From Maximum Common Submaps to Edit Distances of Generalized Maps

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Abstract

Generalized maps are widely used to model the topology of nD objects (such as images) by means of incidence and adjacency relationships between cells (vertices, edges, faces, volumes, ...). In this paper, we introduce distance measures for comparing generalized maps, which is an important issue for image processing and analysis. We introduce a first distance measure which is defined by means of the size of a largest common submap. This distance is generic: it is parameterized by a submap relation (which may either be induced or partial), and by weights to balance the importance of darts with respect to seams. We show that this distance measure is a metric. We also introduce a map edit distance, which is defined by means of a minimum cost sequence of edit operations that should be performed to transform a map into another map. We relate maximum common submaps with the map edit distance by introducing special edit cost functions for which they are equivalent. We experimentally evaluate these distance measures and show that they may be used to classify meshes.

Keywords:
generalized map, edit distance, partial submap isomorphism, metric distance

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1. Introduction

Generalized maps are very nice data structures to model the topology of nD objects subdivided in cells (*e.g.*, vertices, edges, faces, volumes, ...) by means of incidence and adjacency relationships between these cells. In 2D, they are an extension of plane graphs (i.e., a planar graph which is embedded in a plane), and a generalization for higher dimensions. Generalized maps can be used for examples to represent, for example, 3D meshes or Region Adjacency Graphs. In particular, generalized maps are very well suited for scene modeling [1], for 2D and 3D image segmentation [2], and there exist efficient algorithms to extract maps from images [3].

In [4], we have defined two basic tools for comparing 2D combinatorial maps, *i.e.*, map isomorphism (which involves deciding if two maps are equivalent) and submap isomorphism (which involves deciding if a copy of a pattern map may be found in a target map), and we have proposed efficient polynomial time algorithms for solving these two problems. This work has been generalized to open nD combinatorial maps in [5].

However, (sub)map isomorphism are decision problems which cannot be used to measure the similarity of two maps as soon as there is no inclusion or equivalence relation between them. Therefore, we have introduced in [6] a first distance measure to compare generalized maps. This distance measure is defined by means of the size of a largest common submap, in a similar way as a graph distance measure is defined by means of the size of a largest common subgraph in [7].

Contributions of the paper. The distance defined in [6] is based on induced submap relations, such that submaps are obtained by removing some darts and all their seams (just like induced subgraphs are obtained by removing some vertices and all their incident edges). In this paper, we introduce a new kind of submap relation, called partial submap: partial submaps are obtained by removing not only some darts (and all their seams), but also some other seams, just like partial subgraphs are obtained by removing not only some vertices (and their incident edges), but also some other edges.

We introduce a generic distance measure, which is defined by means of the size of a largest submap which may either be an induced or a partial submap, and which is parameterized by two weights which allow to balance the importance of darts with respect to seams. This distance is more general than the one introduced in [6], and we show that it is a metric.

We also introduce an edit distance, which defines the distance between two maps G and G' in an operational way, by means of a minimum cost sequence of edit operations that should be performed to transform G into G'. We relate maximum common submaps with the map edit distance by introducing special edit cost functions for which they become equivalent, in a similar way as Bunke has related graph edit distances with maximum common subgraphs in [8].

Outline. In Section 2, we recall definitions related to generalized maps and to (induced) submap isomorphism. In Section 3, we introduce partial submap isomorphism and we define a generic distance measure based on maximum common (induced or partial) submaps. In Section 4, we introduce a map edit distance and we relate it to the generic distance based on maximum common submaps.

In Section 5, we illustrate our distance measures on some practical examples, and show preliminary results on an application to 2D Mesh classification.

2. Recalls and basic definitions

When objects are modelled by graphs, we need to define graph similarity measures to compare objects. This problem has been widely studied, and different approaches have been proposed based, for example, on graph matching [9, 10], graph kernels [11, 12], or graph embeddings [13, 14]. Graphs describe binary relationships between nodes by means of edges. However, they cannot be used to model faces (which appear when embedding a planar graph in a plane), or higher dimension cells such as volumes. Generalized maps are better suited data structures for describing adjacency and incidence relationships between cells (nodes, edges, faces, volumes, ...). We refer the reader to [15] for more details.

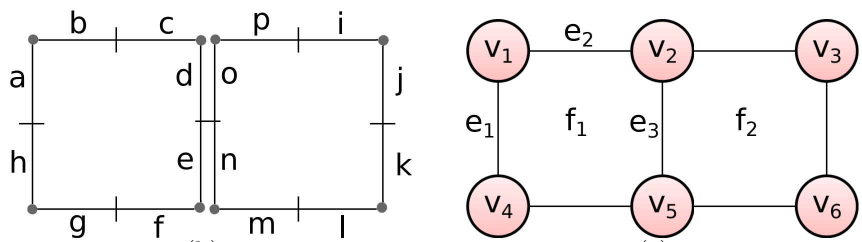
Definition 1. (nG-map) Let $n \geq 0$. An n-dimensional generalized map (or nG-map) is defined by a tuple $G = (D, \alpha_0, \ldots, \alpha_n)$ such that

1. D is a finite set of darts;
2. $\forall i \in [0, n]$, α_i is an involution[2] on D;

[2] An involution f on D is a bijective mapping from D to D such that $f = f^{-1}$.

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p
α_0	h	c	b	e	d	g	f	a	p	k	j	m	l	o	n	i
α_1	b	a	d	c	f	e	h	g	j	i	l	k	n	m	p	o
α_2	a	b	c	o	n	f	g	h	i	j	k	l	m	e	d	p

(a)



(b) (c)

Figure 1: Example of 2G-map. (a) describes α_i involutions of a 2G-map composed of 16 darts denoted by letters from a to p. (b) displays a graphical representation of this 2G-map: darts are represented by grey segments labelled with the associated letter; incident darts separated by little black segments are 0-sewn (e.g., $\alpha_0(b) = c$ and $\alpha_0(c) = b$); incident darts separated with a dot are 1-sewn (e.g., $\alpha_1(a) = b$ and $\alpha_1(b) = a$); adjacent darts are 2-sewn (e.g., $\alpha_2(d) = o$ and $\alpha_2(o) = d$). (c) displays the plane graph modelled by the 2G-map: vertices, edges, and faces of this graph correspond to sets of darts of the 2G-map (e.g., vertex v_1 corresponds to darts $\{a, b\}$, vertex v_2 to darts $\{c, d, o, p\}$, ..., edge e_1 to darts $\{a, h\}$, edge e_3 to darts $\{d, e, o, n\}$, ..., face f_1 to darts $\{a, b, c, d, e, f, g, h\}$, and face f_2 to darts $\{i, j, k, l, m, n, o, p\}$).

3. $\forall i, j \in [0, n]$ such that $i + 2 \leq j$, $\alpha_i \circ \alpha_j$ is an involution.

Fig. 1 displays an example of 2G-map which models a plane graph composed of two adjacent square faces. The 2G-map is composed of sixteen darts which are sewn by the α_i involutions: α_0 sews every dart to a dart which belongs to the next 0-cell vertex (e.g., $\alpha_0(b) = c$, thus allowing us to reach vertex v_2 from vertex v_1), α_1 sews every dart to a dart which belongs to the next 1-cell edge (e.g., $\alpha_1(a) = b$, thus allowing us to reach edge e_2 from edge e_1), and α_2 sews every dart to a dart which belongs to the adjacent 2-cell face (e.g., $\alpha_2(d) = o$, thus allowing us to reach face f_2 from face f_1).

