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Generation and conversion of optical vortices in long-period twisted elliptical fibers

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We theoretically demonstrate that long period twisted elliptical fibers have the ability to change in a certain wavelength range the topological charge of the incoming field by two units. We also show that such fibers can generate charge 2 optical vortices from the incoming Gaussian beams.

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1. Introduction

Generation of optical fields with singularities has become a topical problem of singular optics since first discussions of optical vortices (OVs) [1,2]. At present, several methods of OV generation are used for such a purpose: generation by lens converters [3], spiral phase plates [4], and phase holograms [5]. In past years other new methods have been suggested [6,7,8]. The method of OV generation by so-called q -plates has shown great promise [9,10].

Among the variety of the existing methods, one can single out the special group concerned with OV generation with optical fibers. Though some of them exploit the semblance of stress-applied fibers with lens converters [11], the majority are based on the effect of mode coupling in chiral fiber gratings. Historically, first such an effect of vortex generation (without its recognition) from the Gaussian mode was presented in [12]. Similar phenomena have been observed in other types of helical fiber gratings [13–15]. A theoretical explanation of the observed results, though, has been presented quite recently [2,16,17]. In those papers it has been pointed out that a helical pertur-

bation of refractive index brings forth the coupling between fiber modes with orbital numbers differing by unity. The effect of such mode coupling is insensitive to a particular technique of creation of a helical perturbation and leads to changing the topological charge of the incident field by unity [18]. At present such helical-core fibers are no longer some bizarre objects but are within the reach of state of the art technology [19,20].

However, all such waveguides with a helical perturbation of refractive index have a common limitation: they can change the topological charge of the incoming field only by unity. Meanwhile, it is desirable to have the possibility of changing this charge in somewhat wider limits. In this paper we propose the method of all-fiber changing the topological charge of the incoming field by two units. An inspiring hint on the nature of the class of fibers, which could be the candidates for such systems, can be found in the papers of Kopp *et al.* on twisted fibers. In one of their early papers on that topic the authors refer to effectively elliptic twisted fiber as to a double-helix fiber [21]. Indeed, in a way, such fibers feature π -shifted helices of larger refraction index n . Though, generally speaking, actual distribution of n is more complicated, this notion proves to be sufficient to focus attention on such class of fibers.

The aim of this paper is to demonstrate that long-period twisted elliptic fibers can change the topological charge of the incoming field by two units. In particular, we show that such fibers can generate charge-2 OV from the incoming Gaussian beams.

2. Basic Equations and Coupled Modes

Elliptical twisted fibers are manufactured by simultaneously drawing and twisting the fiber from a preform with an elliptically deformed core. During such technological process no elastic strains appear in the fiber and the effect of twisting is reduced to a mere geometrical modification of refractive index distribution. As is shown in [22], for weakly guiding fibers this leads to the following distribution of the refractive index:

$$n^2(r, \varphi, z) = n_{\text{co}}^2(1 - 2\Delta f(r) - 2n_{\text{co}}^2\Delta\delta r f'_r \cos(2(\varphi - qz)), \quad (1)$$

where Δ is the height of the profile f , $\delta \ll 1$ is dimensionless parameter of ellipticity, n_{co} is the core's refractive index, and $q = 2\pi/H$, with H being the pitch of the fiber [see Fig. 1]. Here the axial-polar coordinates (r, φ, z) are implied and are introduced in the standard way.

In the scalar approximation, which proves to be sufficient for our purposes, the transverse electric field \vec{E}_t satisfies the following equation [23]:

$$\Delta\vec{E}_t + k^2n^2\vec{E}_t = 0, \quad (2)$$

where k is the wave number in vacuum and Δ is the Laplace operator. The change of variables $\tilde{r} = r$, $\tilde{z} = z$, $\tilde{\varphi} = \varphi - qz$ enables one to obtain the translational invariant in \tilde{z} equation:

