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Topology optimization ~~Synthesis~~ of a magnetic field in a three-dimensional finite region

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This article describes the method of magnetic field topology optimization in an axisymmetric three-dimensional finite region. It is assumed that the region of interest is surrounded by a cylindrical solenoid with an electrical current. The solenoid's inner and outer surfaces are built-up by rotating plane Bezier curves around the symmetry axis. As a global minimizer a genetic algorithm method is used. Optimal configurations are provided under given constraints.

Keywords: magnetic field synthesis; topology optimization; nonlinear inverse problems; Bezier curves; active shields; genetic algorithms

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~~Topology optimization Synthesis of magnetic field in three-dimensional finite region~~

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~~This paper describes the method of magnetic field synthesistopology optimization in axisymmetric three-dimensional finite region. It is assumed that the region of interest is surrounded by a cylindrical solenoid with electrical current. The solenoid's inner and outer surfaces are built up by rotating of the plane Bezier curves around symmetry axis. As a global minimizer a genetic algorithm method is used. Optimal configurations are provided under given constraints.~~

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Introduction

Magnetic field topology optimization (TO) ~~synthesis~~ problems are well known to ~~the~~ engineers, as many different applications require applying a magnetic field of a specified distribution. For example, such electromagnetic arrangements are necessary in various biomedical imaging problems, in physics when making high sensitivity measurements, and in ~~common~~ electrical engineering. The problem is still vast and ~~often~~ remains as a topic of contemporary research (Adamiak 1977, Adamiak 1980, Di Barba *et al*. 1995, Di Barba and Savini 1995, Di Barba and Farina 2002, Girdinio and Nervi 2001, Gottvald 1992, Guimaraes 2006, Kim *et al.* 2004, Kim *et al*. 2007, Kim *et al*. 2010, Lopez *et al*. 2004). The cited references should only be taken as illustrative examples.

Usually, the sources of the magnetic field are ~~coils'~~ sets of coils and different types of solenoids. Another application of such an electromagnetic system can be a compensation of harmful external magnetic fields in order to protect sensitive electronic apparatus. In such a case, the opposite field must have the same frequency and amplitude as the external field. If the incident field presents a wide bandwidth, the final aim is to generate an opposite field in the same frequency range or at least in a range as large as possible (Celozzi *et al*. 2008, Sergeant 2003). Such a technique is called *active shielding*. From the theoretical point of view it belongs to ~~, the problem belongs to~~ magnetic field synthesis theory (Adamiak 1977, Adamiak 1980, Tikhonov *et al*. 1995) and topology optimization problems (Kim *et al*. 2007).

In the past the research activities were mostly focused on the finding of a current distribution in a sectional solenoid (Adamiak 1980), a shape of a solenoid which generates a uniform magnetic field on its axis or very close to the axis (Adamiak 1977, Di Barba *et al*. 1995, Di Barba and Savini 1995, Gottvald 1992). In ~~papers~~ ~~(~~Kim *et al.* (2004) the authors proposed the Design Sensitivity Analysis (DSA) based method in order to optimize the salient pole face shape of a high-temperature superconducting synchronous generator. The ~~paper~~article by ~~(~~Kim *et al*. (2007) refers to ~~the~~ finding an optimum permanent magnet distribution and pole shape design problem using DSA combined with commercial EM software. A novel approach which utilizes the second-order sensitivity information in order to optimize a SMES device has been proposed in ~~, Kim *et al*. 2007, (~~Kim *et al*. 2010) ~~the authors proposed a novel approach which utilizes the second order sensitivity information~~. In the

considered problem the design variable vector consisted of a total of eight design parameters (six parameters describing the dimensions of the magnet and two referring to the current densities).

This ~~paper~~article is a continuation of (Adamiak 1977, Adamiak 1980, Ziolkowski and Gratkowski 2008, Ziolkowski and Gratkowski 2011, Ziolkowski 2011) and discusses a problem of designing a shape of a solenoid, which produces a specified magnetic field magnitude in an~~the~~ axisymmetric three-dimensional finite region. The considered optimization problem takes into account two components of the magnetic field intensity vector, and the final objective function is defined with the help of magnetic vector potential. The method of finding an optimal shape is based on a Genetic Algorithm (GA) coupled with Bezier curves. Exa~~e~~mple~~cary~~ results along with the optimal solenoid~~'s~~ shapes under given constraints are presented.

Problem description

A~~Let us a~~ssume it is necessary~~we need~~ to produce a magnetic field free region in a static uniform magnetic field. The external field is in the direction of the z-axis and is given by the equation $\boldsymbol{H} = -H_0\,\mathbf{1}_z$. The protected region is in the shape of cylinder of radius R and height h with the z-axis as a symmetry axis in a cylindrical coordinate system (r, φ, z). The task is to find the shape of a solenoid, so that the magnetic field in the protected region is uniform but in the opposite direction to the external field ($\boldsymbol{H} = H_0\,\mathbf{1}_z$). The common and very basic sets for generating such a uniform magnetic field are well known Helmholtz and Maxwell coils. More complex arrangements have been considered in the past as well (Adamiak 1977, Adamiak 1980, Di Barba *et al.* 1995), however, the considered shapes and numerical procedures have been defined in a different way. Additionally, the considered topology in (Adamiak 1980) could lead to difficulties in practical realization (many current sources must be provided).

