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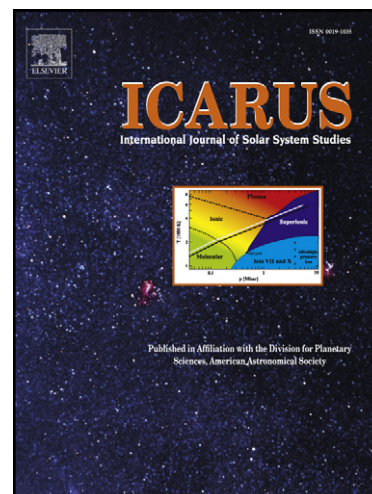
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CONSTRAINING THE LUMINOUS EFFICIENCY OF METEORS

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Abstract

We propose a new approach for studying the radiation of a fireball, one of the main processes which occur when the meteor body enters the planetary atmosphere. The only quantities which directly follow from the available observations are the fireball brightness, its height above sea level, the length along the trajectory, and as a consequence its velocity as a function of time. Other important parameters like meteoroid's mass, its shape, bulk and grain density, temperature remain unknown. The present study takes recent results in fireball aerodynamics and considers them together with the classical postulate that a fraction of the meteoroid kinetic energy is transformed into radiation during its flight. This gives us a new analytical dependence, which in particular shows that the fireball luminosity in general is proportional to the body pre-entry mass value, its initial velocity to the power of 3, and the sine of the slope between horizon and trajectory. Research helps in finding an answer to the general important question: Which fraction of the fireball kinetic energy is transformed into light during meteoroid drag and ablation in the atmosphere?

1. Introduction

Up to now practical results in Meteor Physics are only derived with some uncertainty mainly due to the possible variation of the meteoroid's shape, mass (and also the way they change during the flight), and bulk density. Thus, results with an error in the order of a magnitude are often accepted as sufficient (Öpik, 1958).

The main processes which in general are supposed to be taken into account are meteor body drag, radiation, ionization and ablation. The ablation rate depends on the meteoroid's velocity while deceleration is determined by its mass. Therefore the mass loss and the drag equations for a meteoroid should be solved simultaneously. The ablation of the meteoroid can occur because of evaporation, fusion and spraying of the liquid film by the oncoming air flow, sputtering and fragmentation.

The study of meteor radiation should be based on the three basic sources generating this process: compressed high-temperature atmospheric gas in the shock layer around the body, vapor of body material and other products of destruction, and the body surface. The first and third sources have been considered by Gritsevich and Stulov (2007). The luminosity values were determined as a function of the body radius, its velocity and density of the atmosphere at the height of the flight. The model of an equilibrium mixture of radiating components allowed calculating a radiation flux in the optical frequency range. However such an approach is approximate and we still keep some uncertainty due to unknown geometrical shape of the studied objects.

The radiation from the meteor body surface is significant for micrometeors or for early stage of events with larger scale while we still can consider the free molecular air flow around meteoroid, i.e. when the mean free path of the air molecules is larger than the characteristic body size. As the spectral lines of the elements which are present in majority of collected meteorites dominate in the observed meteor spectrum (the identified atoms and ions are the elements like Fe, Mg, Na, Ca, Ni, Si, Al, Co, Cr or Mn, etc., see e.g. Bronshten, 1983) the suggestion that the dominating contribution to the meteor luminosity comes from the emission of the atoms vaporized from the body surface is often generalized to be valid along the whole luminous trail even for large meteorite-producing fireballs, which for sure have experienced all possible flow regimes during atmospheric entry. This assumption means that atmospheric lines and bands are considered of secondary significance, while the body's incandescent surface may be neglected (Bronshten, 1983). Thus the subsequent research under such assumptions leads to the special case (called luminous equation or photometric formula, depending on the way how it is written) relating the luminosity process to ablation only. This case we will discuss in more details in section 4.

An alternative way to study meteor luminosity is based on the fact that its value should be proportional to the loss of meteoroid kinetic energy as there are no other possible sources for that. Then again the assumption that kinetic energy loss is determined by meteoroid ablation only (while the deceleration term can be ignored) leads to the number of studies (e.g. Bellot Rubio et al., 2002; Campbell Brown and Koschny, 2004; McAuliffe and Christou, 2006; and older papers discussed in section 4). However in the present study we consider the general case when the change in meteoroid mass and velocity are both significant and should necessarily be involved in the mechanism of energy transfer. Throughout this work we also permit changes in meteoroid shape during the whole luminous part of an atmospheric trajectory. We start from the drag and mass loss equations and the geometrical relation along a meteor trajectory in the atmosphere. Using dimensionless parameters and assuming an isothermal atmosphere, we solve the equations for the dynamical behavior of the meteoroid. We then compare the results with the observed drag rate and the light curve of the fireball and derive from this comparison the luminous efficiency.

2. Basic differential equations and first integrals

Let us start from the drag and mass loss equations which in physical theory of meteors are usually (see e.g. Bronshten, 1983; Stulov et al., 1995) given by:

$$M \frac{dV}{dt} = -\frac{1}{2} c_d \rho_a V^2 S, \quad (1)$$

$$H^* \frac{dM}{dt} = -\frac{1}{2} c_h \rho_a V^3 S, \quad (2)$$

The explaining of used variables and parameters are given in the appendix of this paper. Variations in the entry angle γ of the meteoroid are not significant and usually they are not taken into account. Therefore the change in height with time is given as:

$$\frac{dh}{dt} = -V \sin \gamma, \quad (3)$$

This can be used to introduce in Eqs. (1)-(2) the new variable h instead of t . Excluding t we get h as

the independent variable, and we still have 4 unknown variables, namely M , V , S , and ρ_a , depending on h in two equations. That is why two additional dependencies between these quantities are necessary for solving the equations. Thus the analytical solution of Eqs. (1) and (2) is obtained under the assumption of an isothermal atmosphere

$$\rho_a = \rho_0 \exp(-h/h_0), \quad (4)$$

which within the considered range of heights (Tab. 1) is sufficient. Following Levin (1956), we also introduce the parameter μ taking into account variations in the meteoroid shape as

$$\frac{S}{S_e} = \left(\frac{M}{M_e}\right)^\mu. \quad (5)$$

To solve these equations, we introduce dimensionless parameters. As a general rule, we use capital letters for the parameters with dimensions (e.g. M for the mass in kg) and small letters for the dimensionless parameters (e.g. m for the mass between 0 and 1). Detailed naming conventions for all variables and parameters are given in the appendix. Applying these assumptions together with initial conditions $y = \infty$, $v = 1$, $m = 1$, the first integrals of differential Eqs. (1) and (2) appear as:

$$m = \exp\left(-\beta \frac{1-v^2}{1-\mu}\right) \quad (6)$$

$$y = \ln 2\alpha + \beta - \ln(\bar{E}i(\beta) - \bar{E}i(\beta v^2)) \quad (7)$$

For a more detailed derivation, see Gritsevich (2007). In Eqs. (6)-(7), α is the ballistic coefficient describing the drag properties of the meteoroid; β is the mass loss parameter; and μ is the shape change coefficient (again, see the appendix for detailed definitions).