$$\left\{ \frac{\partial^2}{\partial\tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial}{\partial\tilde{r}} + \frac{1}{\tilde{r}^2} \frac{\partial^2}{\partial\tilde{\varphi}^2} + \left(\frac{\partial}{\partial\tilde{z}} - q \frac{\partial}{\partial\tilde{\varphi}} \right)^2 + k^2\tilde{n}^2(\tilde{r}) - 2k^2n_{\text{co}}^2\tilde{r}\Delta\delta f'_r \cos 2\tilde{\varphi} \right\} \vec{E}_t = 0, \quad (3)$$

which after the substitution $\vec{E}_t = \vec{e}_t(\tilde{r}, \tilde{\varphi}) \exp(i\beta\tilde{z})$, β being the propagation constant, is reduced to

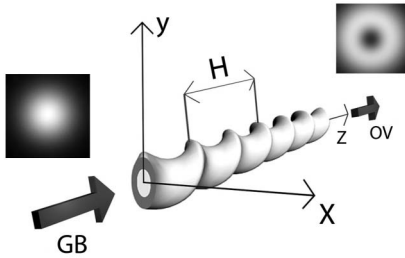


Fig. 1. Geometry of the problem: Schematically shown generation of the optical vortex OV from the incident Gaussian beam GB. Insets show intensity distribution of the corresponding fields.

$$\left\{ \frac{\partial^2}{\partial\tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial}{\partial\tilde{r}} + \frac{1}{\tilde{r}^2} \frac{\partial^2}{\partial\tilde{\varphi}^2} + \left(i\beta - q \frac{\partial}{\partial\tilde{\varphi}} \right)^2 + k^2\tilde{n}^2 - 2k^2n_{\text{co}}^2\tilde{r}\Delta\delta f'_r \cos 2\tilde{\varphi} \right\} e_t(\tilde{r}, \tilde{\varphi}) = 0. \quad (4)$$

In the basis of linear polarizations $|e\rangle = \begin{pmatrix} e_x \\ e_y \end{pmatrix}$ this equation can be recast as

$$(\hat{H}_0 + \hat{V})|e\rangle = \beta^2|e\rangle, \quad (5)$$

where

$$\hat{H}_0 = \left(\frac{\partial^2}{\partial\tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial}{\partial\tilde{r}} + \frac{1}{\tilde{r}^2} \frac{\partial^2}{\partial\tilde{\varphi}^2} + k^2\tilde{n}^2 - 2i\beta q \frac{\partial}{\partial\tilde{\varphi}} + q^2 \frac{\partial^2}{\partial\tilde{\varphi}^2} \right),$$

$\hat{V} = -2k^2n_{\text{co}}^2\tilde{r}\Delta\delta f'_r \cos 2\tilde{\varphi}$. Zero approximation eigenvalue equation $\hat{H}_0|e\rangle = \beta^2|e\rangle$ readily yields eigenvectors given by circularly polarized OVs:

$$|\sigma, l\rangle = \begin{pmatrix} 1 \\ i\sigma \end{pmatrix} \exp(il\tilde{\varphi})F_l(r), \quad (6)$$

where $\sigma = \pm 1$ determines the circular polarization, $l = 0, \pm 1, \pm 2, \dots$ is the topological charge of the vortex solution, and radial functions satisfy [23]

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + k^2\tilde{n}^2 - \frac{l^2}{r^2} - \tilde{\beta}_l^2 \right) F_l(r) = 0. \quad (7)$$

For the spectrum of propagation constants, one obtains

$$\beta_l^{(1,2)} = \pm\tilde{\beta}_l + lq. \quad (8)$$

As is seen from Eq. (8), in general, at $q \neq 0$ there is no degeneracy in the spectra (at $q = 0$ one should take account of the vector term in the waveguide equation [22]). However, at certain points the curves plotted as functions of q may intersect. In such points of the so-called accidental degeneracy, one has to use the perturbation theory with degeneracy to allow for the influence of the perturbation term \hat{V} in Eq. (5). We will demonstrate the application of this technique at the example of $l = 0, 2$ families of spectral curves.

