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Behavioral Heterogeneity in the Option Market

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Abstract

This paper develops and tests a heterogeneous agents model for the option market. Our agents have different beliefs about the future level of volatility of the underlying stock index and trade accordingly. We consider two types of agents: fundamentalists and chartists, who are able to switch between groups according to a multinomial logit switching rule. The model simplifies to a GARCH-type specification with time-varying parameters. Estimation results for DAX30 index options reveal that different types of traders are actively involved in trading volatility. Our model improves frequently used standard GARCH-type models in terms of pricing performance.

Keywords: Heterogeneous Agents, Option Markets, Fundamentalists, Chartists, GARCH.

JEL-Classification: G12, C15

1 Introduction

Volatility is priced and traded in the options market. For example, if option traders believe that markets will become more volatile, they buy at-the-money puts and calls (a long straddle) because option values usually increase when volatility increases. Their buying activity drives option prices up, which in turn causes the implied volatility to increase (Garleanu et al., 2009). Vice versa, if they believe that volatility is overpriced, they short a straddle (sell puts and calls) and their selling decreases the price and lowers implied volatility. Therefore, traders' expectations about future volatility affect their demand for options and their different expectations creates uncertainty about the 'fair value' of volatility and causes volatility itself to become volatile. Models that capture such volatility in volatility, such as stochastic volatility models, are particularly

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important for the pricing performance of the volatility model (Christoffersen and Jacobs, 2004). However, so far no model has been introduced that models these different expectations about future volatility.

In this paper, we propose an alternative model to price options. This model assumes that option traders have heterogeneous beliefs about future volatility and it is this heterogeneity that affects the pricing of options. To implement this model we follow the heterogeneous agents literature, elaborately surveyed by Hommes (2006), which often assumes that two different types of traders are active in the market, where some traders (termed fundamentalists) trade on the long-term mean-reversion towards its fundamental value while other traders (called chartists) trade on short-term patterns. In our case, fundamentalists trade on long-run mean-reversion of conditional volatility to the unconditional volatility, while chartists trade on exogenous shocks from the level process. Additionally, in our model, each trader can decide on her/his strategy based on its performance relative to the other trader's strategy. Since the strategies do not demand any particular skill or knowledge, traders can change their type conditional on the relative performance. As such, the market consists of a weighted average of two simple, boundedly rational, strategies.

Interestingly, when combining the strategies of both trader types into a single model we find that our model reduces to an asymmetric GARCH model with time-varying coefficients, where the time-variation is due to changes in the proportions of fundamentalists and chartists present. This time variation introduces an interesting feature from a GARCH modeling perspective, because it allows the volatility process to be locally unstable (as in an IGARCH model) while guaranteeing global stability. When chartists dominate the market, their persistence may cause the volatility process to be unstable in the short run. When the proportion of fundamentalists increases, their presence ensures that the volatility process becomes stable again. As long as sufficient fundamentalists are present, the volatility process will remain stable and the process can switch between stable and unstable phases depending on the preferred trading strategy. Unconditionally, since the market is divided evenly between fundamentalists and chartists, stability is guaranteed in the long run as long as standard stability conditions are met. Interestingly, this provides an economic and behavioral interpretation to the notion of volatility clustering and GARCH-dynamics often observed in financial market volatility.

When empirically testing our model on option prices using the GARCH-option pricing methodology of Duan (1995), we find evidence that supports the presence of both types of traders. Over time, the fractions of fundamentalist and chartist traders change conditional on how well each trading strategy performs. Subsequently, we present evidence that our model outperforms a standard model without switching in terms of pricing performance both in-sample and out-of-sample, for all maturity - strike combinations.

We contribute to the literature on three different grounds. First, we contribute to the heterogeneous agents literature by focusing on volatility traders; specifically, by assuming heterogeneity in beliefs about the volatility process and by allowing agents to switch between different strategies instead of assuming fixed proportions. In addition, empirical studies on heterogeneous agents models are scarce; we try to fill this void. Second, we contribute to the GARCH literature by giving an economic interpretation of GARCH-dynamics. By proposing this specific setup of a GARCH, we provide an intuitive and parsimonious model-

ing approach to time varying parameters. Third, we contribute to the literature on option pricing as our model comfortably outperforms a standard GARCH model often used to price options.

The remainder of the paper is organized as follows. Section 2 reviews the literature on heterogeneous agent models. In Section 3, we present the econometric framework of our model. Section 4 presents the data and methodology of the estimation procedure. In Section 5, we show the results, and Section 6 concludes.

2 Heterogeneous Agents Models

Evidence against the Efficient Market Hypothesis (EMH, see Fama, 1971) has been mounting in the past decades. In a broad range of markets and using a similarly broad range of techniques, researchers have found evidence against the notion of rationality in financial markets. Examples of these are studies using observed (survey) expectations which frequently reject the notion of rationality (see e.g., Frijns et al., 2008 for experimental evidence or MacDonald, 2000 for an overview). A second example is the existence of numerous anomalies in financial markets. Phenomena like excess volatility, small firm effects, overshooting, and the January effect cannot be explained by representative agent rational expectation models. Theoretically, one can also cast doubt on the EMH. No trade theorems (Milgrom and Stokey, 1982) hypothesize that rational agents will never trade because expectations are equal and all information is discounted in the current market price. Therefore, price changes occur without trade taking place. This is hard to combine with the observation of enormous trading volumes.

One of the responses to the demise of the EMH is the emergence of the behavioral finance literature (Kahneman and Tversky, 1979 or Barberis et al., 1998, *inter alii*). Behavioral finance relaxes the notion of rationality and introduces elements from social psychology to economic decision making. Divergence from the assumption of rationality implies that one can introduce heterogeneity in expectations, i.e. there is only one way of behaving rationally, while there are infinite ways of behaving irrationally or boundedly rational. In general, the literature provides three compelling reasons for heterogeneity in expectations. First, there may be an asymmetry in the information available to market participants, where it is often assumed that part of the information is common to all market participants and part is private. This concept of asymmetric information was first introduced in the New Classical Theory of the macro economy, where agents were unable to obtain information that is public in other parts of the economy. The idea of informational asymmetry among market participants has been particularly well received in the market microstructure literature (e.g., Kyle, 1985). Second, even with symmetric information, agents may interpret this information differently. One argument for this explanation is provided by Kurz (1994), which suggests that economic agents do not know the underlying structural relations of the economy. Agents only have ‘information’ or ‘empirical knowledge’ that is readily observable from the economy. Finally, heterogeneity in expectations may exist due to the existence of fundamentally different types of agents. De Long et al. (1990), for example, propose a model that illustrates that different types of traders can coexist, and that rational investors do not necessarily drive out noise traders. Frankel and Froot (1986, 1990) popular-

ize the view that the (foreign exchange) market is dominated by two types of market participants which differ in the information they use to form their expectations. Fundamentalists think of the exchange rate as an economic model, while chartists predominantly use the exchange rate's history to form their expectations.

