



HAL
open science

Optical vortices routing in coupled elliptical spun fibers

Constantine N. Alexeyev, Tatiana A. Fadeyeva, Yurii A. Fridman, Maxim A. Yavorsky

► **To cite this version:**

Constantine N. Alexeyev, Tatiana A. Fadeyeva, Yurii A. Fridman, Maxim A. Yavorsky. Optical vortices routing in coupled elliptical spun fibers. *Applied optics*, 2012, 51 (10), pp.C17-C21. 10.1364/AO.51.000C17. hal-00731808

HAL Id: hal-00731808

<https://hal.science/hal-00731808>

Submitted on 13 Mar 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - ShareAlike 4.0 International License

Optical vortices routing in coupled elliptical spun fibers

Constantine N. Alexeyev,^{1,*} Tatiana A. Fadeyeva,¹
Yurii A. Fridman,¹ and Maxim A. Yavorsky^{1,2}

¹Taurida National V.I. Vernadsky University, Vernadsky Prospekt, 4, Simferopol, 95007, Crimea, Ukraine

²University of Bordeaux, LOMA, UMR 5798, F-33400 Talence, France;
CNRS, LOMA, UMR 5798, F-33400 Talence, France

*Corresponding author: c.alexeyev@yandex.ua

We have studied the tunneling of a circularly polarized optical vortex (OV) in parallel strongly spun elliptical fibers. In this case it is possible to route the OV in a pure state from one of the fibers to another. We have determined the power efficiency of this process and have shown that such a directional coupler can serve for inversion of the topological charge of the incoming vortex.

OCIS codes: 060.1810, 060.1155, 260.6042.

1. Introduction

Among numerous promising applications of optical vortices (OVs)—singularities of phase fronts [1–3]—one can single out the application connected with their use as information carriers [4–7]. The possibility to store in an OV a greater amount of information closely relates to the fact that OVs are the eigenstates of orbital angular momentum (OAM) operators and in this way allow more degrees of freedom in encoding information [8–10]. This explains the interest in the topics concerned with OV transmission via optical fibers. In our recent paper we have studied the question of OV tunneling in parallel ideal fibers [11], where we have demonstrated that the problem of OV routing in such a system may encounter essential difficulties of the fundamental nature. Because of the complicated structure of hybrid modes, no tunneling of a pure OV is possible in coupled ideal fibers. This may essentially handicap routing the OV in a directional coupler. Meanwhile, the ability to route the streams of information encoded in OVs is desirable and can be considered as one of the key features of the optical network based on information encoding in OAM states.

In the present paper we address the problem of OV routing in coupled spun elliptical fibers. The aim of our paper is to study the tunneling of a circularly polarized OV (CV) from one of a couple of spun fibers to the other and find out whether it is possible for the OV to tunnel into the adjacent fiber in a pure OAM eigenstate. We show that, indeed, this process is possible and determine the tunneling length and effectiveness of OV transfer.

2. Hybrid Modes of Coupled Spun Fibers

As has been established by earlier studies, propagation of OVs in ideal fibers [12] is very sensitive to external perturbations such as induced ellipticity of the cross section [13] and material anisotropy [14]. In general, perturbations make OVs decompose into other fields, which leads to conversion of polarization and topological charge of the initial vortex [15,16]. This situation can be mended by introducing strong periodic perturbation into the system. Technically, this can be made by twisting anisotropic or elliptical fiber. As has been theoretically demonstrated, this leads to stabilization of OVs in fibers [17–19]. Suppression of the spin-orbit interaction (SOI) results in establishing a novel mode structure in the fiber, at which the modes are given by CVs (in spun elliptical fibers) or linearly polarized OVs (in spun anisotropic fibers). As has been shown in those papers, to

overcome the influence of the SOI the pitch of the grating induced in the spun fiber should be on a sub-millimeter scale. Such orders of twist pitch have been reported for spun monomode fibers in [20–22]. For example, [21] reports fabrication of chiral fiber grating with approximately 20 μm pitch.

