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# Determining the number of new employees with learning, forgetting and variable wage with a Newsvendor model in pull systems

Yufei Huang, Feng Chu, Chengbin Chu, Yingluo Wang

**Abstract** This paper develops a new quantitative model to find the optimal number of new employees with a Newsvendor model in a pull production system. This model allows learning, forgetting and variable wage. This paper also provides numerical results on sensitivity analysis, and compares the numerical results in three different situations: the situation with both learning and forgetting effect, that with learning effect but without forgetting effect and the situation with neither learning nor forgetting effect. The conclusions drawn from the comparison may offer theoretical insight for human resource managers to make appropriate employment decisions.

**Keywords** Workforce planning · Learning · Forgetting · Newsvendor model

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## Introduction

Pull strategies first appeared in the production system at Toyota company as Kanban systems, which are now widely implemented in practice and extensively studied in the literature (Ip et al. 2007). Pull strategies underline the importance of controlling material flow to limit WIP (Work In Progress) so as to smooth production flow, improve quality and reduce cost (Hopp and Spearman 2004). For style goods, which are defined as “have a finite selling period during which the sales rate varies in a seasonal and to some extent, predictable fashion” (Murray and Silver 1966), the production is “pulled” by a coming selling period. As described by the well-known Newsvendor model, the goal is to find an optimal production quantity to balance the overage and shortage in the selling period. In real life, because of the fluctuant customer demand, fast update of the style goods must be achieved. For every generation of the products, Newsvendor model is applicable. Besides, as many style-good factories are able to produce different kinds of products with the same equipments to meet the diversified demand of customers, they may carry on different production tasks at the same time. For each kind of products, Newsvendor model is also applicable.

As many papers focus on material supply and production control with Pull strategies (Gauray et al. 2000), workforce supply to fulfill the demand has often been neglected. Facing the coming demand on style goods, besides the supply of materials, another important issue is how to organize workforce to produce suitable quantity of products to meet the demand. The first step of organizing workforce is to decide how many employees to be involved in the production.

To find an optimal number of employees, two main factors should be considered: the productivity and the wage of the employees, namely how many units the employees can

produce and how much they cost. If the tasks are new to an employee, he/she needs some time to get used to these tasks and then his/her productivity will improve with time. But if an employee takes a long time break, he/she may forget what he/she has learnt from his/her previous experience, thus his/her productivity will decrease. This productivity variation can be modeled by learning/forgetting curves. In the meantime, due to the varying productivity of employees, employers often offer them varying wages.

The situation described above is very common in practice. For instance, many countries implement two-day weekend after five weekdays. In this situation, employees will experience several cycles of working and resting, corresponding to cycles of learning how to produce and forgetting the experience they have got. Temporary leave or overtime work also bring employees irregular learning and forgetting cycles. In a more complex case, where a factory begins to produce a new generation of products or a different kind of products while the old products are still produced, some employees may be shifted from one production line to another from time to time due to the fluctuant demand and varying due date of these different products. In this case, they may have multiple learning and forgetting curves corresponding to different products. Although the employees keep working, they will simultaneously experience learning to produce one kind of products while forgetting to produce the other kind.

The problem we address here is to determine the number of employees to hire in a style-good production factory, considering learning-forgetting phenomenon and variable wages to maximize total profit. In order to explore this workforce planning problem which is very complex and must be studied in a step by step manner, this paper considers a simplified situation. We focus on one production season with several production cycles followed by one selling season, like one newspaper selling day in the classical Newsvendor model. In fact, style goods change from one season to another and from one year to the next one. Every season can be considered as a new production horizon, the knowledge that the employees acquire during previous seasons becomes obsolete. Therefore, the seasons are independent one from another and all employees can be considered as new, namely without any initial experience, from modeling point of view, even though they are not actually fired by considering that the fixed firing and hiring costs are negligible compared to other costs.

We also assume that every employee produces only one type of products. As a consequence, the product types can also be considered to be independent one from another. Therefore, this assumption is equivalent to considering a single product model. We further assume that all employees have the same learning and forgetting curve because the individual differences will make the model complex while an average learning or forgetting rate does not alter the generality

of our discussion, although individual difference may be a good topic for future research indeed. Thus, some explanatory accuracy will be neglected for ease of presentation. Besides, in view of the character of style goods, the products are either sold or disposed after they are produced, instead of being conserved in the stock, we further suppose that the initial stock level is zero. As much as we know, this workforce planning problem has rarely been addressed in the literature based on learning, forgetting and Newsvendor model such as ours.

In the following section, we briefly review the related literature; in “Mathematical model”, we provide a mathematical model to solve the problem; this model is then analyzed by providing some numerical results in “Numerical results”; and “Conclusions” gives some concluding remarks and extensions for future research.

## Literature review

Workforce planning has been developing in many industries and public service departments. Ebert (1976) investigated optimal aggregate schedules under changing productivity in manufacturing organizations. Zanakis and Maret (1981) used a Markov model to plan the coming year manpower under conflicting socio-econo-organizational objectives. Lee and Vairaktarakis (1997) addressed the workforce planning problem in serial assembly systems with a same production cycle in all stations. Herer and Harel (1998) analogized the size of temporary workforce with order quantity in the classical Newsvendor model, and used the model to determine the size of temporary workforce with fixed wage and productivity. Vairaktarakis et al. (2002) provided an integer programming model for workforce scheduling problem in synchronous production systems. Recently, Bard et al. (2003) and Jódice et al. (2005) focused on workforce planning problem in postal service departments. Fry et al. (2006) investigated the planning of fire fighters in a fire department by taking into account temporary absences and wastage. Chang et al. (2007) concentrated on the maintenance staff planning process and studied the tradeoff between maintenance personnel staffing levels and the throughput of a production line.

In the meantime, learning and forgetting effects have been widely considered in production systems. Salameh et al. (1993) presented a mathematical model for the learning effect in a finite production inventory model. Jaber and Salameh (1995) extended this paper by taking into account shortages and back orders. Jaber and Bonney (1996) explored the forgetting phenomenon in production processes. Afterward, several papers were published to investigate the effects of learning and forgetting on EMQ, EOQ and setup cost (Jaber and Bonney 1997a, 1999, 2003). As learning and forgetting effects gained more and more attention, the full

understanding of different learning and forgetting models became more important. Jaber and Bonney (1997b) presented a comparative study among the variable regression to invariant forgetting (VRIF) model, the variable regression to variable forgetting (VRVF) model and the learn-forget curve model (LFCM). After testing the accuracy of LFCM based on Globerson et al. (1989)'s experiment, they indicated the superiority of LFCM. Nembhard and Osothsilp (2001) also compared different learning and forgetting models, including 11 statistical models and 3 deterministic models, and found that the recency model (RC) performed consistently well in terms of all criteria they chose. But Jaber and Sikström (2004a) pointed out that the poor performance of LFCM in this study might be attributed to an error, and they proved that LFCM performed better than RC. After finding that LFCM satisfied many characteristics of forgetting (Jaber et al. 2003), Jaber and Sikström (2004b) conducted another comparison among LFCM, RC and the power integration diffusion (PID) model, and suggested that these three models produced very close predictions, but could be differentiated with different initial conditions.

But in the literature, few papers focus on the effect of learning and forgetting on workforce planning in production systems. As far as we know, the earliest paper investigating workforce planning with learning effect was from Ebert (1976), who presented a mathematical model to approximate optimal aggregate schedules under learning curve productivity. Anderson (2001) developed an optimal staffing policy to cope with non-stationary stochastic demand for both apprentice employees and experienced employees. Gans and Zhou (2002) incorporated learning and turnover by using Markov Decision Process to find the optimal hiring policy. Most of these published papers concentrate on workforce planning or scheduling problems during the production process after the employment decisions are made.

This paper aims at determining the optimal number of new employees using LFCM and variable wage with a Newsvendor model in pull systems. Our work is complementary to the papers cited above. Firstly, this paper is the first one that considers the human resource problem with a Newsvendor model, while other papers about Newsvendor model focus on non-human factors, such as pricing, discounting and multi-products (Lau and Lau 1995; Khouja 1995; Lin and Kroll 1997). Secondly, we take into account both learning and forgetting effect of workforce. Gans and Zhou (2002) also considered learning and turnover in service organization. However, we present a deterministic mathematical method to forecast the productivity of employees in a production system using LFCM. Both the perspective and the technique are different. Finally, we consider a workforce forecasting problem before the production actually starts, and we also investigate three different wage strategies in our model.

## Mathematical model

Incorporating productivity and wage into classical Newsvendor model

In this subsection, we modify the classical Newsvendor model and incorporate into our model human resource factors, productivity and wage. The following notation will be used throughout this paper:

Parameters:

- $F(\cdot)$  cumulative distribution function of demand
- $P$  unit selling price of the product
- $C$  unit cost of the product
- $C_0$  cost per unit excluding wage cost
- $V$  salvage value per unit
- $S$  shortage penalty cost per unit
- $\hat{q}$  production quantity per employee during the whole production horizon
- $\hat{w}$  total wage per employee during the production horizon

where  $\hat{q}$  is computed by considering learning and forgetting effect and  $\hat{w}$  is computed according to the wage strategy considered. Other parameters except  $C$  are input data and assumed to be known.