We say that a dart d is i-free if $\alpha_i(d) = d$ and that it is i-sewn otherwise. For example, dart a in Fig. 1 is 2-free because $\alpha_2(a) = a$, whereas dart a is 0-sewn because $\alpha_0(a) = h$. A dart d is said isolated if $\forall i \in [0, n], d$ is i-free. A seam is a tuple (d, i, d') such that d' is i-sewn to d. For example, $(a, 0, h)$ is a seam of the map displayed in Fig. 1 because $\alpha_0(a) = h$.

Definition 2. (seams of a set of darts in an nG-map) Let $G = (D, \alpha_0, \ldots, \alpha_n)$ be an nG-map and E $\subseteq$ D be a set of darts. The set of seams associated with E in G is:
$seams_G(E) = \{(d, i, \alpha_i(d)) | d \in E, i \in [0, n], \alpha_i(d) \in E, \alpha_i(d) \neq d\}$.

In this paper, we modify nG-maps by adding or removing seams : when the seams (d, i, d') and (d', i, d) are removed from (resp. added to) a G-map G, we modify the α_i involution by setting $\alpha_i(d)$ to d and $\alpha_i(d')$ to d' (resp., $\alpha_i(d)$ to d' and $\alpha_i(d')$ to d). In other words, we i-unsew d and d' (resp. i-sew d and d').

Throughout the paper, G and G' denote two nG-maps such that $G = (D, \alpha_0, \ldots, \alpha_n)$ and $G' = (D', \alpha'_0, \ldots, \alpha'_n)$.

Map isomorphism has been defined in [15] to decide of the equivalence of two maps.

Definition 3. (nG-map isomorphism [15]) G and G' are isomorphic, denoted $G \simeq G'$, if there exists a bijection $f : D \rightarrow D'$, such that $\forall d \in D, \forall i \in [0, n], f(\alpha_i(d)) = \alpha'_i(f(d))$.

In [4], induced submap has been defined: G is an induced submap of G' if G preserves all seams of G', i.e, for every couple of darts (d_1, d_2) of G, d_1 is i-sewn to d_2 in G' if and only if d_1 is i-sewn to d_2 in G.

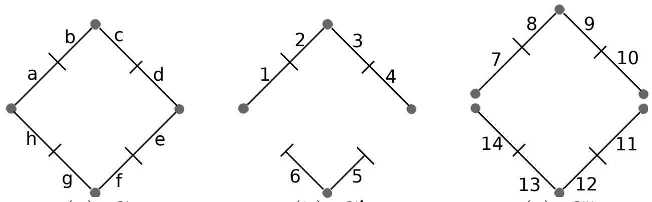
Definition 4. (induced submap) G' is an induced submap of G if $D' \subseteq D$ and $seams_{G'}(D') = seams_G(D')$.

Definition 5. (induced submap isomorphism) There is an induced submap isomorphism from G to G', denoted $G \sqsubseteq^i G'$ if there exists an induced submap of G' which is isomorphic to G.

In [5] we have shown that if there exists an induced submap isomorphism from G to G', then there exists an injection $f : D \rightarrow D'$ called *induced subisomorphism function* such that $\forall d \in D$ and $\forall i \in [0, n]$:

- if d is i-sewn, then $f(\alpha_i(d)) = \alpha'_i(f(d))$;
- if d is i-free, then either $f(d)$ is i-free, or $f(d)$ is i-sewn with a dart which is not matched by f to another dart of D, *i.e.*, $\forall d_k \in D, f(d_k) \neq \alpha'_i(f(d))$.

Fig. 2 displays examples of induced submap isomorphism.



(a) G (b) G' (c) G”

$1 \to a$ $2 \to b$
$3 \to c$ $4 \to d$
$5 \to f$ $6 \to g$

(d) Example of submap isomorphism function from G' to G

Figure 2: Submap isomorphism example. We have $G' \sqsubseteq^i G$ and (d) displays a submap isomorphism function. We also have $G' \sqsubseteq^i G$”. However we don't have G” $\sqsubseteq^i G$. Indeed, darts 7 and 14 are 1-free and cannot be matched to darts a and h respectively as they are 1-sewn.

3. Generic distance measure based on maximum common submap

In [6], we have defined a first distance measure based on induced submap isomorphism. The distance between G and G' is defined by means of the size of the largest nG-map G” such that G” $\sqsubseteq^i G$ and G” $\sqsubseteq^i G'$. In this case, the size of an nG-map is defined by its number of darts.

This definition may lead to surprising results. Let us consider, for example, the three maps G, G', and G” displayed in Fig. 2. The maximum common submap of G and G' is isomorphic to the maximum common submap of G and G'' (and is also isomorphic to G'). As a consequence, the distance between G and G' is equal to the distance between G and G'' when considering the distance defined in [6] whereas G seems more similar to G” than to G'. Indeed, G” is obtained from G by 1-unsewing darts a and b and darts d and e, whereas G' is obtained from G by not only 1-unsewing these darts, but also 0-unsewing darts h and g and darts f and e and finally removing darts h and e.

In this paper, we extend definitions and theoritical results of [6] by defining a new kind of submap relation, called partial submap by analogy with existing work on graphs. Indeed, induced subgraphs are obtained by removing some nodes (and all their incident edges) whereas partial subgraphs are obtained by removing not only some nodes (and all their incident edges) but also some edges. In our map context, partial submaps are obtained by removing not only some darts (and all their seams) but also some other seams.

Definition 6. (partial submap) G' is a partial submap of G if $D' \subseteq D$ and $seams_{G'}(D') \subseteq seams_G(D')$.

Submap isomorphism is extended to the partial case in a straightforward way.

Definition 7. (partial submap isomorphism) There is a partial submap isomorphism from G to G', denoted $G \sqsubseteq^p G'$ if there exists a partial submap of G' which is isomorphic to G.

Note that $G \sqsubseteq^i G' \Rightarrow G \sqsubseteq^p G'$. Note also that if $G \sqsubseteq^p G'$ then there exists an injective function $f : D \to D'$, called *partial subisomorphism function*, such that $\forall d \in D$ and $\forall i \in [0, n]$: if d is i-sewn, then $f(\alpha_i(d)) = \alpha'_i(f(d))$.

For example, let us consider the maps displayed in Fig. 2. We have $G' \sqsubseteq^p G$ and $G' \sqsubseteq^p G$" as $G' \sqsubseteq^i G$ and $G' \sqsubseteq^i G$". We also have G" $\sqsubseteq^p G$. Indeed, we simply have to remove seams $(h, 1, a)$, $(a, 1, h)$, $(d, 1, e)$ and $(e, 1, d)$ from G to obtain a map isomorphic to G".

Throughout the paper, $*$ will denote either p or i (i.e., $* \in \{p, i\}$) so that $G \sqsubseteq^* G'$ will denote either an induced submap isomorphism ($G \sqsubseteq^i G'$) or a partial one ($G \sqsubseteq^p G'$).