Since the perturbation term \hat{V} cannot provide any coupling of fields with opposite polarizations, it is sufficient to study only spectral curves of zero-approximation modes of the same polarization. The spectra of $\sigma = 1$ modes at $l = 0, 2$ are

$$\beta_{1,2} = \pm\tilde{\beta}_0, \quad \beta_{3,4} = \pm\tilde{\beta}_2 + 2q, \quad \beta_{5,6} = \pm\tilde{\beta}_2 - 2q. \quad (9)$$

The plots of these curves are given in Fig. 2. At the points (a) and (b) (at $q = q_0 \equiv (\tilde{\beta}_0 - \tilde{\beta}_2)/2$) the curves of $l = 0$ and $l = 2$ modes intersect. In such

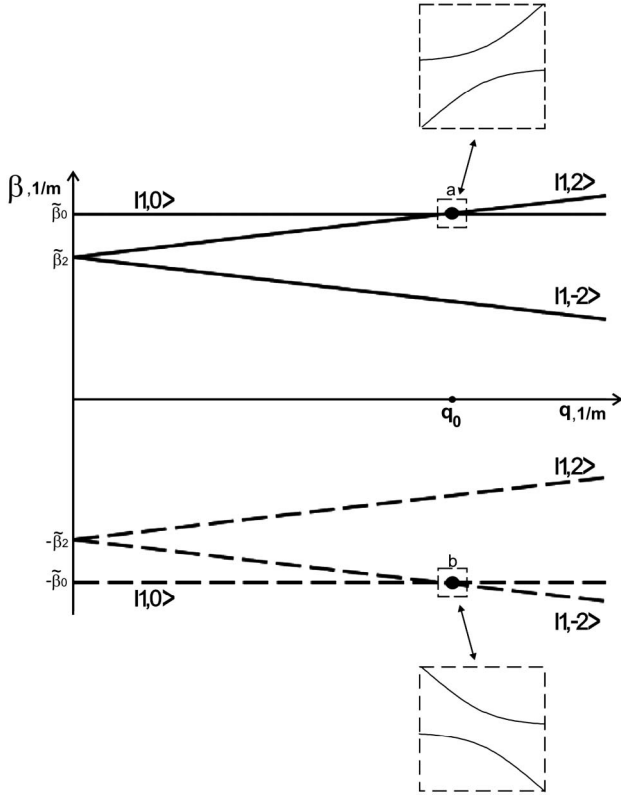


Fig. 2. Zero approximation spectra of twisted elliptical fiber modes versus lattice vector q . The type of the mode is indicated at the corresponding curve. Insets show repulsion of spectral branches due to the effect of mode coupling; the fiber's parameters are: n_{co} 1.5, Δ 0.01, δ 0.05, r_0 $8\lambda_0$, λ_0 632.8 nm, $q \approx q_0$ 11216.845 m^{-1} .

points there takes place intensive hybridization of forward-propagating (a) and backward-propagating (b) zero-approximation modes. For example, at point (a), forward-propagating modes $|1, 0\rangle$ and $|1, 2\rangle$ get coupled, whereas other modes do not interact. According to [24], to obtain the exact form of coupled modes, one has to build the matrix of the total Hamiltonian $\hat{H}_0 + \hat{V}$ in the basis of those zero-approximation eigenvectors of \hat{H}_0 , whose spectra coincide at $q = q_0$. Then the eigenvector problem is reduced to [3,16]

$$\begin{pmatrix} \tilde{\beta}_0^2 - \beta^2 & A \\ A & \tilde{\beta}_2^2 - (\beta - 2q)^2 \end{pmatrix} \vec{x}_a = 0, \quad (10)$$

where $A = -k^2 n_{co}^2 \Delta \delta / N_0 N_2$, normalization factor $N_i = \sqrt{\int_0^\infty x F_i^2 dx}$. Here the vector $\vec{x}_a = (x_1, x_2)$ corresponds to the field $x_1|1, 0\rangle + x_2|1, 2\rangle$. Introducing detunings $\varepsilon = q - q_0$ and $\delta = \beta - \tilde{\beta}_0$, one can further simplify the eigenvalue problem:

$$\begin{pmatrix} 2\tilde{\beta}_0\delta & A \\ A & 2\tilde{\beta}_2(2\varepsilon - \delta) \end{pmatrix} \vec{x}_a = 0. \quad (11)$$