Decision variables:

- $Q$  order quantity, a decision variable in the classical Newsvendor model
- $m$  the number of new employees, a new decision variable in our model

From the classical Newsvendor model, the optimal order quantity can be obtained in the following famous fractile formula (Khouja 1999):

$$F(Q^*) = \frac{P + S - C}{P + S - V} \quad (1)$$

The superscript \* denotes optimality in the above equation. And from the definition of  $\hat{q}$ ,  $C$ ,  $C_0$  and  $\hat{w}$ , we can easily get

$$Q = \hat{q} \cdot m \quad (2)$$

$$C = \frac{\hat{w}m + QC_0}{Q} = \frac{\hat{w}}{\hat{q}} + C_0 \quad (3)$$

Then we substitute (2) and (3) into (1), and let the superscript \* denote optimality. So we have

$$F(m^* \cdot \hat{q}) = \frac{P + S - C_0 - \frac{\hat{w}}{\hat{q}}}{P + S - V} = \frac{P + S - C_0}{P + S - V} - \frac{1}{P + S - V} \cdot \frac{\hat{w}}{\hat{q}} \quad (4)$$

Solving  $m^*$  in (4) yields:

$$m^* = \frac{F^{-1}(\alpha - \beta \cdot \frac{\hat{w}}{\hat{q}})}{\hat{q}} \quad (5)$$

where  $\alpha = \frac{P+S-C_0}{P+S-V}$ ,  $\beta = \frac{1}{P+S-V}$ ,  $m^*$  is the optimal number of employees, and  $F^{-1}(\cdot)$  is the inverse function of the cumulative distributed function  $F(\cdot)$ .

In Eq. 5,  $\alpha$  and  $\beta$  are known parameters. In order to find out the optimal number of employees  $m^*$ , we need to discover the values of  $\hat{q}$  and  $\hat{w}$ . ‘‘Productivity with learning and forgetting’’ provides a mathematical model to calculate  $\hat{q}$  considering learning and forgetting phenomenon; and ‘‘Variable wage’’ discusses  $\hat{w}$  under three different wage strategies.

### Productivity with learning and forgetting

This subsection shows how to compute the total production quantity  $\hat{q}$  of every employee during a multi-cycle production horizon with LFCM.

In the considered production horizon, each cycle is divided into two periods: production period and break period. During production time, employees experience learning, and during break time they experience forgetting. Our goal is to find out how many units one employee can produce in the whole production horizon with alternating learning and forgetting periods.

For the whole production horizon, we define:

- $n$  number of production cycles in the considered production horizon
- $t_i$  duration of the  $i$ th cycle
- $t_{ip}$  production time in  $t_i$
- $t_{is}$  break time in  $t_i$
- $t_S$  duration of break time which will cause total forgetting
- $T_{i,j}$  time required to produce the  $j$ th unit of the  $i$ th cycle, especially we use  $T_1$  to denote the time required to produce the first unit in the first cycle for convenience
- $q_i$  quantity produced in  $t_{ip}$
- $s_i$  quantity produced in  $t_{is}$  if production were continuous
- $\lambda_i$  equivalent number of units continuously produced corresponding to the experience acquired until the beginning of  $t_i$ , with  $\lambda_1 = 0$
- $f_i$  forgetting slope in  $t_i$ , with  $0 \leq f_i \leq 1$
- $\ell$  learning slope

From the above definitions, we have  $t_i = t_{ip} + t_{is}$ ;  $t_{ip}$  and  $t_{is}$  can be different from one cycle to another, which means that every cycle may have different durations;  $t_S$  is a constant;  $\lambda_i$  is the experience remembered from the  $(i - 1)$ th cycle, which is counted in the equivalent number of units continuously produced. The value of  $\ell$  is defined as  $-\ell = \frac{\log(\text{learning rate})}{\log 2}$ .

Based on Wright (1936), the learning curve can be expressed in the following form:

$$T_x = T_1 \cdot x^{-\ell} \quad (6)$$

with  $T_x$  being the time to produce the  $x$ th unit,  $T_1$  the time to produce the first unit.

By introducing the forgetting curve from Carlson and Rowe (1976) into the learning curve, Jaber and Bonney (1996) built the LFCM, and considered a fixed batch size  $q$  in each cycle. The problem addressed here is a multi-cycle production system with undecided production quantity and duration in every cycle, which is more universal. After extending their work to  $n$  cycles, we get the equivalent units of experience at the beginning of the  $(i + 1)$ th cycle (see ‘‘Appendix 1’’ for details):

$$\lambda_{i+1} = (q_i + \lambda_i)^{(\ell+f_i)/\ell} \cdot (q_i + s_i + \lambda_i)^{-f_i/\ell} \quad (7)$$

and the forgetting slope in the  $i$ th cycle:

$$f_i = \frac{\ell(1 - \ell) \cdot \log(q_i + \lambda_i)}{\log \left[ \frac{t_S(1-\ell)}{T_1(q_i + \lambda_i)^{1-\ell}} + 1 \right]} \quad (8)$$

As  $\lambda_i$  units of experience is gained from the  $(i - 1)$ th cycle, the time to produce the first unit in the  $i$ th cycle equals to the time to produce the  $(\lambda_i + 1)$ th unit in the first cycle. During production time  $t_{ip}$  of the  $i$ th cycle, the learning curve is applicable and by following Eq. 6 we have:

$$T_{i,1} = T_1 \cdot (\lambda_i + 1)^{-\ell} \quad (9)$$

Similarly, the time to produce the  $j$ th unit in the  $i$ th cycle is:

$$T_{i,j} = T_1 \cdot (\lambda_i + j)^{-\ell} \quad (10)$$

If  $q_i$  units are produced in the  $i$ th cycle, it is easy to find out the production time  $t_{ip}$  in  $t_i$ : (See ‘‘Appendix 2’’ for details about the approximation of the following equation.)

$$\begin{aligned} t_{ip} &= T_{i,1} + T_{i,2} + \dots + T_{i,q_i} = T_1 \sum_{j=1}^{q_i} (\lambda_i + j)^{-\ell} \\ &\simeq T_1 \frac{(\lambda_i + q_i)^{1-\ell} - (\lambda_i + 1)^{1-\ell}}{1 - \ell} \end{aligned} \quad (11)$$

Similarly, the duration of the  $i$ th cycle is:

$$t_i = t_{ip} + t_{is} = T_1 \frac{(\lambda_i + q_i + s_i)^{1-\ell} - (\lambda_i + 1)^{1-\ell}}{1 - \ell} \quad (12)$$

Solving Eq. 11 for  $q_i$  and Eq. 12 for  $q_i + s_i$ , yields:

$$q_i = \left[ \frac{t_{ip}}{T_1} \cdot (1 - \ell) + (\lambda_i + 1)^{1-\ell} \right]^{1/(1-\ell)} - \lambda_i \quad (13)$$

$$q_i + s_i = \left[ \frac{t_{ip} + t_{is}}{T_1} \cdot (1 - \ell) + (\lambda_i + 1)^{1-\ell} \right]^{1/(1-\ell)} - \lambda_i \quad (14)$$

Then substituting (13) and (14) into (7) gives:

$$\lambda_{i+1} = \left[ \frac{t_{ip}}{T_1} \cdot (1 - \ell) + (\lambda_i + 1)^{1-\ell} \right]^{(\ell+f_i)/\ell(1-\ell)} \cdot \left[ \frac{t_{ip} + t_{is}}{T_1} \cdot (1 - \ell) + (\lambda_i + 1)^{1-\ell} \right]^{-f_i/\ell(1-\ell)} \quad (15)$$

Note that, in Jaber and Bonney's model, as  $\lambda_i$  is the equivalent units of experience remembered at the beginning of the  $i$ th cycle,  $q_i + \lambda_i$  denotes the equivalent units experienced in the  $i$ th cycle, which Jaber and Bonney used to calculate  $f_i$  and  $\lambda_{i+1}$ . But actually, only  $q_i$  units are produced in this  $i$ th cycle. (See Fig. 10 for the explanation of  $\lambda_i$ )

From Eqs. 8, 13 and 15, we obtain the following recursive equations:

$$\begin{cases} q_i &= \left[ \frac{t_{ip}}{T_1} \cdot (1 - \ell) + (\lambda_i + 1)^{1-\ell} \right]^{1/(1-\ell)} - \lambda_i \\ f_i &= \ell(1 - \ell) \cdot \log(q_i + \lambda_i) \log \left[ \frac{t_S(1-\ell)}{T_1(q_i + \lambda_i)^{1-\ell}} + 1 \right] \\ \lambda_{i+1} &= \left[ \frac{t_{ip}}{T_1} \cdot (1 - \ell) + (\lambda_i + 1)^{1-\ell} \right]^{(\ell+f_i)/\ell(1-\ell)} \cdot \left[ \frac{t_{ip} + t_{is}}{T_1} \cdot (1 - \ell) + (\lambda_i + 1)^{1-\ell} \right]^{-f_i/\ell(1-\ell)} \end{cases} \quad (16)$$

