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# A new crowd movement modeling for pedestrians who hold hands

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## Abstract

The aim of this communication is to propose a new crowd movement modeling considering a new particular subgroup behavior: pedestrians holding hands. The first step in this study is to improve our 2D crowd movement modeling [1-3] introducing the orientation of each pedestrian, rarely used in literature. The next step is to describe the particular subgroup behavior of the pedestrians who hold hands. An at-a-distance velocity of deformation allows one to control the interaction between linked pedestrians. The proposed model takes into account the effects of the subgroup as a continuous deformation within the particular subgroup of pedestrians who hold hands while existing methods in literature describe the cohesion of the “normal” subgroup with external forces. Finally, numerical simulations will be presented to study the influence of the pedestrian’s orientation on his movement and to compare subgroup behavior models.

Keywords: discrete model, crowd movement modeling, oriented pedestrian, subgroup behaviour, pedestrians holding hands, emergency evacuation ;

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## 1. Introduction

Crowd movement modeling is currently a challenging problem in various fields, such as in architecture and civil engineering to study flows of people in confined spaces, in the entertainment and visual effects industry to create realistic models of crowds for use in movies, and in psychology/sociology to validate human behavior models under different environmental conditions like panic situations. Over the last fifty years, many studies have been performed to describe the behavior of walking pedestrians. In literature, several discrete models of crowd movements [1-8] exist and allow to well reproduce some observed self organization phenomena (counter flow lines, formation of arches, etc.).

Authors have proposed in [1-3] a new 2D microscopic approach in which the movement of each pedestrian is represented in time and in space. To control the crowd movement on the ground, three targets have been managed: (i) multiple simultaneous collisions, i.e. to detect and to treat each local interaction, such as pedestrian-pedestrian and pedestrian-obstacle; (ii) the desire of each pedestrian to move in a particular direction with a specific speed at

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each time; (iii) the possibility to add forces to make the behavior of the pedestrians more realistic (social forces, subgroup forces, etc.).

Pedestrian-pedestrian and pedestrian-obstacle collisions have been modeled using an approach devoted to the movements of particles. The non-smooth approach proposed by Frémond [9-11] that has mainly been applied to the movement of granular media was selected to treat the contact and to control the type of collisions among particles. This original approach is based on the theory of collisions of rigid bodies, first proposed by Frémond [9, 10] in a rigorous thermodynamic frame, along the lines of the works of [12]. Frémond introduces the concept of the coefficient of dissipation to handle the rebound and the velocities of particles after each collision. A further constraint condition is applied to velocities to avoid overlap. The description of the behavior of a collection of discrete bodies is based on the consideration that the global system is deformable even if particles are rigid.

To represent a pedestrian, a willingness is given to each particle to move according to a given possibly time varying target. Social forces as well as a desired direction/velocity are introduced in order to simulate the behavior of pedestrians [1-3].

The next step is how to include the behavior of groups or sub-groups in this modeling. A “group” is defined as a physical collection of people following the same route, but who may or may not be part of the same social group and a “subgroup” is defined as people within the same physical “group” who want to stay together [13] like those of friends or a family. Several studies have revealed that smaller subgroups constitute the majority of the people in a crowd [13, 14]. But very few studies are able to model the “subgroup” behavior [13, 15]. A particular subgroup concerns pedestrians who hold hands. It is recognized that holding hands is the most effective way of keeping children safe from traffic injury.

The aim of this communication is then to propose a new modeling of the behavior of pedestrians who hold hands. To describe the particular subgroup behavior of the pedestrians who hold hands, we were inspired by the article [16] describing the interactions between rigid particles as an at-a-distance velocity of deformation. The proposed model takes into account the effects of the subgroup as a continuous deformation within the subgroup of pedestrians who hold hands while existing methods in literature describe the cohesion of the subgroup with external forces [13, 15]. This new subgroup model needs that pedestrians be oriented, so the rotation of the pedestrians on themselves will be integrated in the model to obtain an improvement of the 2D microscopic approach [1-3]. It is important to note that the pedestrian’s orientation is rarely used in literature [7, 8].