The author of this~~e~~ ~~paper~~article proposes another approach in which ~~it is assumed~~ one current source~~,~~ only is assumed,~~,~~ and a specified magnetic field distribution is obtained by adjusting a solenoid's winding shape. Moreover, it is required to model non-linear shapes described in a simple parametric form represented by mathematical formulas for modification purposes. With such an approach it will be possible to write an algorithm using CAD built-in language that automatically generates the obtained curves. Taking into account the mentioned requirements it is assumed that ~~This paper considers much more general case, where~~ the inner and outer surfaces of the solenoid are built-up by rotating plane Bezier curves $r = f_1(z)$ and $r = f_2(z)$ around the z-axis, respectively. The solenoid contains a large number of tightly wound turns of wire carrying current I, and then there is an ~~ea~~ffective current density $\boldsymbol{J}$ within the solenoid. In such a case, the actual solenoid can be replaced by a region carrying a constant current density $\boldsymbol{J}$ (the assumption becomes more accurate as the number of turns is increased). The considered arrangement is shown in Fig. 1.

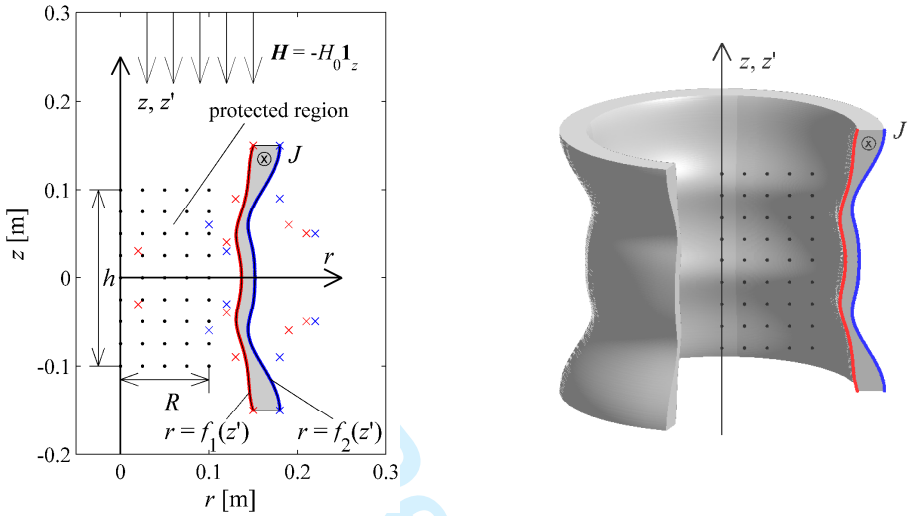

Figure 1. a) On the left - a definition sketch for an exemplary solenoid with protected region (black dots); x, x - f_1 and f_2 Bezier curves' control points; b) on the right - 3D visualization.

Magnetic field and magnetic potential vectors

In cylindrical coordinate system (r, φ, z) a current I flowing in an infinitely thin circular loop generates a magnetic field which can be described by a magnetic vector potential $\boldsymbol{A}$ (only the φ-component exists) (Shill 2003):

$$A_\varphi(r,z)=\frac{\mu_0 I}{\pi k_c}\left(\frac{r'}{r}\right)^{1/2}\left[\left(1-\frac{k_c^2}{2}\right)K(k_c)-E(k_c)\right], \tag{1}$$

where: $k_c^2=\dfrac{4r'r}{(r+r')^2+(z-z')^2}$, K and E are, the complete elliptic integral functions of the first and the second kind, respectively, I is the source current, μ_0 is the permeability of free space.

Here, primed variables z' and r' refer to source points within the solenoid, and the unprimed variables z and r denote field points in the protected region. The magnetic field components are related to the curl of the magnetic vector potential:

$$\boldsymbol{H}=\frac{1}{\mu_0}\mathrm{rot}\boldsymbol{A}=\frac{1}{\mu_0}\left(-\frac{\partial A_\varphi}{\partial z}\mathbf{1}_r+\frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)\mathbf{1}_z\right). \tag{2}$$

Substitution of (1) into (2) yields (Shill 2003):

$$H_r(r,z)=\frac{I}{2\pi r}\frac{(z-z')}{\left[(r+r')^2+(z-z')^2\right]^{1/2}}\left[-K(k_c)+\frac{r'^2+r^2+(z-z'^2)}{(r-r')^2+(z-z')^2}E(k_c)\right]. \tag{3}$$

$$H_z(r,z)=\frac{I}{2\pi\left[(r+r')^2+(z-z')^2\right]^{1/2}}\left[K(k_c)-\frac{r'^2-r^2+(z-z'^2)}{(r-r')^2+(z-z')^2}E(k_c)\right].$$