The distance introduced in [6] is based on the size of a largest common induced submap, where the size is defined by the number of darts. To extend the distance to the partial case, we have to reconsider the definition of the size of an nG-map. Let us consider for example the two nG-maps G and G" displayed in Fig. 2. These two nG-maps have the same number of darts. However G has four more seams than G". To integrate this information, we define the size of an nG-map as a combination of both the number of darts and the number of seams, respectively weighted by two parameters ω_1 and ω_2.

Definition 8. (parameterized size of an nG-map) Given $(\omega_1, \omega_2) \in \mathbb{R}^{+2}$ such that $(\omega_1, \omega_2) \neq (0, 0)$, the size of an nG-map G is $size_{\omega_1,\omega_2}(G) = \omega_1.|D| + \omega_2.|seams_G(D)|$.

Note that when $\omega_1 = 1$ and $\omega_2 = 0$, this size actually corresponds to the one introduced in [6]. Let us now define maximum common submap.

Definition 9. (maximum common submap) Given $(\omega_1, \omega_2) \in \mathbb{R}^{+2}$ such that $(\omega_1, \omega_2) \neq (0, 0)$, a maximum common submap of G and G', denoted $mcs^*_{\omega_1,\omega_2}(G, G')$, is an nG-map such that:

- $mcs^*_{\omega_1,\omega_2}(G, G') \sqsubseteq^* G$;

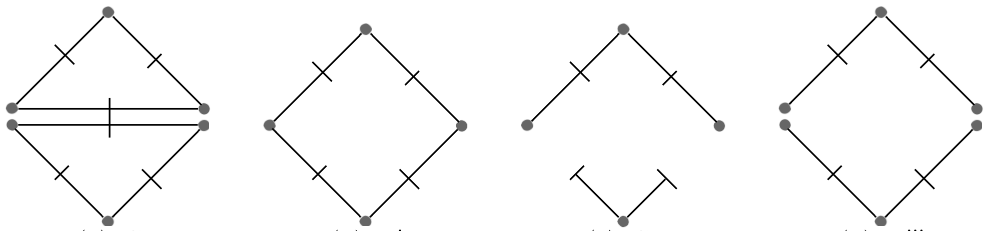


Figure 3: Maximum common submap examples. The maximum common induced submap of G and G' is isomorphic to G" when $\omega_1 = \omega_2 = 1$, i.e., $mcs^i_{1,1}(G, G') \simeq G$", and $size_{1,1}(G") = 6+8$. The maximum common partial submap of G and G' is isomorphic to G''' when $\omega_1 = \omega_2 = 1$, i.e., $mcs^p_{1,1}(G, G') \simeq G'''$ and $size_{1,1}(G''') = 8 + 12$.

- $mcs^*_{\omega_1,\omega_2}(G, G') \sqsubseteq^* G'$;
- $size_{\omega_1,\omega_2}(mcs^*_{\omega_1,\omega_2}(G, G'))$ is maximal.

$mcs^i_{\omega_1,\omega_2}(G, G')$ is called the maximum common induced submap, and $mcs^p_{\omega_1,\omega_2}(G, G')$ the maximum common partial submap.

Fig. 3 displays examples of maximum common submaps.

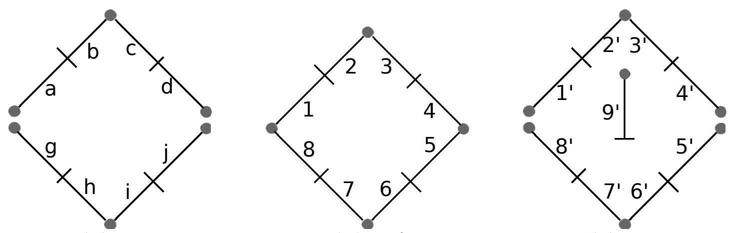
One can easily show that $mcs^*_{\omega_1,\omega_2}(G, G') = mcs^*_{\omega_1,\omega_2}(G', G)$. Also, the size of a maximum common submap is smaller than or equal to the size of original maps, *i.e.*, $size_{\omega_1,\omega_2}(mcs^*_{\omega_1,\omega_2}(G, G')) \leq size_{\omega_1,\omega_2}(G)$ (this is a direct consequence of the fact that $mcs^*_{\omega_1,\omega_2}(G, G') \sqsubseteq^* G$).

Let us now define a distance measure based on the size of the maximum common submap.

Definition 10. (parameterized distance between two nG-maps) Given $\omega_1, \omega_2 \in \mathbb{R}^+$ such that $(\omega_1, \omega_2) \neq (0, 0)$, the distance between G and G' is defined by: $d^*_{\omega_1,\omega_2}(G, G') = 1 - \frac{size_{\omega_1,\omega_2}(mcs^*_{\omega_1,\omega_2}(G,G'))}{max(size_{\omega_1,\omega_2}(G), size_{\omega_1,\omega_2}(G'))}$

Note that the maximum common submap defined in [6] corresponds to $mcs^i_{1,0}$ and that the distance defined in [6] is equivalent to $d^i_{1,0}$.

In [6], we have shown that $d^i_{1,0}$ is a metric, and this result may be extended to $d^i_{\omega_1,0}$ in a straightforward way. Fig. 4 shows us that $d^p_{\omega_1,0}$ and d^*_{0,ω_2} do not satisfy the isomorphism of indiscernibles property so that our distance measure is not a metric in these two particular cases. Let us now show that $d^*_{\omega_1,\omega_2}$ is a metric in all other cases, *i.e.*, whenever $\omega_1 > 0$ and $\omega_2 > 0$.



(a) G (b) G' (c) G"

Figure 4: Examples of non isomorphic maps for which the distance may be null. $d^p_{\omega_1,0}(G, G') = 0$ because $G \sqsubseteq^p G'$ and G and G' have the same number of darts. However G and G' are not isomorphic. Therefore $d^p_{\omega_1,0}$ is not a metric. Also, $d^*_{0,\omega_2}(G, G") = 0$ because $G \sqsubseteq^p G''$ and G and G" have the same number of seams (dart 9 is not sewn). However G and G" are not isomorphic. Therefore $d^*_{0,\omega_2}(G, G')$ is not a metric.

Theorem 1. *Let $n \geq 1$, $\omega_1 > 0$ and $\omega_2 > 0$. The distance $d^*_{\omega_1,\omega_2}$ is a metric on the set $\mathcal{G}$ of all nG-maps so that the following properties hold:*

1. *Non-negativity:* $\forall G_1, G_2 \in \mathcal{G}, d^*_{\omega_1,\omega_2}(G_1, G_2) \geq 0$;
2. *Isomorphism of indiscernibles:*
   $\forall G_1, G_2 \in \mathcal{G}, d^*_{\omega_1,\omega_2}(G_1, G_2) = 0$ *iff* $G_1 \simeq G_2$;
3. *Symmetry:* $\forall G_1, G_2 \in \mathcal{G}, d^*_{\omega_1,\omega_2}(G_1, G_2) = d^*_{\omega_1,\omega_2}(G_2, G_1)$;
4. *Triangle inequality:* $\forall G_1, G_2, G_3 \in \mathcal{G}, d^*_{\omega_1,\omega_2}(G_1, G_3) \leq d^*_{\omega_1,\omega_2}(G_1, G_2) + d^*_{\omega_1,\omega_2}(G_2, G_3)$.

Proof. Properties 1, and 3 are direct consequences of Def. 10.

