolution is predicted in (9) thanks to flatness. Figure 1(a) considers the trajectory planning when the flight envelope is restricted to  $|\theta_{max}| = 10^\circ$ . The total time of the mission is  $t_s = 5.46 \text{ s}$ . It is clear that when the system is forced to follow the reference trajectory, the amplitude of the pitch angle  $\theta$  does not exceed  $10^\circ$  as required. One can see that due to model mismatch, the experimental application does not provide one extrema as found in the theoretical development. One can also notice that the height tracking is not affected during the mission since the LQR controller provides good performance when the pitch angle is restricted to  $|\theta_{max}| = 10^\circ$ . Figure 1(b) considers the trajectory planning when the actuator constraints are considered. The maximal PWM input is  $u_{max} = 0.028$  and the total time of the mission is  $t_s = 4.46 \text{ s}$ . This experiment shows a degradation in the height tracking performance due to the limitation of the baseline controller.



**Fig. 1 Trajectory planning under flight envelope and actuator constraints**

The trajectory re-planning is illustrated in Figure 2(a) where a 40% loss of control effectiveness is taking place in the second motor ( $w_2 = 0.4$ ) at time instant  $t = 31.5 \text{ s}$ . The fault makes the system to loose its height and to fall behind the reference trajectory. This sudden jump in the tracking error along  $x$  and  $z$  directions indicates an eventual occurrence of a fault. Once the fault is detected, the trajectory is temporarily re-planned at about  $t = 32 \text{ s}$  with a  $t_s = 900 \text{ s}$ . Figure 2(b) shows that the fault is successfully identified where  $1 - w_2 = 0.6$  and  $1 - w_1 = 1 - w_3 = 1 - w_4 = 1$ . The temporary trajectory lasts (for illustration purpose) for about  $30 \text{ s}$  where the system is held in

place before re-planning the trajectory with  $t_s = 7$  s. The re-planned trajectory is slower than the initial pre-fault trajectory to take into consideration the new actuator limits after fault occurrence.



**Fig. 2 Trajectory re-planning and fault identification**

## V. Conclusion

This paper proposes a trajectory planning/re-planning approach where reference trajectories are defined as second-order systems. The main advantage of the proposed approach is that solutions are obtained in a straightforward manner by solving simple equations in the fault-free and fault conditions. Therefore, it has very few calculation requirements and it is very suitable for systems with limited calculation capabilities such as small-scale UAVs with single-board microcomputers.

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## References

- [1] Tsourdos, A., White, B. A., and Shanmugavel, M., *Cooperative Path Planning of Unmanned Aerial Vehicles*, John Wiley & Sons, 2010, Progress in Astronautics and Aeronautics Series.
- [2] Lai, L. C., Yang, C. C., and Wu, C. J., “Time-optimal Control of a Hovering Quad-rotor Helicopter,” *Journal of Intelligent and Robotic Systems*, Vol. 45, No. 2, 2006, pp. 115–135.

- [3] Meister, O., Frietsch, N., Ascher, C., and Trommer, G. F., “Adaptive Path Planning for a VTOL-UAV,” *IEEE Aerospace and Electronic Systems Magazine*, Vol. 24, No. 7, 2009, pp. 36–41.
- [4] Kuwata, Y., *Trajectory Planning for Unmanned Vehicles Using Robust Receding Horizon Control*, Ph.D. Dissertation, Massachusetts Institute of Technology, 2007.
- [5] Bouktir, Y., Haddad, M., and Chettibi, T., “Trajectory Planning for a Quadrotor Helicopter,” *16th Mediterranean Conference on Control and Automation*, Ajaccio, Corsica, France, 25-27 June 2008, pp. 1258–1263.
- [6] Cowling, I. D., Whidborne, J. F., and Cooke, A. K., “Optimal Trajectory Planning and LQR Control for a Quadrotor UAV,” *Proceedings of the UKACC International Conference on Control*, Glasgow, UK, 30 August-1 September 2006.
- [7] NAV Laboratory: <http://users.encs.concordia.ca/~ymzhang/UAVs.htm> [cited 15 October 2011].
- [8] Amoozgar, M. H., Chamseddine, A., Zhang, Y. M., “Fault-Tolerant Fuzzy Gain Scheduled PID for a Quadrotor Helicopter Testbed in the Presence of Actuator Faults,” *IFAC Conference on Advances in PID Control*, Brescia, Italy, 28-30 March, 2012.
- [9] Ren, W., and Beard, R. W., “Virtual Structure Based Spacecraft Formation Control with Formation Feedback,” *AIAA Guidance, Navigation, and Control Conference and Exhibit*, Monterey, California, USA, 5–8 August 2002.
- [10] Henrion, D., Tarbouriech, S., and Vladimír, K., “Control of Linear Systems Subject to Input Constraints: a Polynomial Approach,” *Automatica*, Vol. 37, 2001, pp. 597–604.
- [11] Fliess, M., Lévine, J., Martin, P., and Rouchon, P., “Flatness and Defect of Nonlinear Systems: Introductory Theory and Example,” *International Journal of Control*, Vol. 61, No. 6, 1995, pp. 1327–1361.
- [12] Faiz, N., Agrawal, S. K., and Murray, R. M., “Trajectory Planning of Differentially Flat Systems with Dynamics and Inequalities,” *AIAA Journal of Guidance, Control and Dynamics*, Vol. 24, No. 2, 2001, pp. 219–227, doi: 10.2514/2.4732.
- [13] Julier, S. J., and Uhlmann, J. K., “A New Extension of the Kalman Filter to Nonlinear Systems,” *Proceedings of AeroSense: the 11th International Symposium on Aerospace/Defence Sensing, Simulation and Controls*, Orlando, Florida, USA, 20-25 April 1997, pp. 182–193.