Objective function definition

In the protected region, the field generated by a solenoid must compensate the external magnetic field. It means that the magnetic field components must satisfy the following conditions:

$$\begin{cases} H_r=\dfrac{\partial A_\varphi}{\partial z}=0 \\ H_z=\dfrac{1}{\mu_0}\dfrac{1}{r}\dfrac{\partial}{\partial r}(rA_\varphi)=H_0 \end{cases}. \quad (4)$$

Using (4) the optimization problem can be formulated to find the "best" vector $\mathbf{x}$ = [Bezier curves' control points coordinates]^T which:

- minimizes the absolute value of the radial magnetic field component in the protected region: $|H_r(\mathbf{x})|$ = min,
- minimizes the absolute value of the difference between the axial magnetic field component and desired magnetic field: $|H_z(\mathbf{x}) - H_0|$ = min.

From (4) it is also evident that the magnetic vector potential must be a function of one variable r only, $A_\varphi = f(r)$. Taking this assumption into account the optimization problem (4) can be reformulated, and gives~~we obtain~~ the differential equation:

$$\frac{1}{\mu_0}\frac{1}{r}\frac{d}{dr}\left[r f(r)\right]=H_0, \quad (5)$$

with the solution for a desired magnetic vector potential A_φ:

$$A_\varphi = f(r)=\frac{\mu_0 H_0}{2}r, \quad (6)$$

where: H_0 is the~~-~~ desired magnetic field.

In the optimization process, the magnetic vector potential is calculated in discrete points of protected region (marked by black dots in Fig. 1). The used space grid is defined by the following vectors of r and z-coordinates (in cm):

$$\begin{aligned} r &= [10^{-5}\ \ 2.5\ \ 5\ \ 7.5\ \ 10], \\ z &= [-10\ \ -7.5\ \ -5.0\ \ 0\ \ 5.0\ \ 7.5\ \ 10]. \end{aligned} \quad (7)$$

In all these points desired and calculated magnetic vector potentials are determined. Next, the difference is obtained, and finally the maximal value is minimized. Therefore, the objective function $F(\mathbf{x})$ to be minimized is defined as follows:

$$F(\mathbf{x}) = \left| \max\left(A_{\text{desired, k}} - A_{\text{calculated, k}} \right) \right| = \min. \quad (8)$$

where: k relates to a discrete point in protected region, k = 1, ..., N, and N is the number of discrete points.

Bezier curves

In general the sought~~earched~~ optimal shape is absolutely unknown, so it can be of any arbitrary form. Bezier curves were publicized in 1962 by the French engineer Pierre Bézier, who used them to design automobile bodies. Bezier curves are present in almost all CAD software and have been widely used in computer graphics to model smooth curves. Such types of curves are flexible, simple to describe, efficient to calculate and easily controlled. Bezier curves of practically arbitrary shape can be defined entirely by providing few points, only (Internet). Therefore, in the author's opinion they can~~ould~~ be useful in topology and shape optimization problems.~~they have been used as a tool for generating the surfaces of the solenoid.~~

The method which is applied for finding the functions $f_1(z')$ and $f_2(z')$, couples the basic formula (7) with a Genetic Algorithm and Bezier curves. It is assumed that the inner and outer surfaces of the solenoid are generated by rotating ~~a~~ plane Bezier curves about the z-axis. A plane Bezier curve can be defined by the following equation (Piegl 1993):

$$\begin{Bmatrix} r(p) \\ z(p) \end{Bmatrix} = \sum_{i=0}^{n} \begin{Bmatrix} r_i \\ z_i \end{Bmatrix} B_{i,n}(p), \quad (9)$$

where: r_i, z_i are coordinates of Bezier curve control points $C_i(r_i, z_i)$, $B_{i,n}(p)$ are Bernstein polynomials and p belongs to the set [0, 1].

The first few Bernstein polynomials are:

$$B_{0,0}(p) = 1,\ B_{0,1}(p) = 1-p,\ B_{1,1}(p) = p,\ B_{0,2}(p) = (1-p)^2,\ B_{1,2}(p) = 2(1-p)p,$$
$$B_{2,2}(p) = p^2,\ B_{0,3}(p) = (1-p)^3,\ B_{1,3}(p) = 3(1-p)^2 p,\ B_{2,3}(p) = 3(1-p)p^2,$$
$$B_{3,3}(p) = p^3. \quad (10)$$

Coordinates r_i and z_i of the Bezier curve control points C_i are the design parameters. The optimization process is a determination of the parameters r_i and z_i, which ensure a minimum of the objective function, which is obtained from the desired magnetic potential values and the calculated ones in points of the protected region. It is known that in a uniform axi-symmetrical magnetic field only half of the solenoid has to be optimized. In this case, only the upper part of the solenoid is considered for optimization purposes, while the lower part of the solenoid is built taking into account the symmetry plane of the solenoid.

