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# Modeling of PM Synchronous Machines Under Inter-turn Fault

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**Abstract** — This paper presents a theoretical approach as far as faulty Permanent Magnet Synchronous Machines (PMSM) are concerned. The knowledge of PMSM's behavior under inter-turn fault is very useful in order to check drive performances under fault conditions. Control, stability and fault detection often require faulty models. The aim of this paper is to develop a dynamical model of a PMSM under fault conditions without using circuit models coupled with finite element analysis (FEA). The method exposed in this paper only needs some preliminary FE computations that can be used in order to take into account of different winding distributions.

## I. INTRODUCTION

One way to study PMSM under inter-turn short-circuits is to build a improved model. Many authors have developed PMSM models including electrical faults [1],[2] or mechanical faults [3],[4]. Such models often require strong hypotheses. Core saturation, leakage inductances are often neglected and winding arrangements are simplified [5]-[9]. This leads to a lack of accuracy on results. The results can be strongly improved by using time-stepping finite element method (TSFEM). However, this approach needs to perform a new simulation for every fault configuration and requires too much CPU time. In addition, TSFEM cannot be easily coupled with a faulty controlled model implemented under MATLAB/SIMULINK<sup>®</sup> environment. It is also not very appropriate for parametric studies [3],[6],[10].

The presented work concerns non-salient PMSM. The authors present a weak coupled model using FEA to determine elementary inductances of the different coils. An appropriate method gives the right self and mutual inductances of the equivalent external circuit of the faulty machine. Then, thanks to a precise identification of the various parameters (inductances, no-load back-EMF), the study of the behavior of the faulty machine can be easily performed whatever the fault configuration is.

## II. MODEL OF PM MOTOR UNDER FAULT CONDITIONS

We assume that saturation of iron materials is negligible. In addition, only the non-salient PM machines are considered. Under these hypotheses, the following model can be adopted.

Consider a 3-phases PM motor with inter-turn short circuit. The main idea consists in considering an additional circuit in the model as shown on Fig. 1. Each phase is composed of series connected coils which need to be properly modeled [6]. The electrical equation of such model can be written of the form:

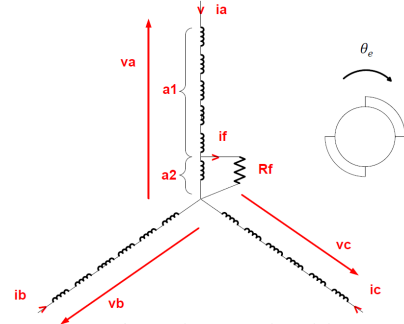


Fig. 1. The external model

$$v_{abcf} = R_{abcf} \cdot i_{abcf} + L_{abcf} \cdot \frac{di_{abcf}}{dt} + e_{abcf} \quad (1)$$

$$\begin{cases} i_{abcf} = [i_a & i_b & i_c & i_f]^T \\ v_{abcf} = [v_a & v_b & v_c & v_f]^T \\ e_{abcf} = [e_a & e_b & e_c & e_f]^T \end{cases} \quad (2)$$

where,  $R_{abcf}$  and  $L_{abcf}$  can be expressed as follows:

$$R_{abcf} = \begin{bmatrix} R_s & 0 & 0 & -R_{a2} \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_s & 0 \\ -R_{a2} & 0 & 0 & (R_{a2} + R_f) \end{bmatrix} \quad (3)$$

$$L_{abcf} = \begin{bmatrix} L_c & 0 & 0 & -(L_{a2} + M_{a1a2}) \\ 0 & L_c & 0 & -M_{a2b} \\ 0 & 0 & L_c & -M_{a2c} \\ -(L_{a2} + M_{a1a2}) & -M_{a2b} & -M_{a2c} & L_{a2} \end{bmatrix} \quad (4)$$