The literature on heterogeneous agents models, or HAMs, continues on the line of thought that there can be fundamentally different types of agents (see Brock and Hommes, 1997, 1998, Chiarella and He, 2001, 2002, Lux, 1998, Lux and Marchesi, 2000, De Grauwe and Grimaldi, 2005, 2006, LeBaron et al., 1999). See also Hommes (2006), LeBaron (2006), Chiarella et al. (2009) and Lux (2009) for overviews. This literature, applied to financial markets, aims to describe stock price evolution by relaxing the assumption of homogeneity among investors. By allowing for heterogeneity among investors, different trader types can be classified along with their potential strategies. Given those classifications, one can evaluate the likelihood of these traders being active and the consequences of their trading for the price and volatility process. As first introduced by Frankel and Froot (1986) traders are typically classified into two categories: fundamentalists, who trade based on fundamentals; and chartists, who trade on observed patterns in past prices. Revolutionary in the models, first introduced by Brock and Hommes (1997, 1998), is that agents are not only different, but are also able to switch between types conditional on performance. This switching introduces a non-linearity that mixes different regimes, based on economic foundations.

Up until now, the majority of studies on HAMs have been conducted in experimental settings. Using either deterministic or stochastic simulation techniques, the presence of different trader types in financial markets can explain some stylized facts of returns in financial markets (see e.g. De Grauwe and Grimaldi, 2005, 2006, Lux, 1998, Alfarano et al., 2005, and He and Li, 2007, 2008). The irregular switching between types induces volatility clustering, heavy tails, slow mean-reversion (persistence), and excess volatility. However, to our knowledge there are only a handful of empirical applications that attempt to estimate a HAM with full-fledged switching mechanism. Those attempts that have been made consider the S&P 500 market index (see Boswijk et al., 2007), commodity markets (Reitz and Westerhoff 2003, 2005, 2007) and EMS exchange rates and Asian equity markets (De Jong et al., 2009a,b). All these studies report significant evidence of heterogeneity among traders and switching between strategies.

Several studies have looked at the impact of heterogeneous beliefs on options prices. In a theoretical setting, Shefrin (2001) shows how heterogeneity in beliefs affects option prices (i.e. leading to different prices than Black-Scholes), and can explain the smile observed in implied volatilities. In a two-trader example, he shows how traders with different beliefs come up with different option prices and that the equilibrium option price is a weighted average of the two different option prices. Several papers focusing on heterogeneity suggest that traders with more pessimistic (optimistic) views will be attracted to out of the money put (call) options (e.g. Benninga and Mayshar, 2000 and Buraschi and Jilstov, 2006). More specifically, studies that focus on heterogeneity among option traders have focused on the heterogeneous beliefs they hold regarding the fundamentals that determine the option price. For example, Benninga and Mayshar (2000) focus on heterogeneity in traders' risk aversion and find that their model

explains the empirically observed smile in implied volatility. Buraschi and Jilstov (2006) propose a theoretical model with model uncertainty and differences in beliefs about dividend growth. Using this model, they empirically investigate the impact of differences in beliefs regarding market fundamentals on the dynamics of option trading and open interest. They find that their model cannot be rejected by the data, produces smaller hedging errors than those produced by the Black-Scholes and Heston (1993) model and can explain the observed smile in option prices. Rzepkowski (2001) proposes a model with heterogeneous expectations in the foreign exchange market and finds empirical evidence of the presence of traders with heterogeneous beliefs about future exchange rates and different levels of confidence in their forecasts in an option pricing context. Ziegler (2002) models two types of agents who differ in their initial beliefs on the dividend process and investigates the effect on option prices.

The literature on heterogeneous expectations in volatility is less developed. Trippi and Harriff (1993) note that “traders who are not interested in the underlying asset except for the relative misalignment of implied volatility across options are significant participants in most option market” (p. 343). They therefore focus on the dispersion in implied volatility and find that it is an important variable for explaining traded volume in the S&P 100 stocks. Guo (1998) introduces heterogeneity in his model by assuming that option traders have heterogeneous expectations about the parameters of the lognormal process of the underlying asset price. Estimation results for S&P 500 index call options indicate that there are two groups: bulls and bears. However, there are no studies on full-fledged HAMS applied to the options market, to our best knowledge.

3 The Econometric Framework

In this section, we develop an econometric model based on the notion that different types of traders are active in the options market. The complete derivation of the theoretical model is in the Appendix. We first define the different strategies option traders may follow and subsequently combine these strategies to define the conditional volatility process. We show that the combined volatility process reduces to an asymmetric GARCH model with time-varying coefficients.

Let S_t be the value of an underlying asset at time t . Then, in a Gaussian discrete-time economy, the (log) return process of the asset (r_t) is given as

$$\begin{aligned} r_t &= \ln \left(\frac{S_t}{S_{t-1}} \right) + d_t = \mu + \sqrt{h_t} \varepsilon_t, \\ \varepsilon_t | \Omega_{t-1} &\sim N(0, 1) \text{ under probability measure } P, \end{aligned} \quad (1)$$

where d_t is the asset’s dividend yield, μ is the mean of r_t , h_t is the conditional volatility of the asset, ε_t is a standard normal random variable and Ω_{t-1} is the information set up to time $t - 1$. In this paper, we focus on explaining the process for h_t .

Assume that traders with purely speculative motives populate the market for options, and that these traders can be classified into two different groups that have different expectations about the future evolution of h_t . Fundamentalists, who trade on the principle of mean reversion and chartists, who trade on

exogenous shocks. While individuals can enter and exit the market at any time, the total population of active traders is always separable into these two groups.

Let $E_t^F(h_{t+1})$ be the prediction of the conditional volatility for the fundamentalists. These fundamentalists are assumed to trade based on mean-reversion, where they expect the conditional volatility to mean-revert to the unconditional volatility. Their best prediction for the volatility process is

$$E_t^F(h_{t+1}) = h_t - (1 - \alpha)(h_t - \bar{h}_t), \quad (2)$$

where \bar{h}_t is the slow moving long-run unconditional volatility and α measures the speed at which the fundamentalists expect the volatility process to mean-revert¹. Since volatility needs to remain positive with probability 1, α can range between $(-1, 1)$ for (2) to be stable, but is expected to be between $[0, 1)$ such that h_t reverts monotonically to \bar{h}_t . When $\alpha \rightarrow 1$ the process becomes very persistent and little mean-reversion takes place. When $\alpha \rightarrow 0$ the process reverts to the unconditional volatility almost instantaneously. Equation (2) reveals that the expectations of fundamentalists essentially follow a GARCH(1,0), not taking into account any shocks in the volatility process. Since volatility is the only (unknown) determinant of option prices and our traders are pure speculators, fundamentalists' demand for options is solely driven by their expectations about the change in volatility. When they expect volatility to increase, they increase their demand for options and vice versa. Hence, fundamentalists' demand for options is positively related to the change in their volatility expectations.