Proof of property 2 We have $d^*_{\omega_1,\omega_2} = 0$ then using Def. 10 we obtain: $size_{\omega_1,\omega_2}(mcs^*_{\omega_1,\omega_2}(G_1, G_2)) = max(size_{\omega_1,\omega_2}(G_1), size_{\omega_1,\omega_2}(G_2))$. We also know using Def. 9 that $mcs^*_{\omega_1,\omega_2}(G_1, G_2) \sqsubseteq G_1$ and $mcs^*_{\omega_1,\omega_2}(G_1, G_2) \sqsubseteq G_2$. Then it follows that $size_{\omega_1,\omega_2}(mcs^*_{\omega_1,\omega_2}(G_1, G_2)) = size_{\omega_1,\omega_2}(G_1) = size_{\omega_1,\omega_2}(G_2)$.

From the fact that $mcs^*_{\omega_1,\omega_2}(G_1, G_2) \sqsubseteq G_1$ and $mcs^*_{\omega_1,\omega_2}(G_1, G_2) \sqsubseteq G_2$ we deduce that there exists an injection between $mcs^*_{\omega_1,\omega_2}(G_1, G_2)$ and a submap of G_1 and also an injection from $mcs^*_{\omega_1,\omega_2}(G_1, G_2)$ and a submap of G_2. Furthermore we have shown that the sizes are equal, and because the $mcs^*_{\omega_1,\omega_2}(G_1, G_2) \sqsubseteq^* G_1$ and $mcs^*_{\omega_1,\omega_2}(G_1, G_2) \sqsubseteq^* G_2$, we deduce that the darts and the seams of $mcs^*_{\omega_1,\omega_2}(G_1, G_2)$ are also in G_1 and also in G_2. In consequence, the two injections are also bijections. Then, there exists a bijection from G_1 to G_2 that preserves all the darts and all the seams. Finally from Def. 3 it follows that $G_1 \simeq G_2$.

Proof of property 4 Let us denote $m_{ij} = mcs^*_{\omega_1,\omega_2}(G_i, G_j)$, $size_{\omega_1,\omega_2}(G) = size(G)$ and $S_{ij} = max(size(G_i), size(G_j))$, and let us show Property 4 by considering separately the two following cases.

(Case 1): $d^*_{\omega_1,\omega_2}(G_1, G_2) + d^*_{\omega_1,\omega_2}(G_2, G_3) \geq 1$.
In this case, the triangle inequality trivially holds as $d^*_{\omega_1,\omega_2}(G_1, G_3) \leq 1$.

(Case 2): $d^*_{\omega_1,\omega_2}(G_1, G_2) + d^*_{\omega_1,\omega_2}(G_2, G_3) < 1$.
In this case, let us first show that there exists at least a common part of G_2 which belongs both to m_{12} and m_{23}, *i.e.*,

$$size(G_2) < size(m_{12}) + size(m_{23}) \tag{1}$$

This inequation can be proven by considering all possible order relations between nG-map sizes. For example, if $size(G_1) \geq size(G_3) \geq size(G_2)$, then:
(Case 2) $\Leftrightarrow 1 - \frac{size(m_{12})}{size(G_1)} + 1 - \frac{size(m_{23})}{size(G_3)} < 1$ (by Def. 10, and as $S_{12} = size(G_1)$ and $S_{23} = size(G_3)$)
$\Leftrightarrow size(G_3) < \frac{size(G_3)}{size(G_1)} size(m_{12}) + size(m_{23})$ (by multiplying by $size(G_3)$)
$\Rightarrow size(G_3) < size(m_{12}) + size(m_{23})$ (as $\frac{size(G_3)}{size(G_1)} < 1$)
$\Rightarrow size(G_2) < size(m_{12}) + size(m_{23})$ (as $size(G_3) \geq size(G_2)$).
Ineq. (1) can be proven in a very similar way for the five other possible order relations between nG-map sizes.

Ineq. (1) shows that the sum of the sizes of the two common submaps m_{12} and m_{23} is always strictly greater than the size of G_2 so that there is at least a common part that both belong to m_{12} and m_{23}. Therefore, the nG-map $mcs^*_{\omega_1,\omega_2}(m_{12}, m_{23})$ is a common submap of G_1, G_2, and G_3 which has at least a size of $size(m_{12}) + size(m_{23}) - size(G_2)$. This nG-map gives a lower bound on the size of the maximum common submap of G_1 and G_3, *i.e.*,

$$size(m_{13}) \geq size(m_{12}) + size(m_{23}) - size(G_2) \tag{2}$$

Let us use this lower bound to show that the triangle inequality holds. When developing the triangle inequality w.r.t. Def. 10, it becomes:

$$size(m_{13}) \geq \frac{S_{13}}{S_{12}} size(m_{12}) + \frac{S_{13}}{S_{23}} size(m_{23}) - S_{13} \tag{3}$$

Let us prove (3) by considering all order relations between nG-map sizes:

(Case 2.1): $size(G_1) \geq size(G_2) \geq size(G_3)$ so that $S_{13} = size(G_1), S_{12} =$

$size(G_1), S_{23} = size(G_2)$. Ineq. (3) becomes $size(m_{13}) \geq size(m_{12}) + \frac{size(G_1)}{size(G_2)} size(m_{23}) - size(G_1)$. As $size(m_{13}) \geq size(m_{12}) + size(m_{23}) - size(G_2)$ (Ineq. (2)), we have to show that $size(m_{23}) - size(G_2) \geq \frac{size(G_1)}{size(G_2)} size(m_{23}) - size(G_1)$, *i.e.*, $size(m_{23}) \leq size(G_2)$ (as $size(G_2) - size(G_1) < 0$). This inequality trivially holds by Def. 9.

(Case 2.2): $size(G_2) \geq size(G_1) \geq size(G_3)$ so that $S_{13} = size(G_1), S_{12} = size(G_2), S_{23} = size(G_2)$. Ineq. (3) becomes $size(m_{13}) \geq \frac{size(G_1)}{size(G_2)} size(m_{12}) + \frac{size(G_1)}{size(G_2)} size(m_{23}) - size(G_1)$. As $\frac{size(G_1)}{size(G_2)} \leq 1$, Ineq. (2) implies that $size(m_{13}) \geq \frac{size(G_1)}{size(G_2)} (size(m_{12}) + size(m_{23}) - size(G_2)$. Therefore, Ineq. (3) holds.

(Case 2.3): $size(G_1) \geq size(G_3) \geq size(G_2)$ so that $S_{13} = size(G_1), S_{12} = size(G_1), S_{23} = size(G_3)$. Ineq. (3) becomes $size(m_{13}) \geq size(m_{12}) + \frac{size(G_1)}{size(G_3)} size(m_{23}) - size(G_1)$. As $size(m_{13}) \geq size(m_{12}) + size(m_{23}) - size(G_2)$ (Ineq. (2)), we have to show that $size(m_{23}) - size(G_2) \geq \frac{size(G_1)}{size(G_3)} size(m_{23}) - size(G_1)$, *i.e.*, $size(m_{23}) \leq size(G_3) \frac{size(G_2) - size(G_1)}{size(G_3) - size(G_1)}$ (as $size(G_3) - size(G_1) < 0$). This inequality trivially holds by Def. 9 because $\frac{size(G_2) - size(G_1)}{size(G_3) - size(G_1)} \geq 1$.

