Coordinated Path following Control of Multiple Nonholonomic Vehicles

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***Abstract*— This paper addresses the problem of coordinated path following control of multiple nonholonomic vehicles. The control laws are derived based on the leader-follower strategy, driving unicyle-type nonholonomic vehicles at kinematic level onto predefined parallel paths, while keeping an in-line formation. Due to the spatial-temporal decoupling characteristics of individual path following controller, the velocity of the follower can be adapted only based on the information of generalized along-path length from the leader, which keeps the inter-vehicle communication to a minimum. Simulation results illustrate the efficacy of the solution to coordinated control proposed here. Moreover, the theoretical analysis in this paper reveals some important issues raising that the path following control on the first-order unicyle-type nonholonomic systems can be extended to underactuated AUVs in future work.**

I. Introduction

The problems of motion control of an autonomous underwater vehicle (AUV) can be classified in three categories: point stabilization [1], trajectory tracking [2] and path following [3]. The objective of point stabilization is to position the vehicle at a fixed location with desired orientation. Trajectory tracking is required to enable the vehicle tracking a time parameterized reference path. Path following drives the vehicle to converge to and follow a desired spatial path, without strict temporal specifications. The underlying assumption of path following control is that the vehicle's forward speed tracks a desired speed profile, while the controller acts on the vehicle orientation to drive it onto the path. Typically, smoother convergence to a path is achieved when path following strategies are used instead of trajectory tracking control laws, and the control signals are less likely pushed to saturation [4]. Benefited from these advantages, path following control has received more attentions than the other two problems.

In addition, as a group of cooperative multiple AUVs dealing with tasks typically provides flexibility, robustness and efficiency beyond what is possible with single AUV, there is currently considerable interest in the problem of coordinated motion control of a team of autonomous vehicles. One envisioned task is utilizing a formation of AUVs to construct 3D images of an underwater oil pipeline or even to take a snapshot covering a large spatial area of interested seabed [5]. In these cases, parallel paths with spatially shifted characteristics must be constructed, and coordinated path following control has to be respected to keep the formation. In this paper, coordinated parallel paths following control of multiple vehicles is explored, and each autonomous vehicle will be modeled as a first-order nonholonomic constrainted system, which means that velocity constraints are imposed on the motion (null sway velocity herein). Based on the coordinated path following control of first-order nonholonomic system, path following of underactuated vehicles which are subject to second-order nonholonomic constraints on acceleration could be extended in future research work.

The main idea in the paper will be realized in two steps. In the first step, each vehicle will be steered to converge to the desired path, under the classic ”virtual target guidance approach” control method based on Lyapunov design [6]. For each individual vehicle, a virtual target as a ”rabbit” runs in the desired path, which is elaborated selected but not traditionally defined as the orthogonal projection of the actual vehicle on the path. The running speed of the ”rabbit” to be tracked along the path can be explicitly controlled, and the singularities and stringent initial condition constraints that are presented in other path following strategies can be bypassed [7]. Therefore, the actual vehicle as a hunting ”fox” is globally attracted, which has the opportunity to be well stuck with the predefined path.

After the vehicles have converged to their paths, we can deal with the problem of coordinated control and formation keeping. Apparently, each generalized length traveling along the paths for different vehicles should be identical (or with fixed offsets) to the same reference, so that the vehicles are able to keep the formation. Interestingly, the inherent characteristic of path following control laws adopted here, is that the single vehicle is driven onto the path (spatial assignment in path following design), is throughly separated from speed convergence (temporal assignment in path following design) [8]. This important characteristic of spatial-temporal decoupling in path following control, endows the coordinated path following controllers with the ability to adapt speeds among multiple vehicles, without degrading the performance of individual vehicles converging to the path.

Naturally, in the second step, the velocity of each vehicle will be regulated based on the shared information of all travelled lengths along the paths via the communication channels. A leader-follower strategy is instrumental to keep the generalized along-path distance [9]. Upon the velocities of different vehicles becoming the desired ones under the control

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law of velocity regulation, disagreements among the traveled length along corresponding paths will be approach to zero (or fixed deviation), and the objective of coordinated control of multiple vehicles is reached.