It is worth recalling that in ideal fibers, modes with orbital number $l = 1$ are given by TE, TM modes and (in an alternative classification, see [12]) two CVs. Modes with $l > 1$ can be represented by four CVs. It should be emphasized that the spectrum of propagation constants in ideal fibers is partially degenerate, which is the reason for instability of OVVs in ideal fibers. Since the mode structure in twisted fibers is simpler, it is natural to suppose that the tunneling of OVVs in such fibers may prove to be more pronounced than in ideal ones. To study the evolution of OVVs in coupled twisted fibers, let us consider the case of parallel elliptical spun fibers. Here and throughout, the term “spun” means that no elastic stresses arise in the fiber while drawing it from a rotating preform. Also in the context of the cited papers, the twisted fiber is equivalent to the spun fiber. To allow for mechanical stresses in twisted fibers, one has to use more elaborate techniques (see, e.g., [23]). The fibers are supposed to be identical (see Fig. 1).

Prior to studying the system of two parallel fibers, one should specify the mode regime in which the fibers operate. As is known [17], if the influence of twisting prevails over the influence of ellipticity, the $l = 1$ modes of spun elliptical fibers are represented by four CVs. Mathematically, the condition of strong spinning is expressed as $q\tilde{\beta} \gg D_1$, where

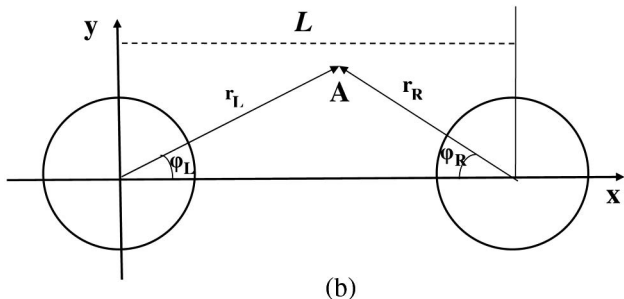
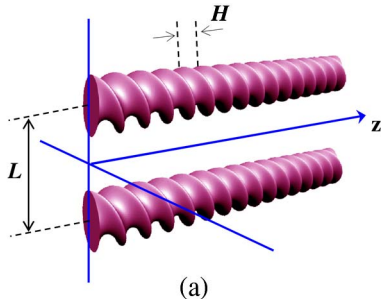


Fig. 1. (Color online) Geometry of coupled fibers: (a) general configuration of the system and its main parameters: pitch H and spacing L and (b) axial polar coordinates for individual fibers in the transverse cross section.

$\tilde{\beta}$ is the scalar propagation constant, $q = 2\pi/H$; H is the pitch of the fiber; and D_1 is some constant that characterizes the influence of ellipticity. In this case one can speak of the strongly spun fiber. The mode structure of such fibers is very simple: it consists of four CVs. In the basis of linear polarizations $|e\rangle = |e_x^e\rangle$, where it is implied that the standard representation of the electric field $\rightarrow E(x, y, z) = \rightarrow e(x, y) \exp(i\beta z)$ is used, their fields can be expressed as

$$|\sigma, l\rangle = \begin{pmatrix} 1 \\ i\sigma \end{pmatrix} e^{il\varphi} F_l(r). \quad (1)$$

Here $\sigma = \pm 1$, F_l satisfies the standard equation in the radial function [24], and the axial-polar coordinates (r, φ) are implied. For $l = 1$, the spectra of these modes are

$$\begin{aligned} \beta(|1, 1\rangle) &= \beta(|-1, 1\rangle) \equiv \beta_+ = \tilde{\beta} + \frac{D_1}{4q\tilde{\beta}} \cdot \frac{D_1}{2\tilde{\beta}}, \\ \beta(|1, -1\rangle) &= \beta(|-1, -1\rangle) \equiv \beta_- = \tilde{\beta} - \frac{D_1}{4q\tilde{\beta}} \cdot \frac{D_1}{2\tilde{\beta}}. \end{aligned} \quad (2)$$

The last expression conveys the idea of suppression of mode dispersion. Indeed, as follows from the corresponding results of [13], $D_1/2\tilde{\beta}$ is the intermodal dispersion caused by ellipticity of a straight fiber. Since $D_1/4q\tilde{\beta} \ll 1$ the splitting of propagation constants in Eq. (2) is strongly reduced by spinning the fiber—a fact well known for fundamental modes [25].