By knowing  $t_{ip}$ ,  $t_{is}$ ,  $t_S$ ,  $\lambda_i$  and  $\ell$ , we can take turns to solve  $q_i$ ,  $f_i$  and  $\lambda_{i+1}$ , and if we know  $t_{i+1,p}$ ,  $t_{i+1,s}$ , we can further get  $q_{i+1}$ ,  $f_{i+1}$  and  $\lambda_{i+2}$ . Repeating this procedure for  $n$  cycles, we can find out the production quantity  $q_i$  in every cycle. So the total production quantity  $\hat{q}$  can be easily calculated out in the following equation:

$$\hat{q} = \sum_{i=1}^n q_i \quad (17)$$

When there is learning effect but no forgetting effect, learning curve is applicable for the whole production process and  $\hat{q}$  can be solved from the following equation: (The subscript of  $T$  denotes the sequence numbers of the produced units.)

$$\begin{aligned} \sum_{i=1}^n t_{ip} &= T_1 + T_2 + \dots + T_{\hat{q}} \\ &= T_1 \cdot (1 + 2^{-\ell} + \dots + \hat{q}^{-\ell}) \end{aligned} \quad (18)$$

where  $T_k$  ( $k = 1, 2, \dots$ ) is the time to produce the  $k$ th unit. When there is no learning effect nor forgetting effect,  $\hat{q}$  can be easily deduced from the following equation:

$$\sum_{i=1}^n t_{ip} = \hat{q} \cdot T_1 \quad (19)$$

Now we find out how to calculate  $\hat{q}$ . In order to get the optimal number of employees  $m^*$  from Eq. 5, we will show how to calculate  $\hat{w}$  in the next subsection.

## Variable wage

This subsection shows how to compute the expected total wage per employee during the production horizon under different wage strategies.

Wage is one of the most important and complex issues in Human Resource Management. It has been proved that wage and performance have strong relations (Brown 1990; Ewing 1996). But the goal of our paper is to make a decision on how many employees should be hired before the production starts. So the actual performance is unknown. This requires that the employers choose a proper wage strategy to optimize the number of employees before they can measure the performance. Considering the characteristics of style goods production, we take into account the following three wage strategies, all of which are commonly used in real world, especially in labor-intensive production factories.

- Fixed wage strategy  
In this strategy, all employees are paid a fixed wage, no matter how many units he or she produces. This is a simple strategy, and is widely used in temporary employment. Under this strategy, the average wage cost of every product will decrease as the total quantity of products increases. So the more products are produced, the lower the average wage cost will be.
- Pay-by-unit strategy  
In this strategy, all employees are paid according to the number of units they produce. This strategy is also widely used in temporary employment. For the employers, compared to the fixed wage strategy, this strategy is not economical when the total quantity of products is large.
- Mixed wage strategy  
In this strategy, employees are paid with a basic fixed wage and a bonus depending on how many units they can produce. This strategy combines the fixed wage strategy and pay-by-unit strategy, and often appears in a long time employment, but its complexity increases the difficulty in wage management.

Without loss of generality, we only consider the third strategy. The other two can be considered as special cases. To calculate the expected wage, the following notations will be used:

- $w_i$  fixed wage in the  $i$ th cycle
- $\delta_i$  bonus per unit in the  $i$ th cycle

From the definition of  $\hat{w}$ , we have:

$$\hat{w} = \sum_{i=1}^n (w_i + \delta_i q_i) \quad (20)$$

As from (20), when  $w_i$  is constant and  $\delta_i = 0$ , it is a fixed wage strategy, when  $w_i = 0$ , and  $\delta_i > 0$ , it is a pay-by-unit strategy and when  $w_i$  is constant and  $\delta_i > 0$ , it is a mixed wage strategy. If the values of  $w_i$  and  $\delta_i$  are known in every cycle, by substituting the values of  $q_i$  from “Productivity with learning and forgetting” into Eq. 20, we can get  $\hat{w}$ .

### Model extensions

The model above can also be extended and implemented in some more complex situations. Firstly, this model is applicable when there is initial experience. In our model, we pre-defined that the initial experience  $\lambda_1 = 0$ . If  $\lambda_1 > 0$ , which denotes the existence of initial experience, by following Eq. 9, the time to produce the first product with initial experience can be derived. Secondly, this model is also applicable when there is a nonzero initial stock  $Q_0$ , if the products in the stock have the same value of  $P$ ,  $S$ ,  $C$  and  $V$  as the products from production. In this simple case, the optimal quantity of products  $Q^*$  will be equal to  $Q_0 + m^* \hat{q}$ , so the optimal employee size is  $m^* = \frac{F^{-1}(\alpha - \beta \cdot \frac{\hat{w}}{\hat{q}}) - Q_0}{\hat{q}}$ . Also note that, if the products from the stock have different values of  $P$ ,  $S$ ,  $C$  or  $V$ , the fractile formula of Newsvendor model must be improved since it does hold any more in this situation.

### Numerical results

From “Mathematical model”, we get the values of  $\hat{q}$  and  $\hat{w}$ , so the optimal number of employees  $m^*$  can be easily calculated by implementing Eq. 5. This section presents numerical results for our mathematical model. It first gives a basic example, and based on this example, it analyzes the sensitivity of some key parameters, such as  $t_{ip} - t_{is}$ ,  $n$ ,  $t_S$ ,  $\ell$  and different wage strategies.

#### A basic example

This subsection considers an example with detailed values. During the production horizon, we consider one-week production cycles where the employees work from Monday to Friday and rest from Saturday to Sunday ( $t_{ip} = 5$  days and  $t_{is} = 2$  days). The whole production horizon  $n$  is 26 weeks, namely half a year. The time it takes to produce the first unit  $T_1$  is 0.05 days. We assume that total forgetting will occur when the break time is longer than 300 days as is assumed by Jaber and Bonney. The learning slope  $\ell$  is 0.152 (90% learning rate). The selling price is  $P = 10$ ; the cost excluding wage is  $C_0 = 3$ ; the shortage penalty cost is  $S = 2$ ; the salvage value is  $V = 3$ , so we have  $\alpha = 1$  and  $\beta = 1/9$  and the demand is normally distributed with mean  $\mu = 1,000,000$

**Table 1** Process of calculating  $\hat{q}$

Cycle no.	$q_i$	$f_i$	$\lambda_i$	Cumulative $\hat{q}_i$
1	191	0.165	0	191
2	229	0.211	124	420
3	246	0.242	251	666
4	257	0.266	367	924
5	265	0.285	473	1,189
6	272	0.301	568	1,461
7	277	0.315	655	1,737
8	281	0.327	733	2,018
9	284	0.337	804	2,302
10	287	0.346	869	2,589
11	290	0.354	927	2,879
12	292	0.361	980	3,171
13	294	0.367	1,028	3,465
14	295	0.373	1,071	3,760
15	297	0.378	1,111	4,057
16	298	0.382	1,147	4,355
17	299	0.386	1,179	4,654
18	300	0.390	1,209	4,955
19	301	0.393	1,235	5,256
20	302	0.396	1,260	5,558
21	303	0.398	1,282	5,861
22	303	0.401	1,302	6,164
23	304	0.403	1,320	6,468
24	305	0.405	1,337	6,773
25	305	0.407	1,352	7,077
26	305	0.408	1,366	7,383

and standard deviation  $\sigma = 100,000$ . And for convenience, we use fixed wage strategy with  $w_i = 200$  and  $\delta_i = 0$ .

By substituting these values into the system of equations (16), we can get  $q_i$ , the quantity produced in every cycle. And from (17), we can further obtain the cumulative quantity that every employee produces during the entire production horizon. Table 1 illustrates the iterative process of calculating  $\hat{q}$  by using a program coded in C language. Note that, as the production time becomes long, the value of  $q_i$  becomes stable. From Table 1, we get  $\hat{q} = 7,383$ , which is the quantity that one employee can produce during the production horizon. From Eq. 20, we easily get  $\hat{w} = 5,200$ . Finally, we substitute the values of  $\hat{w}$ ,  $\hat{q}$ ,  $\alpha$  and  $\beta$  into Eq. 5, the optimal number of employees is  $m^* = 155$ . The corresponding optimal order quantity  $Q^*$  is 1,141,688. From Eq. 18, the optimal number of employees under the situation with learning but without forgetting is 131, and from Eq. 19, the optimal number of employees is 414 if there is no learning nor forgetting.