Chartists do not believe in mean reversion, but trade on recently observed unexpected shocks. Given the current level of volatility, they use recently observed unexpected shocks to the return process to predict the future level of conditional volatility. Given that conditional volatility behaves differently in the presence of positive or negative shock, and chartists may interpret positive and negative shock differently, we allow for an asymmetric impact of these shocks as in the GJR-GARCH model, see Glosten, Jagannathan and Runkle (1993). We define their prediction of the volatility process as

$$E_t^C(h_{t+1}) = h_t + \beta_0(\sqrt{h_t}\varepsilon_t^+)^2 + \beta_1(\sqrt{h_t}\varepsilon_t^-)^2, \quad (3)$$

where $E_t^C(h_{t+1})$ is the volatility prediction of the chartists, ε_t^+ (ε_t^-) is the past positive (negative) shock in the volatility process and β_0 (β_1) measures the extent to which chartists incorporate positive (negative) shocks into their prediction. The asymmetry between positive and negative shocks captures the leverage effect, hence we expect $\beta_0 < \beta_1$. Following the same reasoning as for fundamentalists, chartists' demand for options is a positive function of their change in expectations.

Because we have defined a market where only two types of traders are present, the conditional volatility that is observed in the market (h_{t+1}) is a function of the predictions of chartists and fundamentalists and the proportion by which each trader type is present in the market. Since both strategies involve no particular skill or information from traders, traders can switch to either strategy at any point in time without incurring transaction costs. Let w_t be the fraction of fundamentalists present in the market. Then a natural choice for w_t

¹When empirically implementing the model, we assume that \bar{h}_t is equal to the variance of the underlying return series calculated over the previous 250 trading days with a moving window.

is a rule that considers the relative accuracy or pricing error of following a fundamentalist strategy². We define w_t as a multinomial logit switching rule (see Brock and Hommes, 1997, 1998), where the switching depends on the absolute forecast error of fundamentalists versus chartists. The switching rule is given as

$$w_t = \left(1 + e^{\gamma \left(\left| \frac{\ln(E_{t-1}^F(h_t)) - \ln(h_t)}{\ln(h_t)} \right| - \left| \frac{\ln(E_{t-1}^C(h_t)) - \ln(h_t)}{\ln(h_t)} \right| \right)} \right)^{-1}, \quad (4)$$

where γ measures the sensitivity of market participants (fundamentalists or chartists) to their respective percentage forecast errors in terms of log volatility and is expected to be between 0 and infinity. This sensitivity parameter can be interpreted as the status quo bias of traders, as it governs the reaction speed of traders to profit differences, or, in other words, the aversion of people to give up what they have and know well; see Kahneman et al. (1991). With $\gamma = 0$, agents are distributed uniformly across trader types. As γ increases agents become increasingly sensitive to differences in forecasting performance between the strategies. In the limiting case ($\gamma \rightarrow \infty$) all agents directly switch to the more profitable rule, such that w_t is either 0 or 1. Given this definition, w_t will always be strictly bounded between 0 and 1.

The particular specification of w_t ensures that the more accurate strategy in the previous period attracts more market participants in the following period. Therefore, if fundamentalists predict volatility more accurately than the chartist in period t , the weight w_{t+1} , will increase. Consequently, a better prediction of the chartists reduces w_{t+1} .

Note that the switching rule does not imply that each individual changes its strategy every single day. Theoretically, traders have the opportunity to do so, but the frequency at which strategies are actually updated depends on the magnitude of γ . The advantage of the current setup is that it provides the most flexibility in terms of behavior. Furthermore, the weights reflect the behavior of the population as a whole, and therefore do not necessarily imply anything about the behavior of individual traders. The fact that individuals can enter and exit the market at any time has an effect on the relative magnitude of the population, not the distribution of individuals over groups.

With the given weights and the different trading strategies, we can now establish the process for the conditional volatility. Since the conditional volatility is a consequence of buying and selling of options, and the demand for options is a function of volatility expectations, it can be computed as a weighted average of the fundamentalist and the chartist volatility prediction. This mechanism can be seen as the excess demand of the two groups being combined into market excess demand for options. The market maker transforms market excess demand into changes in the level of volatility, as in Chiarella and He (2002)³. Hence, there is a direct mapping from expectations to volatility, i.e.,

$$h_{t+1} = w_t E_t^F(h_{t+1}) + (1 - w_t) E_t^C(h_{t+1}) \quad (5)$$

²An example for the definition of w_t is the profits fundamentalists make relative to the chartist on an option strategy that involves straddles. When fundamentalists expect volatility to increase, they will go long in a straddle and vice versa. If their strategy pays off well relative to the chartists' strategy, more traders may be inclined to follow this fundamental strategy and hence the proportion of fundamentalists will increase. If their strategy performs poorly relative to chartists, more traders will be inclined to follow a chartist strategy in the future.

³See the Appendix for the exact derivation of this mechanism.

Equation (5) defines the conditional volatility process and reveals that this is an average of the volatility predictions of chartists and fundamentalists, weighted by the proportion of market participants following each strategy. Subsequently, we provide an economic interpretation of (5), by substituting (2) and (3) into (5). After rewriting, we obtain

$$h_{t+1} = -w_t \alpha \bar{h}_t + (1 + w_t \alpha) h_t + (1 - w_t) \left(\beta_0 (\sqrt{h_t} \varepsilon_t^+)^2 + \beta_1 (\sqrt{h_t} \varepsilon_t^-)^2 \right), \quad (6)$$

or

$$h_{t+1} = \bar{h}'_t + \alpha_t h_t + \beta_{0,t} (\sqrt{h_t} \varepsilon_t^+)^2 + \beta_{1,t} (\sqrt{h_t} \varepsilon_t^-)^2, \quad (7)$$

where $\bar{h}'_t = -w_t \alpha \bar{h}_t$, $\alpha_t = (1 + w_t \alpha)$, $\beta_{0,t} = (1 - w_t) \beta_0$ and $\beta_{1,t} = (1 - w_t) \beta_1$. Equation (7) shows that the model essentially reduces to a GJR-GARCH(1,1) model with time-varying coefficients. The time variation in these coefficients is driven by the profitability of being a fundamentalist or a chartist.

Apart from this time variation, there are several interesting features about our model. First, we base our model on behavioral considerations and reduces to a time-varying GJR-GARCH, we can provide an economic interpretation of the GARCH model. Up until now, the GARCH model has mainly been motivated by the empirical observation of time variation in conditional volatility. The model proposed provides an economic interpretation for the source of time variation in volatility and GARCH effects. The model shows that fundamentalists drive the mean reversion of the conditional volatility, and that chartists cause the persistence in volatility. When very few chartists are present in the market, mean reversion occurs at a faster rate than when many chartists are present. Also, the impact of news shocks on the conditional volatility is solely driven by the presence of chartists, who expect recent news to be informative about the future level of volatility. We can therefore explain the GARCH and ARCH effects by the presence of these two types of traders.