In this way, for strongly spun elliptical fibers one has a simple effective description. One can disregard the spinning and treat the fiber as some effective waveguiding optical system whose modes and propagation constants are given by Eqs. (1) and (2). Of course, the effective Hamiltonian $H_{0,\text{eff}}$ of such a notional fiber on the basis of modes, numbered as

$$\begin{aligned} |1\rangle &\equiv |1, 1\rangle, & |2\rangle &\equiv |1, -1\rangle, & |3\rangle &\equiv |-1, -1\rangle, \\ |4\rangle &\equiv |-1, 1\rangle, \end{aligned} \quad (3)$$

is represented by a diagonal matrix, $\text{diag}(\beta_+^2, \beta_-^2, \beta_-^2, \beta_+^2)$.

This notion of an effective fiber essentially simplifies the problem. Indeed, as has been pointed out in [26], the propagation in coupled fibers is governed by the equation

$$(\nabla_t^2 + V_l + V_r) \rightarrow e_t = \beta^2 \rightarrow e_t, \quad (4)$$

where $\nabla_t = (\partial/\partial x, \partial/\partial y)$ and t is the transverse component of the electric field. Here $V_{l(r)}$ are created by left (right) fibers taken separately. In our case, operator $\nabla_t^2 + V_l$ should be identified with the effective Hamiltonian $H_{0,\text{eff}}$ of the left fiber, whereas the perturbation operator should be taken in the form

$$V_r = 2k^2 n_{\text{co}}^2 \Delta \theta (1 - r/r_0). \quad (5)$$

Here $k = 2\pi/\lambda$, λ is the wavelength in vacuum, Δ is the height of the refractive index profile, r_0 is the core's radius, θ is the unity step, and n_{co} is the core's refractive index.

To implement the perturbation theory approach suggested in [26], one has to expand the basis Eq. (3) over left- and right-localized OVs. Then each vector in Eq. (3) should acquire an additional index $-L$ or R —depending on localization of the functions. For example, $|1, 1, L\rangle \equiv \binom{1}{1} \exp(i\varphi_L) F_l(r_L)$, where φ_L , r_L are axial-polar coordinates in the frame associated with the left fiber (see Fig. 1). Then the basis for our problem will be

$$\begin{aligned} &|1, 1, L(R)\rangle, & |1, -1, L(R)\rangle, \\ &|-1, -1, L(R)\rangle, & |-1, 1, L(R)\rangle. \end{aligned} \quad (6)$$

Following the scheme used in quantum mechanics [27], to obtain the modes of identical interacting systems one has to use perturbation theory with degeneracy. The presence of degeneracy is evident because for separated fibers, each propagation constant will correspond to two waves associated with the left or right fiber; that is, there will be two eigenfunctions that belong to the same eigenvalue. The matrix elements are calculated as

$$\langle i|\hat{H}|j\rangle = \iint_S \left(\psi_i^*, \varphi_i^* \right) \hat{H} \begin{pmatrix} \psi_i \\ \varphi_i \end{pmatrix} dS, \quad (7)$$

where S is the total cross section and $|i\rangle = \text{col}(\psi_i, \varphi_i)$.