Global minimizer - Genetic Algorithm

The flowchart of the optimization procedure is shown in Fig. 2.

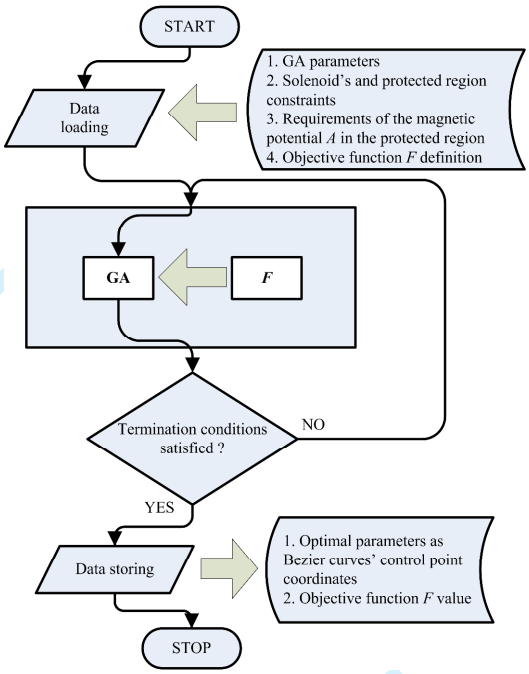

Figure 2. A flowchart of the optimization procedure based on genetic algorithms and Bezier curves.

A modified version of GA presented in (Haupt and Haupt, 2004) has been implemented in MATLAB. At the beginning of the optimization process, GA parameters, requirements of the magnetic potential vector, objective function F, and constraints of the solenoid and protected region are defined. Next, the initial population is randomly generated. Cost functions are calculated for all the candidates. A selection process then takes place~~. It is~~ based on roulette-wheel selection. In all the cases, the selection coefficient is equal to 0.5, which means that half of the population size goes to the crossover process. After the crossover process, mutation takes place. This corresponds to random changes in candidates. The aim of the mutation is to improve candidates, taking them into better solutions. The mutation coefficient should not be very high and in all the cases has been set to 0.3. Best candidates go to the next generation and become a new set of candidate solutions. This process repeats until it stops after reaching a maximum number of iterations.

There are two Bezier curves in the current optimization problem. It is assumed that each Bezier curve is built from six control points C_i (r_i, z_i), i = 0, 1, 2, 3, 4, 5. This~~It~~ means that the total number of optimized parameters is equal to 24. Possible values (the search space) of the parameters for the f_1 curve and f_2 curve are the same and have been shown in Table. 1. The population size was equal to 32, and the total number of generations was equal to 500.

Table 1. Search space (minimal and maximal values of all parameters under consideration)

Parameter		Minimal value [cm]	Maximal value [cm]
C_0	r_0	2	25
	z_0	7.5	15
C_1	r_1	2	25
	z_1	6	10.5
C_2	r_2	2	25
	z_2	4.5	9
C_3	r_3	2	25
	z_3	3	7.5
C_4	r_4	2	25
	z_4	1.5	6
C_5	r_5	2	25
	z_5	0	4.5

It is known that for bigger coils' diameter the field homogeneity increases, however, the total field intensity value decreases (taking into account constant current density). Additionally, for extremely large~~big~~ diameters the resulting~~ed~~ arrangement can be impractical.

For practical reasons, it is assumed that the optimal shape of the solenoid must be close to the protected region and as small as possible (~~i.e.~~*i.e.* the solenoid must be possible to build) and assure the generation magnetic field of high intensity values. In order to satisfy such conditions the maximal possible values of r_i and z_0 coordinates have been set to 25 cm and to 15 cm, respectively (Table 1).

Results

Figures 3-10 show ex~~a~~ample~~ary~~ results after five hundred generations of GA for the successive desired magnetic field H_0 equal to 35 kA/m, 45 kA/m 55 kA/m, 70 kA/m, respectively. In each case a constant current density J equal to 10 A/mm^2 was assumed (*e.g.* I = 10 A is the~~-~~ constant current in ~~one turn for~~ a wire of cross section equal to 1 mm^2). Figures 3, 5, 7 and 9 on the left, show the optimal shapes of the solenoids for each desired H_0, and on the right the objective function values F after succeeding generations of ~~A~~GA. Optimal positions of the Bezier curve control points are marked by blue and red cross. In figures 4, 6, 8 and 10 on the left, relative errors δ [%] for desired magnetic field are presented. The relative error δ is defined as follows:

$$\delta[\%] = \frac{|H_{\text{calculated}} - H_0|}{H_0} \cdot 100. \qquad (11)$$

Distribution for the magnetic field radial component H_r [kA/m] has been presented in the same figures, on the right.

