



**HAL**  
open science

# High-dimension polynomial chaos expansions of effective constitutive equations for hyperelastic heterogeneous random microstructures

A. Clément, Christian Soize, Julien Yvonnet

## ► To cite this version:

A. Clément, Christian Soize, Julien Yvonnet. High-dimension polynomial chaos expansions of effective constitutive equations for hyperelastic heterogeneous random microstructures. Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2012), Vienna University of Technology, Sep 2012, Vienna, Austria. pp.1-2. hal-00734170

**HAL Id: hal-00734170**

**<https://hal.science/hal-00734170v1>**

Submitted on 20 Sep 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# High-dimension polynomial chaos expansions of effective constitutive equations for hyperelastic heterogeneous random microstructures

A. Clément<sup>†\*</sup>, C. Soize<sup>†</sup>, J. Yvonnet<sup>†</sup>

<sup>†</sup>Université Paris-Est, Laboratoire Modélisation et Simulation Multi Echelle, MSME UMR 8208  
CNRS

5 Bd Descartes, 77454 Marne-la-Vallée, France  
{alexandre.clement,christian.soize,julien.yvonnet}@univ-mlv.fr

**Keywords:** uncertainty quantification, computational stochastic mechanics, multiscale modeling, high dimension.

## ABSTRACT

In this research, we address the construction of random effective constitutive equations by nonlinear stochastic homogenization of hyperelastic heterogeneous materials for which a high number of random parameters is needed to characterize the uncertainties at the microscopic scale. We base our new approach on a non-concurrent multiscale method recently proposed for computing the homogenization of nonlinear heterogeneous materials [1] which still represents a difficult task. This technique, based on a numerical construction of the strain energy density function associated with a microstructure, allows this difficulty to be encountered in a deterministic framework. However, in order to obtain an efficient mechanical model, one must take into account the different sources of uncertainties related to the material at the microscopic level.

In [2] we have proposed to extend this method to the stochastic framework for microstructures characterized by random geometrical parameters. The first step of the so-called Stochastic Numerical EXplicit Potential method (S-NEXP method) consists in building a stochastic model associated with a random generator and then performing a statistic reduction of the stochastic field through a principal component analysis. The second step consists in numerically calculating the potential with a classical FEM method for a set of points belonging to the space spanned by the reduced variables and the macroscopic strains. The last step consists in using this database in order to obtain an explicit expression of the potential in terms of the reduced random variables and the macroscopic strains with a separated variables representation associated with an interpolation scheme. This approach appears to be efficient but its use in high dimension (*i.e.* when the number of reduced random variables is high) seems quite infeasible since the number of nonlinear FEM computations needed to compute the random potential becomes very high and involves drastic computational times and memory storage.

We thus propose a reformulation of the problem into an identification problem of polynomial chaos expansions. The steps of the methodology are the following: (1) definition of a random generator to obtain a set of realizations of the geometry of the microstructure, (2) introduction of a discretization of the bounded space spanned by the macroscopic strains, (3) computation of the strain energy density function according to the prescribed macroscopic strains for each realization of the geometry of the microstructure, (4) determination of a reduced-order model of the discretized potential through a principal component analysis, (5) identification of the reduced random variables on a high-dimension polynomial chaos expansions, (6) use of the separated variables representation associated to an interpolation scheme in order to obtain a continuous explicit form of the potential with respect to the macroscopic strains. Since a high number of deterministic coefficients are involved in the polynomial chaos expansions used

in step (5), we use and adapt the methodology introduced in [3] where new algorithms have been proposed to circumvent this issue. As it is shown on figure 1, the issue of the identification of multi-modal random variables through polynomial chaos expansions is also addressed.

Numerical examples allow analyzing the efficiency of the proposed technique. Solutions at both microscopic and macroscopic scales (see figure 1) are compared to the ones obtained with classical methods.

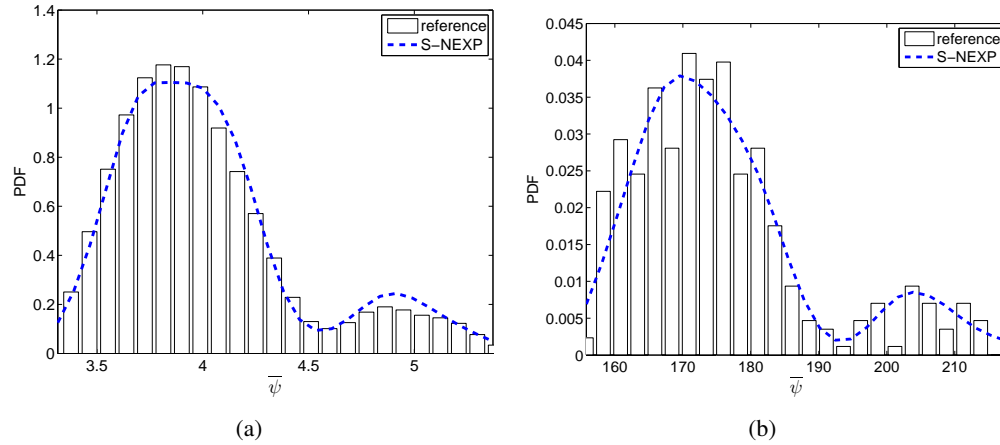


Figure 1: Plot of PDFs of the the strain energy density functions for two different strain states: comparisons between the proposed approach solutions (S-NEXP) and the reference solutions.

## References

- [1] J. Yvonnet, Q.C. He: A non-concurrent multiscale method for computing the response of nonlinear elastic heterogeneous structures. *European Journal of Computational Mechanics*, 19 (2010) 105–166.
- [2] A. Clément, C. Soize, J. Yvonnet: Computational nonlinear stochastic homogenization using a non-concurrent multiscale approach for hyperelastic heterogeneous microstructures analysis. *International Journal for Numerical Methods in Engineering*, accepted with minor revisions.
- [3] C. Soize: Identification of high-dimensional polynomial chaos expansions with random coefficients for non-Gaussian tensor-valued random fields using partial and limited experimental data. *Computer Methods in Applied Mechanics and Engineering*, 199 (2010), 2150–2164.