Although in the basis Eq. (6) the dimension of the matrix of the total Hamiltonian is 8, actually this matrix can be represented as a Cartesian sum of two matrices of lesser dimensions. Indeed, since perturbation operators $V_{l(r)}$ are scalar, that is, they are proportional to the unity matrix, they do not couple the states with orthogonal polarizations: $\langle \sigma, l, F | V_{l(r)} | -\sigma, l', F' \rangle = 0$, where F, F' are either L or R . Therefore, it is sufficient to build the matrix of the total Hamiltonian in the subspace of, say, right circularly polarized (RCP) OVs:

$$\{|1, 1, L\rangle, \quad |1, -1, L\rangle, \quad |1, -1, R\rangle, \quad |1, 1, R\rangle\}. \quad (8)$$

Further, for convenience we will use the shortened notation: $|1, l, L(R)\rangle \equiv |l, L(R)\rangle$, indicating only the orbital number and localization. The other subspace is obtained from Eq. (8) by changing $\sigma \rightarrow -\sigma$.

Using the corresponding results of [11], one can bring the matrix H of the total Hamiltonian to the form:

$$H = \begin{pmatrix} \beta_+^2 & 0 & c_1 & d_1 \\ 0 & \beta_-^2 & d_1 & c_1 \\ c_1 & d_1 & \beta_+^2 & 0 \\ d_1 & c_1 & 0 & \beta_-^2 \end{pmatrix}, \quad (9)$$

where the coupling constants are given by overlap integrals

$$\begin{aligned} c_l &= 2k^2 n_{\text{co}}^2 r_0^2 \Delta N_l^{-1} \sum_{n=0}^{n=l} C_l^n \Lambda^{l-n} \\ &\times \int_0^{2\pi} \cos[(l-n)\varphi] d\varphi \int_0^1 \frac{R^{n+1}}{\tilde{R}^l} K_l(\tilde{R}) J_l(R) dR, \end{aligned} \quad (10)$$

$$\begin{aligned} d_l &= 2k^2 n_{\text{co}}^2 r_0^2 \Delta N_l^{-1} \sum_{n=0}^{n=l} C_l^n \Lambda^n \\ &\times \int_0^{2\pi} \cos[(2l-n)\varphi] d\varphi \int_0^1 \frac{R^{l-n+1}}{\tilde{R}^l} K_l(\tilde{R}) J_l(R) dR. \end{aligned} \quad (11)$$

Here $\tilde{R}^2 = R^2 + \Lambda^2 + 2R\Lambda \cos \varphi$, $\Lambda = L/r_0$, $R = r/r_0$, L is the distance between fiber centers and C_l^n are the binomial coefficients. The normalization factor $N_l = 2\pi r_0^2 \int_0^\infty R F_l^2(R) dR$, K_l is the modified Bessel function, J_l is the Bessel function. Then the solution of the eigenvalue problem $H \rightarrow x = \beta \rightarrow x$ gives the spectrum β^2 of modes and their decomposition over the basis Eq. (8) by the rule:

$$\begin{aligned} \rightarrow x(x_1, x_2, x_3, x_4) \rightarrow |\psi\rangle &= x_1 |1, L\rangle + x_2 |-1, L\rangle \\ &+ x_3 |1, R\rangle + x_4 |-1, R\rangle. \end{aligned}$$

If one assumes that spinning is strong enough, that is $\beta_+^2 - \beta_-^2 \ll c_1, d_1$, then one can set in Eq. (9) $\beta_\pm^2 \approx \tilde{\beta}^2$, which leads to essential simplification of the eigenvalue problem. In this case the eigenvectors are easily found and the corresponding modes are

$$\begin{aligned} |\psi_1\rangle &= |1, L\rangle + |-1, L\rangle + |1, R\rangle + |-1, R\rangle, \\ |\psi_2\rangle &= |1, L\rangle + |-1, L\rangle - |1, R\rangle - |-1, R\rangle, \\ |\psi_3\rangle &= -|1, L\rangle + |-1, L\rangle - |1, R\rangle + |-1, R\rangle, \\ |\psi_4\rangle &= -|1, L\rangle + |-1, L\rangle + |1, R\rangle - |-1, R\rangle. \end{aligned} \quad (12)$$