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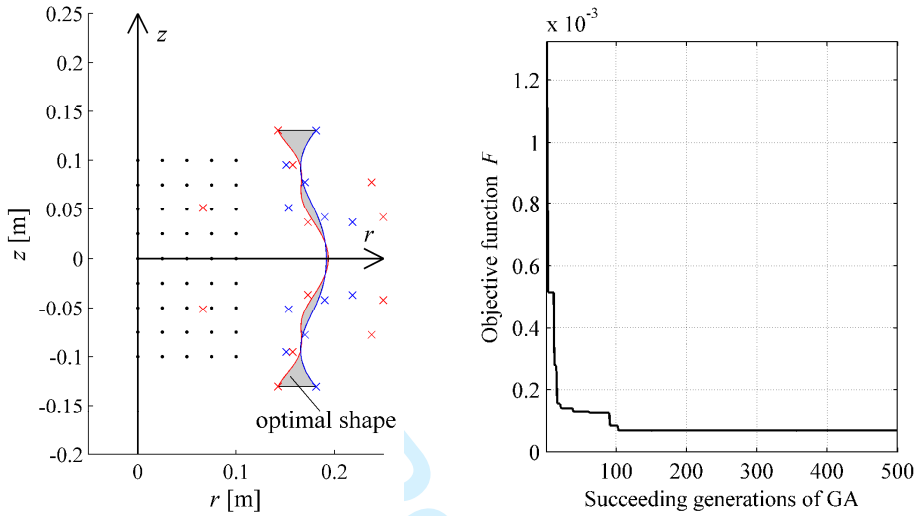

Figure 3. a) On the left - optimal shape of the solenoid for desired $H_0 = 35$ kA/m, $R = 10$ cm, $h = 20$ cm; b) on the right - objective function F values after succeeding generations of GA.

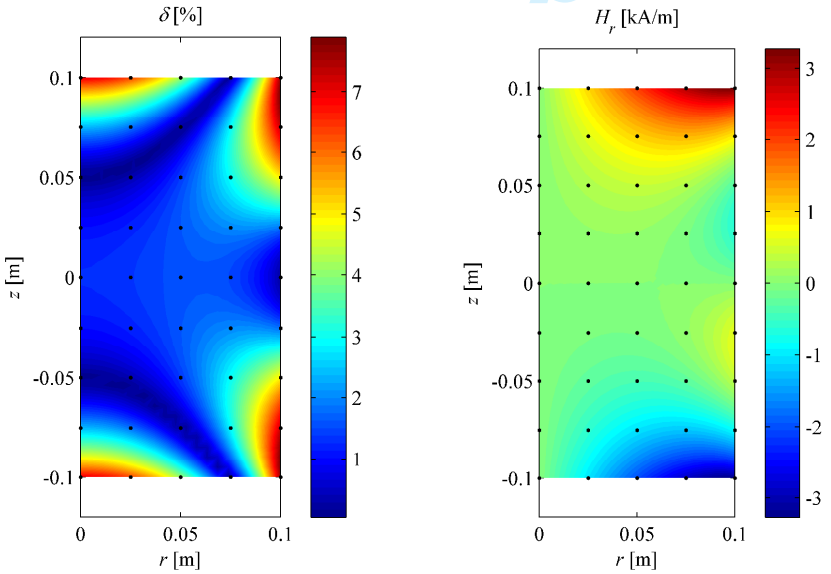

Figure 4. a) On the left - relative error δ [%] for desired ($H_0 = 35$ kA/m) and calculated magnetic field; b) on the right - distribution for the magnetic field radial component H_r [kA/m].

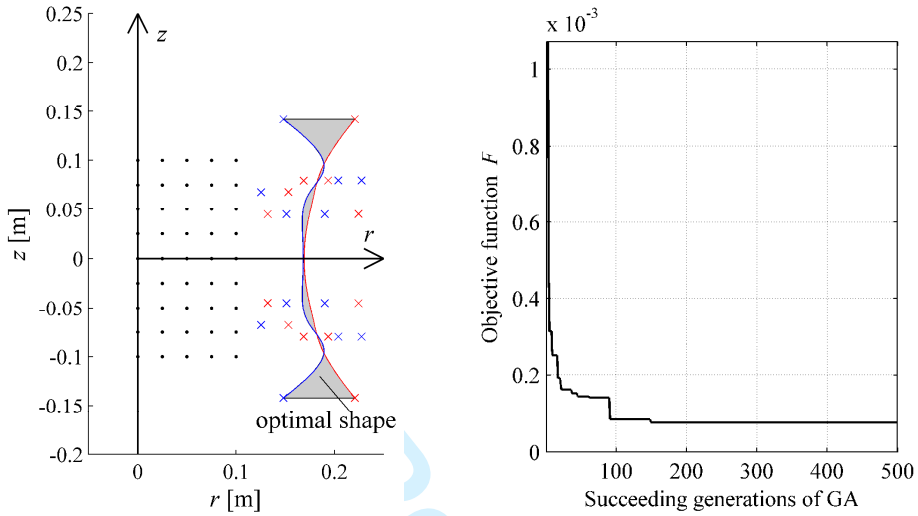

Figure 5. a) On the left - optimal shape of the solenoid for desired $H_0 = 45$ kA/m, $R = 10$ cm, $h = 20$ cm; b) on the right - objective function F values after succeeding generations of GA.