Table 2 shows the comparison among the three situations. From Table 2, we can see that by considering learning and forgetting effect, the optimal number of employees can

**Table 2** Comparison between the three situations

	NLF <sup>a</sup>	LC <sup>b</sup>	LFCM <sup>c</sup>
$\hat{q}$	2,600	8,766	7,383
$m^*$	414	131	155
GAP	62.56% <sup>d</sup>	18.32% <sup>e</sup>	–

<sup>a</sup> NLF denotes the situation without learning nor forgetting

<sup>b</sup> LC denotes the situation with learning but without forgetting

<sup>c</sup> LFCM denotes the situation with learning and forgetting

<sup>d</sup>  $GAP = 1 - m_{LFCM}^*/m_{NLF}^*$

<sup>e</sup>  $GAP = m_{LFCM}^*/m_{LC}^* - 1$

theoretically reduce about 62.56% compared to NLF, and by comparing LFCM to LC, there should be 18.32% additional employees due to forgetting effect in our basic example.

### Sensitivity analysis

In this subsection, we present a series of numerical results to analyze the impact of  $n$ ,  $t_S$ ,  $t_{ip} - t_{is}$ ,  $\ell$  and different wage strategies on the optimal number of employees  $m^*$ . For this purpose, the value of  $P$ ,  $S$ ,  $V$ ,  $C_0$ , namely the value of  $\alpha$  and  $\beta$  should be defined. By considering the practical situation of style goods, we assume that  $P > C_0 \geq V \geq 0$  and  $P > S \geq 0$ . We follow this rule to randomly choose the values of  $P$ ,  $S$ ,  $V$  and  $C_0$ . We first randomly generate a value of  $P$  from  $(0, 100]$ . Then in the range of  $[0, P)$ , we randomly choose the values of  $C_0$  and  $S$ . At last, the value of  $V$  is randomly chosen in  $[0, C_0]$ . For calculating convenience,  $P$ ,  $S$ ,  $V$  and  $C_0$  are assumed to be integers. The 30 groups of values we choose to use in the following sections are listed in “Appendix 3”. The optimal numbers of new employees in three situations are considered: with both learning and forgetting (LFCM), with learning but without forgetting (LC) and without learning nor forgetting (NLF).

### Impact of $n$

Based on the basic example,  $n$  takes its value from 6, 13, 26, 39 or 52. Note that the basic example corresponds to  $n = 26$ . Table 3 shows the computational results under different situations. From the average values in Table 3, it is clear that learning effect (the situation of “LC”) reduces significantly the optimal number of employees. Taking into account the forgetting effect (the situation of “LFCM”) increases the optimal number of employees but still which, however, remains smaller than the situation of “NLF”.

In Fig. 1, we analyze the impact of  $n$  by comparing the number of days that an employee needs to finish the whole production task. This value is obtained by multiplying the average number of employees with the corresponding number of working days under different  $n$ . For example, under LFCM,

when  $n = 13$ , the number of days is  $13(\text{weeks}) \times 5(\text{days}) \times 314(\text{employees}) = 20,410$ . Theoretically, if there were no learning nor forgetting, facing the same task, the number of days is constant no matter how  $n$  changes. See in Fig. 1 the columns marked as “NLF”. From this figure, we find out that due to learning effect, the number of days decreases as  $n$  increases, see columns marked as “LC”. And forgetting effect “slows down” the speed of decrease, see columns marked as “LFCM”. This group of columns is getting shorter but always higher than the “LC” group.

Then we use PAE (percentage of additional employees,  $PAE = m_{LFCM}^*/m_{LC}^* - 1$ ) to denote the gap between the situation LC and the situation LFCM, and PSE (percentage of saved employees,  $PSE = 1 - m_{LFCM}^*/m_{NLF}^*$ ) to denote the gap between the situation NLF and the situation LFCM. Figure 2 shows how the average PAE and PSE change when  $n$  increases. Note that, as the coming demand remains the same, without learning or forgetting, the optimal number of new employees will decrease linearly with  $n$ . But when taking account of learning and forgetting, this relation will no longer be linear. Figure 2 illuminates this change. From Fig. 2, as  $n$  increases, both PSE and PAE increase. The increase of PSE indicates that as the production duration increases, learning effect becomes more significant, thus can lead to more employee saving, and the increase of PAE suggests that the forgetting effect, as an inhibitor of learning, will also be more active. Although the value of additional employees ( $m_{LFCM}^* - m_{LC}^*$ ) due to forgetting decreases (see Table 3), the value of PAE increases contrarily.

### Impact of $t_S$

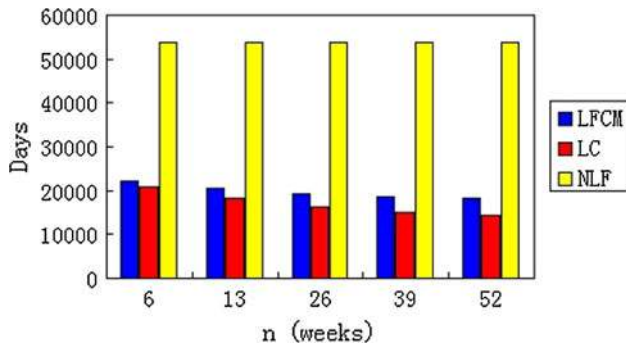
From the definition of  $t_S$ , it is like an intrinsic character of employees, which, together with  $\ell$  and  $\lambda_i$ , decides the forgetting rate  $f_i$  in LFCM (Jaber and Sikström 2004b). Based on our basic example, we consider four values of  $t_S$ : 2, 300, 3,000 and  $\infty$ . Note that when  $t_S = 300$ , it is the basic example; when  $t_S = 2$  we have  $t_{is} = t_S = 2$  which means that total forgetting will occur in every cycle. Table 4 shows the computational results in different situations, and Fig. 3 shows how the average PAE and PSE change when  $t_S$  increases. From the average values in the last row of Table 4, the optimal number of employees under “LFCM” approaches the optimal number of employees under “LC”, as  $t_S$  approaches infinity.

Note that, as  $t_S$  means the time to achieve total forgetting, the larger  $t_S$  is, the smaller the forgetting effect will be. So in Fig. 3, as  $t_S$  increases, PSE increases more and more slowly, and PAE decreases more and more slowly. In other words, the longer time it takes for employees to totally forget what they have learnt, the less additional employees are needed due to forgetting.

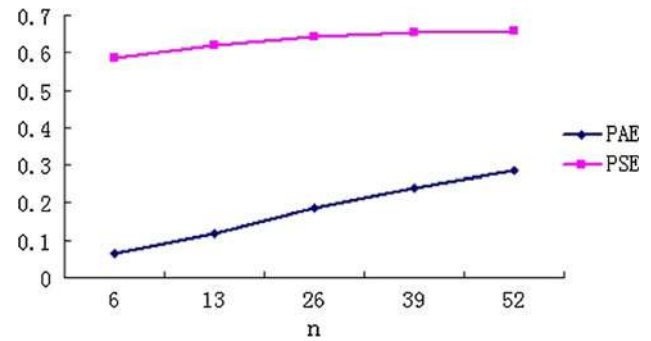


**Table 3** Number of employees under different  $n$

Example no.	$n = 6$			$n = 13$			$n = 26$			$n = 39$			$n = 52$		
	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF
1	741	695	1,786	313	280	824	147	124	412	95	77	275	70	55	206
2	720	676	1,742	304	272	804	143	120	402	93	75	268	68	53	201
3	812	762	1,949	343	307	900	161	136	450	104	84	300	77	60	225
4	769	722	1,837	325	291	848	153	129	424	99	80	283	73	57	212
5	754	708	1,818	318	285	839	149	126	420	97	78	280	72	56	210
6	747	701	1,805	315	282	833	148	125	416	96	77	278	71	55	208
7	737	692	1,785	311	278	824	146	123	412	95	76	275	70	54	206
8	733	688	1,775	309	277	819	145	122	410	94	76	273	70	54	205
9	736	690	1,779	310	278	821	146	123	411	95	76	274	70	54	205
10	753	707	1,813	318	285	837	149	126	418	97	78	279	72	56	209
11	759	713	1,789	321	288	826	151	127	413	98	79	275	72	56	206
12	744	699	1,779	314	282	821	148	125	411	96	77	274	71	55	205
13	738	693	1,774	312	279	819	146	123	409	95	77	273	70	55	205
14	724	679	1,743	305	273	805	143	121	402	93	75	268	69	53	201
15	755	709	1,828	318	285	844	149	126	422	97	78	281	72	56	211
16	750	704	1,817	316	283	839	148	125	419	96	78	280	71	55	210
17	749	703	1,816	316	283	838	148	125	419	96	78	279	71	55	210
18	751	705	1,821	317	284	840	149	125	420	97	78	280	71	55	210
19	761	714	1,841	321	287	850	151	127	425	98	79	283	72	56	212
20	766	721	1,761	324	292	813	153	130	406	99	81	271	73	58	203
21	741	695	1,788	313	280	825	147	124	413	95	77	275	70	55	206
22	735	690	1,777	310	277	820	145	123	410	94	76	273	70	54	205
23	726	681	1,756	306	274	810	144	121	405	93	75	270	69	53	203
24	719	674	1,738	303	271	802	142	120	401	92	74	267	68	53	201
25	713	670	1,726	301	269	796	141	119	398	92	74	265	68	53	199
26	705	662	1,701	298	266	785	140	118	392	91	73	262	67	52	196
27	692	650	1,653	292	262	763	137	116	381	89	72	254	66	51	191
28	783	735	1,858	331	296	858	155	131	429	101	81	286	75	58	214
29	762	715	1,831	321	288	845	151	127	423	98	79	282	72	56	211
30	760	713	1,832	320	287	846	150	127	423	98	79	282	72	56	211
Average	745	699	1,791	314	281	826	147	124	413	96	77	275	71	55	207