The propagation constants of these hybrid modes are

$$\begin{aligned} \beta_1 &= \tilde{\beta} + \frac{c_1 + d_1}{2\tilde{\beta}}, & \beta_2 &= \tilde{\beta} - \frac{c_1 + d_1}{2\tilde{\beta}}, \\ \beta_3 &= \tilde{\beta} + \frac{c_1 - d_1}{2\tilde{\beta}}, & \beta_4 &= \tilde{\beta} - \frac{c_1 - d_1}{2\tilde{\beta}}, \end{aligned} \quad (13)$$

where we have allowed for $\delta\beta \approx \Delta\beta^2/2\tilde{\beta}$ [24]. The other set of hybrid modes has the same structure,

save for the fact that in Eq. (12) all the basis vectors $|l, L(R)\rangle$ should be replaced with the corresponding vectors with the opposite circular polarization. The spectra in Eq. (13) show that initial degeneracy caused by suppression of mode dispersion by spinning the fiber is lifted by the perturbation introduced by the other fiber. As follows from Eq. (12), this simultaneously leads to destruction of a vortex-mode regime in individual spun fibers. The fields of hybrid modes in individual fibers are given by RCP fields $|1, L(R)\rangle \pm |-1, L(R)\rangle$, which are equivalent to Hermite–Gaussian-like fields $\propto \cos \varphi, \sin \varphi$. In this way the OV ceases to be the eigenmode in individual fibers and would decay upon propagation. The details of its evolution will be studied next.

3. Tunneling of Optical Vortices

Consider the evolution of a CV excited at the input end of a coupled spun elliptical fiber. Since this field is no longer an eigenmode of the system, it should be decomposed over the set of modes in Eq. (12): $|\text{CV}(z=0), L\rangle \equiv \sum_k a_k |\psi_k\rangle$. The decomposition coefficients a_k then should be used to obtain the field produced by the left-localized CV $|1, L\rangle$ in the fibers: $|\text{CV}, L\rangle \equiv \sum_k a_k |\psi_k\rangle \exp(i\beta_k z)$. These problems are readily solved; and after a little algebra, one arrives at the following expression for the field:

$$|1, L(z)\rangle = \{\cos(z\Delta\beta)[\cos(z\Delta\beta)|1, L\rangle + i \sin(z\Delta\beta)|1, R\rangle] - \sin(z\Delta\beta)[\sin(z\Delta\beta)|-1, L\rangle - i \cos(z\Delta\beta)|-1, R\rangle]\} \exp(i\tilde{\beta}z), \quad (14)$$

where $\Delta\beta = d_1/2\tilde{\beta}$, $\tilde{\Delta}\beta = c_1/2\tilde{\beta}$.

To study the tunneling of the vortex into the other fiber, one has to study the field $|R\rangle$ in the right fiber:

$$|R\rangle = i[\sin(z\Delta\beta) \cos(z\Delta\beta)|1, R\rangle + \cos(z\Delta\beta) \sin(z\Delta\beta)|-1, R\rangle] \exp(i\tilde{\beta}z). \quad (15)$$

As is evident, the field of the right waveguide is represented by the pure OV $|1, R\rangle$ at such z_0 , where $\cos(z_0\Delta\beta) \sin(z_0\Delta\beta) = 0$. The amplitude A of the

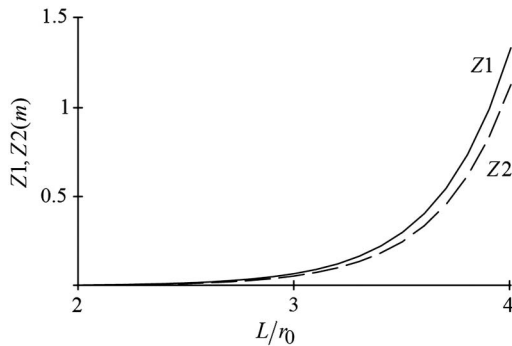


Fig. 2. Dependence of characteristic lengths $Z1$ and $Z2$ on reduced length L/r_0 . Fiber parameters: waveguide parameter $V = 4.16$, $\Delta = 10^{-3}$, $r_0 = 10\lambda_{\text{He-Ne}}$.