Numerical simulations will be presented to study the influence of the pedestrian’s orientation on his movement and to compare subgroup behavior models.

## 2. Crowd model

The proposed crowd model is an improvement of the one presented in [1-3], taking into account the rotation of each pedestrian on himself. First, the granular model allowing to manage multiple simultaneous collisions, is presented. Then, the adaptation of this granular model to the crowd one is done.

### 2.1. A non-smooth granular model based on the theory of Frémond [9, 10]

Let us consider a system of  $N$  circular particles moving in a plane, defined by their mass  $m_i$ , their inertia  $I_i$ , their radius  $r_i$ , their center position (i.e. center of gravity  $G_i$  in this study) described by the vector  ${}^t q_i = (q_i^x, q_i^y) \in R^2$ , their velocity of center position denoted by  $\underline{u}_i(t) = d\underline{q}_i(t)/dt$ , their orientation  $\theta_i$  and their rotational velocity  $\omega_i(t) = d\theta_i/dt$ .

Contacts between particles are assumed to be punctual. Noting  ${}^t \underline{r}_i = ({}^t q_i, \theta_i)$ ,  ${}^t \underline{v}_i(t) = d{}^t \underline{r}_i(t)/dt = ({}^t \underline{u}_i, \omega_i)$  and the generalized displacement vector  $\underline{r}$  of size  $3N$ ,  ${}^t \underline{r} = ({}^t \underline{r}_1, {}^t \underline{r}_2, \dots, {}^t \underline{r}_N)$ , the relative deformation velocity between the  $i^{\text{th}}$  and the  $j^{\text{th}}$  particles colliding at point  $A_{i,j}$  is:

$$\underline{\Delta}_{ij}(\underline{v}(t)) = \underline{u}_i + \underline{\omega}_i \times \underline{G}_i A_{ij} - (\underline{u}_j + \underline{\omega}_j \times \underline{G}_j A_{ij}). \quad (1)$$

The dynamics equation for each particle can be written as the following differential system:

$$\begin{cases} \underline{\underline{M}} \dot{\underline{v}}(t) = -\underline{f}^{\text{int}}(t) + \underline{f}^{\text{ext}}(t) & \text{almost everywhere} \end{cases} \quad (2)$$

$$\begin{cases} \underline{\underline{M}} (\underline{v}^+(t) - \underline{v}^-(t)) = -\underline{p}^{\text{int}}(t) + \underline{p}^{\text{ext}}(t) & \text{everywhere} \end{cases} \quad (3)$$

where  $\underline{\underline{M}}$  is the  $3N \times 3N$  inertial matrix of all the particles;  $\underline{f}^{\text{ext}}$  (resp.  $\underline{f}^{\text{int}}$ ) is the exterior forces vector (resp. interior forces vector) of dimension  $3N$  applied to the deformable system. The existence of a solution of the system given by Eqs. (2) and (3) is proven in [10, 11, 17]. Equation (2) describes the smooth evolution of the multiparticle system, whereas (3) describes its non-smooth evolution during a collision. Hence, Eq. (2) applies almost everywhere, except at the instant of the collision, where it is replaced by Eq. (3). When contact is detected, velocities of colliding particles are discontinuous, and so in Eq. (3), the percussions  $\underline{p}^{\text{int}}$  and  $\underline{p}^{\text{ext}}$ , interior and exterior to the system respectively, are introduced. By definition, percussions have the dimension of a force multiplied by a time. The  $\underline{p}^{\text{int}}$  percussions are unknown; they take into account the dissipative interactions between the colliding particles (dissipative percussions) and the reaction forces that permit the avoidance of overlapping among particles (reactive percussions). Frémond [9, 10] defined the velocity of deformation at the moment of impact  $(\underline{\Delta}(\underline{v}^+) + \underline{\Delta}(\underline{v}^-))/2$  and showed that  $\underline{p}^{\text{int}}$  is defined in duality with  $(\underline{\Delta}(\underline{v}^+) + \underline{\Delta}(\underline{v}^-))/2$  according to the work of internal forces. He then introduced a pseudopotential of dissipation  $\Phi$  that allows us to express  $\underline{p}^{\text{int}}$  as:

$$\underline{p}^{\text{int}} \in \partial \Phi \left( \frac{\underline{\Delta}(\underline{v}^+) + \underline{\Delta}(\underline{v}^-)}{2} \right) \quad (4)$$

where the operator  $\partial$  is the subdifferential that generalizes the derivative for convex functions [10]. The convex function  $\Phi$  [18] is defined as the sum of two pseudopotentials:  $\Phi = \Phi^d + \Phi^r$ ; where  $\Phi^d$  and  $\Phi^r$  characterize the dissipative and reactive interior percussions respectively. The pseudopotential  $\Phi^d$  is chosen to be quadratic:  $\Phi^d(\underline{y}) = K \underline{y}^2/2$ ; where  $K$  is a coefficient of dissipation. This choice allows one to find the classical results when the coefficient of restitution is used. Other choices of  $\Phi^d$  allow one to obtain a large variety of behaviors after impact [10, 17].

In Eq. (3), the problem is to find the velocity  $\underline{v}^+$  after particles' collision. To determine  $\underline{v}^+$ , the following constrained minimization problem has to be solved:

$$\underline{X} = \arg \min_{\underline{Y} \in R^{3N}} \left[ {}^t \underline{Y} \underline{\underline{M}} \underline{Y} + \Phi(\underline{\Delta}(\underline{Y})) - {}^t (2\underline{v}^- + \underline{\underline{M}}^{-1} \underline{p}^{\text{ext}}) \underline{\underline{M}} \underline{Y} \right] \quad (5)$$

where the solution  $\underline{X} = \frac{\underline{v}^+ + \underline{v}^-}{2}$ .

Proof of the existence and uniqueness of the velocity  $\underline{v}^+$  after the simultaneous collisions of several rigid solids, as well as the dissipativity of the collisions, is presented in [9-11].

## 2.2. Adaptation of granular approach to the crowd

A pedestrian can be represented as a circular particle by giving it willingness, i.e. a desire to move in a particular direction with a specific speed at each time. The adaptation [1-3] is improved to take into account the rotational degree of freedom.

The first step of the modified approach is to give a desired trajectory to each particle. Several definitions of the desired trajectory of any one pedestrian are possible: either (i) the most comfortable trajectory for him, where he must exert the least effort, e.g. by avoiding the stairs or making the fewest changes in direction, etc.; (ii) the shortest path; or (iii) the fastest path to move from one place to another [19]. It is possible to combine two strategies in the same simulation or to change the preferred strategy for any reason during the simulation.

The strategy of the shortest path from one point to another is implemented through a Fast Marching algorithm [20] and is used to obtain the desired direction  $\underline{e}_{d,i}$  of an individual  $i$  [1-3]. This direction depends on the environment in which pedestrians walk (obstacles, etc.), the time of day, and also the characteristics of the individual (gender, age, hurried steps or not, etc.). It is defined by:  $\underline{e}_{d,i}(t) = \underline{u}_{d,i} / \|\underline{u}_{d,i}\|$ ; where  $\underline{u}_{d,i}$  is the desired velocity of the  $i$ -th pedestrian.

The amplitude  $\|\underline{u}_{d,i}\|$  of the desired velocity represents the speed at which the  $i$ -th pedestrian wants to move, and it can be influenced by his nervousness. This velocity is chosen by following a normal distribution with an average of  $1.34 \text{ m s}^{-2}$  and standard deviation of  $0.26 \text{ m s}^{-2}$  [21].

