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Replicator dynamics in public goods games with reward funds

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Abstract

Which punishment or rewards are most effective at maintaining cooperation in public goods interactions and deterring defectors who are willing to freeload on others' contribution? The sanction system is itself a public good and can cause problematic "second-order free riders" who do not contribute to the provisions of the sanctions and thus may subvert the cooperation supported by sanctioning. Recent studies have shown that public goods games with punishment can lead to a coercion-based regime if participation in the game is optional. Here, we reveal that even with compulsory participation, rewards can maintain cooperation within an infinitely large population. We consider three strategies for players in a standard public goods game: to be a cooperator or a defector in a standard public goods game, or to be a rewarder who contributes to the public good and to a fund that rewards players who contribute during the game. Cooperators do not contribute to the reward fund and are therefore classified as second-order free riders. The replicator dynamics for the three strategies exhibit a rock-scissors-paper cycle, and can be analyzed fully, despite the fact that the expected payoffs are nonlinear. The model does not require repeated interaction, spatial structure, group selection, or reputation. We also discuss a simple method for second-order sanctions, which can lead to a globally stable state where 100% of the population are rewarders.

Keywords: evolutionary game theory; cooperation; sanction; second-order social dilemma; rock-scissors-paper cycle

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1. Introduction

An enduring conundrum in the biological and social sciences is how cooperation can emerge and be maintained in a sizable group containing exploiters. The conundrum is the so-called social dilemma [1, 2] because its nature is described as follows: groups of cooperators outperform groups of defectors, whereas in a mixed group defectors always outperform cooperators. This represents common conflicts between a social optimum and individual interests very well, and it has traditionally been modeled as the public goods game in many experimental and theoretical studies [3].

In the public goods game (PGG), cooperators confer benefits on others with some cost to themselves, whereas defectors exploit the benefits without such contribution to others. Defection is the selfish choice that results in a decrease in the total benefit to the group, but defection is rational from the evolutionary viewpoint because of a higher individual payoff, with no cost. Thus, natural selection will often drive elimination of cooperation. Classical and evolutionary game studies have, however, identified supportive mechanisms under which cooperation is nonetheless sustained, such as repeated interactions [4, 5], reputation [6, 7], spatial structure [8, 9], and group selection [10, 11].

Punishment of defectors and rewards for cooperators are also major factors that maintain cooperation between self-interested individuals, as suggested by growing experimental and theoretical evidence [12–32]. However, sanctions are costly, and therefore pose the next conundrum: how costly sanctioning can subsist in the presence of those who freeload on others' contributions to sanctions. This issue is the "second-order social dilemma" [12, 14], which has been particularly well addressed, in the case of costly punishment. One of possible solutions is to punish second-order freeloaders as well [13, 15, 24, 32].

At the same time, there is the issue of how costly punishment can emerge [21, 33]. In a population of defectors, a rare punisher suffers enormous costs because of the need to continuously punish defectors. However, recent studies have shown that punishment-based

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cooperation can emerge if participation in the PGG is optional rather than compulsory [20, 21, 26, 32]. We note that optional participation is another way to maintain cooperation [33–39], which can lead to "rock-scissors-paper"-type cyclic domination, well-known in evolutionary game theory [40, 41], among cooperators, defectors, and loners who earn a small but fixed payoff, instead of participating in the PGG [37–39]. Interestingly, Sigmund *et al*. [32] have found that when it comes to punishing second-order freeloaders, natural selection favors pool-punishment rather than peer-punishment. Peer-punishment is a sanctioning technique which has been the most widely used form of punishment in PGGs in which players decide whether to impose fines on exploiters *after* the PGG. By contrast, in pool-punishment, players have to decide whether to contribute to a punishment fund *before* the PGG [14], analogous to forming a volunteer band of watchmen in advance.

While optional participation could be required for a population to evolve from a stalemate where everybody defects to a coercion-based regime, there problems associated with opting out of a public goods project, such as global environmental issues, remain [21]. When participation is compulsory, peer-rewarding can cause cyclical dynamics in infinite populations if reputation alone is important (for pair-wise interactions see Sigmund et al. [16]; for interactions of arbitrary size see Hauert [30]). In contrast, reputation is given less weight in finite populations [29].