A second interesting feature of the model concerns the stability conditions of (7). Under normal circumstances, fundamentalists follow a strategy that ensures that conditional volatility remains bounded. However, the chartist strategy is an unstable strategy when β_0 and β_1 are positive and volatility predicted by chartists will not remain bounded. However, the fact that both types of traders are present and w_t fluctuates over time allows the volatility process (7) to be unstable in the short run, while guaranteeing stability of the GARCH process in the long run. Whether (7) is stable in the long run depends on the values for α , β_0 and β_1 and is an issue we address in the empirical section.

A third feature about the model is the time-varying unconditional volatility \bar{h}'_t . This time variation in unconditional volatility is not only caused by slow-moving change in the underlying unconditional volatility (as suggested by Engle and Lee, 1999), but also driven by the fraction of fundamentalists or chartists present. Fundamentalists believe in reversion of conditional volatility to the mean, whereas chartists do not believe in mean reversion. Hence, when only chartists are present, there is no unconditional volatility (as in an IGARCH model). When fundamentalists are present, the unconditional volatility enters the model, and becomes more influential the larger the percentage of fundamentalists actively trading.

The model presented here could essentially be applied to any type of security in financial markets. However, in the current paper we estimate the model on option prices using the GARCH option pricing methodology of Duan (1995). From Black and Scholes (1973) we know that the volatility of the underlying asset is the only unknown variable in the option pricing model. Hence trading in options is essentially trading on the expectations about the future volatility of the underlying. The following section discusses the data and methodology followed in estimating the model and subsequently we summarize the empirical results.

4 Data and Methodology

We use daily closing DAX 30 index options and futures prices for a one-year period from January until December 2000. The raw data set is directly obtained from EUREX, the European Futures and Options Exchange. The market for DAX index options and futures is the most active index options and futures market in Europe.

For index options the expiration months are the three nearest calendar months, the three following months within the cycle March, June, September and December, as well as the two following months of the cycle June, December. For index futures, the expiration months are the three nearest calendar months within the cycle March, June, September and December. The last trading day is the third Friday of the expiration month if that is an exchange trading day; otherwise, it is the first possible day prior to that Friday.

We exclude options with less than one week and more than 25 weeks until maturity and options with a price of less than two Euros to avoid liquidity related issues and because of less useful information on volatilities. In Table 1 we report the number of observations for the call options included in our study (we also use put options, so the total number of options actually used in the empirical study is twice the number reported in the table, i.e. in total we have 42,972 observations). The table shows the different number of options for different ranges of moneyness (M) and different maturities (T). As can be seen, at-the-money options ($0.95 \leq M < 1.05$) are the most heavily traded options. Similarly, we find most options with a maturity between one and three month (21 to 63 trading days). However, the table also shows that a considerable number of deep in-the-money and deep out-of-the-money options are included.

 INSERT TABLE 1 HERE

The DAX index calculation assumes that cash dividends are reinvested. Therefore, when calculating option prices we do not have to adjust the index level for the fact that the stock price drops on the ex-dividend date. But since cash dividend payments are taxed, the reinvestment does not fully compensate for the decrease in the stock price. Therefore, in the conversion from e.g. futures prices to the implied spot rate, one empirically observes a different implied dividend adjusted underlying for different maturities. For this reason, we work with the adjusted underlying index level implied out from futures or option prices.

To estimate the parameters of our volatility models, we use the following procedure to evaluate option prices. First, we compute the implied interest rates and implied dividend adjusted index rates from the observed put and call option prices. We are using a modified put-call parity regression proposed by Shimko (1993). The put-call parity for European options reads:

$$c_{i,j} - p_{i,j} = [S_t - PV(D_j)] - X_i e^{-r_j(T_j-t)}, \quad (8)$$

where $c_{i,j}$ and $p_{i,j}$ are the observed call and put closing prices, respectively, with exercise prices X_i and maturity $(T_j - t)$. $PV(D_j)$ denotes the present value of dividends to be paid from time t until the maturity of the options contract at time T_j and r_j is the continuously compounded interest rate that matches the maturity of the option contract. Therefore, we can infer a value for the implied dividend adjusted index for different maturities, $S_t - PV(D_j)$, and the continuously compounded interest rate for different maturities, r_j . To ensure that the implied dividend adjusted index value is a non-increasing function of the maturity of the option, we occasionally adjust the standard put-call parity regression. Therefore, we control and ensure that the value for $S_t - PV(D_j)$ is decreasing with maturity, T_j .

Second, to evaluate options, the physical process has to be transformed into a risk-neutral process. We make use of the Local Risk Neutral Valuation Relationship (LRNVR) developed in Duan (1995). Under the LRNVR the conditional variance process remains unchanged, but under the pricing measure Q the conditional expectation of r_t is equal to the risk free rate r_f ,

$$E^Q[\exp(r_t)|\Omega_{t-1}] = \exp(r_f). \quad (9)$$

The risk-neutral Gaussian process reads

$$r_t = \ln\left(\frac{S_t - PV_t(D)}{S_{t-1} - PV_{t-1}(D)}\right)_t = r_f - \frac{1}{2}\sigma_t^2 + \sqrt{h_t}\varepsilon_t \quad (10)$$

$\varepsilon_t|\Omega_{t-1} \sim N(0, 1)$ under the risk-neutralized probability measure Q .

In Equation (10), ε_t is not necessarily normal, but to include the Black-Scholes model as a special case we typically assume that is a Gaussian random variable. The locally risk-neutral valuation relationship ensures that under the physical probability measure P and the risk neutral measure Q , the volatility process satisfies

$$Var^Q[r_t|\Omega_{t-1}] = Var^P[r_t|\Omega_{t-1}] = h_t. \quad (11)$$

A European call option with exercise price X and time to maturity T has at time t price equal to

$$c_t = \exp(-rT)E_t^Q[\max(S_t - X, 0)|\Omega_{t-1}]. \quad (12)$$

For these kinds of derivative valuation models with a high degree of path dependency, computationally demanding Monte Carlo simulations are commonly used for valuing derivative securities. We use the simulation adjustment method, the empirical martingale simulation (EMS) of Duan and Simonato (1998), which

has been shown to substantially accelerate the convergence of Monte Carlo price estimates and to reduce the so-called simulation error.

Next, we make use of the cross-section of option prices with different maturities and strike prices. Using Monte Carlo simulations, the model generates volatility dynamics for all the different expiration dates. In other words, it starts off from the observed dividend-adjusted underlying and local volatility of today, and iterates forward until expiration. As a result, for each maturity we obtain a number of terminal stock prices equal to the number of simulation paths. Option prices are calculated by discounting the expected payoffs. We calibrate the parameters of the model by minimizing the root mean squared pricing error between the cross-section of market prices on a particular day and the theoretical option prices

$$RMSE = \sqrt{\frac{1}{N_c} \min \sum_{i=1}^n \sum_{j=1}^{m_i} (\hat{c}_{i,j} - c_{i,j})^2 + \frac{1}{N_p} \min \sum_{i=1}^n \sum_{j=1}^{m_i} (\hat{p}_{i,j} - p_{i,j})^2}, \quad (13)$$

where N_c and N_p are the total number of call ($c_{i,j}$) and put ($p_{i,j}$) options evaluated, i refers to the n different maturities and j to the m_i different strike prices in a particular maturity series i .