In the second step, the desired velocity of each pedestrian is introduced into the original discrete models to simulate crowd movement. Let  $\underline{f}^{\text{int}}(t) = \underline{h}^a(t)$ , where  $\underline{h}^a(t)$  allows one to give a desired direction, amplitude of the velocity and orientation to each pedestrian. Each component  $\underline{h}_i^a(t)$  of the vector of dimension  $3N$ :  ${}^t \underline{h}^a = ({}^t \underline{h}_1^a, {}^t \underline{h}_2^a, \dots, {}^t \underline{h}_N^a)$ , is associated with pedestrian  $i$  and can be expressed as  ${}^t \underline{h}_i^a(t) = ({}^t \underline{f}_i^a(t), l_i^a(t))$ , where  $\underline{f}_i^a(t)$  is the so-called acceleration force [6] giving the desired direction and amplitude of the velocity to the pedestrian, and  $l_i^a(t)$  is an angular torque giving him his desired orientation.  $\underline{f}_i^a(t)$  is defined in [1-3, 6] by:

$$\underline{f}_i^a(t) = m_i \frac{\|\underline{u}_{d,i}\| \underline{e}_{d,i}(t) - \underline{u}_i(t)}{\tau_i} \quad (6)$$

where  $\underline{u}_i$  is the actual velocity and  $\tau_i$  is a relaxation time, which specifies how long the pedestrian will take to recover his desired velocity either after a contact or after he suddenly changes his walking direction. Helbing [6] chose  $\tau = 0.5 \text{ s}$  in his numerical simulations. Smaller values of  $\tau_i$  let the pedestrians walk more aggressively. The influence of the relaxation time parameter  $\tau$  has been studied in [3]. The chosen value of this parameter is less than or equal to  $0.5 \text{ s}$ ; as such, several contacts may occur because the pedestrians walk aggressively.

$l_i^a(t)$  is the combination of a linear spring (with the desired direction) and linear damper, it is defined by:

$$l_i^a(t) = -k(\theta_i - \theta_{d,i}) - c\omega_i \quad (7)$$

where  $\theta_{d,i}$  is the angle between  $\underline{e}_{d,i}$  and a reference direction  $\underline{e}_x = [1; 0]$ ,  $k$  is the stiffness and  $c$  the damping.

The pedestrians' behavior can be enriched by adding other external social forces [3, 6, 15] so as to become more realistic (socio-psychological force, attractive force, group force, etc.).

### 2.3. Influence of the pedestrian's orientation

Two different simulations are presented in this part to study the influence of the pedestrian's orientation. The first one concerns the interaction between two pedestrians and the second one is about an evacuation exercise.

Three cases are studied in the following simulations, function of the coefficients of dissipation for the tangential components of percussions  $K_T$  and of the rotational degree of freedom: (i)  $K_T$  is neglected, so  $K_T = 0 \text{ kg}$ ; (ii)  $K_T = 100 \text{ kg}$  and the rotation of each pedestrian on himself is neglected, so the pedestrian is not oriented; and (iii)  $K_T = 100 \text{ kg}$  and the rotation of each pedestrian on himself is taken into account, so the pedestrian is oriented.

#### 2.3.1. Pedestrian / pedestrian interaction

Two identical pedestrians are moving in opposite direction. After the collision, they try to recover their desired velocities. The numerical simulations are done depending on the value of  $K_T$  and on the rotation of each pedestrian on himself.

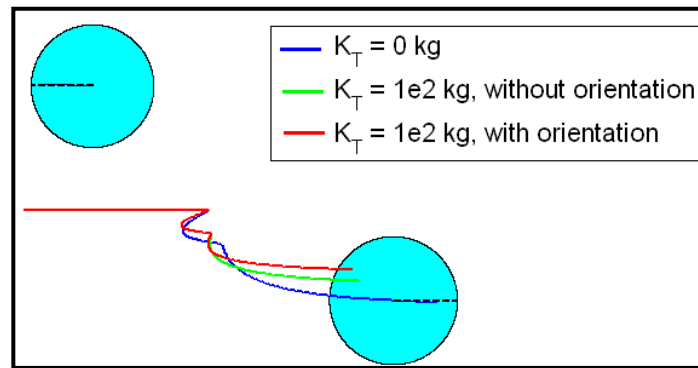


Fig. 1. Different trajectories of one pedestrian after a shock with an other one.