The three others cases can be proven in a similar way. □

4. Map edit distance

The distance defined in the previous section is defined in a denotational way, by means of the size of a largest common submap. In this section, we introduce another distance measure which is defined in a more operational way, by means of a minimum cost sequence of map edit operations that should be performed to transform one map into another map. This second distance measure may be viewed as an adaptation of classical graph edit distances to generalized maps.

In Section 4.1, we define edit operations that are used to transform maps, and we define a map edit distance which is parameterized by edit operation costs. Then, we relate the map edit distance to maximum common submaps by introducing special edit cost functions for which they are equivalent, in a similar way as Bunke has related maximum common subgraphs to graph edit distances in [8].

4.1. Edit operations and edit distance

Let us first define map edit operations. These edit operations allow one to add/delete a whole set of darts or seams, instead of adding/deleting darts or seams one by one. Indeed, the addition/deletion of a single dart or seam may lead to a non valid nG-map. Let us consider for example the nG-map G of Fig. 1. We cannot delete dart e without also removing dart n or dart d (otherwise $\alpha_0 \circ \alpha_2$ no longer is an involution so that Property 3 of Def. 1 no longer is satisfied).

Operations 1 to 4 define four basic edit operations. In the following, we ensure that sets of darts or seams which are added or deleted are consistent so that applying these operations leads to valid nG-maps.

The del_E operation deletes a set of darts E and i-frees every non deleted dart which was i-sewn with a deleted dart.

Operation 1. (del_E) Let $G = (D, \alpha_0, \ldots, \alpha_n)$ be an nG-map and E a set of darts such that $E \subseteq D$. $del_E(G) = (D', \alpha'_0, \ldots, \alpha'_n)$ where $D' = D \setminus E$ and $\forall d' \in D', \forall i \in [0, n]$:

- if $\alpha_i(d') \in D'$, then $\alpha'_i(d') = \alpha_i(d')$;
- otherwise $\alpha'_i(d') = d'$.

The $add_{E,F}$ operation is the inverse operation. It adds a new set of darts E and a new set of seams F. The added seams must sew new darts of E either to darts of G or to other new darts.

Operation 2. ($add_{E,F}$) Let $G = (D, \alpha_0, \ldots, \alpha_n)$ be an nG-map, E a set of isolated darts such that $E \cap D = \emptyset$ and F a set of seams such that $\forall (d_1, i, d_2) \in F$, $\{d_1, d_2\} \cap E \neq \emptyset$: $add_{E,F}(G) = (D', \alpha'_0, \ldots, \alpha'_n)$ where $D' = D \cup E$ and $\forall d' \in D'$, $\forall i \in [0, n]$:

- if $\exists (d', i, d'') \in F$: $\alpha'_i(d') = d''$;
- otherwise if $d' \in E$: $\alpha'_i(d') = d'$;
- otherwise $\alpha'_i(d') = \alpha_i(d')$.

The sew_F operation adds a new set of seams F.

Operation 3. (sew_F) Let $G = (D, \alpha_0, \ldots, \alpha_n)$ be an nG-map and F be a set of seams such that $\forall (d_1, i, d_2) \in F, \alpha_i(d_1) = d_1$ and $\alpha_i(d_2) = d_2$. $sew_F(G) = (D, \alpha'_0, \ldots, \alpha'_n)$ where $\forall i \in [0, n], \forall d' \in D$:

- if $\exists (d', i, d'') \in F$ then $\alpha'_i(d') = d$";
- otherwise $\alpha'_i(d) = \alpha_i(d)$.

The $unsew_F$ operation deletes a set of seams F.

Operation 4. ($unsew_F$) Let $G = (D, \alpha_0, \ldots, \alpha_n)$ be an nG-map and $F \subseteq seams_G(D)$ be a set of seams. $unsew_F(G) = (D, \alpha'_0, \ldots, \alpha'_n)$ where $\forall i \in [0, n], \forall d' \in D$:

- if $\exists (d', i, d'') \in F$ then $\alpha'_i(d') = d''$;
- otherwise $\alpha'_i(d') = \alpha_i(d')$.

Let us finally define an edit path as a sequence of edit operations, and the edit distance as the cost of the minimal cost edit path.

Definition 11. (edit path) Let $\Delta =< \delta_1, \ldots, \delta_k >$ be a sequence of k edit operations. Δ is an edit path for G if $\delta_k(\delta_{k-1}(\ldots(\delta_1(G))))$, denoted $\Delta(G)$, is an nG-map (according to Def. 1).

Edit paths may be combined and we denote $\Delta_1 \cdot \Delta_2$ the concatenation of two edit paths Δ_1 and Δ_2.

Definition 12. (map edit distance) Let c be a function which associates a cost $c(\delta) \in \mathbb{R}^+$ with every edit operation δ. The edit distance between G and G' is $d_c(G, G') = \sum_{\delta_i \in \Delta}(c(\delta_i))$ where Δ is an edit path such that $\Delta(G) = G'$ and $\sum_{\delta_i \in \Delta}(c(\delta_i))$ is minimal.

4.2. Relation with maximum common induced submap

Let us now relate the map edit distance to maximum common induced submaps. To this aim, we first define the edit path that allows one to transform a map G into another map G' such that $G' \sqsubseteq^i G$ or, conversely, to transform G' into G.

Definition 13. (edit path associated with an induced submap isomorphism function) Let G and G' be such that $G \sqsubseteq^i G'$, and let $f : D \to D'$ be an associated induced subisomorphism function. Without loss of generality, we assume that $D \cap D' = \emptyset$. We define the edit paths:

- $i\Delta_f^{G' \to G} =< del_E >$ (*i.e.*, $i\Delta_f^{G' \to G}$ removes all darts which are not matched by f);
- $i\Delta_f^{G \to G'} =< add_{E,F} >$ (*i.e.*, $i\Delta_f^{G \to G'}$ adds all darts which are not matched by f, and all seams that sew these darts either together or to darts of D).

Where

- $E = \{d' \in D' | \nexists d \in D, f(d) = d'\}$;
- $F = \{(d'_1, i, d'_2) \in seams_{G'}(D')\} | d'_1 \in E$ or $d'_2 \in E\}$.

One can easily show that $i\Delta_f^{G' \to G}(G')\ simG$ and $i\Delta_f^{G \to G'}(G) \simeq G'$.

Let us consider for example maps G and G' and the induced subisomorphism function f of Fig. 2. We have

- $i\Delta_f^{G \to G'} =< del_{\{h,e\}} >$;
- $i\Delta_f^{G' \to G} =< add_{\{e,h\},\{(h,1,a),(a,1,h),(h,0,g),(g,0,h),(e,1,d),(d,1,e),(e,0,f),(f,0,e)\}} >$.

Let us now show that there exists a cost function such that an edit path that transforms G into $mcs^i_{\omega_1,\omega_2}(G, G')$ and then $mcs^i_{\omega_1,\omega_2}(G, G')$ into G' has a minimal cost, *i.e.*, its cost is equal to the edit distance in this case. This relates the distance introduced in Section 2 with the edit distance.