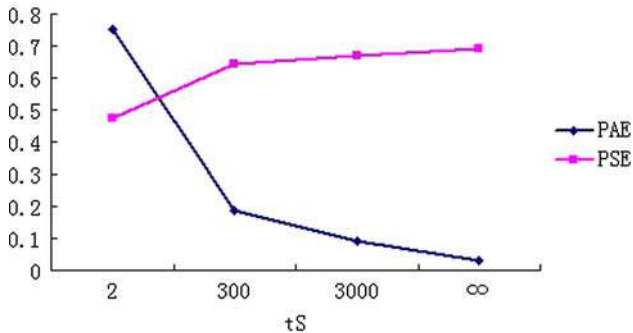
**Fig. 1** Number of days for one employee under different  $n$



**Fig. 2** The impact of  $n$  on PAE and PSE

**Table 4** Number of employees under different  $t_S$ 

Example no.	$t_S = 2$			$t_S = 300$			$t_S = 3,000$			$t_S = \infty$		
	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF
1	217	124	412	147	124	412	135	124	412	127	124	412
2	211	120	402	143	120	402	131	120	402	124	120	402
3	237	136	450	161	136	450	148	136	450	140	136	450
4	225	129	424	153	129	424	141	129	424	132	129	424
5	221	126	420	149	126	420	138	126	420	129	126	420
6	219	125	416	148	125	416	136	125	416	128	125	416
7	216	123	412	146	123	412	134	123	412	126	123	412
8	215	122	410	145	122	410	134	122	410	126	122	410
9	215	123	411	146	123	411	134	123	411	126	123	411
10	220	126	418	149	126	418	138	126	418	129	126	418
11	221	127	413	151	127	413	139	127	413	131	127	413
12	217	125	411	148	125	411	136	125	411	128	125	411
13	216	123	409	146	123	409	135	123	409	127	123	409
14	212	121	402	143	121	402	132	121	402	124	121	402
15	221	126	422	149	126	422	138	126	422	130	126	422
16	220	125	419	148	125	419	137	125	419	129	125	419
17	220	125	419	148	125	419	137	125	419	129	125	419
18	220	125	420	149	125	420	137	125	420	129	125	420
19	223	127	425	151	127	425	139	127	425	131	127	425
20	222	130	406	153	130	406	141	130	406	133	130	406
21	217	124	413	147	124	413	135	124	413	127	124	413
22	215	123	410	145	123	410	134	123	410	126	123	410
23	212	121	405	144	121	405	132	121	405	125	121	405
24	210	120	401	142	120	401	131	120	401	123	120	401
25	209	119	398	141	119	398	130	119	398	122	119	398
26	206	118	392	140	118	392	129	118	392	121	118	392
27	202	116	381	137	116	381	127	116	381	119	116	381
28	228	131	429	155	131	429	143	131	429	135	131	429
29	223	127	423	151	127	423	139	127	423	131	127	423
30	222	127	423	150	127	423	139	127	423	130	127	423
Average	218	124	413	147	124	413	136	124	413	128	124	413

**Fig. 3** The impact of  $t_S$  on PAE and PSE

Now, pay more attention to the situations with  $t_S = 2$  and  $t_S = \infty$ , respectively. As  $t_S = 2$  means after every break, employees forget all they learnt from their previous production, namely in every cycle, the time to produce the first unit constantly is  $T_1$ , and as  $t_{ip}$  and  $t_{is}$  are constant, the product quantity  $q_i$  becomes the same in every cycle, as if at the beginning of every cycle the production refreshes. So the theoretical result in this situation can be derived by implementing learning curve separately in every cycle without considering forgetting effect. For  $t_S = \infty$ , as is demonstrated by Jaber and Bonney, “the learn-forget curve coincides with the original learning curve.” This means that the whole multi-cycle production process with learning and forgetting can be transformed into one cycle production

**Table 5** Model veracity

	$t_S = 2$	$t_S = \infty$	$t_{ip} - t_{is} = 7 - 0$
Model results	218	128	84
Theoretical results	219	124	84
Gap	0.46%	3.23%	0.00%

process with learning. By implementing learning curve for the entire production process, the theoretical result can be easily obtained. The above two extreme situations offer chances to verify our model. In the first column and second column of Table 5, when  $t_S = 2$  and  $t_S = \infty$ , according to Table 4, the results of our model are 218 and 128 respectively, and by merely applying learning curve as we mentioned above, the theoretical results of these two situations are respectively 219 and 124. The gaps between the results of our model and the theoretical results are small and acceptable. The inaccuracy comes from the approximation of Eq. 11.

#### Impact of $t_{ip}$ and $t_{is}$

In our model, learning refers to employees improving their productivity during production time  $t_{ip}$ , and forgetting refers to employees losing experience during break time  $t_{is}$ . According to Jaber and Bonney (1997b), LFCM coincides with the empirical data from Globerson et al. (1989), if the break time is between 16 and 82 days. But a work schedule with weekly break time is more practical.

Based on our basic example, we adjust the work schedule within every week from 1 working day and 6 rest days to 7 working days and 0 rest day, particularly when  $t_{ip} = 7$  and  $t_{is} = 0$ , there will be no rest time, thus no forgetting will occur. In this case, the result of our model should be theoretically equal to the situation with learning but no forgetting. See the last column of Table 5 for comparison between the results of our model and the theoretical results. We can see that the results are exactly the same. When  $t_{ip} = 1$  and  $t_{is} = 6$ , under the situation with no learning nor forgetting, we cannot find optimal values for some of the examples, because the productivity of the employees is too low to profit. So the value of PSE under  $t_{ip} = 1$  and  $t_{is} = 6$  is undefined. Table 6 shows a part of the computational results of different working schedules.

Figure 4 illustrates the result of the similar analysis on the number of days for one employee in "Impact of  $n$ ". From this figure we notice the same trend as the impact of  $n$ . This is because both of them describe the trend of the number of days when the working time increases. But the difference is that the increase of  $n$  implies more production cycles and the increase of  $t_{ip}$  indicates more working days in every cycle. Particularly when  $t_{ip} = 7$  and  $t_{is} = 0$ , the column marked as

"LC" and the column marked as "LFCM" have equal height which demonstrates the veracity of our model.

Figure 5 shows how PAE and PSE change when  $t_{ip}$  increases. Note that, as  $t_{ip}$  means the working time per cycle and  $t_{is}$  means the rest time per cycle, the increase of  $t_{ip}$  and the decrease of  $t_{is}$  will lead to a smaller optimal number of employees. This trend coincides with the computational results in Table 6. The average values in the table denote the decrease of both "LFCM" and "LC" values, and when  $t_{ip} = 7$  and  $t_{is} = 0$ , the optimal numbers of employees are identical under "LFCM" or "LC".

#### Impact of $\ell$

Based on the basic example, let  $\ell$  vary from 100% learning rate to 80% learning rate. Table 7 shows a part of the computational results, and Fig. 6 shows how PAE and PSE change when  $\ell$  varies.

Note that, as  $\ell$  means the effect of learning, the smaller the learning rate is, the more significant the learning effect will be. So in Fig. 6, as the learning rate decreases, PSE convexly increases and PAE increases concavely. In other words, as the learning rate decreases, the learning effect becomes more and more significant, employees' productivity may improve several times than before. Although the forgetting effect can slow down this trend, the optimal number of employees is still quickly reduced. Normally, 90% learning rate is widely used when considering learning curve. And when the learning rate is under 80%, the theoretical results seem to be quite unrealistic. In Table 7, when learning rate is 90%, the optimal number of employee is 147 under "LFCM". The number of employees saved is 266, compared to the optimal number of employee 413 under "NLF", and the number of additional employees due to forgetting is 23, compared to the optimal number of employee 124 under "LC".

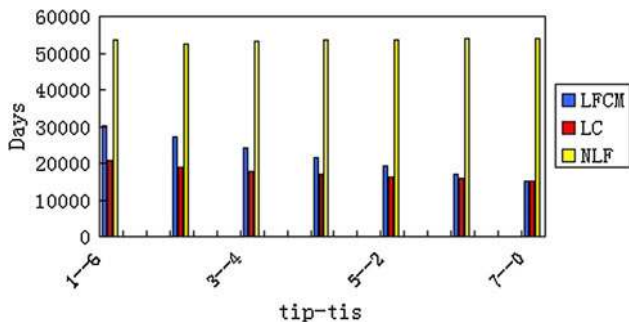
Specially, when learning rate is 100% ( $\ell = 0$ ), our model is not applicable as  $\ell = 0$  cannot be a denominator. But as this means there is no learning effect, we can still get the results by using Eq. 19.