Figure 1 shows the behavior of the pedestrian after a shock is different when an orientation is given to the pedestrian (red curve) or not (other curves). In literature, crowd models rarely take into account this criterion. It seems to be interesting to consider it now.

#### 2.3.2. Evacuation of a square room

The aim of this part is to compare an evacuation situation for the three different cases, considering only the way of treating the local pedestrian-pedestrian contact and the local pedestrian-obstacle contact.

We consider a square room of side 5 m, where 20 pedestrians want to escape by a door of 1 m width (Fig. 2). The parameters used in simulations are given in Table 1.

Table 1. Evacuation of a classroom - parameters used in simulations (\* uniformly distributed within their range). The response time is the time needed by pedestrian  $i$  to start evacuating after the triggering of the evacuation movement.

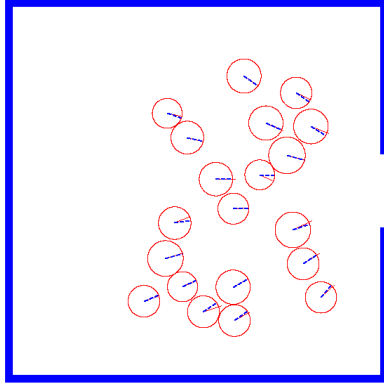


Fig. 2. Evacuation of a square room.

Parameters	Symbol	Value	Unit
Walking speed*	$\ \vec{U}_{d,i}\ $	$[1.2, 2]$	$m s^{-1}$
Radius*	$r_i$	$[0.2, 0.25]$	$m$
Mass*	$m_i$	$[60, 100]$	$kg$
Response time	$t_{r,i}$	0	$s$
Relaxation time*	$\tau_i$	$[0.1, 0.5]$	$s$
Normal coefficient of dissipation	$K_N$	$10^4$	$kg$
Tangential coefficient of dissipation	$K_T$	$\{0, 10^2\}$	$kg$
Time step	$h$	$10^{-2}$	$s$

For each studied case, 50 simulations are performed (Fig. 3). The initial conditions of these runs are the same for each case. Figure 3 a, b and c shows the linear regression of the 50 simulations in each case. The slope allows us to obtain the average flow through the door. The obtained linear regressions can be compared in Figure 3 d. The orientation of each pedestrian which is usually neglected in literature, when  $K_T = 100 kg$  and when the rotation of each pedestrian on himself is taken into account, influences the results of this evacuation exercise. So, this rotational degree of freedom has to be considered in crowd movement in the future.