In this work, we explore the effects of pool-rewarding in compulsory PGGs with infinite populations. Similar to pool-punishment, players first decide whether to contribute to a reward fund. After a one-shot PGG among all group members, the common fund is divided equally among those players who contributed, irrespective of their contribution to the fund. While the list of real-world examples of reward funds is too long to list, we shall consider a generous voluntary fund which may be threatened with collapse by second-order freeloaders. We propose a minimalistic model for infinite populations that does not require repeated interactions, reputation, spatial structure, group selection, or optional participation. We also compare two types of benefit-sharing models, which differ on whether or not a

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contributor in the PGG may oneself benefit, thus corresponding to "weak altruism" and "strong altruism" [42, 43]. The evolution of cooperation is investigated by means of the replicator dynamics [40, 41].

2. The game-theoretical model

Consider an infinitely large, well-mixed population of constant size. From time to time, a group of N players is randomly formed from the population (where $N \geq 2$). The PGG is of a one-shot version. Each player is asked to contribute $c_1 > 0$ to the public good. The contributions are then distributed in the following different ways: in the case of weak altruism (**WA**), the contribution, c_1, will be multiplied by $r_1 > 1$ and then equally shared among all N players in the group, but in the case of strong altruism (**SA**), it will be shared among $N-1$ other co-players only. In both cases, if all group members contribute, they obtain a payoff of $(r_1 - 1)c_1 > 0$. The PGG with **SA** is a social dilemma for any rate of r_1, and the PGG with **WA**, also for $r_1 < N$. Indeed, in each case, a player that does not contribute to the public goods can get an improved payoff by c_1 with **SA**, and by $c_1\left(1 - r_1 / N\right) > 0$ with **WA**, no matter what the other players do. For the PGG with **WA**, we assume $r_1 < N$, as the social dilemma would otherwise be completely relaxed due to the benefits by switching to a contributor.

Next, we introduce the following pool-rewarding mechanism. Before participating in the PGG, each player is first asked to contribute $c_2 > 0$ to a fund to reward cooperative behaviors in the PGG. The integrated contribution to the reward fund is multiplied by $r_2 > 1$, and after the PGG distributed equally to those who have contributed to the public good, if any.

We consider the following three strategies: rewarders (R) who contribute both to the PGG and to the reward fund, cooperators (C) who contribute to the PGG but not to the reward fund, and defectors (D) who contribute neither to the PGG nor to the reward. If all S contributors in the PGG are R-players, they each obtain a net reward of $(r_2 - 1)c_2 > 0$, and if all of them are C-players, they obtain nothing. The rewarding system is a second-order social dilemma for $r_2 < S$ because withdrawing one's contribution to the reward fund can increase individual payoff by $c_2\left(1 - r_2 / S\right) > 0$.

We note that pool-rewarding itself is another case of weak altruism: an R-player is allowed to obtain a return from contributing to the reward fund. We do not eliminate a return for individuals who choose to contribute to rewards. R-players would be more likely to evolve with it than without it. In the latter case D-players dominate (See Appendix A.1 for details).

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Nevertheless, it is not clear whether or not such weakly altruistic, reward system can subsist in the presence of second-order freeloaders. Indeed, the funding stage is set up before the PGG and thus R-players cannot avoid the risk of being exploited by C-players.

We denote the expected payoff values for R-, C-, and D-players with P_R, P_C, and P_D, respectively. The frequencies of the three strategies are expressed as x, y, and z ($x+y+z=1$). The average payoff for the population is given by $\overset{G}{P} = xP_R + yP_C + zP_D$. The strategy's expected payoff is supposed to be the sum of the payoff from the PGG and from the reward fund. The replicator equations are written as

$$\dot{x} = x\left(P_R - \overset{G}{P}\right),\ \ \dot{y} = y\left(P_C - \overset{G}{P}\right),\ \ \dot{z} = z\left(P_D - \overset{G}{P}\right). \tag{1}$$

We first calculate the expected payoffs from the PGG. In the case of **WA**, a D-player in a group with S contributors obtains a benefit of r_1c_1S/N ($0 \le S \le N-1$). Hence, the expected payoff is given by

$$\begin{aligned} P_D^1 &= \sum_{S=0}^{N-1}\binom{N-1}{S}(1-z)^S z^{N-S-1}\frac{r_1c_1S}{N} \\ &= r_1c_1\left(1-\frac{1}{N}\right)(1-z), \end{aligned} \tag{2a}$$

where $\binom{N-1}{S}(1-z)^S z^{N-S-1}$ is the probability that S of $N-1$ co-players in the PGG are contributors. In the case of **SA**, a D-player in the group obtains a benefit of $r_1c_1S/(N-1)$, and calculating the expected payoff as in Eq. (2a),

$$P_D^1 = r_1c_1(1-z). \tag{2b}$$

Both the expected payoffs for R- and C-players (denoted by P_R^1, resp. P_C^1) are reduced from P_D^1, by the cost for a contributor σ: $\sigma = c_1(1-r_1/N)$ in the case of **WA** and $\sigma = c_1$ in the case of **SA**.