#### Impact of different wage strategies

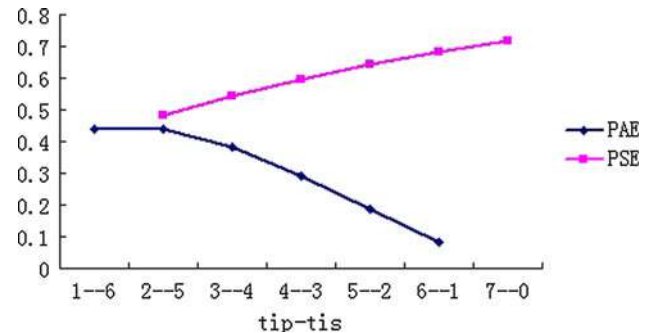
Based on the basic model, we use different wage strategies to evaluate the wage effect on both the optimal number of employees and the total wage cost. To facilitate the comparison, we give values to  $w_i$  and  $\delta_i$  in such a way that these three strategies give employees the same total wage in the situation of NLF. In the mixed wage strategy, the fixed part is a half of the whole wage. For example, in the basic model,  $w_i$  is 200 and  $\hat{w}$  is 5,200, so we consider a pay-by-unit strategy with  $w_i = 0$  and  $\delta_i = 2$ , and a mixed wage strategy with  $w_i = 100$ , and  $\delta_i = 1$ . And then based on the basic model, we respectively let  $w_i$  equal to 100, 150, 200, 250 and

**Table 6** Number of employees under different  $t_{ip} - t_{is}$

Example no.	4 - 3			5 - 2			6 - 1			7 - 0		
	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF
1	208	161	513	147	124	412	108	100	344	83	83	296
2	202	156	501	143	120	402	105	97	336	81	81	288
3	227	176	560	161	136	450	119	110	376	91	92	323
4	215	167	526	153	129	424	113	104	355	87	87	305
5	211	164	522	149	126	420	110	102	351	85	85	301
6	209	162	519	148	125	416	109	101	348	84	84	299
7	207	160	514	146	123	412	108	99	344	83	83	295
8	206	159	511	145	122	410	107	99	342	82	82	293
9	206	160	512	146	123	411	107	99	343	82	83	294
10	211	163	521	149	126	418	110	102	350	85	85	301
11	212	165	510	151	127	413	112	103	347	86	86	299
12	208	162	510	148	125	411	109	101	344	84	84	296
13	207	160	509	146	123	409	108	100	342	83	83	294
14	203	157	501	143	121	402	106	98	336	81	81	289
15	212	164	526	149	126	422	110	102	352	85	85	302
16	210	163	523	148	125	419	109	101	350	84	84	300
17	210	162	523	148	125	419	109	101	350	84	84	300
18	211	163	524	149	125	420	110	101	351	84	84	301
19	213	165	530	151	127	425	111	102	355	85	85	304
20	213	167	498	153	130	406	114	106	343	89	89	297
21	208	161	514	147	124	413	108	100	345	83	83	296
22	206	159	511	145	123	410	107	99	342	82	83	294
23	204	157	505	144	121	405	106	98	338	81	82	290
24	202	156	500	142	120	401	105	97	335	81	81	287
25	200	155	496	141	119	398	104	96	332	80	80	285
26	198	153	489	140	118	392	103	95	328	79	79	282
27	194	150	473	137	116	381	101	94	320	78	78	275
28	219	170	531	155	131	429	115	106	360	89	89	310
29	213	165	526	151	127	423	111	103	353	86	86	304
30	213	165	527	150	127	423	111	102	353	85	85	303
Average	209	161	514	147	124	413	109	100	345	84	84	297



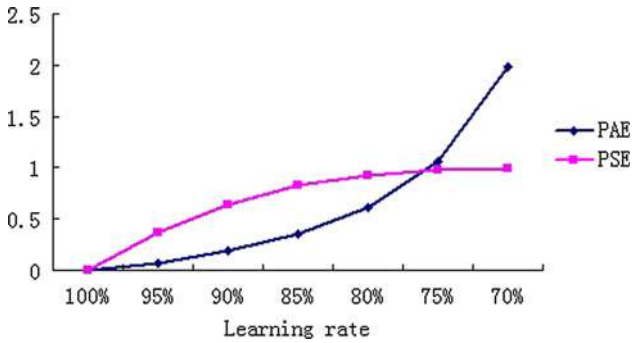
**Fig. 4** Number of days for one employee under different  $t_{ip} - t_{is}$



**Fig. 5** Impact of  $t_{ip}$  and  $t_{is}$  on PAE and PSE

**Table 7** Number of employees under different  $\ell$

Example no.	100%			95%			90%			85%			80%		
	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF
1	412	412	412	258	240	412	147	124	412	72	53	412	29	18	412
2	402	402	402	251	234	402	143	120	402	70	52	402	28	17	402
3	450	450	450	282	263	450	161	136	450	80	59	450	31	20	450
4	424	424	424	267	249	424	153	129	424	76	56	424	30	19	424
5	420	420	420	263	245	420	149	126	420	74	54	420	29	18	420
6	416	416	416	260	243	416	148	125	416	73	54	416	29	18	416
7	412	412	412	257	240	412	146	123	412	72	53	412	28	18	412
8	410	410	410	256	238	410	145	122	410	72	53	410	28	18	410
9	411	411	411	256	239	411	146	123	411	72	53	411	28	18	411
10	418	418	418	262	244	418	149	126	418	74	54	418	29	18	418
11	413	413	413	262	244	413	151	127	413	75	55	413	30	19	413
12	411	411	411	258	241	411	148	125	411	73	54	411	29	18	411
13	409	409	409	257	239	409	146	123	409	72	53	409	29	18	409
14	402	402	402	252	235	402	143	121	402	71	52	402	28	17	402
15	422	422	422	263	245	422	149	126	422	74	54	422	29	18	422
16	419	419	419	262	244	419	148	125	419	73	54	419	29	18	419
17	419	419	419	261	244	419	148	125	419	73	54	419	29	18	419
18	420	420	420	262	244	420	149	125	420	73	54	420	29	18	420
19	425	425	425	265	247	425	151	127	425	74	55	425	29	18	425
20	406	406	406	261	245	406	153	130	406	78	58	406	32	20	406
21	413	413	413	258	241	413	147	124	413	72	53	413	29	18	413
22	410	410	410	256	239	410	145	123	410	72	53	410	28	18	410
23	405	405	405	253	236	405	144	121	405	71	52	405	28	17	405
24	401	401	401	251	233	401	142	120	401	70	52	401	28	17	401
25	398	398	398	249	232	398	141	119	398	70	51	398	27	17	398
26	392	392	392	246	229	392	140	118	392	69	51	392	27	17	392
27	381	381	381	240	224	381	137	116	381	68	50	381	27	17	381
28	429	429	429	271	253	429	155	131	429	77	57	429	31	19	429
29	423	423	423	265	247	423	151	127	423	75	55	423	29	18	423
30	423	423	423	265	247	423	150	127	423	74	55	423	29	18	423
Average	413	413	413	259	241	413	147	124	413	73	54	413	29	18	413



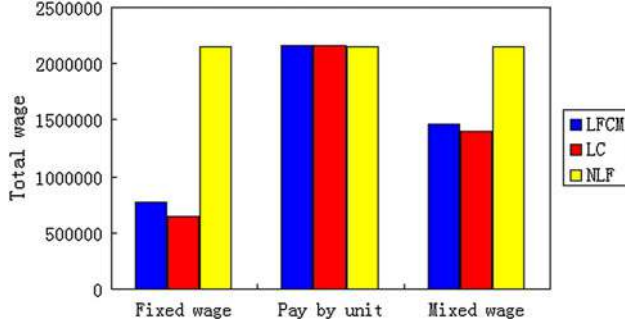
**Fig. 6** Impact of  $\ell$  on PAE and PSE

300. Table 8 shows the computational results. In Table 8, SET 1 to SET 5, respectively, denote the situations of  $w_i = 100$  to  $w_i = 300$ . The values corresponding to each SET are the average values of 30 different groups of  $\alpha$  and  $\beta$ . From the results of Table 8, as the wage increases from SET 1 to SET 5, the number of employees decreases slowly.