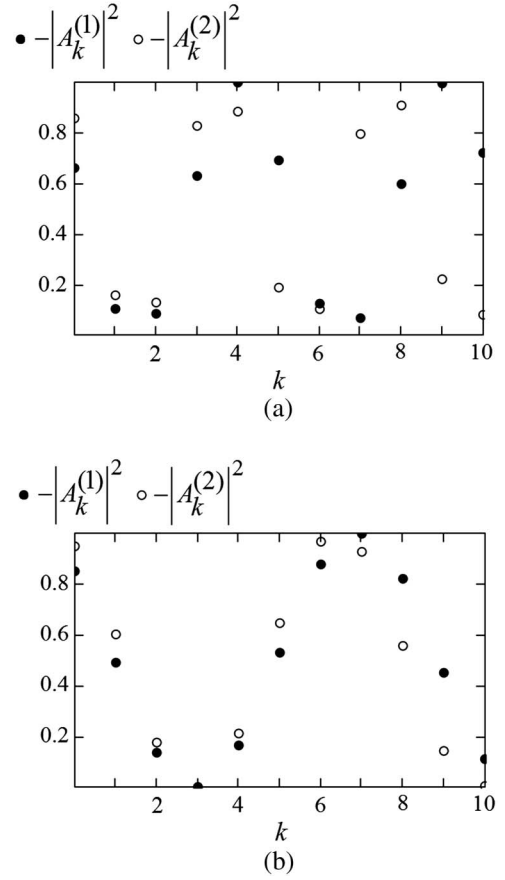


Fig. 3. Dependence of squared modules $|A_k^{(i)}|^2$ of tunneled $l = 1$ OV on k at fixed distances L between the centers of the fibers: (a) $L = 3r_0$ and (b) $L = 5r_0$; $V = 4.16$, $\Delta = 10^{-3}$, $r_0 = 10\lambda_{\text{He-Ne}}$.

vortex then is given by $\sin(z_0\Delta\beta) \cos(z_0\Delta\beta)$. From this condition, one obtains two sets of coordinates of cross sections where the left-localized vortex goes over into the right-localized vortex of the same charge without any impurities:

$$z_k^{(1)} = \frac{(2k+1)\pi}{2\Delta\beta}, \quad z_k^{(2)} = \frac{(k+1)\pi}{\Delta\beta}. \quad (16)$$

The plots of characteristic lengths $Z1 \equiv \frac{\pi}{\Delta\beta}$ and $Z2 \equiv \frac{\pi}{\tilde{\Delta}\beta}$ as functions of the distance between the fibers are given in Fig. 2.

The amplitudes $A_k^{(i)}$ of the tunneled vortex are

$$A_k^{(1)} = \sin \frac{(2k+1)\pi d_1}{2c_1}, \quad A_k^{(2)} = \cos \frac{(k+1)\pi c_1}{2d_1}, \quad (17)$$

where $k = 0, 1, 2, \dots$. Effectiveness of energy transfer is determined by $|A_k|^2$. The plots of successions in Eq. (17) for two values of parameter Λ are given in Fig. 3. As is seen in the figure, at certain k one can achieve rather effective tunneling of the vortex in a pure state. The same problem can be solved for a given k by changing the distance between the fibers. Figure 4 shows the dependence of tunneling efficiency as a function of Λ for $k = 0, 1$.

