Uncertainty quantification for post-buckling analysis of cylindrical shells with experimental comparisons

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1 Introduction

The present work concerns the experimental identif cation of an uncertain nonlinear computational model in the context of the post-buckling analysis of a cylindrical shell. It proposes an alternative approach to existing methodologies for which only system parameter uncertainties are modeled [5]. This methodology is adapted to the analysis of large static deformations of geometrically nonlinear structural systems in the presence of both system parameters uncertainties and model uncertainties. The available experimental data is made up of the nonlinear static def ection of a cylindrical shell. First, the deterministic nonlinear computational model is constructed using the f nite element method and the corresponding nonlinear response is used as a reference deterministic solution for which a reduced-order basis is deduced using the POD (Proper Orthogonal Decomposition) analysis. The mean reduced-order nonlinear computational model is then explicitly constructed in the context of three-dimensional solid f nite elements. Moreover, a positive-def nite operator related to the nonlinear stiffness of the structure is def ned, allowing the use of the nonparametric probabilistic methodology for constructing the uncertain nonlinear reduced-order computational model. Finally, the experimental identif cation of the uncertain nonlinear computational model is carried out in order to validate the proposed methodology.

2 Methodology of analysis

The structure under consideration is a cylindrical shell for which an experimental characterization of its post-buckling response has been conducted [2]. First, the experimental description of the cylindrical shell allows a deterministic computational f nite element model to be constructed with the f nite element method and the post-buckling deterministic nonlinear response, solution of

$$[K^{(1)}]\,\mathbf{u} + \mathbf{f}^{NL}(\mathbf{u}) \;=\; \mathbf{f} \quad , \tag{1}$$

is calculated as a reference solution.The projection basis $\boldsymbol{\varphi}^{\alpha}$, $\alpha = \{1, \ldots , N\}$, required for the construction of the mean reduced-order nonlinear computational model is deduced from this reference calculation using the Proper Orthogonal Decomposition (POD) method which is known to be particularly eff cient in nonlinear static cases [4]. Decomposing the reference solution on this POD basis yields the following set of nonlinear equations

$$\mathcal{K}^{(1)}_{\alpha\beta}\, q_\beta \,+\, \frac{1}{2}\Big(\hat{\mathcal{K}}^{(2)}_{\alpha\beta\gamma} \,+\, \hat{\mathcal{K}}^{(2)}_{\beta\gamma\alpha} \,+\, \hat{\mathcal{K}}^{(2)}_{\gamma\alpha\beta}\Big)\, q_\beta\, q_\gamma \,+\, \mathcal{K}^{(3)}_{\alpha\beta\gamma\delta}\, q_\beta\, q_\gamma\, q_\delta \,=\, \mathcal{F}_\alpha \quad , \tag{2}$$

in which the expressions of $\mathcal{K}^{(1)}_{\alpha\beta}$, $\hat{\mathcal{K}}^{(2)}_{\alpha\beta\gamma}$ and $\mathcal{K}^{(3)}_{\alpha\beta\gamma\delta}$ and its symmetry properties can be found in [3]. Using the f nite element method, the elementary contributions of the linear, quadratic and cubic internal forces projected on this POD basis are calculated for each f nite element before assemblage. The mean reduced-order nonlinear computational model described by Eq.(2) is then explicitly obtained as shown in [1]. From the knowledge of these terms and using a reshaping operation, a positive-def nite stiffness operator is constructed [3], allowing the nonparametric probabilistic model of uncertainties to be used for modeling both system parameter uncertainties and model uncertainties. The random response is then obtained by solving the random equation

$$\boldsymbol{\mathcal{K}}^{(1)}_{\alpha\beta}\, \mathbf{Q}_\beta + \frac{1}{2}\Big(\hat{\boldsymbol{\mathcal{K}}}^{(2)}_{\alpha\beta\gamma} + \hat{\boldsymbol{\mathcal{K}}}^{(2)}_{\beta\gamma\alpha} + \hat{\boldsymbol{\mathcal{K}}}^{(2)}_{\gamma\alpha\beta}\Big)\, \mathbf{Q}_\beta\, \mathbf{Q}_\gamma + \boldsymbol{\mathcal{K}}^{(3)}_{\alpha\beta\gamma\delta}\, \mathbf{Q}_\beta\, \mathbf{Q}_\gamma\, \mathbf{Q}_\delta \,=\, \mathcal{F}_\alpha \quad , \text{with } \mathbf{U} \,=\, [\Phi]\, \mathbf{Q} \quad . \tag{3}$$

3 Experimental validation

The experimental validation consists in formulating the identif cation of the uncertain nonlinear computational model from the experimental post-buckling nonlinear response. The random post-buckling response is considered as a function of one scalar dispersion parameter δ controlling the uncertainty level in the random stiffness operator. The experimental identif cation requires to solve an optimization problem. The cost function to be minimized contains penalty terms only in the areas for which the experimental data is not within the conf dence region described by the uncertain nonlinear computational model. Despite of the sensitivity of the post-buckling behaviour of the cylindrical shell, a good agreement between the experimental response and between the nonlinear uncertain model is observed.

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