Fourth, having estimated the parameters in-sample, we turn to out-of-sample valuation performance and evaluate how well each day's estimated models value the traded options at the end of the following day. We filter the available option prices according to our criteria for the in-sample calibration. For the out-of-sample exercise, we make use of the observed futures prices in order to derive the implied dividend adjusted underlying index level⁴. Given a futures price F_j with time to maturity T_j , spot futures parity is used to determine $S_t - PV(D_j)$ from

$$S_t - PV(D_j) = F_j e^{-r_j T_j}, \quad (14)$$

where $PV(D_j)$ denotes the present value of dividends to be paid from time t until the maturity of the options contract at time T_j and r_j is the continuously compounded interest rate (the interpolated EURIBOR rate) that matches the maturity of the futures contract (or time to expiration of the option). If a given option price observation corresponds to an option that expires at the time of delivery of a futures contract, then the price of the futures contract can be used to determine the quantity $S_t - PV(D_j)$ directly.

However, the maturities of DAX index options do not always correspond to the delivery dates of the futures contracts. In particular, for index options the two following months are always expiration months, but not necessarily a delivery month for the futures contract. When an option expires on a date other than the delivery date of the futures contract, then the quantity $S_t - PV(D_j)$ is computed from various futures contracts. Let F_1 be the futures price for a contract with the shortest maturity, T_1 and F_2 and F_3 are the futures prices for contracts with the second and third closest delivery months, T_2 and

⁴The futures market is the most liquid market and the options and the futures market are closely integrated. Therefore, it can also be assumed that the futures price is more informative for option pricing than just using the value of the index. For every observed futures closing price we can derive the implied dividend adjusted underlying index level and evaluate the option.

T_3 , respectively. Then the expected future rate of dividend payment d can be computed via spot-futures parity by

$$d = \frac{r_3 T_3 - r_2 T_2 - \log(F_3/F_2)}{(T_3 - T_2)}. \quad (15)$$

Hence, the quantity $S_t - PV(D) = S_t e^{-dT}$ associated with the option that expires at time T in the future can be computed by⁵

$$S_t e^{-dT} = F_1 e^{-(r_1 - d)T_1 - dT} \quad (16)$$

This method allows us to perfectly match the observed option price and the underlying dividend adjusted spot rate. Given the parameter estimates and the implied dividend adjusted underlying we can calculate the by the model predicted option prices and compare them to the observed option prices of traded index options. For the out-of-sample part, the same loss functions are used; i.e. the prediction performance of the various models are evaluated and compared by using the root mean squared valuation error criterion. We compare the predicted option values with the observed prices for every traded option. We repeat the whole procedure over the out-of-sample period and conclude, which model minimizes the out-of-sample pricing error.

In the empirical part of the paper, we model the expectations of conditional volatility of fundamentalists (and chartists) in an EGARCH setting. Our choice is motivated by the results of Lehnert (2003) who finds that applying a standard GARCH framework results in numerous violations of parameters in their permissible parameter space. The EGARCH setting resolves these issues, as it imposes only minor restrictions on the parameter space (see Nelson, 1991).

5 Results

This section presents the empirical results of the option pricing application of our heterogeneous agents model for the second moment. First, we focus on the estimation results, and second we look at the pricing errors of our model, both in-sample and out-of-sample. Finally, we present a number of further insights into the exact working of the model, like the variability over time, the behaviour of the two forecasting rules, and the differences with a standard volatility process. The estimation exercises are conducted in a setting with and without switching. This allows us to examine the direct effect of introducing more flexibility in the model; in other words, it allows us to see the advantage of our model over a standard GARCH.

5.1 Estimation Results

 INSERT TABLE 2 HERE

Table 2 presents the estimation results for the option pricing model with heterogeneous agents. Focusing first on the static setup, with $w_t = w = 1/2$,

⁵See e.g. the appendix in Poteshman (2001) for details.

(reported in Panel A)⁶, we find that the mean-reversion parameter α is just below one throughout the sample (0.957), implying that mean-reversion takes place, but is very slow. The absolute magnitude of the mean estimate of α indicates that on average a little over 4% of the excess volatility to disappear in the next period. The value for α is always between zero and one, which is reassuring as it would reject the stability conditions. The value for α is consistent with findings on GARCH-models that are frequently applied to financial time series data, where the coefficient on the GARCH-term is often larger than 0.9. Parameter estimates for the shock terms, β_0 and β_1 , behave as expected as well; the results for this asymmetric setup imply that there is a clear leverage effect: Positive shocks in the level result in a reduction of the variance $\beta_0 < 0$; negative shocks in the level result in an increase of the variance $\beta_1 > 0$. Therefore, negative shocks in the level have a destabilizing effect to the variance process. In absolute terms β_0 and β_1 are of similar magnitude and one can verify that the stability conditions for the GARCH hold ($1 + \alpha + \beta_0 + \beta_1 < 1$). However, we do see that there is quite some variation in β_0 and β_1 over the sample period. The average estimated local volatility, i.e. the starting value for the volatility dynamics, is equal to 0.22 and fluctuates between 0.14 and 0.34 over the sample period.

The results for the switching model, reported in Panel B of Table 2, are generally consistent with the static model. The average mean reversion parameter α tends to be slightly smaller than for the static model, indicating slightly more mean reversion on average, but this could well be driven by the lower minimum value observed for α . The difference with the static model is that the switching model allows for switching between fundamentalist and chartist strategies, which is captured by the sensitivity of choice parameter γ . We find that γ is positive and of considerable magnitude throughout the sample. This implies that the switching mechanism functions as a positive feedback rule. In other words, the positive sign of γ indicates that agents switch towards the group with the smallest forecasting error. The magnitude of γ is conditional on the functional form of the profit function (in our case, a loss function consisting of the percentage forecasting error, Equation (4)). Therefore, it is not possible to make any statements about the sensitivity to profit differences of traders in the option market at this time. We will, however, be able to say something about the evolution of individual's behaviour over time in the sensitivity analysis below.

Overall, we observe that all coefficients have the sign and magnitude as hypothesized consistent throughout the sample. Both fundamentalists and chartists appear to be active in the market, and their individual effects on the variance process are as expected (stabilizing and destabilizing, respectively). Also, we find significant evidence of switching between the two rules.