Simulation results illustrate the performance of the coordinated control system proposed here, and the theoretical analysis in this paper reveals some important issues that are valuable for further research on path following control for multiple underactuated AUVs.

II. Nonholonomic system

In the last decades, there has been an increasing interest in the control of nonholonomic systems, which are subject to nonholonomic constraints. These so-called nonholonomic constraints most commonly arise in mechanical systems where some constraints are imposed on the motion. This class of nonholonomic systems are abundant in real life, which have been involved in all kinds of intelligent mechanical systems, including manipulators, mobile robots, surface vessels, underwater vehicles, helicopters, spacecrafts, etc [10].

In this section, the definitions of nonholonomic system and some related concepts, are firstly recalled here. And then, the connection between nonholonomic mobile vehicle and underactuated vehicle will be explained. This is the theoretic root inspiring us to address the problem of coordinated control of multiple nonholonomic mobile vehicles, before going to underactuated underwater vehicles. At the same time, we can benefit from convenient experiments on nonholonomic mobile vehicles at the early stage, other than the high-cost and exhaustive experiments on underwater vehicles, especially when we deal with multiple vehicles.

A. Concepts of Nonholonomic Constraints and Nonholonomic Systems

1) Holonomic Systems: Consider a system of generalized coordinates q, with the dynamics $\ddot{q} = f(q, \dot{q}, u)$,where u is a vector of external generalized inputs. If the conditions of constraints limiting the motion of the system, can be expressed as the time-derivative of some functions of the generalized coordinates with the form $\Phi(q, t) = 0$, then the constraints are said to be *holonomic* [11]. This type of constraint is so-called integrated, since the holonomic constraint can be solved by integration.

2) Nonholonomic Systems : In classic mechanics, systems with nonholonomic constraints, which are defined as linear constraints w.r.t. generalized coordinates q, having the form $\Phi(q, t)\dot{q}(t) = 0$. That means the equations of motion constraints are irreducible, and can not be expressed as time derivative of some function of the state. Therefore, the constraints are non-integrable, which are called as nonholonomic constraints [11]. Within nonholonomic systems, the generalized coordinates are not independent of each other.

Moreover, the nonholonomic constraints can be classified into two categories, the first-order nonholonomic constraints and the second-order nonholonomic constraints.

The first-order nonholonomic constraints are defined as constraints on the generalized coordinates and velocities of the form $h(q, \dot{q}) = 0$ that are non-integrable, i.e. can not be written as $\Phi(q, t) = 0$. These constraints include nonholonomic constraints arising in classical mechanics and nonholonomic constraints arising from kinematics.

The second-order constraints are defined as constraints on the generalized coordinates, velocities and accelerations of the form $h(q, \dot{q}, \ddot{q}) = 0$, which are non-integrable, i.e. can not be written as the time derivative of some function of the generalized coordinates and velocites, i.e. $\Phi(q, \dot{q}) = 0$. These nonholonomic constraints can not be solved by integration, as they are an essential part of the dynamics.

3) Underactuated Systems : Consider systems that can be written as $\ddot{q} = f(q, \dot{q}) + G(q)u$, where q is the state vector of independent generalized coordinates, $f(.)$ is the vector field representing the dynamics of the systems, $\dot{q}$ is the generalized velocity vector, G is the input matrix, and u is a vector of generalized force inputs. The dimension of q is defined as the degrees of freedom. System is said to be underactuated if the external generalized forces are not able to command instanteneous accelerations in all directions in the configuration space, i.e. $rank(G) < dim(q)$ [11]. The definition figure out that the underactuated systems are with fewer independent control actuators than degree of freedom to be controlled.

4) Relationship between Nonholonomic and Underactuated Systems: The first-order nonholonomic constraints, or velocity constraints, most commonly occur in, for example, wheeled mobile robots and wheeled vehicles, including tractor with trailer systems. The second-order nonholonomic constraints, or acceleration constraints, most commonly occur in, for example, manipulators, surface vessels, underwater vehicles, spacecrafts.