$R_s$  and  $L_c = L - M$  are respectively the phase resistance and the self inductance of the healthy machine.  $R_{a2}$  and  $L_{a2}$  are the resistance and the self inductance of the faulty winding  $a_2$  and  $e_f$  is its no-load EMF.  $M_{a1a2}$ ,  $M_{a2b}$  and  $M_{a2c}$  are respectively the mutual inductances between the winding  $a_2$  and the windings  $a_1$ ,  $b$  and  $c$ .  $i_f$  and  $R_f$  are the fault current and the fault resistance (Fig. 1.).  $\theta_e$  is the electrical position of the rotor. It has been shown that the parameters of the different parts affected by the short-circuit are directly linked to the ratio  $\mu = N_f/N_s$  between the number of shorted turns and the number of total turns of one phase [7],[8]. This assumption is exact in the case of windings with one slot per pole and per phase and supposing one pole pair machines. In fact, the previous equations (3), (4) depend on the winding arrangement. In the case of non conventional windings or windings with more than one slot per pole and per phase, a more rigorous approach need to be adopted. The aim of this paper is to develop a general method based on FEA and winding

arrangement modeling which leads to determine the parameters of the faulty model for various fault configurations.

### III. INDUCTANCE MATRIX CALCULATION

The main idea presented concerns the use of a minimum number of FE simulations in order to obtain the inductance matrix of the equivalent scheme of a PM motor.

#### A. Healthy conditions

A  $2p$ -poles,  $N_e$ -slots PM non-salient machine is considered with  $N_c$  turns per coil in the case of a concentric winding arrangement. Fig. 2. shows the example of a 18-slots 16- poles PMSM. Field calculation is performed with linear FEM. If only one coil  $k$  is supplied by the current  $i_k$ , the fluxes  $\phi_i$  in the different coils can be determined by:

$$\phi_i = \frac{L_z}{S_c} \cdot \left[ \iint_{S_+} A. ds - \iint_{S_-} A. ds \right] \quad i = 1, \dots, N_e \quad (5)$$

where  $L_z$  is the axial length of the machine,  $A$  is the potential vector,  $S_c$  is the cross section area of the coil.  $S_+$  and  $S_-$  denote the go and return areas of the coil. The computation the various inductances are performed under the assumptions presented in section II. The fluxes of the different coils obtained with the supply currents  $i_k$  allow us to determine the inductance  $L_k$  of the coil  $k$  and the different mutual inductances  $M_{k,i}$ . Self and mutual inductances can be divided in two parts:  $L_k, M_{k,i}$  and  $L_\sigma, M_\sigma$  matching respectively with common flux  $\phi_c$  and leakage flux  $\phi_l$  as shown on Fig.2. Even if  $\phi_l$  does not cross the air-gap, it contributes to the mutual inductance between the two coils. This is specific to concentric windings. Hence, assuming that the leakage flux does not depend on current level, the energy  $W$  in a whole slot with currents  $i_1$  and  $i_2$  is given by:

$$W = 1/2 \cdot L_\sigma \cdot i_1^2 + 1/2 \cdot L_\sigma \cdot i_2^2 + M_\sigma \cdot i_1 \cdot i_2 \quad (6)$$

Consequently,  $L_\sigma$  and  $M_\sigma$  can be easily determined using  $W$ ,  $i_1$  and  $i_2$ . In the case of concentric windings and considering the symmetry of the machine, the previous calculation is performed once and permutation technique leads to obtain the inductance matrix of the healthy system of 18 independent coils:

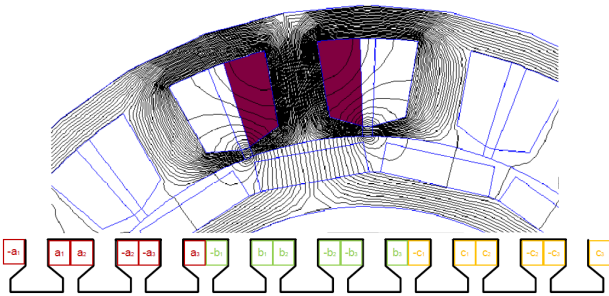


Fig. 2. Considered PM motor and the winding arrangement for a half machine

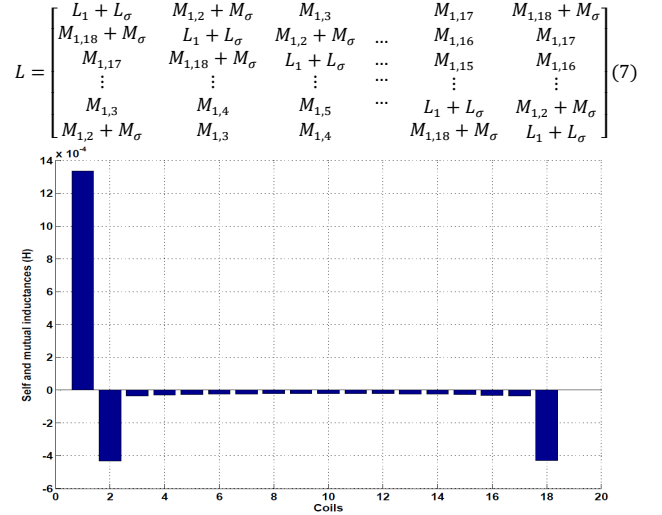


Fig. 3. First row of the inductance matrix (7)

$L$  is a symmetrical and circulant matrix. Its first row representation is given on Fig.3. The shape of this matrix depends on the coils arrangement. It is useful in order to define the various parameters of the PMSM under fault conditions which will be done in paragraph III.C. Investigations about the effect of the winding arrangement are presented in the next paragraph.

#### B. Windings arrangements

The approach presented above can be performed to obtain the inductance matrix of both single or double layer distributed windings for healthy machines. Here, we present the case of a double layer winding assuming that the same approach can be done for single layer arrangement.

We consider a 48-slots 8-poles PMSM with a double layer winding including shortened coils with a coil pitch 5/6 as shown on Fig.6. The whole inductance matrix  $L$  is a symmetric and circulant matrix. As each slot contains the wires of two different coils, each coil has two leakage mutual inductances with two other coils sharing the same slot. On Fig.4, we can see that coil 1 has two leakage mutual terms with coils 6 and 44. This can be clearly seen on Fig.5 which shows the representation of the first row of the inductance matrix (8). The location of the mutual leakage terms ( $M_\sigma$ ) on a row is depending on the coil pitch. The proposed approach can also be applied for other more complicated winding arrangements such as fractional arrangements.

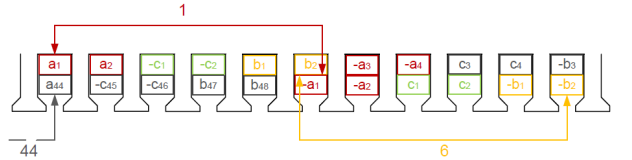


Fig. 4. Double layer winding arrangement for one pole

$$L = \begin{bmatrix} L_1 + L_\sigma & M_{1,2} & M_{1,6} + M_\sigma & M_{1,44} + M_\sigma & M_{1,48} \\ M_{1,48} & L_1 + L_\sigma & \dots & M_{1,2} & \dots & M_{1,46} & \dots & M_{1,47} \\ M_{1,17} & M_{1,18} & \dots & L_1 + L_\sigma & \dots & M_{1,45} & \dots & M_{1,46} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots \\ M_{1,3} & M_{1,4} & \dots & M_{1,5} & \dots & L_1 + L_\sigma & \dots & M_{1,2} \\ M_{1,2} & M_{1,3} & \dots & M_{1,4} + M_\sigma & \dots & M_{1,44} + M_\sigma & \dots & L_1 + L_\sigma \end{bmatrix} \quad (8)$$