Figure 7 shows how the total wage changes under different wage strategies. The total wage is defined as the sum of every employee's wage during the whole production process and is obtained by multiplying the number of employees  $m^*$  with the corresponding wage  $\hat{w}$ . From Fig. 7, the "NLF" columns have the same height. This is due to the presupposition that the three strategies will give employees the same total wage in the situation of NLF. As a mixed wage strategy

**Table 8** Number of employees under different wage strategy

SET no.	Fixed wage			Pay by unit			Mixed wage		
	LFCM	LC	NLF	LFCM	LC	NLF	LFCM	LC	NLF
Set 1	148	125	417	147	124	417	148	124	417
Set 2	148	125	415	146	123	415	147	124	415
Set 3	147	124	413	146	123	413	146	123	413
Set 4	147	124	411	145	122	411	146	123	411
Set 5	147	124	409	144	121	409	145	123	409
Average	148	124	413	146	123	413	146	123	413

**Fig. 7** Impact of different wage strategies

includes two parts of wage, the fixed part and the bonus part, the values under mixed wage strategy are between the corresponding values from the other two strategies. Under pay-by-unit strategy, the height of the three columns are nearly the same, because the optimal order quantity is determined by the coming demand, no matter how many employees are employed. Under both fixed wage strategy and mixed wage strategy, because of forgetting effect, more employees are needed, so the “LFCM” columns are higher than the “LC” columns. And from Fig. 7, pay-by-unit strategy leads to the most wage cost, while fixed wage strategy leads to the least here. This is because  $w_i < \delta_i \cdot Q^*$ . If production quantity  $\hat{q}$  is much smaller, we will have  $w_i > \delta_i \cdot Q^*$ , fixed wage strategy will cost more.

## Conclusions

This paper incorporates learning, forgetting and variable wage into the classical Newsvendor model to determine the optimal number of new employees to achieve maximum profit. The impacts of several parameters on the optimal number of employees are analyzed, and some extreme situations are discussed.

The numerical results give some interesting insights. Firstly, learning effect can significantly improve the productivity, and by taking into account forgetting effect, the

forecast of employee’s productivity will be closer to practice, thus the decision on how many employees to hire will be more accurate. Secondly, although the forgetting effect, as an inhibitor of learning effect, can become more significant as the production time gets longer (for example, more production cycles or longer production time in every cycle), the learning effect is dominant during the whole process. Thirdly, the optimal number of employees is sensitive to learning rate, which indicates that a proper learning rate is an important parameter when deciding the number of employees. Finally, different wage strategies have less impact on the optimal employee size but can influence the total wage cost.

As we investigate the workforce planning problem in pull systems, this paper focuses on how to make decisions when employing workforce, which is the first step of a whole workforce management. Direct extensions to this paper are numerous. The first extension might be taking into account the relation between wage and productivity. As in our paper, the production does not start before the recruiting finishes, productivity are supposed to be determined by LFCM. The impact of wage as incentive on productivity is not considered. Second, as the production time  $t_{ip}$  and the break time  $t_{is}$  can be different in every production cycle, this model is applicable when the work schedule is irregular. Besides the break time of weekend, a much more realistic and complex work schedule with temporary overtime or halt of production can be considered. Third, if some of the employees are novice while some may already have experience, it is common sense that experienced employees will have higher salary, therefore how to balance between productivity and wage becomes worthy of investigation.

Extending this single product model to a multi-product one is also an interesting topic. When considering multi-product models, employees may have multiple learning and forgetting curves corresponding to different products, and experience learning and forgetting when they are assigned to different tasks. If the products are dependent in production, experience transmission must be considered.

Furthermore, as this paper investigated a “one day” case of the newsboy, what will happen in the next several periods?

After the first period, if the factory continues in producing the same or similar products and the interval between two periods is smaller than  $t_S$ , some previous experience may be remembered. If the factory chooses producing a different kind of products, will there be experience transmission between two kinds of products? What is the employment policies in the next period? Considering learning and forgetting, how does the factory fire people or hire people? Will all employees be fired or only some of them? Can employees that are fired be available to be hired in the next period? Will people who are fired forget faster than those who just take a break during season intervals? Thus, much more effort needs to be devoted into this area, especially when the firing and hiring costs may be non-negligible due to more and more restrictive legislative constraints.

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## Appendix 1: LFCM and its extension

According to Jaber and Bonney (1996), for the first learning and forgetting cycle (see Fig. 8), we have

$$\lambda = q^{(\ell+f)/\ell} \cdot (q+s)^{-f/\ell} \quad (21)$$

$\lambda$  represents the experience remembered from the first cycle expressed in the equivalent number of units continuously produced (therefore without forgetting). Note that, Eq. 21 is a special case of Eq. 7, by letting  $i = 1$  in Eq. 7. An extreme situation called total forgetting was also proposed by Jaber and Bonney. Suppose that total forgetting appears when the break time reaches  $t_S$ , and the total forgetting occurs at the point of  $(q + R)$  units. Under this situation, the time to produce the  $(q + R)$ th unit equals to  $T_1$  (see Fig. 9). The following equation is used to calculate  $f$  in Jaber and Bonney's paper.

$$f = \ell \cdot \frac{\log q}{\log(q+R) - \log q} \quad (22)$$

Now we extend the above results to  $n$ -cycle horizon. In the  $i$ th cycle,  $\lambda_i$  is the equivalent units of the experience gained from the  $(i - 1)$ th cycle; the forgetting slope is  $f_i$ , which can be calculated from the  $i$ th cycle. We use the same notations of "Mathematical model", such as  $q_i$ ,  $s_i$  etc. Taking into account the experience gained from the  $(i - 1)$ th cycle  $\lambda_i$ , when the production of the  $i$ th cycle finishes, experience of  $q_i + \lambda_i$  units are gained. Figure 10 explains how  $\lambda_i$  changes

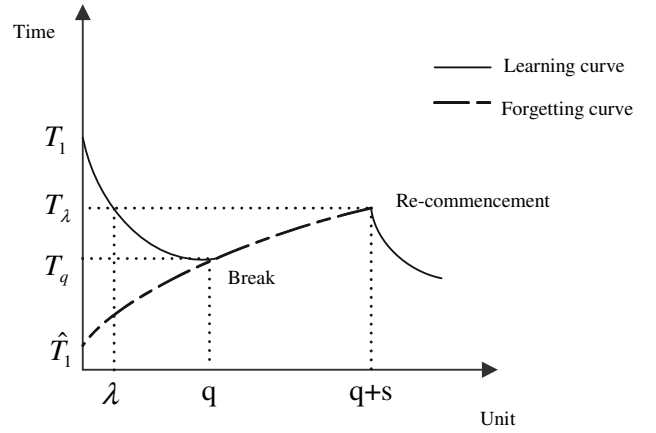


Fig. 8 The learn-forget curve

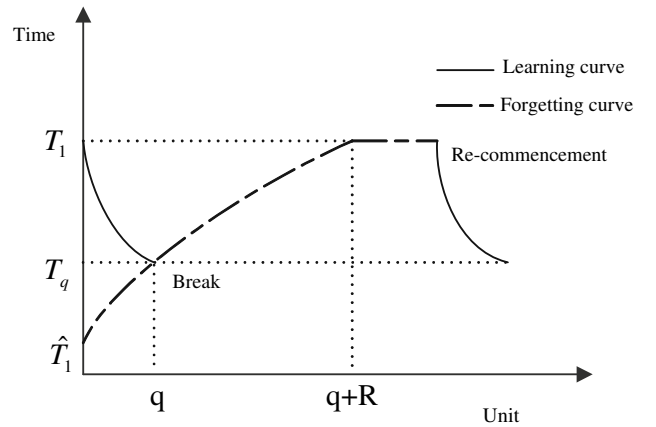


Fig. 9 The learn-forget curve with total forgetting

in the first three cycles. In this figure, colored periods denote the working time and white periods denote the rest time. For example, after the first cycle, namely after the first colored period and white period, the equivalent experience is  $\lambda_2$ , the values of  $\lambda_2$  under four different situations are given in the figure. Following the same procedure in the first cycle, we substitute  $q$  with  $q_i + \lambda_i$ ,  $q + s$  with  $q_i + s_i + \lambda_i$  and  $q + R$  with  $q_i + R + \lambda_i$  in (21) and (22), respectively. We then get:

$$f_i = \ell \cdot \frac{\log(q_i + \lambda_i)}{\log(q_i + \lambda_i + R) - \log(q_i + \lambda_i)} \quad (23)$$

$$\lambda_{i+1} = (q_i + \lambda_i)^{(\ell+f_{i+1})/\ell} \cdot (q_i + s_i + \lambda_i)^{-f_i/\ell} \quad (24)$$

Following the approximation method of "Appendix 2", we get the break time to cause total forgetting

$$\begin{aligned} t_S &= \int_{q_i + \lambda_i}^{q_i + \lambda_i + R} T_1 \theta^{-\ell} d\theta \\ &= \frac{T_1}{1 - \ell} \cdot [(q_i + \lambda_i + R)^{1-\ell} - (q_i + \lambda_i)^{1-\ell}] \end{aligned} \quad (25)$$

We define  $t(q)$  as the theoretical time to produce  $q$  units and  $t(q) = T_1 + T_2 + \dots + T_q$ , the subscripts denote the