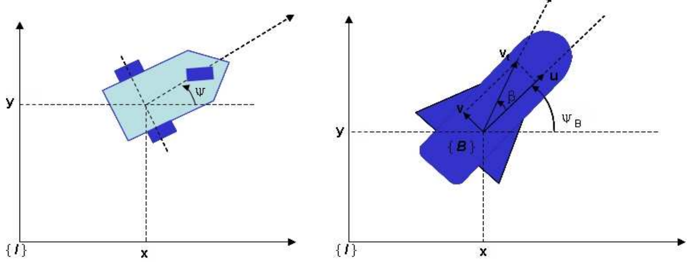
Due to lack of actuator in certain directions, acceleration constraints exist in certain directions inside underactuated systems. That means, underactuation leads to a constraint on the acceleration, such that the underactuated systems can be taken as a special case of the second-order non-holonomic systems.

An important remark is the fact that the research on underactuated system is an extension of the research on the first-order nonholonomic systems. Indeed, nonholonomic systems have constraints on the velocity and only kinematics equations of the system are considered. Underactuated systems have constraints on the acceleration, and both kinematics and dynamics have to be considered in the control design.

B. Control on Nonholonomic Unicycles vs. Underactuated Underwater Vehicles

In this section, motion control of unicycle-type mobile vehicles with first-order nonholonomic constraints on velocity, and control of underactuated underwater vehicles with second-order nonholonomic constraints on acceleration are discussed here. Furthermore, the connection between motion control of both systems in kinematic level is figured out.

1) Control on Nonholonomic Unicycles: In figure 1(a), a unicycle-type of wheeled mobile robot has two identical parallel, non-deformable rear wheels and a passive front wheel. It is assumed the contact between the wheels and the ground is pure rolling and non-slipping. The wheels control provides the forward force F and angular torque N applied on the robot's center of mass. The robot mass and moment of inertia are denoted m and I, respectively. Let v and r denote the forward and rotational speed of the robot.



(a) Unicyle-type vehicle (b) Underwater vehicle

Fig. 1. Unicycle-type vehicle and underwater vehicle

The kinematic model of the unicycle-type vehicle is given by

$$\begin{cases} \dot{x} = vcos\psi \\ \dot{y} = vsin\psi \\ \dot{\psi} = r \end{cases} \quad (1)$$

The dynamic model of the unicycle-type vehicle is obtained by augmenting (1) with the equations

$$\begin{cases} \dot{v} = F/m \\ \dot{r} = N/I \end{cases} \quad (2)$$

This system is subject to a constraint on the velocity as follows.

$$\dot{x}sin\theta - \dot{y}cos\theta = 0$$

Obviously, this constraint can not be integrated, i.e., it can not be expressed as the time-derivative of some function of the state (x, y, θ). Therefore, the system is first-order nonholonomic system, which coincides with the truth that vehicle is suffered from lateral zero-speed constraints.

2) Control on Underactuated Underwater Vehicles: In figure 1(b), an autonomous underwater vehicle (AUV) is equipped with two identical back thrusters, mounted symmetrically with respect to its longitudinal axis. Recruiting two different kinds of working mode of the thrusters, i.e. common and differential outputs, a force F along the vehicle's longitudinal axis and a torque Γ on its vertical axis are generated, respectively. As there is no lateral thruster and only two actuators for motion in three degrees of freedom, the vehicle is indeed underactuated.

Let u and v are the longitudinal (surge) and transverse (sway) velocities, respectively. Let r is the angular speed (yaw rate). The kinematic equations of the AUV can be written as

$$\begin{cases} \dot{x} = ucos\psi_B - vsin\psi_B \\ \dot{y} = usin\psi_B + vcos\psi_B \\ \dot{\psi}_B = r \end{cases} \quad (3)$$

The dynamical model of the underactuated vehicle based on the model of INFANTE AUV [12], is obtained by augmenting (3) with the dynamic equations, which are given by

$$\begin{cases} F = m_u\dot{u} + d_u \\ 0 = m_v\dot{v} + m_{ur}ur + d_v \\ \Gamma = m_r\dot{r} + d_r \end{cases} \quad (4)$$

with

$$\begin{array}{ll} m_u = m - \dot{X}_{\dot{u}} & d_u = -X_{uv}u^2 - X_{vv}v^2 \\ m_v = m - Y_{\dot{v}} & d_v = -Y_v uv - Y_{v|v|}v \mid v \mid \\ m_r = I_z - N_{\dot{r}} & d_r = -N_v uv - N_{v|v|}v \mid v \mid \\ m_ur = m_Y - r & -N_r ur \end{array}$$

where m denotes the system mass, I_z is the moment of inertia w.r.t. the z-axis. $X_{..}$, $Y_{..}$, and $Z_{..}$ are hydrodynamic derivatives. F and Γ define the inputs of force and torque that is applied to AUV, respectively.