Proposition 1. Let $\omega_1 \in \mathbb{R}^+ \setminus \{0\}$ and c be the cost function such that

- $c(del_E) = c(add_{E,F}) = \omega_1.|E|$;

- $c(sew_F) = c(unsew_F) = +\infty$.

Let $G" = mcs^i_{\omega1,0}(G, G') = (D", \alpha"_0, \ldots, \alpha"_n)$, and $f : D" \rightarrow D$ (resp. $f' : D" \rightarrow D'$) be an induced subisomorphism function associated with the subisomorphism relation $G" \sqsubseteq^i G$ (resp. $G" \sqsubseteq^i G'$).

We have $d_c(G, G') = \sum_{\delta_j \in i\Delta_f^{G\rightarrow G"} \cdot i\Delta_{f'}^{G"\rightarrow G'}} (c(\delta_j))$.

Proof. According to Def. 13, $i\Delta_f^{G\rightarrow G"} \cdot i\Delta_{f'}^{G"\rightarrow G'}$ is an edit path that transforms G into G' passing though the nG-map $G"$ which is a maximum common induced submap of G and G'. To prove that the cost of this edit path is equal to the edit distance, we have to prove that this edit path has a minimum cost.

Suppose that there exists an edit path Δ_2 such that $c(\Delta_2) < c(i\Delta_f^{G\rightarrow G"} \cdot i\Delta_{f'}^{G"\rightarrow G'})$. Δ_2 is a composition of deletions $del_{Dr_0}, \ldots del_{Dr_n}$ and additions $add_{Da_0,S_0}, \ldots add_{Da_n,S_n}$ such that $\Delta_2(G) \simeq G'$. Note that a dart cannot belong to two operations, otherwise the dart is involved in a deletion and an addition operation. Then it would be possible to remove the dart from the two operations and in consequence get a better cost. It follows that $\forall D_i, D'_i \in \{Dr_0, \ldots Dr_n, Da_0, \ldots, Da_n\}, D_i \neq D'_i \Rightarrow D_i \cap D'_i = \varnothing$. This observation allows us to reorganize operations of Δ_2 starting with deletions followed by additions: $\Delta_2 =< del_{Dr_0}, \ldots del_{Dr_n}, add_{Da_0,S_0}, \ldots add_{Da_n,S_n} >$. We can also compact the deletions in a unique operation, and the same can be done for the additions such that: $\Delta_2 =< del_{D'r}, add_{D'a,S'} >$, with $D'r = Dr_0 \cup \ldots \cup Dr_n$, $D'a = Da_0 \cup \ldots \cup Da_n$ and $S' = S_0 \cup \ldots \cup S_n$.

Note that the nG-map $del_{D'r}(G)$ is an nG-map that is composed of darts that are in G and also in G' then using Def. 13, it follows that $del_{D'r}(G) \sqsubseteq^i G$ and $del_{D'r}(G) \sqsubseteq^i G'$.

Let us consider the two parts of the edit path separatly. If we have $c(del_{D'r}) < c(\Delta_f^{G\rightarrow G"})$ then $size_{\omega1,0}(del_{D'r}(G)) > size_{\omega1,0}(G")$, which contradicts the maximum common submap definition. On the other side if $c(add_{Da',S'}) < c(\Delta_{f'}^{G"\rightarrow G'})$ we can look to the inverse operation of $add_{D'a,S'}$ that would remove the darts from G' to obtain a common nG-map of G and G'. It also contradicts the maximum common submap definition because it would imply that $size_{\omega1,0}(del_{D'a}(G')) > size_{\omega1,0}(G")$ and in consequence it would be the $G"$. Then it follows that $c(del_{D'r}) \geq c(i\Delta_f^{G\rightarrow G"})$ and $c(add_{Da'_n,S'_n}) \geq c(i\Delta_{f'}^{G"\rightarrow G'})$ and therefore that $c(\Delta_2) \geq c(i\Delta_f^{G\rightarrow G"} \cdot i\Delta_{f'}^{G"\rightarrow G'})$. $\square$

4.3. Relation with maximum common partial submap

Let us now relate the map edit distance to maximum common partial submaps. Like in Section 4.2, we first define an edit path that allows one to transform a map G into another map G' such that $G' \sqsubseteq^p G$ or, conversely, to transform G' into G.

Definition 14. (edit path associated with a partial submap isomorphism function) Let G and G' be such that $G \sqsubseteq^p G'$, and let $f : D \rightarrow D'$ be an associated partial subisomorphism function. Without loss of generality, we assume that $D \cap D' = \emptyset$. We define the edit paths:

- $p\Delta_f^{G' \rightarrow G} =< unsew_F, del_E >$ (*i.e.*, $p\Delta_f^{G' \rightarrow G}$ first unsews all darts which are not matched by f and all darts which are matched by f but not sewn in G', and then removes all darts which are not matched by f);
- $p\Delta_f^{G \rightarrow G'} =< add_{E,\emptyset}, sew_F >$ (*i.e.*, $p\Delta_f^{G \rightarrow G'}$ first adds all darts which are not matched by f, and then sews them and sews all the matched darts which are not sewn in G').

Where

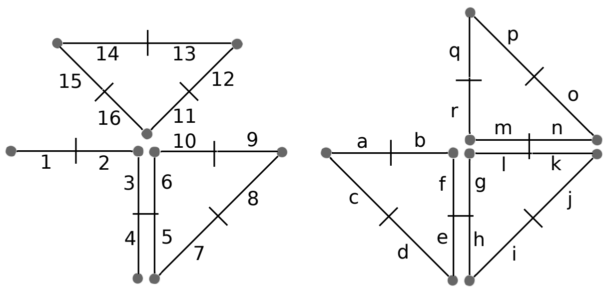
- $E = \{d' \in D' | \nexists d \in D, f(d) = d'\}$;
- $F = \{(d_1, i, d_2) \in seams_{G'}(D') | (d_1, i, d_2) \notin seams_{G'}(D)\}$.

One can easily show that $p\Delta_f^{G' \rightarrow G}(G') \simeq G$ and $p\Delta_f^{G \rightarrow G'}(G) \simeq G'$. Fig. 5 displays an example of edit path.

Let us now show that there exists a cost function such that an edit path that transforms G into $mcs^p_{\omega_1,\omega_2}(G, G')$ and then $mcs^p_{\omega_1,\omega_2}(G, G')$ into G' has a minimal cost, *i.e.*, its cost is equal to the edit distance in this case. This relates the distance introduced in Section 2 with the edit distance.

Proposition 2. Let ω_1, ω_2 in $\mathbb{R}^+$ such that $\omega_1 \neq 0$, $\omega_2 \neq 0$ and let c be the cost function such that :

- $c(del_E) = \omega_1.|E| + \omega_2.|\{(d_1, i, d_2) \in seams_{G'}(D'), d_i \in E \text{ or } d_2 \in E\}|)$;
- $c(add_{E,F}) = \omega_1.|E| + \omega_2.|F|$;
- $c(sew_F) = c(unsew_F) = \omega_2.|F|$.