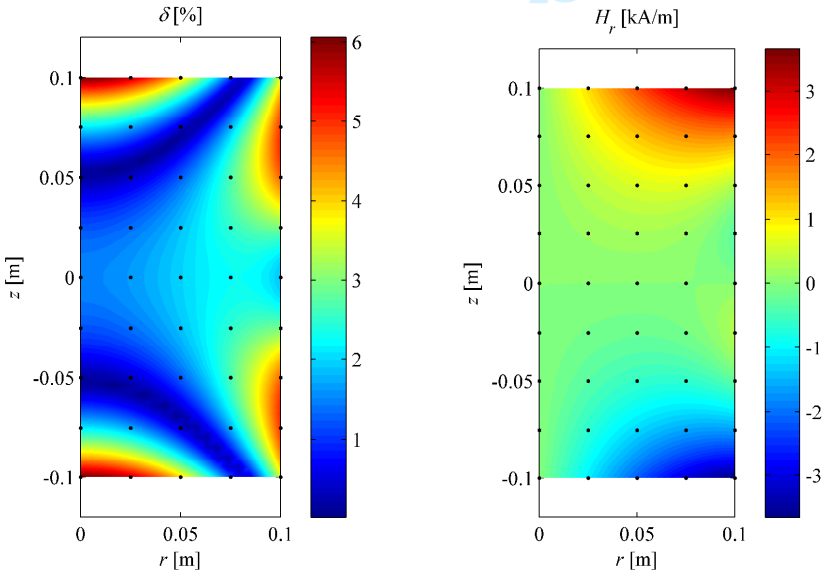

Figure 6. a) On the left - relative error δ [%] for desired ($H_0 = 45$ kA/m) and calculated magnetic field; b) on the right - distribution for the magnetic field radial component H_r [kA/m].

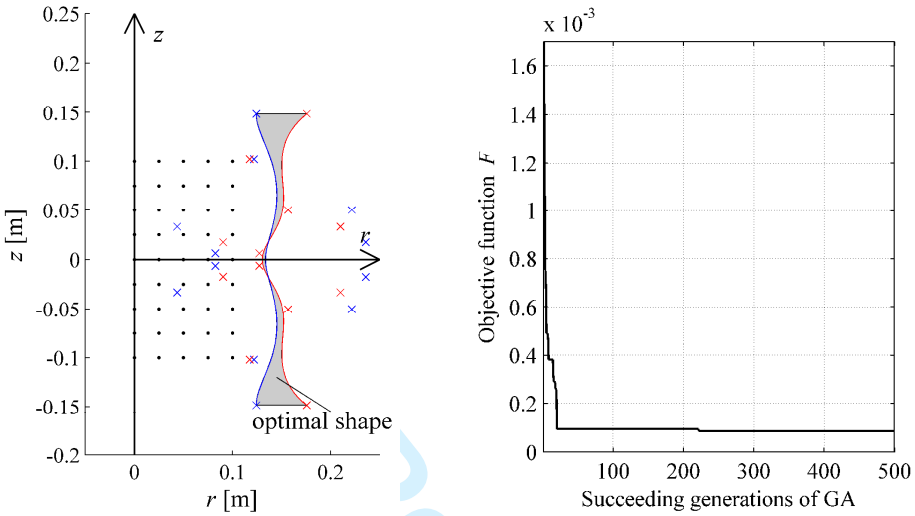

Figure 7. a) On the left - optimal shape of the solenoid for desired $H_0 = 55$ kA/m, $R = 10$ cm, $h = 20$ cm; b) on the right - objective function F values after succeeding generations of GA.

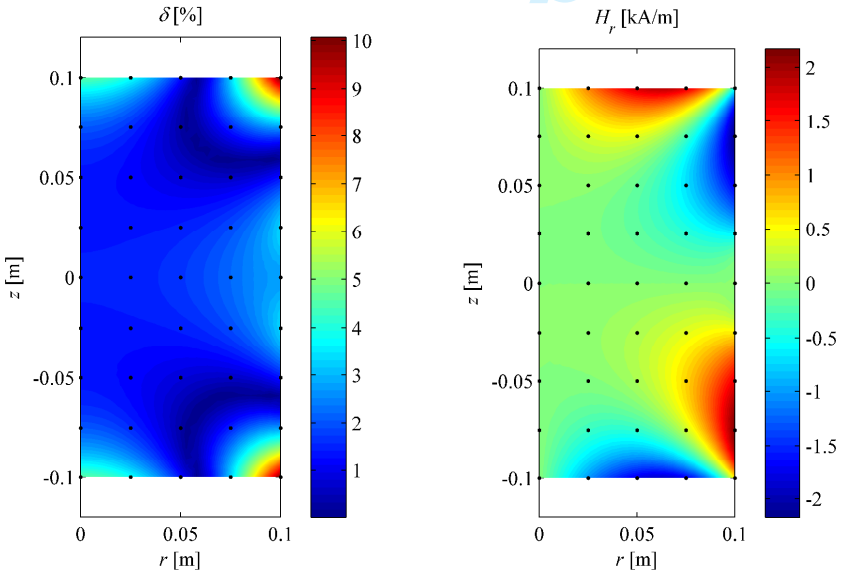

Figure 8. a) On the left - relative error δ [%] for desired ($H_0 = 55$ kA/m) and calculated magnetic field; b) on the right - distribution for the magnetic field radial component H_r [kA/m].