Assuming u is never equal to zero, and $\mid v \mid \ll u$. Then, the sideslip angle β can be defined as $arctan(v/u)$. Consider the flow frame $\{W\}$ that is obtained by rotating body frame $\{B\}$ around the z_B axis through the sideslip angle β, The kinematic equations can then be re-written to yield

$$\begin{cases} \dot{x} = v_t cos\psi_W \\ \dot{y} = v_t sin\psi_W \\ \dot{\psi}_W = r + \dot{\beta} \end{cases} \quad (5)$$

Where $\psi_W = \psi_B + \beta$, and v_t is the total speed expressed in $\{W\}$. Clearly, $v_t = \sqrt{u^2 + v^2}$.

3) Connection of Motion Control between Unicycles and AUVs: Notice how the choice of a new frame simplified the first two kinematic equations in (5) and brought out their similarities with those (2) of a wheeled robot. Although the constraints are different inside unicycle and underactuated AUV system, the control inputs are the same: the forward and yaw speeds. This explains the connection between unicycle-type vehicle and AUV path following control design. The only difference is that the inefficiency of a side thruster due to the underactuated design in the AUV system, make the total speed v_t resulted from both surge and sway components u and v. While the first-order nonholonomic constraint imposing on unicycle-type vehicle, make the total speed is permanently equal to its forward speed v.

In the overall control loop, the kinematic controller actually acts as a reference subsystem, giving the desired signals for the control subsystem based on the dynamics level. Using backstepping techniques [13], the control law in kinematic level can be extended to deal with vehicle dynamics.

Therefore, the control design on unicycle-type mobile robot could be used in the early stage to the control design on underactuated underwater vehicles. Moreover, the experiment on unicycle is more convenient than that of underwater vehicle, regarding the cost and duration, especially when we deal with multiple vehicles. In this paper, we investigate the coordinated path following control of multiple unicycles (first-order nonholonomic system), and then migrate the methodology and extend the control laws to underactuated vehicles (second-order nonholonomic system) in future work.

III. Coordinated Path Following of Multiple Nonholonomic Vehicles

A. *Path Following Controller for a single Nonholonomic Vehicle*

Consider Figure 2, where a nonholonomic unicycle-type of wheeled vehicle follows a predefined spatial path. Let P be an

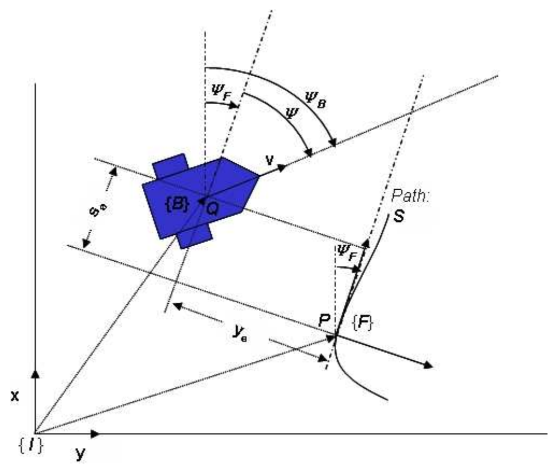


Fig. 2. Path following of nonholonomic vehicle

arbitrary point on the path to be followed and Q be the center of mass of the moving robot. Associated with P, consider the corresponding Serret-Frenet frame $\{F\}$ in figure 2. The path S is parameterized by a moving target P on the path, with curvilinear abscissa (along path length) denoted by s. Let $(s_e,\, y_e)$ denote the coordinates of Q be in $\{F\}$. Let the rotations from $\{F\}$ to $\{I\}$ and $\{B\}$ to $\{I\}$ be denoted by the yaw angles ψ_F and ψ_B, respectively. Further, let $c_c(s)$ and $g_c(s)$ denote the path curvature and its derivative respectively, and then $\psi_F = c_c(s)\dot{s}$. With the denotation of variable $\psi_e = \psi_B - \psi_F$, the kinematic model of unicycle in the Serret-Frenet frame can be derived as

$$\begin{cases} \dot{s}_e = -\dot{s}(1 - c_c y_e) + v cos\psi_e \\ \dot{y}_e = -c_c \dot{s} s_e + v sin\psi_e \\ \dot{\psi}_e = r - c_c \dot{s} \end{cases} \tag{6}$$

where $r = \dot{\psi}_B$

The dynamical model of the nonholonomic unicycle is the same as that of (2).