Regarding the reward system, the expected payoff for D-players is $P_D^2 = 0$. A C-player in a group with S contributors and n_R R-players (and thus $S-n_R$ C-players) receives a reward of $r_2c_2n_R/S$ ($0 \le n_R \le S-1$). Hence, the expected reward for a C-player in a group with S contributors is

$$\begin{aligned} P_C^2(S) &= \sum_{n_R=0}^{S-1}\binom{S-1}{n_R}\left(\frac{x}{1-z}\right)^{n_R}\left(\frac{y}{1-z}\right)^{S-n_R-1}\frac{r_2c_2n_R}{S}, \\ &= r_2c_2\left(1-\frac{1}{S}\right)\left(\frac{x}{1-z}\right), \end{aligned}$$

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(3)

where $\binom{S-1}{n_R}\left(\frac{x}{1-z}\right)^{n_R}\left(\frac{y}{1-z}\right)^{S-n_R-1}$ is the probability that n_R of the other $S-1$ contributors are R-players. Consequently, the expected reward for a C-player is

$$P_C^2=\sum_{S=1}^{N}\binom{N-1}{S-1}(1-z)^{S-1}z^{N-S}P_C^2(S)$$

$$=r_2c_2\left(1-\frac{1-z^N}{N(1-z)}\right)\left(\frac{x}{1-z}\right) \quad .$$

(4)

Among S contributors, switching from R to C yields $c_2(1-r_2/S)$. Thus, the expected net reward for an R-player, P_R^2, is reduced from P_C^2 by

$$c_2\sum_{S=1}^{N}\binom{N-1}{S-1}(1-z)^{S-1}z^{N-S}\left(1-\frac{r_2}{S}\right)=c_2\left(1-\frac{r_2}{N}\frac{1-z^N}{1-z}\right)$$

$$=:F(z)\,. \qquad (5)$$

$F(z)$ has a unique root $\hat{z}$ in the open interval $(0,1)$ if, and only if, $1<r_2<N$, because $F(z)$ is monotonic, $F(0)=c_2(1-r_2/N)>0$, and $F(1)=c_2(1-r_2)<0$. Therefore, the advantage C-players have over R-players will change from positive to negative as z increases across $\hat{z}$.

Integrating the above results, we can determine that $P_R=P_R^1+P_R^2$, $P_C=P_C^1+P_C^2$, and $P_D=P_D^1+P_D^2$, and obtain a simple expression for the average payoff for the population

$$\bar{P}=c_1(r_1-1)(1-z)+c_2(r_2-1)x\,, \qquad (6)$$

both for the **WA** and **SA** cases.

3. Dynamics

The evolutionary dynamics of the three strategies take place in the state space $S_3=\{(x,y,z):x,y,z\geq 0,x+y+z=1\}$. The three homogeneous states in which 100% of the population are R-players ($x=1$), C-players ($y=1$), and D-players ($z=1$) correspond to three vertices of the simplex S_3 (which we denote by R, C, and D, respectively). These are obviously fixed points for the replicator system Eq. (1). There are no other fixed points on the boundary of S_3 for non-degenerate cases. Indeed, on the edge C-D: $x=0$, $\dot{z}=(P_D-P_C)z(1-z)=\sigma z(1-z)>0$, where $\sigma=c_1(1-r_1/N)$ in the case of **WA** and $\sigma=c_1$ in the case of **SA**. Thus, the evolution on the edge C-D is unidirectional from C to D.

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On the edge R-C: $z=0$ and on the edge D-R: $y=0$, resulting in $\dot{y}=\left(P_C-P_R\right)y\left(1-y\right)=c_2\left(1-r_2/N\right)y\left(1-y\right)$ and $\dot{x}=\left(P_R-P_D\right)x\left(1-x\right)=[c_2\left(r_2-1\right)-\sigma]x\left(1-x\right)$, respectively. The evolution on both edges is unidirectional and its direction depends on the magnitude of the relationship between r_2 and N, and between $c_2\left(r_2-1\right)$ and σ, respectively.