The spectra feature the so-called repulsion of spectral curves [24] (see insets in Fig. 2) and are given by the expressions

$$\begin{aligned} \beta_{1,2}^{(a)} &= \tilde{\beta}_0 + \varepsilon \pm \sqrt{\varepsilon^2 + \Gamma^2}, \\ \beta_{1,2}^{(b)} &= -\tilde{\beta}_0 - \varepsilon \pm \sqrt{\varepsilon^2 + \Gamma^2}, \end{aligned} \quad (12)$$

where $\Gamma^2 \approx A^2 / 4\tilde{\beta}_0^2$. After a little algebra, one can obtain the expressions for coupled modes:

$$\begin{aligned} |\Psi_{1a}\rangle &= \{c_1|1, 0\rangle \exp[i(\tilde{\beta}_0 + \varepsilon)z] + c_2|1, 2\rangle \exp[i(\tilde{\beta}_2 - \varepsilon)z]\} \\ &\quad \times \exp\left(iz\sqrt{\varepsilon^2 + \Gamma^2}\right), \\ |\Psi_{2a}\rangle &= \{-c_2|1, 0\rangle \exp[i(\tilde{\beta}_0 + \varepsilon)z] + c_1|1, 2\rangle \\ &\quad \times \exp[i(\tilde{\beta}_2 - \varepsilon)z]\} \exp\left(-iz\sqrt{\varepsilon^2 + \Gamma^2}\right), \end{aligned} \quad (13)$$

where $c_{1,2} = \frac{1}{\sqrt{2}} \sqrt{1 \mp \frac{\varepsilon}{\sqrt{\varepsilon^2 + \Gamma^2}}}$. Analogously, one obtains the formulae for coupled backward-propagating modes:

$$\begin{aligned} |\Psi_{1b}\rangle &= \{-c_2|1, 0\rangle \exp[-i(\tilde{\beta}_0 + \varepsilon)z] + c_1|1, -2\rangle \\ &\quad \times \exp[-i(\tilde{\beta}_2 - \varepsilon)z]\} \exp\left(iz\sqrt{\varepsilon^2 + \Gamma^2}\right), \\ |\Psi_{2b}\rangle &= \{c_1|1, 0\rangle \exp[-i(\tilde{\beta}_0 + \varepsilon)z] + c_2|1, -2\rangle \\ &\quad \times \exp[-i(\tilde{\beta}_2 - \varepsilon)z]\} \exp\left(-iz\sqrt{\varepsilon^2 + \Gamma^2}\right). \end{aligned} \quad (14)$$

It should be emphasized that the fields in Eqs. (13) depend on the azimuthal coordinate φ and not on $\bar{\varphi}$. The remaining OVs: backward-propagating $|1, 2\rangle$ and forward-propagating $|1, -2\rangle$, remain uncoupled and their fields do not alter. The results obtained are sufficient to solve the problem of Gaussian mode's passage through such a fiber.

3. Generation of Double-Charged Optical Vortices

Let us study now the passage of the Gaussian beam through the twisted elliptical fiber with $q = q_0$. If the waist radius of the beam is correlated with the core's radius near the input end [23] the incident Gaussian beam can be approximated by $|1, 0\rangle$ mode. Before the fiber the field is given by the incident and reflected fields:

$$\begin{aligned} |\Phi_1(z \leq 0)\rangle &= |1, 0\rangle e^{ikz} + R_1|1, 0\rangle e^{-ikz} + R_2|1, 2\rangle e^{ikz} \\ &\quad + R_3|1, -2\rangle e^{-ikz}. \end{aligned} \quad (15)$$