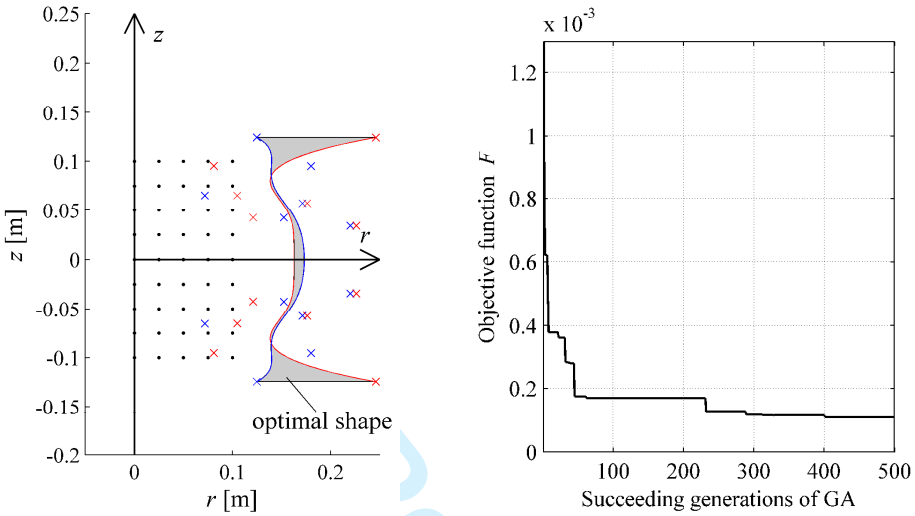

Figure 9. a) On the left - optimal shape of the solenoid for desired $H_0 = 70$ kA/m, $R = 10$ cm, $h = 20$ cm; b) on the right - objective function F values after succeeding generations of GA.

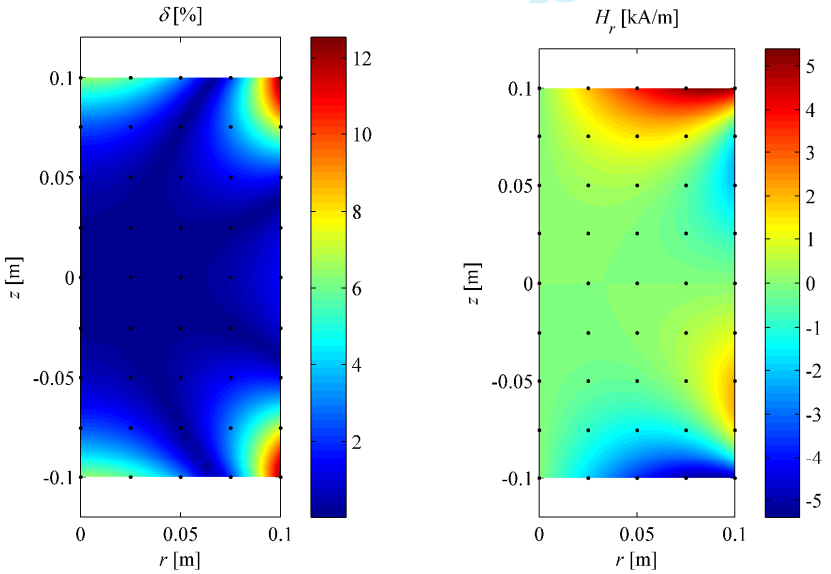

Figure 10. a) On the left - relative error δ [%] for desired ($H_0 = 70$ kA/m) and calculated magnetic field; b) on the right - distribution for the magnetic field radial component H_r [kA/m].

Mean values of relative errors δ (in the whole protected region as well as on the solenoid's axis) for each desired magnetic field value~~s~~ have been summarized in Table 2.

Table 2. Summarized results of optimization

Desired magnetic field Hz [kA/m]	Mean error δ [%] on solenoid's axis	Mean error δ [%] in whole protected region
35	1.85	2.22
45	1.79	2.07
55	1.87	1.77
70	1.63	1.60
85	2.14	2.38

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Manufacturing process of the solenoid

As has been statedtold already, the resultinged solenoids consist of wires of cross section equal to 1 mm^2 with current I = 10 A. Fig. 11 a) shows the optimal shape of the solenoid for desired H_0 = 45 kA/m, and Fig. 11 b) shows the magnified upper part of the same solenoid.