To our best knowledge, a heterogeneous agents model has never been comparably applied to the options market. However, we can compare our results with related literature. First of all, the signs and magnitudes of the chartist expectation formation function are directly comparable to the standard EGARCH-

⁶Although arbitrary, the most natural choice is to put $\gamma = 0$, such that $w_t = w = \frac{1}{2}$. This choice has no effect on the estimation results as w only serves as a scaling parameter. This is also why estimating w as a free parameter is not possible, as it would not be identified. Given that the average w_t in the switching case is approximately $\frac{1}{2}$, the choice of $w = \frac{1}{2}$ in the static case makes the order of magnitude of the coefficients between the two setups directly comparable.

results, due to Nelson (1991). The relative impact of positive versus negative shocks corroborates previous findings; the typical results for the leverage effect indicate that the relative effect of negative shocks on the variance process is larger than the positive shocks. Second, our results are directly in line with previous findings on estimates of heterogeneous agents models for alternative markets. Boswijk et al. (2007) find significant evidence of the co-existence of chartists and fundamentalist for the S&P500 from 1870 to 2006; De Jong et al. (2009a,b) present similar results for the British Pound during the EMS crisis. Our results on the switching mechanism, however, are stronger compared to Boswijk et al. (2007); evidence for switching is limited given their estimate of the switching parameter. This implies that traders in the options market are more prone to change their strategy in response to a difference in profits compared to traders in the S&P500 or foreign exchange market.

5.2 Pricing Errors

As additional empirical evidence for our model, we examine both in-sample and out-of-sample pricing errors. The results for the models with and without switching are reported in the final two columns of Table 2. Results suggest that the assumption of agents switching to the more profitable forecasting rule is very much supported by the data. Comparing Panel A and B reveals that our switching model outperforms the static model by on average $\epsilon 0.83$ in-sample, and $\epsilon 0.52$ out-of-sample in terms of pricing error; the distributional statistics of the pricing errors are generally favourable for the behavioural volatility trading model as well. In other words, next to introducing a more intuitive appeal to volatility models, our heterogeneous agents model also proves to be more effective in explaining and forecasting option prices.

 INSERT TABLE 3 HERE

Since we fit our model on option with different levels of moneyness and different maturities, we can also assess the pricing performance for different levels of moneyness and maturity. In Table 3 we report the pricing performance of both the static and the switching model. In Panel A we report the results for the in-sample pricing errors. Overall, we can make four main observations from this table. First, we find that pricing errors tend to be lowest for options with a maturity between 21 and 63 trading days. Second, there generally is a hump-shaped pattern of pricing errors over the moneyness range; pricing errors tend to be larger for at-the-money options relative to both far out-of-the-money and in-the-money options. Third, we find that pricing errors are always smaller for the switching model, for all levels of moneyness and all maturities. Finally, when considering the difference between the static and switching model we find that the biggest improvement in terms of pricing error is observed for the deep in-the-money and deep out-of-the-money options.

In Panel B we report out-of-sample pricing errors for different levels of moneyness and different maturities. Interestingly, out-of-sample performance for both the static and switching model is best at the shortest maturity, which

contrasts the in-sample pricing errors, which were smallest at intermediate maturities. However, similar to in-sample performance we find that out-of-sample pricing errors are always smaller for the switching model.

5.3 Additional Insights

 INSERT FIGURE 1 HERE

Since the model is estimated for each day of the year, we can examine the stability of the estimated coefficients during the estimation process. By following the course of the estimated coefficients over the year, we will be able to say something about the conditional behaviour of heterogeneous traders. Figure 1 displays the course of the coefficients of the two expectation formation functions, fundamentalists and chartists, and the intensity of choice parameter γ .

Overall, the parameters of the fundamentalist and chartist expectation formation functions are relatively stable; α , β_0 and β_1 move in a relative small band within the region one expects them to be.

At around two-thirds of the sample, at day 160, β_0 and β_1 start moving towards zero and α moves towards one, while γ becomes larger and more volatile. This change in the parameters can be directly explained by the logic of the underlying heterogeneous agents model. The volatility in the underlying is relatively constant in this period, which can be seen from the estimated local volatility. Both groups form their expectations by taking the most recently observed volatility plus or minus some correction term. When this correction term goes to zero (because the β 's move towards zero and α towards one), agents expect a relatively constant volatility. As both fundamentalists and chartists expect small innovations to the volatility process, the difference in the forecasting error between the two strategies will be relatively small as well. Therefore, there is not much to be gained by changing strategy. As the forecasting errors are small, large shifts in γ will not induce large shifts in the distribution of weights over strategies (see Equation (4)). This is exactly why the estimate of γ shows large shifts in this period.

 INSERT FIGURE 2 HERE

Another feature of interest is the degree of co-movement between the level of volatility and the pricing error. Figure 2 presents the evolution of the estimated local volatility (upper plot) and the in-sample pricing error of our model (lower plot). There is a clear positive correlation between the estimated fundamental volatility and the pricing error. Consistent with previous literature, we find that volatility shows distinctive periods of high and low volatility. Interestingly, the local volatility estimates exactly fluctuate around the long-run volatility level estimated from return data.

 INSERT FIGURE 3 HERE

To provide some insights on the exact nature of the model and the behaviour of the volatility generated by the model, we present one single simulation path out of 10,000 in the Monte Carlo simulations in Figure 3. We use the (optimized) coefficients from a random day. A number of observations can be made. As one would expect, the volatility h lies between the expectations of the fundamentalists, $E_t^F(h_{t+1})$, and chartists, $E_t^C(h_{t+1})$. The distance between the three is governed by the weight w_t . Weights continuously fluctuate around the benchmark of one half with a minimum of close to zero and maximum of around 0.80. Weights fluctuate relatively calm, which illustrates that traders tend to stick to their chosen strategy and do not change strategy or trade on a daily basis although they have the opportunity to do so. The autocorrelation in the weight series is equal to 0.73, indicating that almost three quarters of the traders hold on to their choice on a daily basis.

The nature of the two groups is clearly illustrated by the course of the volatility process. Roughly, we observe a negative relation between the level of volatility and the fraction of fundamentalists. High spikes in volatility always coincide with low weights; i.e., a relatively high volatility is caused by the fact that the market is dominated by chartists. The most clear example of this can be seen around observation number 18 and 26 where w_t reaches its minimum and h_t its maximum. The reverse is true as well; when fundamentalist make up over 80% of the market around period 68, volatility drops towards its long-run value. Therefore, fundamentalists are stabilizing, and chartists destabilizing. None of the groups gets driven out of the market, and both groups experience periods of dominance.

 INSERT FIGURE 4 HERE

In Figure 4 we plot the volatility for the simulation path shown in Figure 3 generated by the static model and by the switching model. As can be seen from the plot, the switching model can capture jumps in volatility much better than the static model. This is of course due to the fact that at times of high volatility high weight is given to the chartist strategy for which the volatility process is known to be unstable.