To analyze the dynamics in the interior of S_3, let us introduce a new variable $f=x/(1-z)$, which represents the fraction of contributors in the PGG that are also rewarders. This yields

$$\dot{f}=-\frac{xy}{(1-z)^2}\left(P_C-P_R\right)=-f\left(1-f\right)F(z). \tag{7}$$

Substituting $x=f(1-z)$ and Eq. (6) into $\dot{z}=z\left(P_D-\overset{G}{P}\right)$ yields

$$\dot{z}=-z\left(1-z\right)[c_2\left(r_2-1\right)f-\sigma]. \tag{8}$$

3.1. The global attractor D

Supposing $c_2\left(r_2-1\right)-\sigma<0$, then the direction of evolution on the edge D-R is from R to D. Eq. (8) yields $\dot{z}>0$ in the interior of S_3. Thus, there is no interior fixed point and all interior orbits converge to the vertex D, which is a global attractor (Fig. 1a). If $r_2<N$, the direction of evolution on the edge R-C is from R to C; if $r_2>N$ and otherwise, it is from C to R; and when $r_2=N$, the edge R-C consists of unstable fixed points. We note that if $r_2<1$, then $c_2\left(r_2-1\right)-\sigma<0$ holds. In the boundary case that $c_2\left(r_2-1\right)-\sigma=0$, $\dot{z}=0$ holds when $f=1$ and thus, the edge D-R is a line of fixed points. If $r_2<N$, the edge is separated into an unstable segment ($0\le z<\hat{z}$) and a stable one ($\hat{z}<z\le 1$). Since $\dot{z}>0$ holds in the interior of S_3, all interior orbits converge to the stable segment (Fig. 1b). If $r_2\ge N$, then the edge D-R has no unstable segment. Random drift and occasional invasion of the missing C-player will eventually send the state within the stable segment to the vertex D, in the long run.

[Fig. 1 is around here]

3.2. The global attractor R

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Supposing $c_2(r_2-1)-\sigma>0$ and $r_2>N$, then the direction of evolution on the edge D-R is from D to R, and from C to R on the edge R-C. The fact that $F(z)<0$ in the open interval $(0,1)$ yields $\dot{x}>0$ in the interior of S_3. Thus, there is no interior fixed point and all interior orbits converge to the vertex R, which is a global attractor (Fig. 2). If $r_2=N$, then the edge R-C is a line of fixed points, which consists of an unstable segment ($0\le x<x_{RC}$) and a stable one ($x_{RC}<x\le 1$), where x_{RC} is given by $\sigma/[c_2(r_2-1)]$ as a non-trivial solution of Eq. (8). The fact that all interior states satisfy $\dot{x}>0$ leads the population to evolve towards the stable segment. Thus, random drift and occasional invasion of the missing D-player will eventually bring the population to the vertex R, in the long run.

[Fig. 2 is around here]

3.3. The mixture equilibrium of the three strategies

Supposing that $c_2(r_2-1)-\sigma>0$ and $1<r_2<N$, the direction of evolution on the edge D-R is from D to R, and from R to C on the edge R-C. Thus, the three edges of S_3 form a heteroclinic cycle of a rock-scissors-paper type. We now have a unique interior root $\hat{z}$ of $F(z)$ and $0<\hat{f}:=\sigma/[c_2(r_2-1)]<1$. From Eqs. (7) and (8), we see that there is a unique interior fixed point $Q=(\hat{x},\hat{y},\hat{z})$, with

$$\hat{x}=\hat{f}\left(1-\hat{z}\right),\quad \hat{y}=(1-\hat{f})\left(1-\hat{z}\right). \tag{9}$$

The mixture equilibrium, *Q*, is a center, i.e., it is neutrally stable and surrounded by closed orbits that fill the interior of S_3 (Fig. 3). This results because the equations of Eqs. (7) and (8) can be expressed in the form of a Hamiltonian system, *H*, and now *H* has a strict maximum at the unique fixed point $(\hat{f},\hat{z})$ corresponding to *Q* (See Appendix A.2 and [30] for details).

Given c_1, r_1, and N, which are all original parameters for the PGG, the location of *Q* can be determined by the remaining parameters, c_2 and r_2. According to Eq. (9), *Q* lies on the line $y=(1/\hat{f}-1)x$, independent of the group size, N. As N increases, *Q* moves toward the vertex D along the line y and $\hat{z}\to 1$ as $N\to\infty$. On the other hand, as N decreases, *Q* moves in the opposite direction and $\hat{z}$ decreases to $2/r_2-1>0$, which

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occurs when $N = 2$. In other extreme cases, where $r_2 = 1$, $r_2 = N$, $c_2(r_2 - 1) = \sigma$, and $c_2 = \infty$, Q arrives at the vertex D, the edges R-C, D-R, and C-D, respectively.
[Fig. 3 is around here]