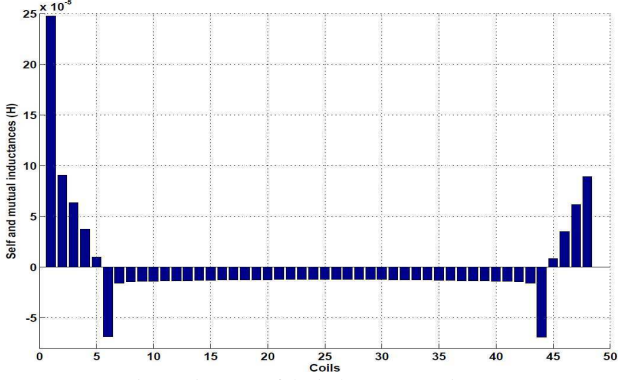


Fig. 5. First row of the inductance matrix (8)

In all cases, the inductance matrix of the 3-phase winding can be obtained according to the winding arrangement:

$$L_{abc} = C_{3,N_e}^T \cdot L \cdot C_{3,N_e} \quad (9)$$

where  $C_{3,N_e}$  is the winding matrix.

### C. Parameters identification under fault conditions

We consider the case of the 18-slots 16- poles PMSM. We assume that  $N_f$  turns of coil 1 are short-circuited. The ratio  $\mu$  becomes  $\mu' = N_f/N_c$ . Using local considerations, we will modify the expression of the healthy inductance matrix (7) in order to obtain a formulation corresponding to the faulty case. Inside the slot which contains the coil 1 and 2 for instance, the short-circuit leads to consider the configuration shown on Fig.8 where  $i_1'' = i_1 - i_f$  and  $i_1' = i_1$  according to Fig.1. We can determine the various leakage inductances by using energy formulation. The whole energy in the slot is given by:

$$W = 1/2 \cdot L_{\sigma 1'} \cdot i_1'^2 + 1/2 \cdot L_{\sigma 2} \cdot i_2^2 + 1/2 \cdot L_{\sigma 1''} \cdot i_1''^2 + (M_{\sigma 1',2} \cdot i_1' \cdot i_2 + M_{\sigma 2,1''} \cdot i_2 \cdot i_1'' + M_{\sigma 1'',1'} \cdot i_1'' \cdot i_1') \quad (10)$$

Using this equation, we can deduce the value of the leakage and mutual inductance terms for a given faulty case.

▪ Additional row and column are inserted in the whole inductance matrix corresponding to the faulty coil. The self and mutual inductances  $L_1, M_{1,i}$  ( $i = 2, \dots, N_e$ ) are affected by  $\mu'$ :

$$L_1' = (1 - \mu')^2 \cdot L_1 ; M_{1,i}' = (1 - \mu') \cdot M_{1,i} \quad (11)$$

The self and mutual inductances of the additional circuit can be given by:

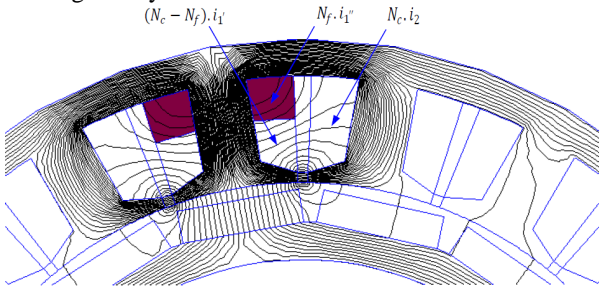


Fig. 6. Faulty case considered

$$\begin{aligned} L_{N_e+1} &= \mu'^2 \cdot L_1 ; M_{N_e+1,i} = \mu' \cdot M_{1i} \\ M_{1,N_e+1} &= \mu' \cdot (1 - \mu') \cdot M_{1i} \end{aligned} \quad (12)$$