**Fig. 10** The explanation of  $\lambda_i$  in the first 3 cycles

	q <sub>1</sub>	s <sub>1</sub>	q <sub>2</sub>	s <sub>2</sub>	q <sub>3</sub>	s <sub>3</sub>	
	↓	↓	↓	↓	↓	↓	↓
	$\lambda_1$	$\lambda_1 + q_1$	$\lambda_2$	$\lambda_2 + q_2$	$\lambda_3$	$\lambda_3 + q_3$	$\lambda_4$
No learning and forgetting	0	0	0	0	0	0	0
Learning	0	$q_1$	$q_1$	$q_1 + q_2$	$q_1 + q_2$	$q_1 + q_2 + q_3$	$q_1 + q_2 + q_3$
LFCM	0	$q_1$	$\lambda_2 < q_1$	$\lambda_2 + q_2$	$\lambda_3 < \lambda_2 + q_2$	$\lambda_3 + q_3$	$\lambda_4 < \lambda_3 + q_3$
With total forgetting	0	$q_1$	0	$q_2$	0	$q_3$	0

sequence number of the produced units. So by implementing “Appendix 2”, the theoretical time to produce  $q_i + \lambda_i$  units is:

$$t(q_i + \lambda_i) = \int_0^{q_i + \lambda_i} T_1 \theta^{-\ell} d\theta = \frac{T_1}{1 - \ell} \cdot (q_i + \lambda_i)^{1 - \ell} \quad (26)$$

Solving Eq. 25 for  $(q_i + \lambda_i + R)$  yields

$$q_i + \lambda_i + R = \left[ \frac{1 - \ell}{T_1} t_S + (q_i + \lambda_i)^{1 - \ell} \right]^{1/(1 - \ell)} \quad (27)$$

and solving Eq. 26 for  $\frac{1 - \ell}{T_1}$  yields

$$\frac{1 - \ell}{T_1} = \frac{(q_i + \lambda_i)^{1 - \ell}}{t(q_i + \lambda_i)} \quad (28)$$

Substituting Eq. 28 into Eq. 27 yields:

$$q_i + \lambda_i + R = \left[ \frac{t_S}{t(q_i + \lambda_i)} + 1 \right]^{1/(1 - \ell)} (q_i + \lambda_i) \quad (29)$$

By substituting Eq. 29 into Eq. 23 and simplifying it, we have

$$f_i = \frac{\ell(1 - \ell) \cdot \log(q_i + \lambda_i)}{\log[t_S/t(q_i + \lambda_i) + 1]} \quad (30)$$

Further substituting Eq. 26 into Eq. 30 yields

$$f_i = \frac{\ell(1 - \ell) \cdot \log(q_i + \lambda_i)}{\log\left[\frac{t_S(1 - \ell)}{T_1(q_i + \lambda_i)^{1 - \ell}} + 1\right]} \quad (31)$$

This is how we get Eq. 7 (namely Eq. 24) and 8 (namely Eq. 31) in “Productivity with learning and forgetting”.

## Appendix 2: Approximation of $\sum_{j=1}^{q_i} (\lambda_i + j)^{-\ell}$

In this section, we discuss the approximation by considering a discrete function as a continuous one in our paper, and present the gaps between the approximate values and the true values.

Consider a common question in the following form:

$$f(b) = \sum_{k=1}^b (k + c)^a = (1 + c)^a + (2 + c)^a + (3 + c)^a + \dots + (b + c)^a \quad (32)$$

In the above equation,  $k = 1, 2, \dots, b$ ,  $a$  and  $c$  are constant, with  $a$  and  $c \in \mathbb{R}$  and  $b \in \mathbb{N}$ . Consider  $f(b)$  as a continuous function rather than a discrete one (Salameh et al. 1993), we will get:

$$f(b) = \sum_{k=1}^b (k + c)^a \simeq \int_1^b (k + c)^a dk \quad (33)$$

Let  $\theta = k + c$ , (33) will be transformed into the following form:

$$f(b) \simeq \int_{1+c}^{b+c} \theta^a d\theta \quad (34)$$

Follow the rules in calculus, we can easily solve  $f(b)$ :

$$\begin{aligned} f(b) &\simeq \int_{1+c}^{b+c} \theta^a d\theta = \frac{\theta^{1+a}}{1+a} \Big|_{1+c}^{b+c} \\ &= \frac{(b+c)^{1+a} - (1+c)^{1+a}}{1+a} \end{aligned} \quad (35)$$

Let  $a = -\ell$ ,  $b = q_i$  and  $c = \lambda_i$ , from the above result:

$$\sum_{j=1}^{q_i} (\lambda_i + j)^{-\ell} \simeq \frac{(q_i + \lambda_i)^{1 - \ell} - (1 + \lambda_i)^{1 - \ell}}{1 - \ell} \quad (36)$$

This is how we get Eq. 11.

Next, we test the veracity of this approximation of Eq. 36 based on the numerical examples in “Numerical results”. It is obvious that the gap between the approximation and the true value of  $f(b)$  is a decreasing function of  $b$ , which means that the larger  $b$  is, the smaller the gap will be. In our paper,



**Table 9** The veracity of the approximation

	Basic model	$n$	$t_S$	$t_{ip}, t_{is}$	$w$	$\ell$
Cycle no.	$i = 1$	$i = 1$	$i = 1$	$i = 1, t_{ip} = 1, t_{is} = 6$	$i = 1$	$i = 1, \ell = 0$
$q_i$	190.571	190.571	190.571	30.139	190.571	100.000
Approximate value	100.000	100.000	100.000	20.000	100.000	99.000
True value	100.931	100.931	100.931	21.320	100.931	100.000
Gap	0.92%	0.92%	0.92%	6.19%	0.92%	1%

the larger  $q_i$  is, the more precise this approximation will be. So the worst case of this approximation will happen in the producing cycles with the smallest production quantity  $q_i$ . Because the productivity of employees increases as the production goes on, the worst case will appear in the first producing cycle with the smallest production quantity. So in the basic example, the worst case happens in the first producing cycle. Under different  $n, t_S$  and  $w$ , the worst cases

also happen in the first producing cycle for the same reason. Under different  $t_{ip} - t_{is}$ , the worst case happens in the first producing cycle when  $t_{ip} = 1$  and  $t_{is} = 6$ , as in this situation, the production quantity is the smallest among all the first cycles due to the shortest production time. Under different  $\ell$ , the worst case happens in the first producing cycle when  $\ell = 0$ , namely 100% learning rate, as no learning leads to less production quantity.

**Table 10** The values of  $P, C_0, S, V(100 \geq P > C_0 \geq V \geq 0, P > S \geq 0)$

$P$	$C_0$	$S$	$V$	$P + S - C_0$	$P + S - V$	$\alpha$	$\beta$
32	10	9	3	31	38	0.815789	0.026316
50	25	8	12	33	46	0.717391	0.021739
98	96	85	94	87	89	0.977528	0.011236
15	2	11	0	24	26	0.923077	0.038462
27	7	18	1	38	44	0.863636	0.022727
36	13	22	4	45	54	0.833333	0.018519
54	29	28	15	53	67	0.791045	0.014925
75	56	29	42	48	62	0.774194	0.016129
86	73	28	62	41	52	0.788462	0.019231
94	88	26	83	32	37	0.864865	0.027027
12	1	1	0	12	13	0.923077	0.076923
18	3	2	0	17	20	0.850000	0.050000
26	6	2	1	22	27	0.814815	0.037037
35	12	0	4	23	31	0.741935	0.032258
44	19	42	8	67	78	0.858974	0.012821
54	29	48	15	73	87	0.839080	0.011494
65	42	54	27	77	92	0.836957	0.010870
76	58	58	44	76	90	0.844444	0.011111
87	76	60	66	71	81	0.876543	0.012346
5	0	2	0	7	7	1.000000	0.142857
33	11	13	3	35	43	0.813953	0.023256
43	18	15	7	40	51	0.784314	0.019608
54	29	15	15	40	54	0.740741	0.018519
64	41	13	26	36	51	0.705882	0.019608
73	53	12	38	32	47	0.680851	0.021277
85	72	7	61	20	31	0.645161	0.032258
94	88	3	82	9	15	0.600000	0.066667
13	1	11	0	23	24	0.958333	0.041667
20	4	17	0	33	37	0.891892	0.027027
29	8	23	2	44	50	0.880000	0.020000

Table 9 shows the worst cases and the gaps of the approximation. The true values are solved by calculating each discrete part of the function using a C program, and the approximate values are obtained with the method presented above. In Table 9, the first two rows explain the conditions of the worst cases; the values in the third row are the corresponding  $q_i$ ; the values in the fourth row are the approximate values we use in our model by implementing the above approximation method; the values in the fifth row are the true values obtained by calculating  $\sum_{j=1}^{q_i} (\lambda_i + j)^{-\ell}$  directly using the C program. The gaps are quite small except when  $t_{ip} = 1$  and  $t_{is} = 6$ , one working day and six rest days in one week, which is an unrealistic extreme situation. This will not lose the generality of our discussion.

### Appendix 3: Values of $P$ , $S$ , $V$ and $C_0$

Table 10 shows the values of  $P$ ,  $S$ ,  $V$  and  $C_0$  that we choose.

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