Finally we assume that a given fraction τ of the meteoroid kinetic energy is converted into visible radiation. Its luminosity then is described by the following formula:

$$I = -\tau \cdot \frac{dE}{dt}, \quad (8)$$

which in terms of meteor body mass and velocity can be rewritten as:

$$I = -\tau \cdot \frac{d(MV^2/2)}{dt} = -\tau \cdot \left(\frac{V^2}{2} \frac{dM}{dt} + MV \frac{dV}{dt} \right) \quad (9)$$

3. Approximation of observed trajectories

As it follows from the previous section the real phenomena can be qualitatively described as soon as values for the non-dimensional parameters α , β , μ , and τ are known. Therefore it is expedient to

separate the problem of the determination of these parameters from observational data into several subtasks, each of which will allow us to identify one of the parameters unambiguously by the approximation of the corresponding physical process. We assume that from the data of observations at certain time points ($i = 1, 2, \dots, n$) the values of the meteor altitude h_i and velocity V_i , as well as the meteor stellar magnitude M_{pan_i} are known. Such kind of data has been published for example in the paper of Halliday et al. (1996). The paper contains the detailed final report on the Meteorite Observation and Recovery Project (MORP), which was prepared by the researchers directly involved in the project.

3.1. Altitude-velocity relation

The analysis of the function (7) shows that there is actually just one pair of the parameters α and β which provide the best fit to the observed meteoroid altitude and velocity values (Gritsevich, 2007). Therefore these two parameters can be easily determined firstly from the altitude and the body's deceleration in the atmosphere. The problem is solved using the least square method (see Gritsevich, 2008a, 2009 for exact formulas and details). The main advantage here is that during this part of the proposed model we do not involve any other assumptions or empirical constants. Thus the exact values of the meteoroid shape factor, meteoroid density and shape change parameter are not required.

As we use Eq. (7) now not in meaning of describing, but for general approximation of real phenomena it should be clear that our model is identically sensitive to different types of meteoroid's mass loss (e.g. it takes into account (i) the fusion of the outer layer, followed by spraying of the liquid layer by the hypersonic air flow, (ii) a vaporization of the solid phase or liquid layer, and removal of mass in the form of vapor, and (iii) a meteoroid fragmentation occurred by minute detachment of secondary solid particles from the parent body). Indeed, as mass loss parameter β fits the actual observational data we assume change in meteoroid mass (6) to be result of ablation mechanisms in reality occurred along trajectory.).

3.2. Analysis of change in kinetic energy

We consider here the general case when both mass and velocity of a meteor body are changing with time along the considered segment of a trajectory. The theoretical curve (7) allows us to define $\frac{dy}{dv}$. On the other hand from the geometrical equation (3) follows that

$$\frac{dy}{dt} = -\frac{V \sin \gamma}{h_0}. \quad (10)$$

Using the definition of the dimensionless velocity and then replacing $\frac{dy}{dv}$ with the derivative of (7) by using the theory of Special Functions (Yanke et al., 1964) yields

$$\frac{dV}{dt} = V_e \frac{dv}{dy} \frac{dy}{dt} = -\frac{V_e^2 v^2 \sin \gamma}{2h_0} \cdot \frac{\bar{\text{Ei}}(\beta) - \bar{\text{Ei}}(\beta v^2)}{\exp(\beta v^2)} \quad (11)$$

Using the definition of the dimensionless mass and replacing $\frac{dm}{dv}$ with the derivative of equation (6) we receive:

$$\frac{dM}{dt} = M_e \frac{dm}{dv} \frac{dv}{dt} = M_e \frac{2\beta v}{1-\mu} \exp\left(-\beta \frac{1-v^2}{1-\mu}\right) \cdot \frac{dv}{dt} \quad (12)$$

Thus the luminosity equation (9) can be then rewritten as:

$$\begin{aligned} I &= -\tau \cdot \frac{d(MV^2/2)}{dt} = -\tau \cdot \left(\frac{V^2}{2} \frac{dM}{dt} + MV \frac{dV}{dt} \right) = \\ &= -\tau \cdot \left(M_e V_e \frac{\beta v^3}{1-\mu} + M_e V_e v \right) \exp\left(-\beta \frac{1-v^2}{1-\mu}\right) \frac{dV}{dt} = \\ &= \tau \cdot \frac{M_e V_e^3 \sin \gamma}{2h_0} v^3 \cdot (\bar{\text{Ei}}(\beta) - \bar{\text{Ei}}(\beta v^2)) \cdot \left(\frac{\beta v^2}{1-\mu} + 1 \right) \cdot \exp\left(\beta \frac{(\mu v^2 - 1)}{1-\mu}\right) = \tau \cdot \frac{M_e V_e^3 \sin \gamma}{2h_0} f(v) \quad (13) \end{aligned}$$

Where

$$f(v) = v^3 \cdot (\bar{\text{Ei}}(\beta) - \bar{\text{Ei}}(\beta v^2)) \cdot \left(\frac{\beta v^2}{1-\mu} + 1 \right) \cdot \exp\left(\beta \frac{(\mu v^2 - 1)}{1-\mu}\right) \quad (14)$$

As the value of mass loss parameter β is known from section 3.1., only the shape change coefficient μ remains unknown in the right side of Eq. (14). So we can now get its value from the condition of the best conformity to the shape of observed light curve, e. g. applying once again the least square method. The multiplier τM_e in (13) then can be defined from the $I(t)$ amplitude according to observations.