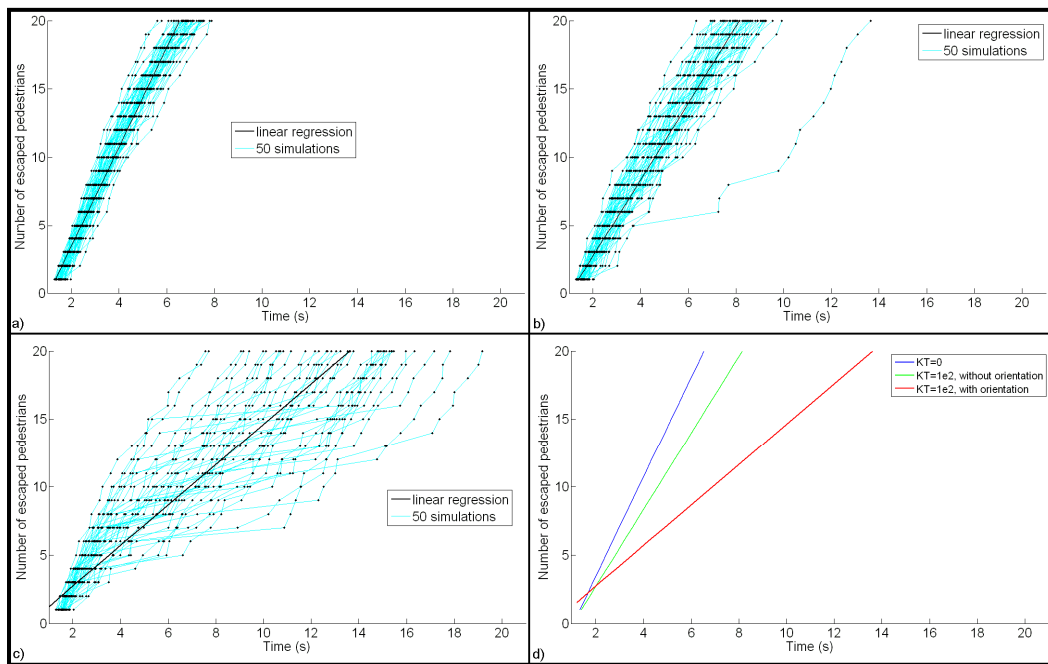


Fig. 3. Evacuation of a square room – 50 simulations and linear regression – a)  $K_T = 0 kg$  ; b)  $K_T = 100 kg$  and the rotation of each pedestrian on himself is neglected; c)  $K_T = 100 kg$  and the rotation of each pedestrian on himself is taken into account; and d) comparison of the three linear regression.

### 3. Pedestrians holding hands

We propose a particular case of subgroup: pedestrians holding hands. While existing methods describe the cohesion of the subgroup with forces [13, 15], the proposed method to describe this original behavior, inspired by [16] proposing an at a distance interaction between rigid particles, is to model the effects of the subgroup as a continuous deformation of the system for both linked pedestrians.

The at a distance velocity of deformation is the derivative with respect to time of the squared distance between linked shoulders, i.e. points  $A_i$  and  $A_j$  for pedestrians  $i$  and  $j$  (Fig. 4).

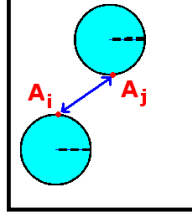


Fig. 4. Two pedestrians holding hands – Example of linked shoulders.

Taking this example, the at a distance velocity of deformation is expressed as:

$$\Delta_{ij}^*(\underline{v}(t)) = 2 \left( \underline{u}_i + \underline{\omega}_i \times \underline{G}_i A_i - \left( \underline{u}_j + \underline{\omega}_j \times \underline{G}_j A_j \right) \right) \underline{A}_j A_i \quad (8)$$

In the pseudopotential of dissipation  $\Phi^d$ , as well as quadratic terms taking into account normal  $K_N$  and tangential  $K_T$  coefficients of dissipation, an other quadratic term  $K_V$  is added for the coefficient of viscous dissipation.

On a same example, a comparison has been done between different subgroup approaches: Singh [13], Moussaïd [15], and pedestrians holding hands with  $K_V = 100 \text{ kg m}^{-1}$ . A subgroups composed of three pedestrians collides with a fourth pedestrians. Figure 5 shows the proposed subgroup approach allows one to re-form the subgroup after a collision faster than the others. Moreover, only the proposed approach allows one to keep the shape of the subgroup throughout the simulation.

### 4. Conclusions

An improvement of the crowd movement model presented in [1-3] is proposed in this communication.

First, the orientation of the pedestrians, usually neglected in literature's models, is introduced. Numerical simulations in an evacuation situation show that this criterion influences the pedestrian's behavior and has to be considered in the future.

Second, a particular case of subgroup is proposed: pedestrians holding hands. It's an original approach using the formalism of pseudopotential of dissipation to control the link between two pedestrians. A comparison of this new proposed approach with two others ones of the literature gives satisfactory results.