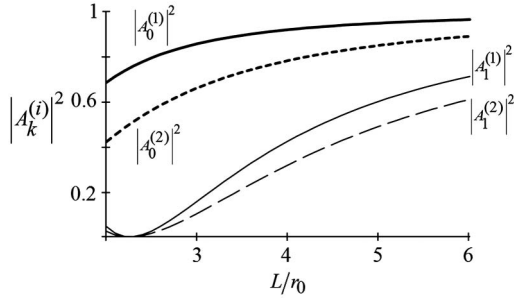


Fig. 4. Dependence of squared modules $|A_k^{(i)}|^2$ of tunneled $l = 1$ OV on reduced length L/r_0 for $k = 0, 1$; $V = 4.16$, $\Delta = 10^{-3}$, $r_0 = 10\lambda_{\text{He-Ne}}$.

In the case of $l \geq 2$ modes, ellipticity does not affect the mode structure essentially, and the modes of separated fibers are given by $l \geq 2$ modes of ideal fibers. This is why one can use for the description of coupled spun fibers the results for tunneling of $l \geq 2$ vortices in parallel ideal waveguides [11]. A simple analysis of the corresponding results shows that for $l \geq 2$ at negligible spin-orbit coupling, the field in the right fiber will be given by

$$|R\rangle \propto \sin(z\Delta\beta_l) \cos(z\Delta\beta_l) |l, R\rangle + \cos(z\Delta\beta_l) \sin(z\Delta\beta_l) |-l, R\rangle, \quad (18)$$

where $\Delta\beta_l = d_l/2\tilde{\beta}$ and $\Delta\beta_l = c_l/2\tilde{\beta}$. This leads to the same expressions for the amplitudes of the tunneled OV $A_k^{(i)}$ as in Eq. (17), save for the only difference: d_1 and c_1 should be replaced with d_l and c_l .

Naturally, along with the process of OV tunneling into the same OAM state, there may take place the tunneling into the state with the inverse topological charge. As follows from Eq. (15), this occurs whenever $\sin(z\Delta\beta) \cos(z\Delta\beta) = 0$. From this condition, one can easily obtain the equations analogous to Eqs. (16) and (17) in positions of such cross-sections and in amplitudes of $|-l, R\rangle$ OV. In this way, such types of directional couplers can perform changing the sign of the topological charge of the incoming OV.

4. Conclusion

In this paper, we have studied the tunneling of a circularly polarized OV in the system of two parallel spun elliptical fibers. We have shown that in the case of strong twisting, where the fiber maintains propagation of circular OVs, it is possible to route the OV in a pure state from one of the fibers to another. We have determined the power efficiency of this process for the set of couplers' length, at which the vortex excited at the input end of one arm tunnels into the pure state in the other arm of the coupler. We have also shown that such directional couplers can serve as inverters of topological charge.