Eq. (13) shows that in general the meteor luminosity is proportional to the body's pre-entry mass value, its initial velocity raised to the power of 3, and sinus of a slope. As it immediately follows from dimension analysis, the luminous efficiency coefficient τ in (13) can be kept as non-dimensional parameter.

Using the definition of the ballistic coefficient α (see appendix) the pre-entry mass value M_e can be written as:

$$M_e = \left(\frac{1}{2} c_d \frac{\rho_0 h_0}{\alpha \sin \gamma} \frac{A_e}{\rho_m^{2/3}} \right)^3 \quad (15)$$

Therefore by substituting Eq. (15) into Eq. (13) the current model allow us to determine the product $\tau(c_d A_e)^3 \rho_m^{-2}$ as

$$\frac{\tau(c_d A_e)^3}{\rho_m^2} = \frac{16I\alpha^3 \sin^2 \gamma}{V_e^3 \rho_0^3 h_0^2 f(v)} \quad (16)$$

The right side of Eq. (16) contains such known parameters as the atmospheric density $\rho_0 = 0.00129 \text{ g/cm}^3$ at sea level, the atmospheric scale height $h_0 = 7160 \text{ m}$, the initial velocity V_e and the slope γ between horizon and trajectory. The values of the ballistic coefficient α and ratio $I/f(v)$ are found by the present technique according to sections 3.1-3.2.

Let us note that no initial assumptions about the meteoroid's shape, mass, or bulk density have been involved in our study so far.

3.3. Application to the real phenomena

The theoretical concept described above is applied to the MORP fireballs # 018, 138 and 219, for which observational data are published by Halliday et al. (1996). The main trajectory data are given in Table 1. In the two last columns the local deceleration estimates resulting from our method (Eq. (11)) and numerical derivatives obtained by Halliday et al. (1996) are shown, respectively. Following Ceplecha and ReVelle (2005) we use the formula

$$M_{pan} = -2.5(\lg I - 10.185) \quad (17)$$

for converting the brightness which is expressed in absolute magnitudes by Halliday et al. (1996) into fireball luminosity, which is supposed to be the total energy emitted across the whole electromagnetic spectrum (Ceplecha et al., 1998). In the right side of Eq. (17) we exchange the multiplier of 2/5 given by Ceplecha and ReVelle (2005) with the value 2.5, which comes from the absolute magnitudes definition (see e.g. Karttunen et al., 1996). According to ReVelle (2009) the value of 2/5 in Ceplecha and ReVelle (2005) should be considered as a typographical error. The fireball luminosity in (17) is given in energetic units $\text{erg/s} = 10^{-7} \text{ W}$, thus the final formula for converting absolute magnitudes we used in the present calculations has the form:

$$I = 10^{-0.4M_{pan} + 3.185} \text{ W} \quad (18)$$

In Table 2 the obtained values of the ballistic coefficient α , mass loss parameter β , shape change coefficient μ and the factor $\tau(c_d A_e)^3 \rho_m^{-2}$ are given. In the last column of Table 2 we give values of the ablation coefficient σ , which are estimated by the formula:

$$\sigma = \frac{c_h}{c_d H^*} = \frac{2\beta}{(1-\mu)V_e^2} \quad (19)$$

The corresponding analytical dependencies $I(v)$ (formula (13)) are shown in figure 1 (a-c), the dots shows experimental results derived from observations (Table 1). This helps us to put tighter margins on the estimate of the luminous efficiency coefficient. However at this stage for determination of the luminous efficiency coefficient τ we need to involve assumptions on the value of the product of the drag coefficient and initial shape factor $c_d A_e$ and meteoroid bulk density. According to a review made by Gritsevich (2008b), $c_d A_e = 1.8$ can be considered as the most reasonable value. It correspond in particular to a body with an initial shape factor $A_e = 1.5$ and a drag coefficient $c_d = 1.2$. Let us note nevertheless that an initial spherical shape ($A_e = 1.21$) and $c_d \sim 1$ are also widely used in meteor physics studies.

The review of meteorite bulk densities reveal that the bulk density of stony meteorites is generally in the range of 3.3 to 3.9 g/cm^3 , with an average of about 3.6 g/cm^3 , which is denser than most

terrestrial rocks (Wood 1963). Most meteorites are ordinary chondrites (Grady 2000), with bulk density from 3.0 g/cm^3 to $3.8\text{-}4.0 \text{ g/cm}^3$. Some carbonaceous meteorites rich in volatiles (CI and CM classes) have bulk densities lower than 3.0 g/cm^3 . The higher bulk density values have stony-iron meteorites ($4\text{-}6 \text{ g/cm}^3$) and iron meteorites ($4\text{-}8 \text{ g/cm}^3$) which are highly differentiated meteorites possibly representing cores of large asteroid parent bodies broken apart by catastrophic collisions. However such a high bulk density iron objects expected to be rare exception among observed meteors (Borovička, 2006).

Consolmagno and Britt (1998) make a note that most meteorites are substantially denser than their suggested asteroid analogous which must have larger porosity because of presence of empty space on scales larger than the size of the meteorites. Also the terrestrial atmosphere acts here as a filter “absorbing” more efficiently the low-density meteoroids and asteroids as well as braking the larger objects along mechanically weaker planes (faults, fractures and more porous areas) allowing the more solid fragments only to reach the surface as meteorites. Some asteroid mass and volume data supporting this conclusion were produced from space-based instruments and ground-based high-performance telescopes. Bulk density measurements for asteroids and asteroid-like bodies were published by Millis and Elliot (1979) for Pallas ($2.6 \pm 0.5 \text{ g/cm}^3$), Millis et al. (1987) for Ceres ($2.7 \pm 0.14 \text{ g/cm}^3$), Scholl et al. (1987) for Hygiea ($2.05 \pm 1.00 \text{ g/cm}^3$), Belton et al. (1995) for Ida ($2.6 \pm 0.5 \text{ g/cm}^3$), by Smith et al. (1995) for Phobos ($1.53 \pm 0.1 \text{ g/cm}^3$), and Deimos ($1.34 \pm 0.83 \text{ g/cm}^3$), by Veverka et al. (1997) for Mathilde ($1.4 \pm 0.4 \text{ g/cm}^3$), by Merline et al. (1999) for Eugenia ($1.20 \pm 0.60 \text{ g/cm}^3$), by Viateau (2000) for Psyche ($1.8 \pm 0.6 \text{ g/cm}^3$), and for Hermione ($1.8 \pm 0.4 \text{ g/cm}^3$), by Viateau and Rapaport (2001) for Vesta ($3.3 \pm 0.5 \text{ g/cm}^3$), and Parthenope ($2.3 \pm 0.2 \text{ g/cm}^3$), by Veverka et al. (2000) and Yeomans et al. (2000) for Eros ($2.67 \pm 0.03 \text{ g/cm}^3$) and by Abe et al. (2006) for Itokawa ($1.95 \pm 0.14 \text{ g/cm}^3$). Thus, asteroid bulk density values are in general in the range of $1\text{-}3 \text{ g/cm}^3$.