(a) G (b) G'

$1 \to a$	$2 \to b$	$3 \to f$
$4 \to e$	$6 \to g$	$5 \to h$
$7 \to i$	$8 \to j$	$9 \to k$
$10 \to l$	$11 \to m$	$12 \to n$
$13 \to o$	$14 \to p$	$15 \to q$
$16 \to r$		

(c) f

$$p\Delta_f^{G\to G'} =< add_{\{c,d\}},\ sew_{\{(1,1,c),(c,1,1),(d,1,4),(4,1,d),(c,0,d),(d,0,c),(11,2,10),(10,2,11),(12,2,9),(9,2,12)\}} >$$

$$p\Delta_f^{G'\to G} =< unsew_{\{(m,2,l),(l,2,m),(n,2,k),(k,2,n)\}\cup\{(x,i,y)\in seams_{G'}(D')|\{x,y\}\cap\{c,d\}\neq\emptyset\}},\ del_{\{c,d\}} >$$

Figure 5: Example of edit path associated with a partial submap isomorphism function. (c) describes a partial submap isomorphism function from darts of G to darts of G'. $p\Delta_f^{G\to G'}$ gives the edit path associated with f which may be used to transform G in G'. $p\Delta_f^{G'\to G}$ gives the edit path associated with f which may be used to transform G' in G.

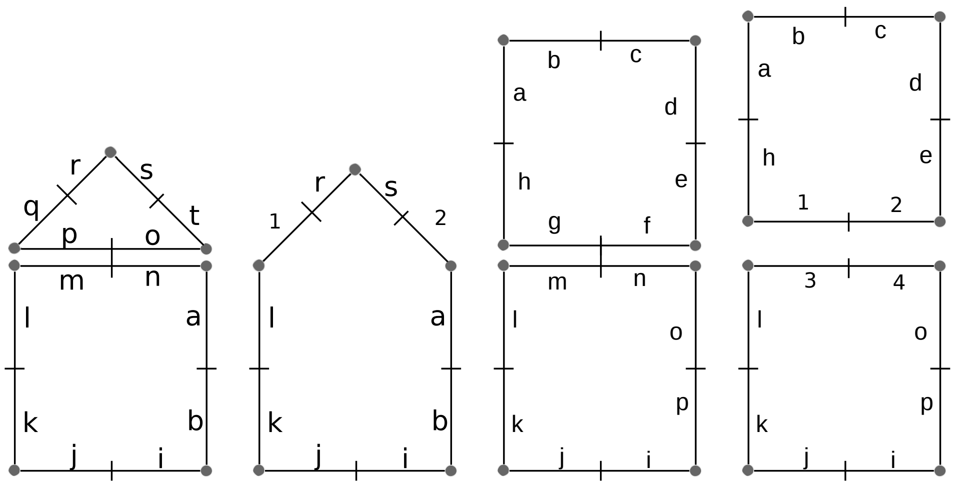
Let $G" = mcs^p_{\omega_1,\omega_2}(G,G') = (D", \alpha"_0, \ldots, \alpha"_n)$, and $f : D" \to D$ (resp. $f' : D" \to D'$) be a partial subisomorphism function associated with the subisomorphism relation $G" \sqsubseteq^p G$ (resp. $G" \sqsubseteq^p G'$).

We have $d_c(G,G') = \sum_{\delta_j \in p\Delta_f^{G\to G"} \cdot p\Delta_{f'}^{G"\to G'}} (c(\delta_j))$.

Proof. In a very similar way that we have proven in Prop. 1 we can show that involved seams and involved darts cannot belong to two different operations. It follows that operations can be reorganized starting by the deletion of seams, deletion of darts, addition of new darts and finally addition of new seams. This order of operations leads to the maximum common partial submap after the deletions, then we can prove that there is no edit path with a smaller cost that would transform G into G' without passing througth the maximum common partial submap and as a consequence it follows that the cost of the edit path is minimum. □

5. Experimental results

In this section, we first illustrate basic differences between our distance measures on a small example. Then, we quickly describe algorithms for computing approximations of our distance measures and we evaluate these



(a) G_1 (b) G'_1 (c) G_2 (d) G'_2

Figure 6: Distance examples: For the two maps G_1 and G'_1 : $d^i_{\omega_1,0}(G_1, G'_1) \approx 0.43$, $d^i_{1,1}(G_1, G'_1) \approx 0.56$ and $d^p_{1,1}(G_1, G'_1) \approx 0.57$. And for the two other maps G_2 and G'_2 : $d^i_{\omega_1,0}(G_2, G'_2) = 0.125$, $d^i_{1,1}(G_2, G'_2) \approx 0.21$ and $d^p_{1,1}(G_2, G'_2) \approx 0.08$

algorithms, and compare our distance measures, on a set of 2G-maps which model a same mesh at different levels of details. Finally, we illustrate the interest of our distance measures on a small mesh classification problem.

In the following experiments, we will consider operations defined in Section 4.1 to degrade and modify G-maps *del* to Op. 1, *add* to Op. 2, *sew* to Op. 3 and *unsew* is referencing to Op. 4 .

5.1. Comparison of induced- and partial-based distance measures on an example

Let us consider maps of Fig. 6 to illustrate the basic differences of these different distances.

Let us first consider the distance based on the maximum common induced submap. In this case, the edit path is composed of two operations: *add* and *del*. The cost of the edit path that transforms G_1 into G'_1 is equal to $8 \cdot \omega_1$ as 6 darts $\{p, m, n, o, q, t\}$ are deleted and 2 darts $\{1, 2\}$ are added. In this case, $d^i_{\omega_1,0}(G_1, G'_1) = 1 - 8/14 \approx 0.43$. When $\omega_2 \neq 0$, the distance also takes into account the number of sewn/unsewn darts (related to the added/deleted darts), so that for example $d^i_{1,1}(G_1, G'_1) = 1 - \frac{8+12}{14+32} \approx 0.56$. The cost of the edit path that transforms G_2 into G'_2 is equal to $8 \cdot \omega_1$ as 4 darts $\{g, f, m, n\}$

are deleted and 4 darts $\{1, 2, 3, 4\}$ are added. In this second example, we have $d^i_{\omega_1,0}(G_2, G'_2) = 1-14/16 = 0.125$. However, the two maps look rather similar (the only difference is that on the right-hand-side map, the two squares are not 2-sewn). This rather counter- intuitive result comes from the fact that we consider the maximum common induced submap or, from an edit distance point of view, we forbid sew and unsew operations.

When considering the distance based on the maximum common partial submap, edit paths are composed of four operations: *unsew*, *del*, *add* and *sew*. The cost of the edit path that transforms G_1 into G'_1 is equal to $16{\cdot}\omega_2{+}4{\cdot}\omega_1{+}4{\cdot}\omega_2$ as we unsew the 16 seams of the 4 deleted darts $\{p, m, n, o\}$ and add 4 new seams $\{(q, 1, l), (l, 1, q), (t, 1, a), (a, 1, t)\}$. In this case $d^p_{1,1}(G_1, G'_1) = 1{-}24/56 \approx 0.57$. The cost of the edit path that transforms G_2 into G'_2 is equal to $4 \cdot \omega_2$ as we just remove the seams $\{(g, 2, m), (m, 2, g), (f, 2, n), (n, 2, f)\}$. In this second example, we have $d^p_{1,1}(G_2, G'_2) = 1 - 48/52 \approx 0.08$.