References

1. J. F. Nye and M. Berry, "Dislocations in wave trains," Proc. Roy. Soc. London A **336**, 165–190 (1974).

2. M. Vasnetsov and K. Staliunas, eds., *Optical Vortices*, Vol. 228 of Horizons in World Physics (Nova Publishers, 1999).
3. M. S. Soskin and M. V. Vasnetsov, "Singular optics," in *Progress in Optics*, Vol. 42 (Elsevier, 2001), pp. 219–76.
4. G. Gibson, J. Courtial, M. Padgett, M. Vasnetsov, V. Pas'ko, S. Barnett, and S. Franke Arnold, "Free space information transfer using light beams carrying orbital angular momentum," Opt. Express **12**, 5448–5456 (2004).
5. Z. Bouchal and R. Chelechovsky, "Mixed vortex states of light as information carriers," New J. Phys. **6**, 131 (2004).
6. Z. K. Su, F. Q. Wang, R. B. Jin, R. S. Liang, and S. H. Liu, "A simple scheme for quantum networks based on orbital angular momentum states of photons," Opt. Commun. **281**, 5063–5066 (2008).
7. I. B. Djordjevic, "Deep space and near Earth optical communications by coded orbital angular momentum (OAM) modulation," Opt. Express **19**, 14277–14289 (2011).
8. L. Allen, S. M. Barnett, and M. J. Padgett, *Optical Angular Momentum* (Institute of Physics Publishing, 2003).
9. A. Bekshaev, M. Soskin, and M. Vasnetsov, *Paraxial Light Beams with Angular Momentum* (Nova Publishers, 2008).
10. G. Molina Terriza and M. Padgett, "Special issue on orbital angular momentum," J. Opt. **13**, 060201 (2011).
11. C. N. Alexeyev, T. A. Fadeyeva, N. A. Boklag, and M. A. Yavorsky, "Tunneling of orbital angular momentum in parallel optical waveguides," J. Opt. **13**, 064012 (2011).
12. K. N. Alexeyev, T. A. Fadeyeva, A. V. Volyar, and M. S. Soskin, "Optical vortices and the flow of their angular momentum in a multimode fiber," Semicond. Phys. Quantum Electron. Optoelectron. **1**, 82–89 (1998).
13. C. N. Alexeyev, M. S. Soskin, and A. V. Volyar, "Spin orbit interaction in a generic vortex field transmitted through an elliptic fiber," Semicond. Phys. Quantum Electron. Optoelectron. **3**, 501–513 (2000).
14. K. N. Alekseev, A. V. Volyar, and T. A. Fadeeva, "Spin orbit interaction and evolution of optical eddies in perturbed weakly directing optical fibers," Opt. Spectrosc. **93**, 639–642 (2002).
15. C. N. Alexeyev, A. N. Alexeyev, B. P. Lapin, and M. A. Yavorsky, "Optical angular momentum and mode conversion in optical fibres with competing form and material anisotropy," J. Opt. A **10**, 055009 (2008).
16. C. N. Alexeyev, A. N. Alexeyev, B. P. Lapin, and M. A. Yavorsky, "Controlling the optical angular momentum by elliptical anisotropic fibres," J. Opt. A **11**, 105406 (2009).
17. C. N. Alexeyev and M. A. Yavorsky, "Optical vortices and the higher order modes of twisted strongly elliptical optical fibres," J. Opt. A **6**, 824–832 (2004).
18. C. N. Alexeyev, A. V. Volyar, and M. A. Yavorsky, "Intensely twisted elliptic optical fibres maintaining propagation of a single optical vortex," J. Opt. A **8**, L5–L9 (2006).
19. K. N. Alekseev and M. A. Yavorskii, "Twisted optical fibers sustaining propagation of optical vortices," Opt. Spectrosc. **98**, 53–60 (2005).
20. V. I. Kopp, V. M. Churikov, J. Singer, N. Chao, D. Neugroschl, and A. Z. Genack, "Chiral fiber gratings," Science **305**, 74–75 (2004).
21. V. S. Oh, K. R. Lee, U. C. Paek, and Y. Chung, "Fabrication of helical long period gratings by use of a CO₂ laser," Opt. Lett. **29**, 1464–1466 (2004).
22. V. I. Kopp, V. M. Churikov, J. Singer, N. Chao, D. Neugroschl, and A. Z. Genack, "Single and double helix chiral fiber sensors," J. Opt. Soc. Am. B **24**, A48–A51 (2007).
23. C. N. Alexeyev, T. V. Borshak, A. V. Volyar, and M. A. Yavorsky, "Angular momentum conservation and coupled vortex modes in twisted optical fibres with torsional stress," J. Opt. A **11**, 094011 (2009).
24. A. W. Snyder and J. D. Love, *Optical Waveguide Theory* (Chapman & Hall, 1985).
25. A. Galtarossa and C. R. Menyuk, eds. *Polarization Mode Dispersion. Optical and Fiber Communications Reports* (Springer, 2005).
26. C. N. Alexeyev, N. A. Boklag, and M. A. Yavorsky, "Higher order modes of coupled optical fibres," J. Opt. **12**, 115704 (2010).
27. A. S. Davydov, *Quantum Mechanics* (Pergamon/Oxford, 1976).