With the above notation, the problem of path following for single robot can be formulated as below:

Path Following Control. *Given a predefined path to be followed by a nonholonomic unicycle-type vehicle, and given a desired speed profile $v_d(t) \geq v_{min} > 0$ for the vehicle speed v, derive kinematic control laws to drive y_e, s_e, ψ_e and $v - v_d$ asymptotically to zero.*

As in [7], define the approach angle

$$\delta(y_e, v) = -sign(v)\theta_a tanh(k_\delta y_e) \tag{7}$$

Where $0 < \theta_a < \pi/2$ and $k_\delta > 0$. The approach angle satisfying $y_e v \sin\delta \leq 0$, is instrumental in shaping transient maneuvers during the path approaching phase.

Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2}[s_e^2 + y_e^2 + (\psi_e - \delta)^2] \tag{8}$$

Resorting to the kinematics model (6), the derivative of V_1 is

$$\begin{aligned} \dot{V}_1 =& s_e(v\cos\psi_e - \dot{s}) + y_e v \sin\delta + \\ & (\psi_e - \delta)(\dot{\psi}_e - \dot{\delta} + y_e v \frac{\sin\psi_e - \sin\delta}{\psi_e - \delta}) \end{aligned}$$

It is straightforward to show that the choice

$$\begin{cases} \dot{s} = k_1 s_e + v cos\psi_e \\ \dot{\psi}_e = \dot{\delta} - y_e v \frac{\sin\psi_e - \sin\delta}{\psi_e - \delta} - k_2(\psi_e - \delta) \end{cases} \tag{9}$$

where k_1 and k_2 are positive gains, lead to

$$\dot{V}_1 = -k_1 {s_e}^2 + y_e v \sin\delta - k_2(\psi_e - \delta)^2$$

With the approaching angle designed in (7), $\dot{V}_1 \leq 0$. That means above control law makes $\dot{V}_1$ negative semi-define.

Considering Lyapunov function candidate $V_v = \frac{1}{2}(v - v_d)^2$, it is trivial to choose the speed control law

$$\dot{v} = \dot{v_d} - k_4(v - v_d) \tag{10}$$

where $k_4 > 0$.

With (10), $\dot{V}_v$ is negative semi-define and the vehicle speed v converge to desired speed v_d assuming $v_d(t) \geq v_{min} > 0$, with performance of global stable.

It indicates that controlling speed v is throughly decoupled with other control behaviors, which means, driving the robot onto the path with error space $\{s_e, y_e, \psi_e\}$ equal to zero (spatial assignment in path following design) no matter how the speed control works, is throughly separated from speed convergence (temporal assignment in path following design). This important characteristic of spatial-temporal decoupling in path following control, endows the coordinated path following controller with a dedicated ability of speed adaptation among multiple vehicles, without degrading the performance of individual vehicle's convergence to the path.

Therefore, the feasible strategy for coordinated path following to build a formation, is that

(1) Elect one vehicle as a leader and other vehicles as followers. Both the leader and the followers recruit their own path following control laws to trace the predefined paths,

(2) and then, adjust the desired speed of follower vehicles, bring generalized along-path length $s_i (i = 1, 2, \cdots, n)$, to be equal for in-line formation or fulfill some geometric conditions for special formation.