Within the fiber, the field can be represented as

$$\begin{aligned} |\Phi_2\rangle &= T_1|\Psi_{1a}\rangle + T_2|\Psi_{2a}\rangle + T_3|\Psi_{1b}\rangle + T_4|\Psi_{2b}\rangle \\ &\quad + T_5|1, 2\rangle e^{i\tilde{\beta}_2 z} + T_6|1, -2\rangle e^{i\tilde{\beta}_2 z}, \end{aligned} \quad (16)$$

whereas the output field looks like

$$|\Phi_3(z \geq d)\rangle = (P_1|1, 0\rangle + P_2|1, 2\rangle + P_3|1, -2\rangle)e^{ik(z-d)}. \quad (17)$$

Here R_i , T_i , and P_i are unknown coefficients. As usual, the linear algebraic equations for these coefficients are obtained from matching fields and their derivatives with respect to z at the boundaries.

The dependence of transmission coefficients $|P_i|^2$ versus wavelength of the incident Gaussian beam is shown in Fig. 3. As follows from numerical results, at certain wavelength range the incident Gaussian beam gets almost completely transformed into charge-2 OV $|1, 2\rangle$. As the fiber's length increases, the area of effective conversion diminishes. Figure 4 shows typical conversion curves at $d = 207$ mm. These results demonstrate that twisted elliptical fibers can be used as all-fiber generators of charge-2 OV.

As is evident, this class of fibers has the ability to change the topological charge of the incoming field by 2 units. For example, such fibers can convert an incident $|1, 1\rangle$ OV into OV $|1, 3\rangle$ [Fig. 5]. Such selectivity can be explained through dynamical properties of the perturbation operator $V \propto \cos 2\varphi$: it can couple only the basic vectors $|\sigma, l\rangle$, whose orbital numbers differ by two units: $\langle \sigma, l | \hat{V} | \sigma, l \pm 2 \rangle \neq 0$. Of course such conversion would take place at other q or wavelength λ than q_0 and λ_0 , where the Gaussian beam gets converted into the OV. Basically, this conversion of the topological charge is closely connected with the

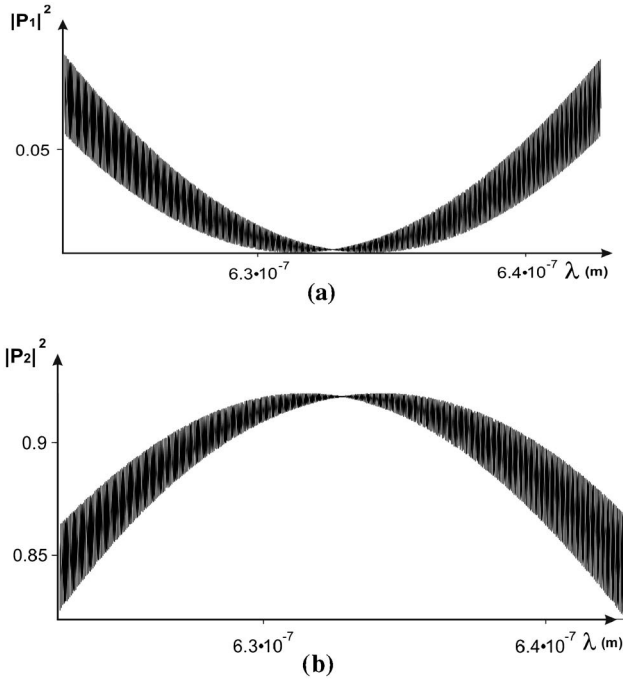


Fig. 3. Transmission coefficients $|P_i|^2$ for the outcoming modes $|1, 0\rangle$ (a) and $|1, 2\rangle$ (b) versus wavelength of the incoming field $|1, 0\rangle$; the fiber's length $d = 2.55$ mm, $\Delta = 0.01$, $\delta = 0.05$, $r_0 = 8\lambda_0$, $\lambda_0 = 632.8$ nm, $H = 0.56$ mm. The coefficient for the outcoming mode $|1, 2\rangle$ is negligibly small (not shown).