In addition to the self and mutual inductances corresponding to the common flux  $\varphi_c$ , the self and mutual inductances corresponding to the leakage flux  $\varphi_l$  have to be properly identified using (10). The leakage inductances  $L_{\sigma 1'}$  and  $L_{\sigma 1''}$  are added in the diagonal terms of the new matrix whereas the mutual leakage inductances  $M_{\sigma 1',1''}, M_{\sigma 1,2}$  and  $M_{\sigma 1'',2}$  are added in the non-diagonal terms. The final matrix (13) for the faulty machine is defined as:

$$L = \begin{bmatrix} (1 - \mu')^2 \cdot L_1 + L_{\sigma 1'} & (1 - \mu') \cdot M_{1,2} + M_{\sigma 1',2} & (1 - \mu') \cdot M_{1,18} + M_{\sigma 1',2} \mu' & (1 - \mu') \cdot L_1 + M_{\sigma 1,1''} \\ (1 - \mu') \cdot M_{1,18} + M_{\sigma 1',2} & L_1 & M_{1,17} & \mu' \cdot M_{1,18} + M_{\sigma 1'',2} \\ (1 - \mu') \cdot M_{1,17} & M_{1,18} & \dots & M_{1,16} & \mu' \cdot M_{1,17} \\ \vdots & \vdots & \dots & \vdots & \vdots \\ (1 - \mu') \cdot M_{1,3} & M_{1,4} & \dots & M_{1,2} + M_{\sigma} & \mu' \cdot M_{1,3} \\ (1 - \mu') \cdot M_{1,2} + M_{\sigma 1',2} & M_{1,3} & \dots & L_1 + L_{\sigma} & \mu' \cdot M_{1,2} + M_{\sigma 1'',2} \\ \mu' \cdot (1 - \mu') \cdot L_1 + M_{\sigma 1,3''} & \mu' \cdot M_{1,2} + M_{\sigma 1'',2} & \dots & \mu' \cdot M_{1,18} + M_{\sigma 1'',2} & \mu'^2 \cdot L_1 + L_{\sigma 1''} \end{bmatrix}$$

By this way, an inter-turn fault leads to modify the healthy matrix (7) of order  $N_e$  into a faulty matrix (13) of order  $N_e + 1$ . The same method can be applied on the double layer presented winding by modifying (8).

▪ Finally, using the a modified winding matrix  $C_{4,19}$ , we can easily identify the various terms of the inductance matrix  $L_{abcf}$  of the faulty equivalent model with the following relation:

$$L_{abcf} = C_{4,19}^T \cdot L \cdot C_{4,19} \quad (14)$$

▪ Other parameters such as  $R_{abcf}$  and  $e_f$  also need to be identified in order to get a complete model.  $R_{abcf}$  can be determined assuming that resistive effects are directly linked to the number of turns. The no-load back-EMF  $e_f$  is also supposed to be proportional to the number of shorted turns. For the considered  $N_f$  turns in fault, we have:

$$e_f = N_f \cdot p \cdot \Omega \cdot \frac{d[\Phi_{rot}(\theta_e)]}{d\theta_e} \quad (15)$$

where  $\Phi_{rot}$  is the magnet flux through each turns of the considered coil,  $\theta_e$  is the electrical rotor position with respect to the faulty coil,  $p$  the number of poles pairs,  $\Omega$  the mechanical speed. Spatial harmonics can also be included.

## IV. COMPARISONS AND DISCUSSIONS

The parameters identification presented above can be validated with a full finite element computation. It will be also compared to another approach presented in [7],[8]. Then, the whole developed model has to be tested on the 18-slots 16-poles PMSM.

### A. Parameters Comparisons

Fig.7 shows the comparison between the results obtained by the proposed approach and a full FEA. They are also compared with another approach which does not

consider the exact winding topology and using the following parameters [7],[8]:

$$\begin{cases} L_{a2} \approx \mu^2 \cdot L \\ M_{a1a2} \approx \mu \cdot (1 - \mu) \cdot M \\ M_{a2c} \approx M_{a2b} \approx \mu \cdot M \end{cases} \quad (16)$$

According to the definition given above,  $\mu'$  and  $\mu$  are related by:

$$\mu = \mu' \cdot N_s / N_c \quad (17)$$

where  $N_s/N_c$  is the number of coils per phase which is equal to 6 in the considered machine. The various computations are performed for different values of the ratio  $\mu'$  on coil 1. A good agreement is observed between the proposed approach and the FEA method whereas formulas in (16) seem to give approximate results. As mentioned in section II, these results concerns some restricted cases. Nevertheless, in order to be more precise on armature reaction and saturation effects, the real dynamic inductances must be considered.