B. *Coordinated Path Following Controller for Multiple Nonholonomic Vehicles*

Coordinated Parallel Paths Following Control. *Given n parallel paths to be followed by n nonholonomic unicycle-type vehicles, and given a desired speed profile $v_1^d(t) \geq v_{min} > 0$ for the leader vehicle speed v_1, derive feedback control laws to drive y_{ei}, s_{ei}, ψ_{ei}, $v_i - v_i^d$, and generalized along-path distance Δs asymptotically to zero.*

1) Paths formulation: Predefined paths are in general not straight lines, but feasible curves. A in-line formation with n vehicles is created by a set of shifted vectors d_i, relative to the baseline path of the virtual leader, as depicted in Figure 3. The individual path for vehicle i is

$$s_i(\mu) = s(\mu) + R_B^I d_i$$

where u is the path parameters, and R_B^I is a rotation matrix from a moving body frame B to the inertial frame I. For unicycle vehicle moving on the 2D plane, the desired path which the virtual leader is following, is then given by $s(\mu) = [x(\mu), y(\mu), \theta(\mu)]^T$. The tangent vector along the path in the (x, y) directions is chosen as the x axis of the moving body frame B. The angle of the tangent vector in the inertial frame I gives the heading $\theta(\mu) = arctan(\frac{y(\mu)'}{x(\mu)'})$. Therefore, the rotation matrix for the parallel paths is given by

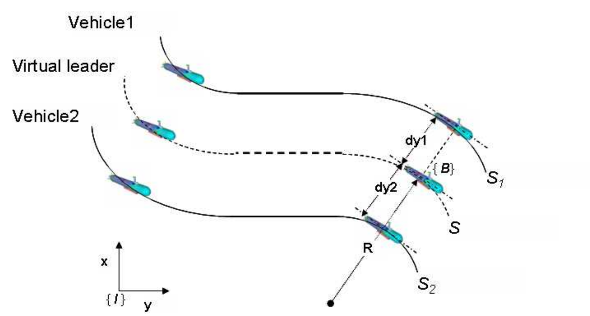


Fig. 3. Illustration of paths setup

$$R_B^I = \begin{pmatrix} cos\theta(\mu) & -sin\theta(\mu) & 0 \\ sin\theta(\mu) & cos\theta(\mu) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2) Strategy of Coordinated Parallel Paths following: In order to simplify the control design, one vehicle is elected as a leader, with the formation shifted vector $d_1 = [0, 0, 0]^T$. This means that the virtual leader coincides with the vehicle 1, and the other vehicle i will be a follower with shifted vector $d_i = [0, d_{yi}, 0]^T, i = 2, 3, ..., n$.

In the case of in-line formation for parallel paths (different from identical paths in [9]) as depicted in figure 3, there is always a relationship between the along-path position of the virtual target of the leader s_1, and the desired along-path position of the virtual target of the follower s_2. That is

$$\dot{s}_2^d(\mu) = \frac{c_{c1}}{c_{c2}} \dot{s}_1(\mu) \tag{11}$$

Since $c_{ci} \in \Re$ and $c_{ci} = 1/R_i$, where R_i is the radii of the tangent circle (i.e. the circle of curvature which is tangent to the curve) at one point of the path.

According to the path formulation, there is $R_2 = R_1 + d_{y2}$, such that

$$\frac{c_{c1}}{c_{c2}} = \frac{R_2}{R_1} = 1 + d_{y2} c_{c1}(\mu) \tag{12}$$

Substitute (11) with (12), and then

$$\dot{s}_2^d(\mu) = (1 + d_{y2} c_{c1}(\mu)) \dot{s}_1(\mu)$$

Therefore

$$s_2^d(t) = s_1(t) + d_{y2} \int_0^t c_{c1}(t, \mu) \dot{s}_1(t, \mu) dt \tag{13}$$

3) Leader Control: In the case of the leader, a path following controller is easily obtained by recruiting laws of (9) and (10). That is,

$$\begin{cases} \dot{s}_1 = k_1 s_{e1} + v_1 cos\psi_{e1} \\ \dot{\psi}_{e1} = \dot{\delta}_1 - y_{e1} v_1 \frac{\sin\psi_{e1} - \sin\delta_1}{\psi_{e1} - \delta_1} - k_2(\psi_{e1} - \delta_1) \\ \dot{v}_1 = \dot{v}_1^d - k_4(v_1 - v_1^d) \end{cases} \tag{14}$$

where v_1^d is desired speed profile, and $\dot{v}_1^d$ is the derivative which is normally equal to zero.