From this review we conclude that in the case we are unable to determine the meteoroid composition, internal structure and porosity either due to lack of sufficient observational data or due to nonexistence of recovered meteorites, the bulk density of such meteoroid most likely is within the range of $1\text{-}4 \text{ g/cm}^3$. Our results are summarized in Table 3.

Based on extensive meteorite density and porosity review done by Britt and Consolmagno (2003), Consolmagno et al. (2008), and more recent results achieved by the authors, Consolmagno et al. (2010) suggest the bulk density value of 3.5 g/cm^3 to be the most appropriate input in meteoroid’s models. This conclusion is in good agreement with existing dynamic mass determination methods (e.g. Wetherill and ReVelle, 1981; Halliday et al., 1996; Gritsevich, 2009) and corresponds to the type I according to the fireball classification and PE-criterion introduced by Ceplecha and McCrosky (1976). Therefore, following the assumption made by Halliday et al. (1996), for the fireballs considered in this study we accept the bulk density value of 3.5 g/cm^3 to be preferred. The luminous efficiency coefficient values are nearly the same if we simultaneously decrease assumed meteoroid’s bulk density and the value of the product of the drag coefficient and pre-entry shape factor $c_d A_e$.

4. Previous investigations concerning the luminous efficiency coefficient

We should note here that there are a large number of publications devoted to the possible ways of determining the value of the luminous efficiency coefficient τ . The starting point is mostly based on the theory of meteor radiation developed by Öpik (1933; 1955; 1963). The theory was created for small meteors not more than a few centimeters in diameter. Then for the special case with

negligible deceleration (when we can assume that the meteoroid velocity is constant, i.e. $\frac{dV}{dt} = 0$) we can derive the photometric formula for the calculation of the mass of a fireball from Eq. (8):

$$M_{ph}(t) = - \int_{t_1}^t \frac{I}{\tau V^2} dt \quad (20)$$

The integral is taken along the entire luminous segment of the trajectory, from the extinction time $t = t_1$ to the corresponding time point t where the photometric mass estimate is done. The formula (20) is appropriate when kinetic energy loss is dominated by mass loss while the deceleration rate is considered being insignificant. Up to now the majority of pre-entry mass estimates and light curve studies are done with the use of the simplified photometric approach according to Eq. (20).

Eq. (20) in fact is an integral form of the luminosity equation used by Whipple (1943) and Jacchia et al. (1965), which is a form of Eq. (9) when $\frac{dV}{dt} = 0$:

$$I = -\frac{\tau}{2} \frac{dM}{dt} V^2 \quad (21)$$

As the spectra of meteors consist mainly of a discrete number of emission lines whose relative intensities vary greatly with meteor velocity, it was suggested that the luminous efficiency coefficient τ should be given by the power function of V : $\tau = \tau_0 V^n$ (Jacchia et al., 1965). So, initially Whipple (1938; 1943) had assumed a linear relationship between luminous efficiency coefficient and meteoroid velocity. Using this assumption, Jacchia (1948) derived from Öpik's data that $\tau_0 = 6.46 \cdot 10^{-19}$ (the authors used cgs scale, while I is expressed in units of the intensity of a zero-magnitude star (Jacchia et al. 1965)). These values of n and τ_0 were used by Jacchia (1948, 1949, 1952) for deriving the masses of meteors photographed in the Harvard Meteor program. Later, the following formula

$$\tau = 1.0 \cdot 10^{-19} V \text{ (in cgs and 0-mag units)} \quad (22)$$

assuming $n = 1$, was adopted by Verniani (1964) and Jacchia et al. (1965) in their study of the meteor luminous efficiency based on the Harvard's photographic data. In usual cgs units, Eq. (22) is equivalent to (Verniani, 1964):

$$\tau = 5.25 \cdot 10^{-10} V \quad (23)$$

For comparison the maximal luminous efficiency coefficient values calculated according to formula (23) are given in Table 4.

Finally, Eq. (20) was applied to the observational data (given in the form of tabulated values) changing the form of the equation to (see e.g. McCrosky and Posen (1968)):

$$M_{ph} = \frac{2}{\tau_0} \sum \frac{I}{V^3} \Delta t \quad (24)$$

The assumed empirical dependence of luminous efficiency coefficient on velocity was experimentally calibrated for ten iron and nickel artificial meteors by Ayers et al. (1970). Assuming

that $\tau = \tau_0 V$ for these artificial events with pre-entry masses ranging from 0.64 to 5.66 g led the authors to the conclusion that τ_0 is around 10^{-18} .

In order to summarize the observational data of Canadian Network fireballs Halliday et al. (1996) proposed the following scheme for calculating luminous efficiency coefficient τ :

$$\tau = 0; V \leq 10 \text{ km/s};$$

$$\tau = 0.04; 10 < V < 36 \text{ km/s and}$$

$$\tau = 0.069(V^*/V)^2; \text{ where } V^* = 36 \text{ km/s, and } 36 \text{ km/s} \leq V \quad (25)$$

Currently, while estimating the mass with the photometric method, researchers mostly use the variable values of τ obtained from the analysis of the observational data of the Prairie and European Fireball Networks calibrated by the Lost City meteorite fall. Depending on the velocity of the fireball (slower/faster than 25.372 km/s), different formulas for calculating τ from the velocity, body mass, and atmospheric density (relative to that at the point of maximal brightness of the fireball) have been suggested by Ceplecha and ReVelle (2005).