### B. Dynamic simulations

The model presented in section II and the parameters identified in III allow to perform the dynamic simulation of faulty operations. We consider that half of the turns of coil 1 are short-circuited ( $\mu' = 0.5$ ) by resistance  $R_f \ll 1\Omega$ . The PMSM is supplied by a Voltage Source Inverter (VSI) and controlled using a speed scheme at  $\Omega = 1000 \text{ Rpm}$ . Fig.6 shows the current waveforms using parameters obtained the proposed method (set 1) and those calculated by (16) (set 2). The phase currents change with the parameter set. The fault current calculated with set 2 is higher than the other. Due to the lower value of the inductances obtained by (16), fault current also contains non negligible high frequency components in this case. Set 2 lead to higher phase currents in order to maintain the speed reference with the considered fault. In fact, results given by set 2 can only be correct where  $R_{a2} \approx L_{a2} \cdot \omega$ . Consequently, we can state that the approximate parameters give wrong estimations of the variables in dynamic conditions. Moreover, conclusions that can be taken with set 2 are biased and can lead to "missed fault" or "false alarms" issues.

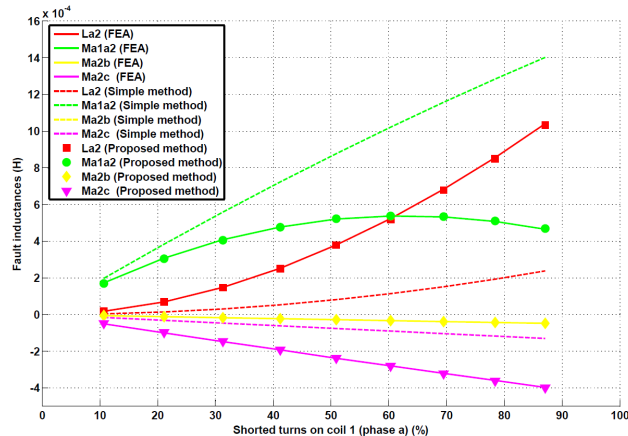


Fig. 7. Fault inductances: comparison between FEA, proposed method and other approaches

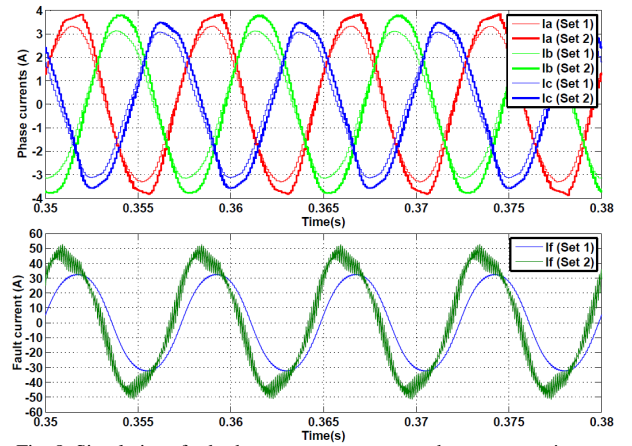


Fig. 8. Simulations for both parameters sets: top: phase currents  $i_{abc}$  - bottom: fault currents  $i_f$

## V. CONCLUSION

The method proposed in this paper provides a new way to simulate PMSM under inter-turn faults. Parameters are properly identified for various fault configurations and the approach can cover any type of windings arrangements. This prevent to consider wrong estimations of the variables regarding dynamic results. Compared to TSFEM, it does not require a large CPU time and the accuracy of the results is not affected. Moreover, thanks to the theoretical expressions, parametric studies can be easily performed.

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