6 Conclusions

In this paper, we introduce a model of behavioral volatility trading. Being the only unobserved variable in an option pricing model, volatility plays a pivotal role in the determination of the value of an option. Our market consists of two types of agents that have different views on volatility and trade accordingly. Fundamentalists are expecting the conditional volatility to mean revert to a long-run volatility level. Chartists on the other hand respond solely on noise from the level process and bid up (down) volatility if they receive a negative (positive) signal from the stock market. Depending on the profitability of their strategy, agents are able to switch between groups according to a multinomial logit switching mechanism. The model is shown to reduce to a GJR-GARCH(1,1) with time varying coefficients. The difference, however, lies

in the fact that we provide a behavioral underpinning and that time variation in the coefficients depends on trader behavior.

In an application of the model to DAX index options, using our GARCH option pricing model, we find evidence that different types of traders are actively involved in trading volatility. Both fundamentalists and chartists are shown to be active in the options market, and both groups are consistently present. Hence, we find evidence that observed option prices are the result of heterogeneity in expectations about future volatility. In addition, we find evidence of switching between the groups, i.e., at certain points in time mean-reverting fundamentalists dominate the market, at other points it is dominated by destabilizing chartists. Introducing the possibility to switch gives a substantial reduction in both in- and out-of-sample pricing errors. In other words, volatility traders indeed change their forecasting behavior dependent on the relative profitability.

It would also be interesting to experiment with alternative specifications of the model, such as alternative profit functions (as is common in the heterogeneous agents literature). Also, the expectation formation functions are flexible to incorporate numerous different specifications, including ones with exogenous information.

The model presented above represents the most simplistic forms of fundamentalist and chartist behavior. There are several extensions possible to the strategies for both types of traders. First, we can extend the fundamentalist strategy by allowing for dynamics in the “unconditional volatility”. Such types of models follow from Engle and Lee (1999) and are often referred to as two-component GARCH models. Allowing for such additional dynamics can be done straightforwardly, and would imply that the model takes the form of a GJR-GARCH(2, 2). Second, chartists may also consider other stochastic variables, such as trading volumes, number of transactions, etc. These stochastic variables can easily be added to the model.

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A Appendix

The Appendix presents the exact derivation of the volatility trading model. Consistent with the general setting of Westerhoff and Dieci (2006), we model the behavior of option traders in a simple behavioral demand setting with a market maker and discrete choice switching. Instead of a multi-market setting with equity as in Westerhoff and Dieci (2006), we focus on a single market with index options.

Let S_t be the value of an underlying asset at time t . Then, in a Gaussian discrete-time economy, the (log) return process of the asset (r_t) is given as

$$\begin{aligned} r_t &= \ln \left(\frac{S_t}{S_{t-1}} \right) + d_t = \mu + \sqrt{h_t} \varepsilon_t, \\ \varepsilon_t | \Omega_{t-1} &\sim N(0, 1) \text{ under probability measure } P, \end{aligned} \quad (17)$$

where d_t is the asset's dividend yield, μ is the mean of r_t , h_t is the conditional volatility of the asset, ε_t is a standard normal random variable and Ω_{t-1} is the information set up to time $t-1$.

Let $E_t^F(h_{t+1})$ be the prediction of the conditional volatility for the fundamentalists. Their best prediction for the volatility process is equal to

$$E_t^F(h_{t+1}) = h_t - (1 - \alpha)(h_t - \bar{h}_t), \quad (18)$$

where \bar{h}_t is the long-run unconditional volatility and α measures the speed at which the fundamentalists expect the volatility process to mean revert.

Since volatility is the only (unknown) determinant of option prices, fundamentalists' (extra) demand for options in period $t+1$ is solely conditional on their forecast of the change in conditional volatility. In a Black-Scholes framework, the first derivative of the option price to volatility is defined as vega. In other words, an option's price change can be related to its underlying volatility by vega; $\Delta P_{K,T,t} = \nu_{K,T} \Delta h_t$, where $\nu_{K,T}$ is the vega of an option with strike price K , expiry T . As such, the expected price change equals vega times the expected change in volatility, $E_t^i(\Delta P_{K,T,t+1}) = \nu_{K,T} E_t^i(\Delta h_{t+1})$. The option demand relation is assumed to be linear in price expectations, such that

$$\begin{aligned} D_t^F &= f(E_t^F(\Delta P_{K,T,t+1})) = f(\nu_{K,T} E_t^F(\Delta h_{t+1})) \\ &= f \nu_{K,T} (-(1 - \alpha)(h_t - \bar{h}_t)) \end{aligned} \quad (19)$$

in which D_t^F is the demand for options by fundamentalists at time t and $f > 0$.

Chartists, on the other hand, trade on innovations from the stock market. We therefore define their prediction of the volatility process as

$$E_t^C(h_{t+1}) = h_t + \beta_0 (\sqrt{h_t} \varepsilon_t^+)^2 + \beta_1 (\sqrt{h_t} \varepsilon_t^-)^2, \quad (20)$$

where $E_t^C(h_{t+1})$ is the volatility prediction of the chartists, ε_t^+ (ε_t^-) is the past positive (negative) shock in the volatility process and β_0 (β_1) measures the extent to which chartists incorporate positive (negative) shocks into their prediction. Following the same reasoning as the for the fundamentalists, the chartist demand for options reads

$$\begin{aligned} D_t^C &= f(E_t^C(\Delta P_{K,T,t+1})) = f(\nu_{K,T} E_t^C(\Delta h_{t+1})) \\ &= f\nu_{K,T}(\beta_0(\sqrt{h_t \varepsilon_t^+})^2 + \beta_1(\sqrt{h_t \varepsilon_t^-})^2) \end{aligned} \quad (21)$$

in which D_t^C is the chartist demand for options at time t and $c > 0$.

The switching rule, based on the discrete choice model (Manski and McFadden, 1981), is given by⁷

$$w_t = \left(1 + e^{\gamma \left(\left| \frac{\ln(E_{t-1}^F(h_t)) - \ln(h_t)}{\ln(h_t)} \right| - \left| \frac{\ln(E_{t-1}^C(h_t)) - \ln(h_t)}{\ln(h_t)} \right| \right)} \right)^{-1}, \quad (22)$$

where γ measures the sensitivity of market participants (fundamentalists or chartists) to their respective percentage forecasting errors in terms of volatility and is expected to be between zero and infinity.