5. Discussion

This study takes into account recent results in fireball aerodynamics which are considered together with the classical postulate that a fraction of the meteoroid kinetic energy is transformed into radiation during its flight. The change in meteoroid kinetic energy is described according to the drag and the mass loss equations, this allow us to consider it as a function of meteoroid's velocity. Instead of using a variety of unknown quantities (namely, c_d , c_h , H^* , S_e , and M_e) we introduce two dimensionless parameters with the following physical meaning:

- the ballistic coefficient α characterizes the aerobraking efficiency. It is proportional to the ratio between the mass of the atmospheric column along the trajectory with cross section S_e , to the meteoroid pre-entry mass M_e ;
- the mass loss parameter β is proportional to the ratio of the fraction of the kinetic energy of the unit body's mass arriving at the body in the form of heat to the effective destruction enthalpy.

These parameters have been found by comparing the theoretical curve (Eq. (7)) with the actual rate of body deceleration in the atmosphere as described in section 3.1.

The other two dimensionless parameters appeared in the study, μ and τ , are found as the best fit of Eq. (13) to photometrical observations. From the definition, the shape change coefficient μ characterizes the role of the meteoroid rotation during the flight and its value affect the shape of luminosity curve (13). The luminous efficiency coefficient τ is the proportionality factor between the loss in meteoroid kinetic energy and fireball luminosity. As τ comes at Eq. (13) as a multiplier only it determines the intensity (amplitude) of described process. All the studies cited in Section 4 agree that the luminous efficiency depends on the instantaneous velocity. In this work the luminous efficiency is assumed to be constant. However, in view of the fits to the light curves, a velocity-dependent luminous efficiency could be considered in further studies.

We can conclude from the comparison of our results (Table 3) that old luminous efficiency coefficient estimates (Verniani, 1964; Jacchia et al., 1965) do not exceed 0.001 (Table 4), which is too low in the view of current results. This fully explains the fact that photometric way always overestimated the real mass calculated from Eq. (20) at that time (see e.g. Bronshten, 1983). At the

same time the prediction that $\tau = \tau_0 V$ was suitable as it leads to the conclusion that $I \sim M_e V_e^3$ (Eqs. (20), (24)) which is in agreement with our results (formula (13)).

Halliday et al. (1996) suggested $\tau = 0.04$ as appropriate value for the considered fireballs (except for the one to two last points where the velocity is less than 10 km/s; there the authors suggest that $\tau = 0$). This is in good correspondence with our results for fireball # 219.

At the same time, assuming that the luminous efficiency τ should vary along the trajectory, Ceplecha and ReVelle (2005) derived a range from 0.006 to 0.097 for its possible apparent values for the Lost City fireball. The intrinsic luminous efficiency for Lost City was found by the authors to be less than 0.015, which does not contradict our results.

Ceplecha et al. (1998) make a note that as the uncertainty in luminosity I derived from observations is of the order of 50 % (as the precision of M_{pan} is ± 0.4 mag). As a consequence the authors assumed that the deceleration term in (9) is significant just for slow meteors ($V < 16$ km/s) if their ablation coefficient does not exceed 0.016 s²/km². However the good fit of experimental points with the obtained theoretical curves (Fig. 1) supports the idea that the actual uncertainty in luminosity is not so large. Thus we can assume that for the three cases under consideration each proceeding step (namely the primary values deduced from observations by Halliday et al. (1996), the formula (18) for converting absolute magnitudes, and further analysis as described in the present study) quite precisely describes the observed phenomena.

In the present study, besides the question of luminous efficiency itself, we have approached another important topic, namely the rate of meteoroid rotation and its influence to the ablation type, drag rate and luminosity. The shape change coefficient μ for fireball # 219 is derived to be 0.18 and apparently is much smaller than the ones for two others fireballs. The coefficient μ value characterizes the rate of the meteoroid rotation during the flight. The case $\mu = 0$ provide the stabilized motion without rotation, when the maximal heating and evaporation occurs in the vicinity of the front critical point of a meteor body (Vislyi et al., 1985). Thus the middle section of a body remains to be constant. This possibility is directly supported by the shape of some fallen meteorites evidently retain the orientation they had along the trajectory (Bronshten, 1983). The assumption most often used is $\mu = 2/3$. In the number of studies μ is replaced by $2/3$ from the very beginning (Wetherill and ReVelle, 1981; Halliday et al., 1996; Ceplecha et al., 1998). In the present study this value fully corresponds to the # 138 fireball. In this case the mass loss of a body occurs uniformly over the whole surface due to its rapid and chaotic rotation, and we can consider that the body shape stayed self similar (see, e.g. Bronshten, 1983). Fireball # 219 demonstrated substantially different behavior during the penetration into the atmosphere comparable to other cases under consideration (Tab. 1). It had almost the same pre-entry velocity, but it reached a much higher peak deceleration of about 12 km/s², and was extremely bright and relatively fast. Surely, this is assumed to be connected with its more vertical entry. On the other hand these effects might be linked to its low shape change coefficient value.

5. Conclusion

This study combines photometric and dynamical observations to better constrain the percentage of kinetic energy emitted as light. Both changes in meteoroid mass and velocity are considered in the described model. No initial assumptions about the exact meteoroid's shape, mass or bulk density were taken. Thus during the fireball entry into the atmosphere the main physical dependencies $M(t)$, $h(t)$, $V(t)$, $I(t)$ can be qualitatively approximated combining standard differential equations of meteor physics and their first integrals. Such an analytical approach allows us to calculate basic non-dimensional parameters α , β , and μ and put more tight margins on the luminous efficiency

coefficient τ . For the cases considered here, we find τ in the range of 0.6 % to 8 % for stony meteoroids while the product of the drag coefficient and initial shape factor $c_d A_e$ is assumed to be in the range from 1.4 to 1.8. These values are an important tool in our understanding of the extensive observational data on the deceleration of meteors and bolides.

