Identification of an elasticity-tensor random field at mesoscopic scale using experimental measurements at mesoscopic and macroscopic scales for complex hierarchical microstructures

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Biomechanical materials are among the most complex materials to be modeled with respect to the complexity level of their constitutive materials at the micro-scale. For instance, it is the case of the cortical bone for which several scales must be defined. At a macro-scale, such a medium is often modeled as a homogeneous material for which the effective mechanical properties can be identified using experimental tests. At micro-scale, this material is not only non homogenous and random but it also cannot be described in terms of mechanical constituents. It is the reason why we consider a meso-scale for which the medium is modeled with apparent properties represented by an elasticity-tensor random field. A complete methodology is proposed for the experimental identification of the random field at meso-scale (1) using image field measurements at macro- and meso-scales, (2) introducing two associated boundary value problems for which the change of scale is performed in the context of stochastic homogenization and (3) using statistical inverse methods. A validation will be presented for cortical bone with experimental simulations which measure displacement (or strain) fields at the macro- and meso-scales for a given sample submitted to a given load. The experimental displacement (strain) field is measured on the whole domain (1x1 cm) at the macro-scale while, at the meso-scale, the displacement (strain) field is measured only on a representative elementary volume (1x1 mm). First, at the macro-scale, the experimental effective elasticity tensor is identified by solving an inverse problem for the sample. Second, the effective elasticity tensor at the macro-scale is computed using the stochastic model of the elasticity-tensor field at meso-scale. Finally, the displacement (strain) field is computed at meso-scale by solving a stochastic boundary value problem using the stochastic model of the elasticity-tensor field and for which the Dirichlet conditions correspond to the experimental displacement field measured on the boundary of a representative elementary volume. Then, the identification of the parameters of the stochastic model of the elasticity-tensor field at meso-scale is carried out by (1) maximizing the likelihood of random displacement (strain) field computed by the stochastic boundary value problem with the experimental displacement (strain) field at meso-scale and by (2) minimizing the distance between the experimental effective elasticity tensor and the

effective elasticity tensor of the material at macro-scale. For the numerical proposed application, the experimental database is computationally constructed.

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