The first two terms in (14) contribute to kinematic control, and the third one contributes to speed control.

4) Follower Control: The follower recruits similar path following control laws to those recruited by the leader.

$$\begin{cases} \dot{s}_2 = k_1 s_{e2} + v_2 cos\psi_{e2} \\ \dot{\psi}_{e2} = \dot{\delta}_2 - y_{e2} v_2 \frac{\sin\psi_{e2} - \sin\delta_2}{\psi_{e2} - \delta_2} - k_2(\psi_{e2} - \delta_2) \\ \dot{v}_2 = \dot{v}_2^d - k_4(v_2 - v_2^d) \end{cases} \tag{15}$$

The only difference between the controller of the leader and that of the follower is that, the follower's forward speed v_2 must be adapted to reduce the generalized along-path distance between the two vehicles to zero.

A solution proposed to speed adaption is

$$v_2^d = (1 + d_{y2} c_{c1}(\mu)) v_1^d + \frac{2}{\pi} k_v \arctan(\Delta s) \tag{16}$$

where $\Delta s = [s_1(t) + d_{y2} \int_0^t c_{c1}(t, \mu)) \dot{s}_1(t, \mu) dt] - s_2(t)$ is the generalized along-path distance between the two vehicles, derivated from (13).

Straightforward computations show that the derivative of the follower's speed is

$$\dot{v}_2^d = d_{y2} g_{c1} v_1^d + (1 + d_{y2} c_{c1}) \dot{v}_1^d + \frac{2}{\pi} k_v \frac{((1 + d_{y2} c_{c1}) v_1^d - \dot{s}_2)}{1 + (\Delta s)^2} \tag{17}$$

where g_{c1} is the derivative of the path curvature, and $k_v > 0$ is a slack variable to impose restrictions on how much the follower's speed is allowed to catch up the leader.

There is one thing highlighted in the controller design, that only the generalized along-path length of the leader $s_1'(= [s_1(t) + d_{y2} \int_0^t c_{c1}(t, \mu)) \dot{s}_1(t, \mu) dt])$ is required for the follower, as c_{c1} and g_{c1} can be estimated by means of the value of s_2' and predefined path information. With the error of along-path distance (Δs) between the leader and the follower, the follower is able to reduce the relative distance, and then keep the relative position according to the leader in the formation. Neither speed nor Cartesian position of the leader is needed, such that the amount of information exchanged between two vehicles are minimized, and the inter-vehicle communication is kept to a minimum.

With control laws proposed here, both the leader and the follower asymptotically converge to the paths, and their relative along-path distance is guaranteed in terms of geometric constraints of the formation. A formal proof of this result heavily relies on Lyapunov-based design, which is similar to the method introduced in [14] for a single vehicle, and in [15] for multiple vehicles.

IV. SIMULATION

This section illustrates the performance of coordinated path following control laws for nonholonomic multi-vehicle system proposed in this paper.