5.2. Approximation algorithms

We have described in [6] an algorithm which efficiently computes an approximation of $d^i_{1,0}(G, G')$. We have extended this algorithm to compute an approximation of our generic distance measure $d^*_{\omega_1,\omega_2}$ in a rather straightforward way. This algorithm basically computes r common submaps in a greedy randomized way, and returns the cost associated with the largest computed submap, among the r computed ones. Each greedy construction is done in polynomial time, in $\mathcal{O}(n \cdot s \cdot \log(s))$ where n is the dimension of the nG-map and $s = size_{1,0}(G) \cdot size_{1,0}(G')$.

In this section, we evaluate the quality of the computed approximations on maps for which we actually know the exact value of $d^p_{1,1}$. This exact value is known by construction: starting from an initial map G_0, we generate a set of 14 maps $\{G_1, G_2, \ldots, G_{14}\}$ such that G_i is a partial submap of G_0 which is obtained by randomly removing from G_0 $i \times 5\%$ of seams or darts. This way, we can compute the exact value of $d^p_{1,1}(G_0, G_i)$ (as $G_i \sqsubseteq^p G_0$ and we know an edit path from G_0 to G_i).

Fig. 7 compares $d^p_{1,1}$ with the approximations of d^p and d^i computed by our algorithm, on average for 60 different sets of 15 maps, starting from 60 different initial 2G-maps which model 60 different meshes selected from the [16] repository. These initial 2G-maps have 4808 darts and 14031 seams, on average.

Fig. 7 shows us that the approximation of d^p computed by our greedy algorithm is rather close to the exact value of d^p. It also shows us that the

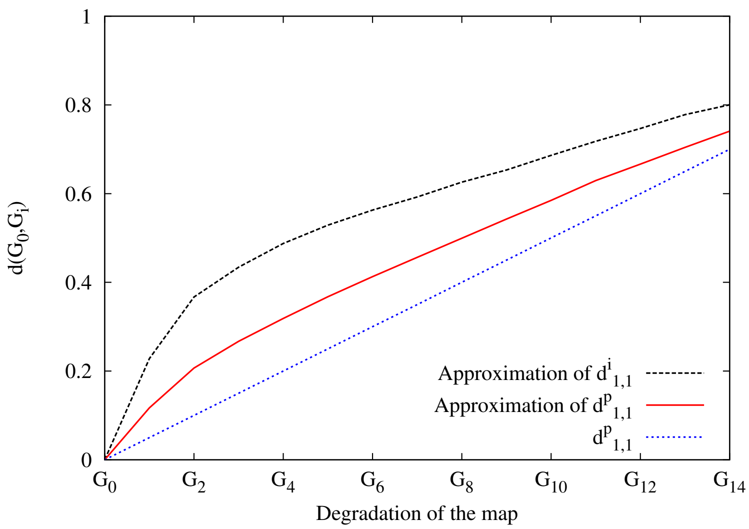
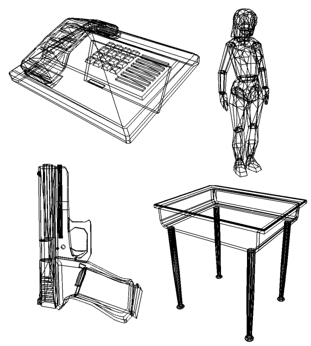


Figure 7: Comparison of $d^p_{1,1}$ with the approximations of $d^p_{1,1}$ and $d^i_{1,1}$ computed by our algorithm (with r = 10). For each $i \in \{1, ..., 14\}$, G_i is a submap of G_0 obtained by removing $i \cdot 5\%$ of its seams or darts. The three curves respectively plot the evolution of $d^p_{1,1}(G_0, G_i)$ and of the approximations of $d^p_{1,1}(G_0, G_I)$ and $d^i_{1,1}(G_0, G_i)$ (average results on a set of 60 maps).

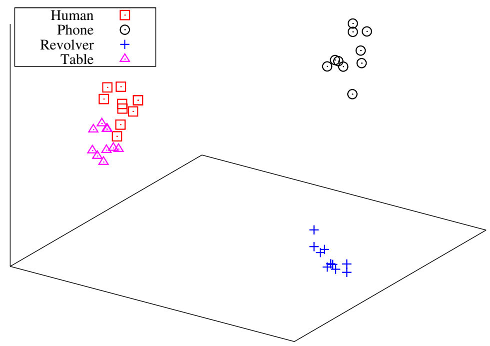
approximation of d^i is greater than the approximation of d^p. This comes from the fact that unsew operations used to transform G_0 in G_i lead to dart deletions and additions when considering d^i (as the only way to unsew a dart is to remove it, with all its seams, and then to add it again with all its seams except the one that had to be removed

5.3. Meshes classification

Let us now show that our distance measures may be used to classify objects modelled by $2G$-maps. We consider $2G$-maps which model meshes extracted from [16] repository. These meshes are triangular meshes so that they do not contain much structural information. To obtain more relevant $2G$-maps, whose faces have different number of edges, we have merged faces with small diedral angles (less than 5 degrees). To ensure that the classification is not influenced by the size of the maps, we have considered four initial $2G$-maps which all have 4808 darts (see Fig. 8(a)). We have generated four classes of ten $2G$-maps: starting from each of the four initial maps of Fig. 8(a), we have generated ten maps by randomly applying *del* and *unsew*



(a) Meshes Examples



(b) Meshes Classification

Figure 8: (a) The orignal meshes used in the classification process. (b) 3D representation of the matrix of dissimilarity. Each point represents a map modelling a mesh.

operations on the initial map, so that the reduced maps contain from 15% to 25% of the darts and seams of the initial map. Therefore, we obtain an experimental set of forty $2G$-maps generated from 4 different initial $2G$-maps (these initial maps are not included in the experimental set as they have more darts and seams).

We have first classified this experimental set by using the k-nearest neighbour (kNN) classification algorithm (with a leave-one-out principle), and by using our algorithm to compute an approximation of $d^p_{1,1}$ for evaluating the dissimilarity of two 2G-maps. The classification rate ranges between 90% and 95% when k is between 1 and 7, the best classification rate being reached when $k = 3$.

We also have computed an embedding of the experimental set in a vector space by computing a matrix of dissimilarity corresponding to the approximations of $d^p_{1,1}$ computed by our algorithm. Fig. 8(b) displays the 3D projection of this vector space by using multidimensional scaling (MDS) [17]. It shows us that the four sets of $2G$-maps are rather well separated. Two classes are rather close, i.e., humans and tables. This may come from the fact that humans have two arms and two legs which may rather well matched with the four table legs.

Note that these results have been obtained by only considering structural informations so that they are independent from any geometrical information and transformation (position of vertices, scale factor, and any rigid transformation (translation, rotation ...).

6. Conclusion

In this paper, we have introduced a generic distance based on the size of maximum common submaps which may either be partial or induced. In addition we introduced a map edit distance with its operations and we related both distance.

Our edit distance may be related to previous work on nG-map pyramids [18, 19] which represent a same object at different levels of details. In a pyramid, the map at level i may be obtained by applying edit operations to the map at level $i-1$. These edit operations may be used to compute the edit distance between two maps at two different levels in a same pyramid. Note however that the edit operations used in pyramids are different from those used in this paper so that we should first define a relationship between edit operations.

Further work will mainly concern the experimental evaluation of the relevancy of our distance measure for classifying or retrieving objects modelled by nG-maps, such as images and meshes. We also plan to extend our distances by integrating geometrical information by means of label substitution costs.

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