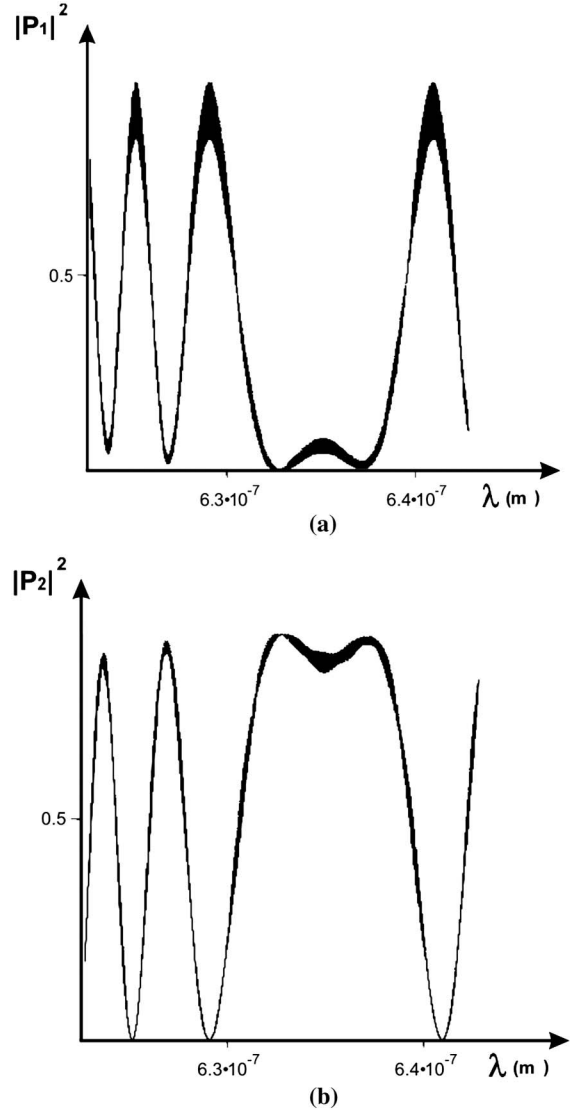


Fig. 4. Transmission coefficients for the outcoming modes $|1, 0\rangle$ (a) and $|1, 2\rangle$ (b) versus wavelength of the incoming field $|1, 0\rangle$; the fiber's length $d = 207$ mm. The other parameters are the same as in Fig. 3.

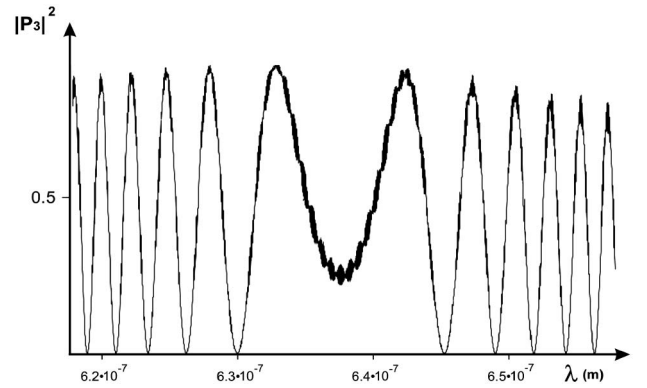


Fig. 5. Transmission coefficient for the outcoming vortex $|1, 3\rangle$ versus wavelength of the incoming field $|1, 1\rangle$. Fiber parameters: $d = 209.3$ mm, $n_{co} = 1.5$, $\Delta = 0.01$, $\delta = 0.05$, $r_0 = 8\lambda_0$, $\lambda_0 = 632.8$ nm, $H = 0.44$ mm.

presence of a double-helix in the structure of lines of equal refractive index.

4. Conclusion

In this paper we have theoretically demonstrated that long-period twisted elliptical fibers possess the ability to change in a certain wavelength range the topological charge of the incoming field by two units. In particular, we have also shown that such fibers can generate charge-2 optical vortices from the incoming Gaussian beams.

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