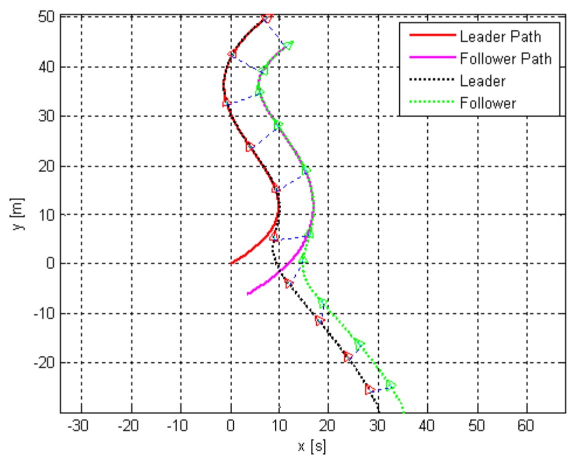


Fig. 4. Desired paths and actual Leader/Follower trajectories

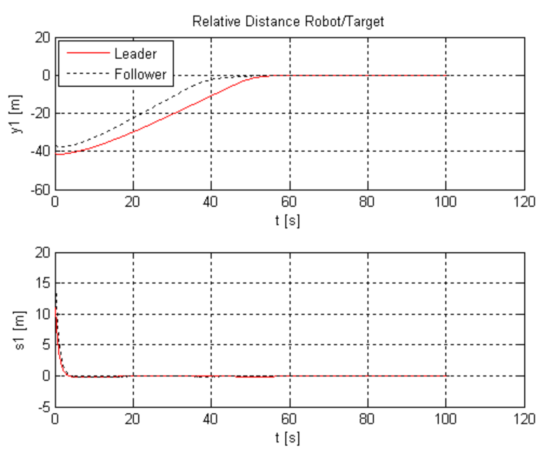


Fig. 5. Distance between the nonholonomic vehicles and their targets

The virtual leader coincided with the unicycle-type vehicle 1, such that the shifted vector of corresponding path was $d_1 = [0, 0, 0]^T$. The vehicle 2 was a follower, whose predefined path was with shifted vector $d_2 = [0, 7, 0]^T$. The follower and the leader were required to keep a in-line formation. The initial positions of the leader and the follower were $(30, -30)$ and $(35, -30)$ respectively. The initial speeds of the leader and the follower were $0.1m/s$. The desired speed v_1^d was set equal to $1m/s$. As illustrated in figure 4, both the leader and follower converged to the predefined paths, and kept the in-line formation. In figure 5, the error space of the two vehicles

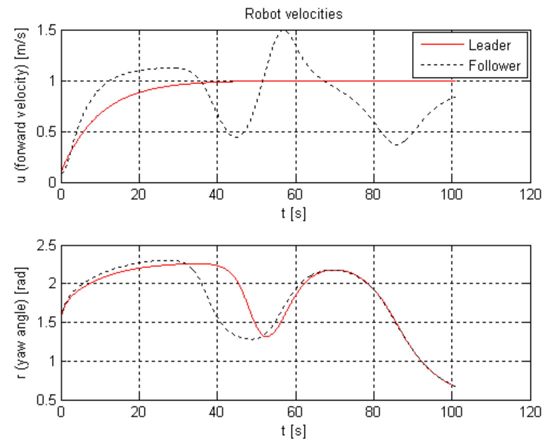


Fig. 6. Forward and rotational speeds of nonholonomic vehicles

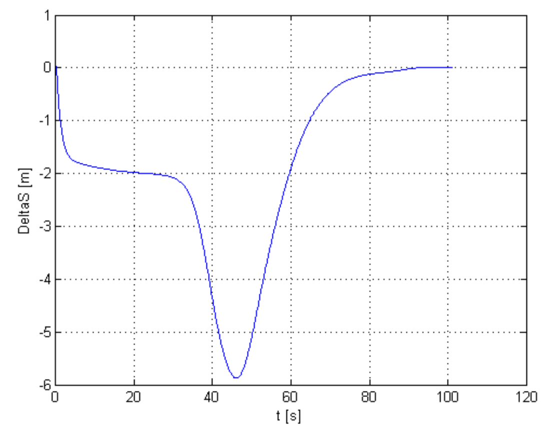


Fig. 7. Generalized along-path distance between Leader and Follower

w.r.t. the respective paths were driven to zero. The forward speed adaptation of the follower is illustrated in figure 6, and the angular speed of the follower was the same as that of the leader in the situation of in-line formation. The generalized along-path distance Δs between the vehicles in figure 7 was decaying to 0. There were only one leader and one follower defined in the simulation, however, more than one vehicle as follower can be coordinated while tracking the paths with the proposed control laws in the paper.

V. CONCLUSION

This paper addressed the problem of coordinated parallel paths following control of nonholonomic vehicles, based on the leader-follower strategy to keep the in-line formation. Both the leader and the follower adopt similar path following control laws. The only difference is that the leader travelling along the predefined path at a desired speed assignment, the follower is then adapting its own speed according to the information of a generalized along path distance between them, and try to catch up with the leader. Simulation results for the in-line formation illustrated the efficacy of the solution herein.

Furthermore, the proposed methodology using for nonholonomic unicycle-type vehicles can be extended to underactuated underwater vehicles, due to the similarity in kinematics between the first-order nonholonomic system and the underactuated system indicated in the second part of the paper. With the reference from the kinematics controller, control laws on different dynamics can be handled by means of backstepping technology in future work.

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