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Appendix

List of symbols

σ	=	ablation coefficient
α	=	ballistic coefficient
β	=	mass loss parameter
γ	=	slope between horizon and trajectory
ρ_0	=	gas density at sea level
ρ_a	=	gas density
ρ_m	=	meteoroid bulk density
A	=	shape factor
A_e	=	pre-entry shape factor of meteoroid
c_d	=	drag coefficient
c_h	=	heat-transfer coefficient
h	=	height
H^*	=	effective destruction enthalpy
h_0	=	scale height
I	=	meteor luminosity
E	=	meteoroid kinetic energy
M	=	meteoroid mass
M_e	=	pre-entry meteoroid mass
M_{pan}	=	absolute panchromatic magnitude
M_{ph}	=	photometric mass
S	=	middle section area
S_e	=	pre-entry middle section area of meteoroid
t	=	time
V	=	velocity
V_e	=	pre-entry velocity
μ	=	shape change coefficient
τ	=	luminous efficiency coefficient

Dimensionless parameters used

$$y = h/h_0$$

$$v = V/V_e$$

$$m = M/M_e$$

$$A_e = \frac{S_e \rho_m^{2/3}}{M_e^{2/3}}$$

$$\alpha = 0.5c_d \frac{\rho_0 h_0 S_e}{M_e \sin \gamma}$$

$$\beta = 0.5(1 - \mu) \frac{c_h V_e^2}{c_d H^*}$$

$$\mu = \log_m \frac{S}{S_e}$$

$$\tau = -I \frac{dt}{dE}$$

Special mathematical function used

The exponential integral $Ei(x)$, which for real, nonzero values of x , can be defined as:

$$\bar{Ei}(x) = \int_{-\infty}^x \frac{e^z dz}{z}.$$

The integral has to be understood in terms of the Cauchy principal value, due to the singularity in the integrand at zero.

Integrating the Taylor series for function $\frac{e^{-z}}{z}$, and extracting the logarithmic singularity, we can

derive the following series representation for $Ei(x)$ for real values of x (see e.g. Abramovitz and Stegun, 1972):

$$\bar{Ei}(x) = c + \ln x + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}, \quad x > 0$$

where c is the Euler–Mascheroni constant (also called Euler's constant). It is defined as the limiting difference between the harmonic series and the natural logarithm:

$$c = \lim_{n \rightarrow \infty} \left(\sum_{k=1}^n \frac{1}{k} - \ln n \right) \approx 0.5772$$

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Table 1. Detailed trajectory data for the chosen Canadian Network fireballs.

MORP No.	t^1 , s	h^1 , km	V^1 , km/s	M_{pan}^1	y	v	I, W	dV/dt , km/s ²	dV/dt^1 , km/s ²
018	0.00	75.5	18.5	-3.5	10.54	1.00	38459	0.00	
	1.00	65.3	18.4	-6.8	9.12	0.99	803526	-0.15	
	1.50	60.4	18.3	-8.6	8.44	0.99	4216965	-0.30	
	2.00	55.0	18.1	-9.4	7.68	0.98	8810489	-0.59	
	2.50	49.9	17.7	-9.6	6.97	0.96	10592537	-1.18	
	3.00	44.9	17.2	-9.7	6.27	0.93	11614486	-1.90	
	3.50	40.1	16.4	-9.6	5.60	0.89	10592537	-3.02	
	4.00	35.8	14.8	-8.9	5.00	0.80	5559043	-5.05	
	4.33	33.1	13.1	-8.1	4.62	0.71	2660725	-6.70	-5.7
	4.67	30.9	10.9	-7.1	4.32	0.59	1059254	-7.77	-7.1
138	5.07	28.7	7.8	-5.3	4.01	0.42	201837	-6.93	-7.4
	5.41	27.5	5.3	-3.2	3.84	0.29	29174	-4.61	
	0.00	72.2	16.9	-2.8	10.08	1.00	20184	0.00	
	1.00	65.4	16.9	-5.1	9.13	1.00	167880	0.00	
	2.00	58.5	16.8	-7.2	8.17	0.99	1161449	-0.10	
	2.83	52.9	16.7	-8.4	7.39	0.99	3507519	-0.19	
	3.50	48.4	16.3	-8.7	6.76	0.96	4623810	-0.60	
4.00	45.1	15.4	-8.7	6.30	0.91	4623810	-1.59	-2.3	

	4.58	41.7	13.7	-7.8	5.82	0.81	2018366	-3.67	-3.6
	5.08	39.2	11.7	-5.9	5.47	0.69	350752	-6.05	-4.4
	5.58	37.0	9.5	-2.9	5.17	0.56	22131	-7.73	
219	0.00	67.7	18.4	-6.4	9.46	1.00	555904	0.00	
	0.58	62.7	18.3	-9.5	8.76	0.99	9660509	-0.20	
	1.08	55.2	18.2	-11.1	7.71	0.99	42169650	-0.40	
	1.58	47.8	17.9	-12.1	6.68	0.97	105925373	-1.01	
	2.08	40.6	17.1	-12.9	5.67	0.93	221309471	-2.66	
	2.58	33.5	15.6	-13.1	4.68	0.85	266072506	-5.78	
	2.83	30.3	14.1	-12.9	4.23	0.77	221309471	-8.67	-7.0
	3.08	28.1	11.6	-12.4	3.92	0.63	139636836	-12.09	-11.5
	3.33	26.1	7.8	-10.1	3.65	0.42	16788040	-11.75	

[†](Halliday et al., 1996)

Table 2. The values of the main parameters found according our model.

MORP No.	V_e km/s	$\sin\gamma$	α^\ddagger	β^\ddagger	μ	$\tau(c_d A_e)^3 \rho_m^{-2}$ cm ⁶ /g ²	σ s ² /km ²
018	18.5	0.57	24.13	1.48	0.75	0.0036	0.034
138	16.9	0.40	38.90	2.89	0.67	0.0041	0.061
219	18.4	0.77	12.51	2.06	0.18	0.0193	0.015

[†](Halliday et al., 1996), [‡](Gritsevich, 2009)