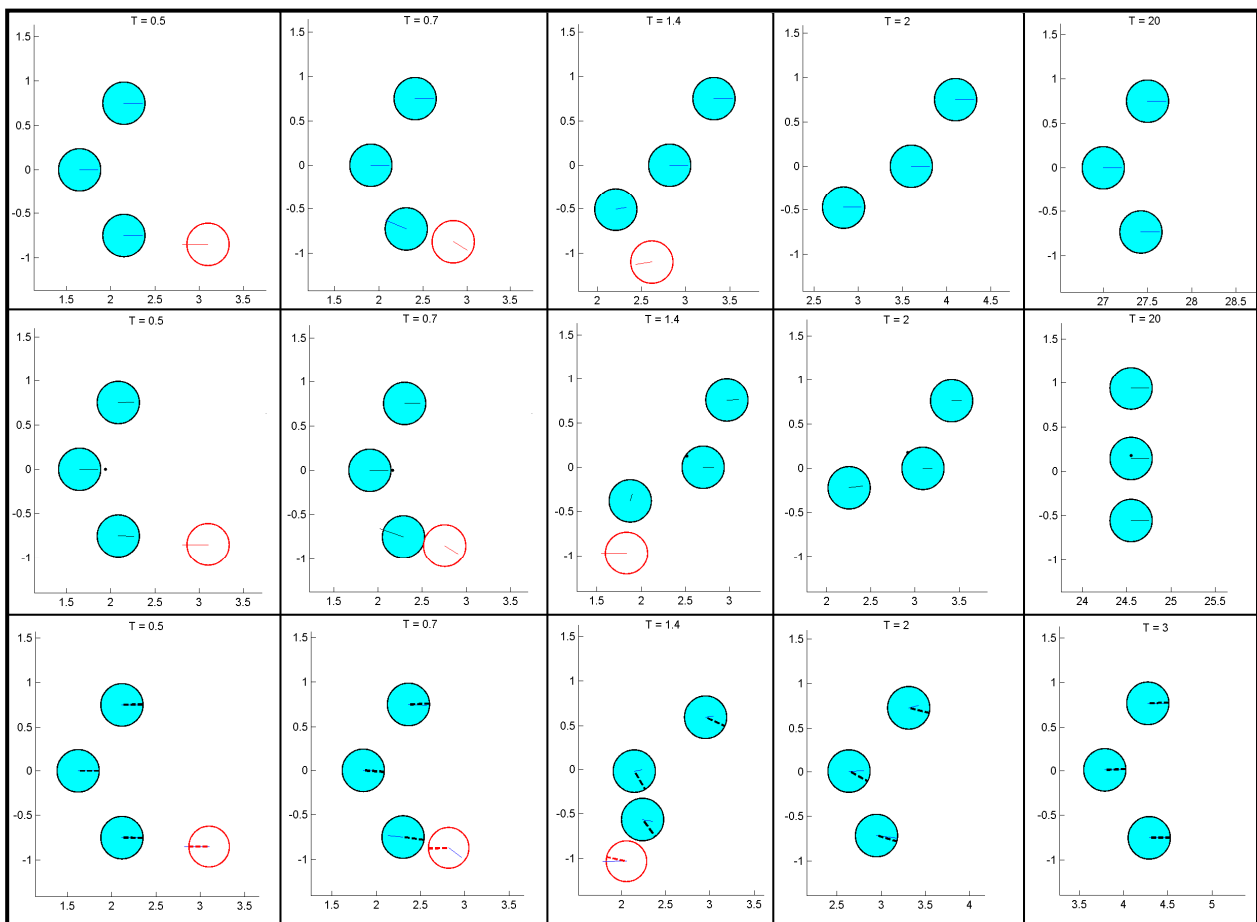


Fig. 5. Example of a three pedestrians' subgroup colliding with a fourth pedestrian – each row correspond to an approach: Singh [13] top line, Moussaïd [15] middle line, and pedestrians holding hands down line.

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