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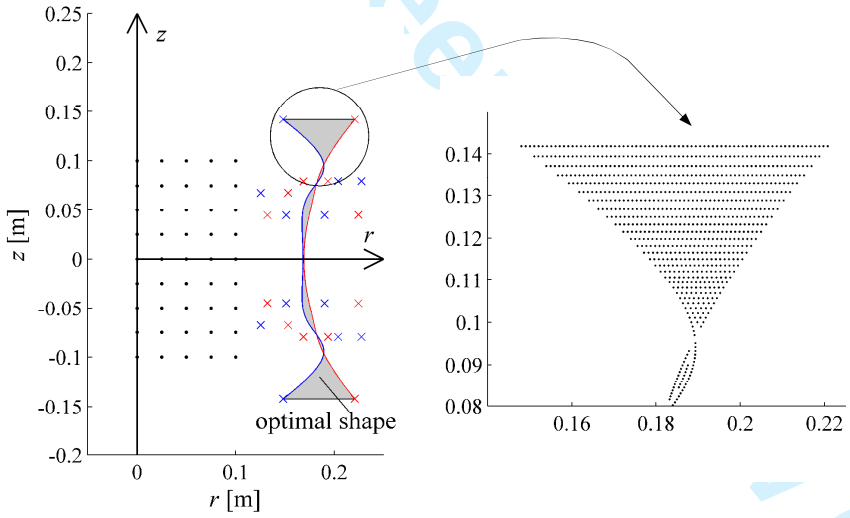

Figure 11. a) On the left - optimal shape of the solenoid for desired H_0 = 45 kA/m,; b) on the right - magnified upper part of the solenoid; black points represent middle of the wires with current equal to 10 A.

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Influence of manufacturing process's accuracy on relative error δ

TIn thisthis section it is examinesd how manufacturing process's accuracy influences on the relative error δ in the protected region. To perform the investigation the exaempleary shape obtained for desired magnetic field H_0 = 45 kA/m has been chosen (Fig. 11 a). In order to study such an effect the optimal values of the Bezier curve control points r-coordinates have been changed by η equal to 5 % of their optimal value. Two cases have been considered. In the first case the change was done randomly (Fig. 12 a), and in the second case all r-coordinates' values have been increased by η (Fig. 12 b).

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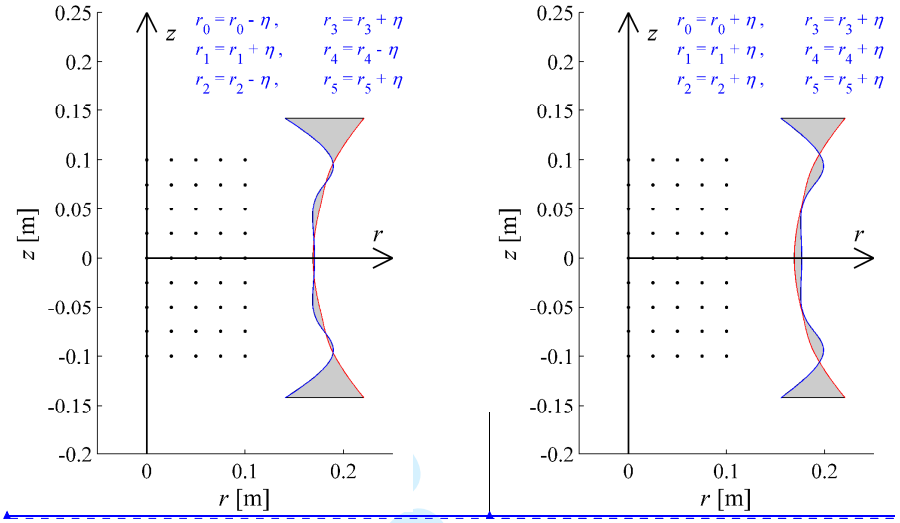

Figure 12. a) On the left - shape of the solenoid in which the optimal values of the Bezier curve control points r-coordinates have been changed randomly by η; b) on the right - shape of the solenoid in which all the optimal values of the Bezier curve control points r-coordinates have been increased by η.

The results of investigations have been summarized in Table 3.

Table 3. Influence of the manufacturing process's accuracy on the relative error δ

Relative error δ [%]	Optimal shape (Fig. 5a)	Exemplary shape (Fig. 11 a)	Exemplary shape (Fig. 11 b)
Minimal value	0.0964	0.2392	0.9744
Maximal value	6.0786	6.2925	11.856
Mean value	2.4393	2.5192	6.072
Median value	1.8684	2.0951	6.372

Conclusions

In this ~~paper~~article the synthesis of a~~the~~ uniform magnetic field in an~~the~~ axisymmetric finite three-dimensional region has been performed. The proposed optimization method utilizes genetic algorithm as a global minimizer and rotated Bezier curves as inner and outer surfaces of the solenoid. For given constraints optimal shapes of the solenoid are presented. The obtained solenoids can also shield external magnetic fields of the same magnitude but opposite direction.

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