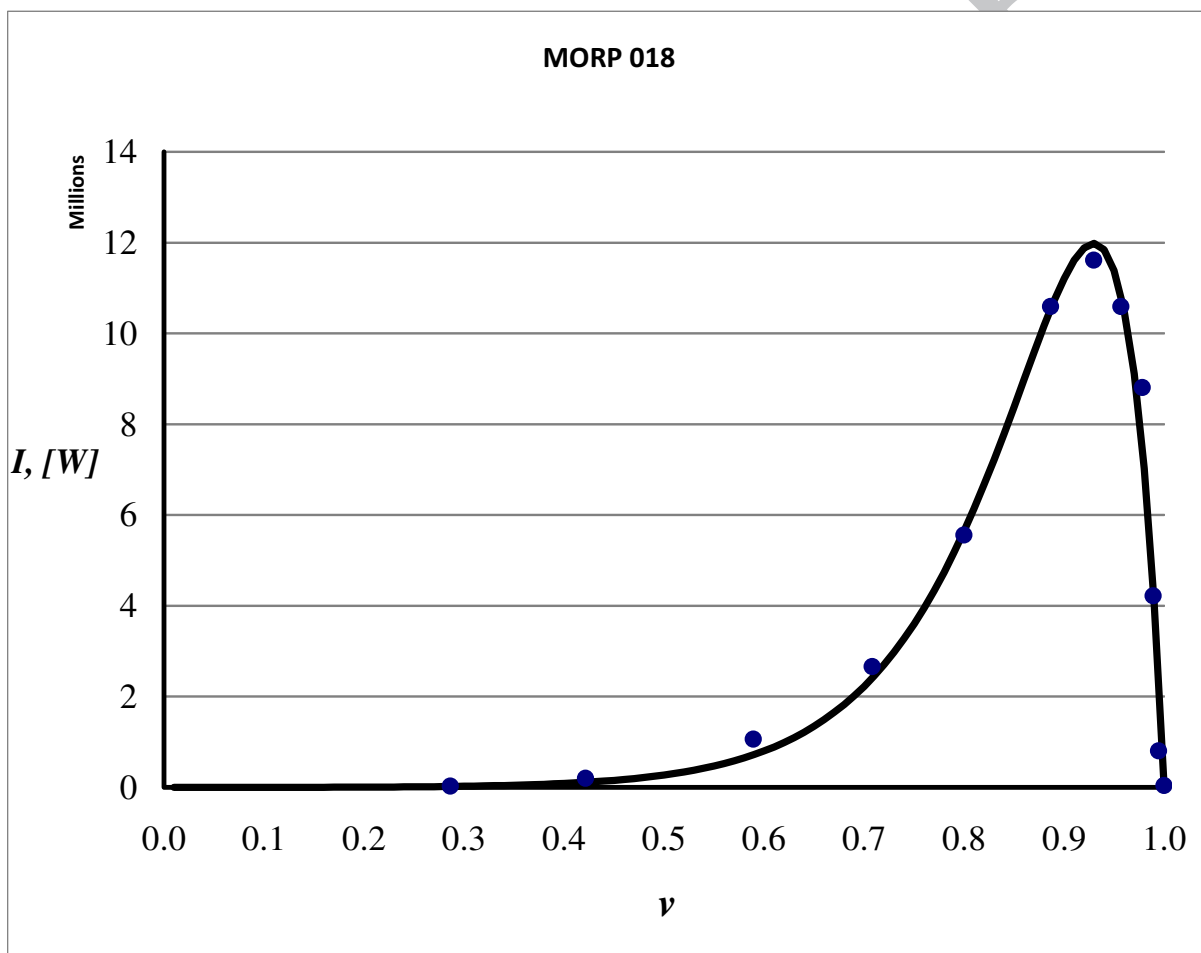
Table 3. The values of the luminous efficiency coefficient τ calculated for different assumptions about meteoroid density and its aerodynamic properties (see discussion in the text).

ρ_m , g/cm ³	1.0	1.5	2.0	2.5	3.0	3.5	4.0
MORP 018							
$c_d A_e = 1.2$	0.0021	0.0046	0.0083	0.0129	0.0186	0.0253	0.0330
$c_d A_e = 1.4$	0.0013	0.0029	0.0052	0.0081	0.0117	0.0159	0.0208
$c_d A_e = 1.6$	0.0009	0.0020	0.0035	0.0054	0.0078	0.0107	0.0139
$c_d A_e = 1.8$	0.0006	0.0014	0.0024	0.0038	0.0055	0.0075	0.0098
$c_d A_e = 2.0$	0.0004	0.0010	0.0018	0.0028	0.0040	0.0055	0.0071
MORP 138							
$c_d A_e = 1.2$	0.0024	0.0053	0.0094	0.0147	0.0212	0.0288	0.0376
$c_d A_e = 1.4$	0.0015	0.0033	0.0059	0.0093	0.0133	0.0181	0.0237
$c_d A_e = 1.6$	0.0010	0.0022	0.0040	0.0062	0.0089	0.0122	0.0159
$c_d A_e = 1.8$	0.0007	0.0016	0.0028	0.0044	0.0063	0.0085	0.0111
$c_d A_e = 2.0$	0.0005	0.0011	0.0020	0.0032	0.0046	0.0062	0.0081
MORP 219							
$c_d A_e = 1.2$	0.0112	0.0251	0.0446	0.0697	0.1004	0.1366	0.1785
$c_d A_e = 1.4$	0.0070	0.0158	0.0281	0.0439	0.0632	0.0860	0.1124
$c_d A_e = 1.6$	0.0047	0.0106	0.0188	0.0294	0.0423	0.0576	0.0753
$c_d A_e = 1.8$	0.0033	0.0074	0.0132	0.0207	0.0297	0.0405	0.0529