With the given weights and the different trading strategies, we can now establish the process for the price of an individual option. Since the market demand is a consequence of the proportion of market participants following each strategy, it is computed as a weighted average of the fundamentalist and the chartist prediction,

$$D_t^M = w_t D_t^F + (1 - w_t) D_t^C. \quad (23)$$

The adjustment of the price volatility process, finally, is conditional on the market's (excess) demand D_t^M and the effect of what can be seen as the market maker's adjustment coefficient $\eta > 0$ ⁸

$$P_{K,T,t+1} = P_{K,T,t} \eta D_t^M. \quad (24)$$

By substituting and subsequently rewriting, we arrive at

$$\Delta P_{K,T,t+1} = \eta \nu_{K,T} \left[\begin{array}{c} -w_t f(1 - \alpha)(h_t - \bar{h}_t) + \\ (1 - w_t)c(\beta_0(\sqrt{h_t \varepsilon_t^+})^2 + \beta_1(\sqrt{h_t \varepsilon_t^-})^2) \end{array} \right]. \quad (25)$$

Finally, dividing left and right by $\nu_{K,T}$ in order to transform back from individual options to the volatility process of the underlying and rewriting yields

$$\begin{aligned} h_{t+1} &= w_t(1 - \alpha)^* \bar{h}_t + (1 - w_t(1 - \alpha)^*) h_t \\ &\quad + (1 - w_t)\beta_0^*(\sqrt{h_t \varepsilon_t^+})^2 + (1 - w_t)\beta_1^*(\sqrt{h_t \varepsilon_t^-})^2, \end{aligned} \quad (26)$$

in which $(1 - \alpha)^* = (1 - \alpha)\eta f$ is the market impact of fundamentalists, and $\beta_0^* = \beta_0 \eta c$, $\beta_1^* = \beta_1 \eta c$ the market impact of chartists from positive and negative shocks, respectively. Equation (26) is equivalent to Equation (6), which we estimate in the empirical section.

⁷Originally, the discrete choice model was derived from an exponential utility function. More recently, though, the mechanism has been applied to a broader range of utility functions, like mean-variance utility as in DeGrauwe and Grimaldi (2005, 2006).

⁸See Chiarella and He (2002) for the effect of a similar type of market maker on price dynamics.

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Figure Legends

Figure 1: Parameter Estimates over Time

Notes: This figure displays the estimated coefficients of the model for the 250 trading days in the year 2000.

Figure 2: Estimated Volatility and in-sample Pricing Error over time

Notes: This figure displays the local volatility (in percentages per year, upper plot) and the pricing error (in Euros, lower plot) over time.

Figure 3: Single Simulation Path

Notes: This figure displays a single simulation path using optimized coefficients of January 26, with coefficient values $\alpha = 0.932$; $\beta_0 = -0.429$; $\beta_1 = 0.367$; $\gamma = 29.222$.

Figure 4: Volatility Generated by the Static and Switching Model

Notes: This figure displays a generated volatility path for January 26, for both the standard and switching model, using optimized coefficients. For the switching case, these are $\alpha = 0.932$; $\beta_0 = -0.429$; $\beta_1 = 0.367$; and $\gamma = 29.222$. For the static case, these are $\alpha = 0.932$; $\beta_0 = -0.483$; and $\beta_1 = 0.399$.

Tables

Table 1: Number of Observations

Moneyness	Trading days to expiration						Total
	$T \leq 21$		$21 < T \leq 63$		$T > 63$		
	Total	Daily Avg	Total	Daily Avg	Total	Daily Avg	
$M < 0.90$	627	2.49	2,204	8.75	1,607	6.38	4,438
$0.90 \leq M \leq 0.95$	783	3.11	1,540	6.11	664	2.63	2,987
$0.95 \leq M \leq 1.00$	1,224	4.86	2,106	8.36	885	3.51	4,215
$1.00 \leq M \leq 1.05$	1,258	4.99	2,194	8.71	965	3.83	4,417
$1.05 \leq M < 1.10$	703	2.79	1,553	6.16	669	2.65	2,925
$M \geq 1.10$	213	0.85	1,122	4.45	1,169	4.63	2,504
Total	4,808		10,719		5,959		21,486

Notes: This table reports the number of observations for different levels of moneyness and different maturities for the period January until December 2000 for call options only. Moneyness is defined as the strike price over the index futures price (X/F). If moneyness < 1 , the option is said to be in-the-money and if moneyness > 1 the option is out-of-the-money. In addition, we report the total number of options for a particular moneyness and maturity.

Table 2: Parameter Estimates

	α	β_0	β_1	h_0	γ	In-sample	Out-of-Sample
Panel A: Static							
Mean	0.957	-0.242	0.240	0.221	-	3.343	7.406
SD	0.023	0.101	0.075	0.047	-	1.066	5.119
Min	0.872	-0.487	0.106	0.140	-	1.171	1.364
Max	1.000	-0.040	0.415	0.336	-	6.426	39.375
1st quartile	0.943	-0.318	0.176	0.177	-	2.755	4.105
3rd quartile	0.971	-0.161	0.300	0.259	-	4.200	9.423
Panel B: Switching							
Mean	0.936	-0.245	0.241	0.220	107.34	2.511	6.891
SD	0.034	0.089	0.070	0.051	89.70	1.138	5.356
Min	0.806	-0.432	0.084	0.130	19.66	0.461	0.859
Max	1.000	-0.027	0.435	0.339	523.90	6.126	42.266
1st quartile	0.919	-0.302	0.184	0.171	50.90	1.736	3.270
3rd quartile	0.959	-0.176	0.290	0.263	135.80	3.138	9.124

Notes: The table presents the average parameters estimates, standard deviation, min, max, 1. and 3. quartile of the daily estimations of the model during the period January until December 2000. h_0 represents the estimated local volatility, or starting value of the volatility process. Additionally, we report in-sample and out-of sample pricing errors. Panel A shows the results without switching, i.e., $\gamma = 0$ such that $w_t = w = 1/2$, and Panel B with switching such that γ is estimated contemporaneously.

Table 3: Pricing Performance for different Moneyness and Maturity

	Static			Switching		
	Panel A: In-sample					
	$T \leq 21$	$21 < T \leq 63$	$T > 63$	$T \leq 21$	$21 < T \leq 63$	$T > 63$
$M < 0.90$	2.468	2.448	3.849	2.050	1.569	2.270
$0.90 \leq M \leq 0.95$	2.676	1.981	2.156	2.380	1.925	1.869
$0.95 \leq M \leq 1.00$	3.827	2.361	2.585	3.403	2.070	1.604
$1.00 \leq M \leq 1.05$	3.912	2.563	3.796	3.729	2.350	1.917
$1.05 \leq M < 1.10$	2.205	4.522	2.978	1.949	2.900	2.335
$M \geq 1.10$	2.360	6.095	4.578	1.578	3.412	2.360
	Panel B: Out-of-Sample					
	$T \leq 21$	$21 < T \leq 63$	$T > 63$	$T \leq 21$	$21 < T \leq 63$	$T > 63$
$M < 0.90$	2.472	3.663	5.965	2.157	3.406	5.714
$0.90 \leq M \leq 0.95$	3.749	5.867	7.885	3.618	5.849	7.957
$0.95 \leq M \leq 1.00$	6.657	7.669	9.571	6.377	7.507	9.291
$1.00 \leq M \leq 1.05$	6.716	7.916	10.396	6.633	7.723	9.822
$1.05 \leq M < 1.10$	3.432	7.283	10.027	3.345	6.335	9.657
$M \geq 1.10$	2.237	6.437	8.537	1.664	4.426	7.211

Notes: This table reports the root-mean-squared pricing errors of the model for both the static (no switching) case and the switching case over the different levels of moneyness (M) and days to maturity (T). Panel A reports the in-sample pricing errors while Panel B presents the out-of-sample pricing errors.