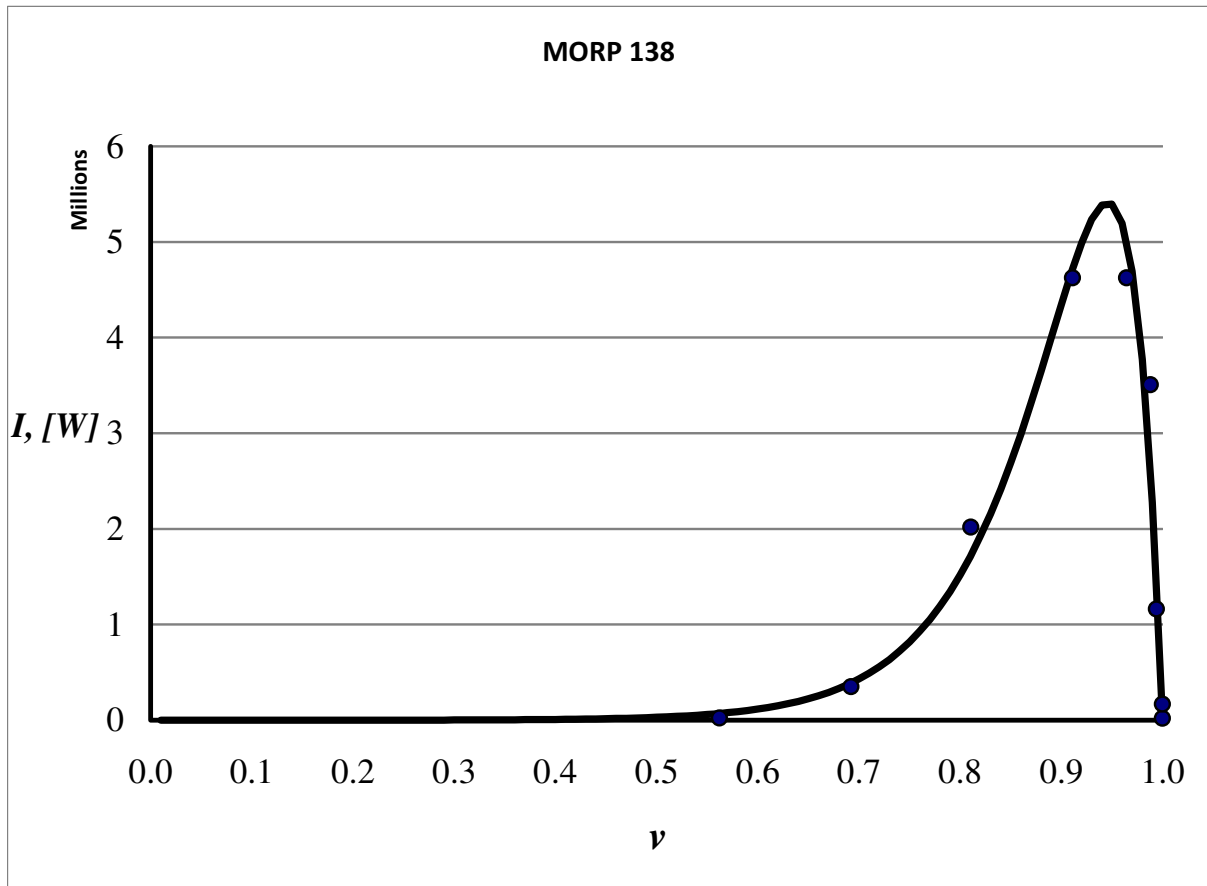
$c_d A_e = 2.0$	0.0024	0.0054	0.0096	0.0151	0.0217	0.0295	0.0385
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Table 4. The maximal (corresponding to the initial velocity) values of the luminous efficiency coefficient τ calculated according formula (23) given by Verniani (1964).

MORP No.	τ
018	0.0010
138	0.0009
219	0.0010

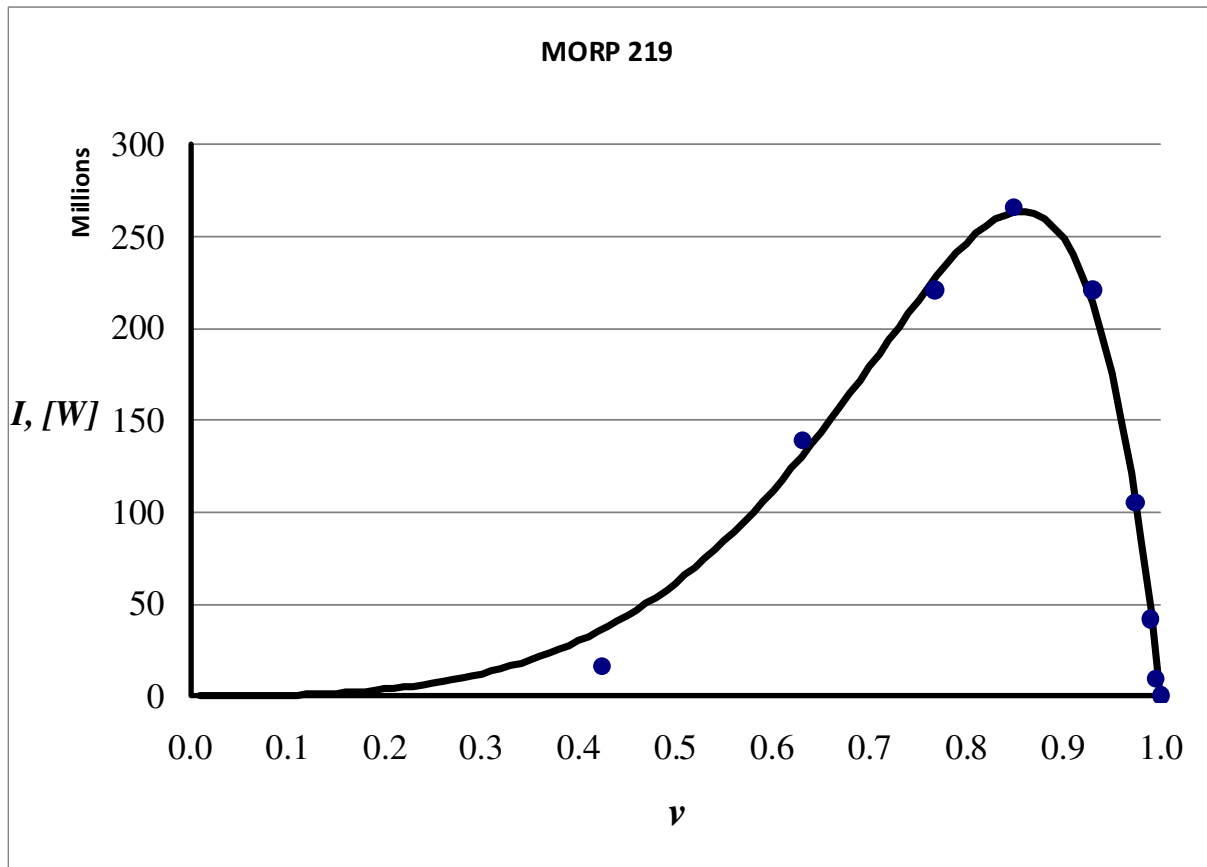


(a)



(b)

ACCEPTED



(c)

Fig. 1 (a-c). Meteor luminosity: the obtained theoretical curves and experimental result (dots).

ACCEPTED

Research Highlights to the ICARUS-11344R1 Manuscript

(Title: CONSTRAINING THE LUMINOUS EFFICIENCY OF METEORS)

Authors: Maria Gritsevich and Detlef Koschny

- We adopt a new technique to interpret brightness of a meteor.
- Both changes in meteoroid mass and velocity are considered.
- Meteor luminosity is expressed analytically as a function of its velocity.
- We test methodology on 3 MORP fireballs.

ACCEPTED MANUSCRIPT