4. Discussion

Conflict between contributors and freeloaders in public-goods interactions is inevitable. How can we avoid conflict between contributors and freeloaders? An effective solution is to set up a reward fund for cooperative behaviors. The key conditions for the reward system necessary to maintain cooperation with free riders in public goods games (PGGs) are given by

$$c_2(r_2 - 1) > \sigma, \tag{10}$$

where $\sigma = c_1(1 - r_1 / N)$ in the case of weak altruism and $\sigma = c_1$ in the case of strong altruism. Eq. (10) means that the optimum group reward should exceed the cost for a contributor in the PGG, which is relaxed by a self-returning benefit of $r_1 c_1 / N$ in the case of weak altruism. In infinite populations, it has been determined that peer-rewarding is a potent motivator, but only if reputation is important [16, 30]. However, in pool-rewarding, this is not the case. With such attractive rewards, cooperative investments in both the PGG and the reward fund can subsist, even when second-order freeloaders can dominate the rewarding system, i.e., for $r_2 < N$. In the case, the replicator dynamics exhibit a rock-scissors-paper cycle among the three strategies: defectors who never contribute (first-order freeloaders), cooperators who contribute only in the PGG (second-order freeloaders), and rewarders who contribute to both.
The cyclical evolutionary scenario can be described as follows. If most players are rewarders, the reward system is actually a second-order social dilemma and thus cooperators spread. If cooperators are prevalent, it is better to become a defector due to the social dilemma. If most players are defectors, the number of beneficiaries of the reward is usually small enough to subvert cooperator dominance over rewarders, and thus the number of rewarders increases. If the number of rewarders increases sufficiently, then the second-order dilemma returns. In this scenario, traditional defectors play a pivotal role in maintaining the cyclic domination among the three strategies. The moderate advantage defectors have over cooperators, given by σ, prevents the second-order dilemma from eliminating rewarders and then ensures that rewarders, not cooperators, dominate.

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Global environmental and energy issues often appear to be compulsory public goods projects, such that in the short-term cooperation will yield only very little benefit and the social optimum is not to cooperate. The situation is not a social dilemma, and has thus remained outside the scope of studies on the evolution of cooperation in large groups. In our model, this may correspond to the case where $0 \le r_1 < 1$. We remark that the results shown hold even when $0 \le r_1 < 1$, and thus pool-rewarding is applicable to a broader range of public-goods interactions.

We note that in the extreme case where $r_1 = 0$, our model is significantly similar to an earlier public goods game with optional participation [37, 38, 39]. Indeed, the PGG degenerates into a game in which there is no longer benefit from contribution c_1. Each player therefore seems to have the option to avoid the participation fee of c_1, instead of taking part in another PGG with a cost of c_2 and a multiplier of r_2. This is just an implementation of the inverse form of the loner's option.

A fascinating extension of this work is to consider second-order sanctions [13, 15, 24, 32]. Indeed, in our model, it looks practical for the rewarding system to mete out punishment on cooperators (second-order freeloaders) in such a way that will reduce rewards for those [12]. Let us see how, for instance, reducing rewards to cooperators by a% changes the dynamics. According to preliminary numerical simulations, the existing interior fixed point Q is destabilized (Fig. 4), and for discount rates a higher than a threshold value, the population can converge to a state of 100% rewarders, irrespective of the initial conditions (Fig. 4b). As increasing a crosses the threshold, a new mixture equilibrium P, of cooperators and rewarders, enters the state space S_3 and is unstable within the rewarder-cooperator boundary (See Appendix A.3 for details). If defectors (first-order freeloaders) are absent, the population cannot avoid the resulting coordination problem: depending on the initial condition, the population evolves to become either 100% rewarders or 100% cooperators. Otherwise, interestingly, the population can make an end run around the bistability and establish the social optimum. It would be a rather intriguing issue for future research to theoretically analyze the result that reward-based cooperation will necessarily become globally stable, whenever it cannot be invaded by second-order freeloaders. By contrast, in the case of pool-punishment, punishment-based cooperation can never become globally stable, even if second-order sanctions are assumed, because a state of 100% first-order freeloaders remain stable [44].

[Fig. 4 is around here]

One important issue we left out is the effects of economies and diseconomies of scale on the provision of sanctions. So far we have focused on linear cost-benefit functions for rewarding, whereby any group of rewarders generates the same per capita group benefit. According to Mathew and Boyd [33], the existing interior fixed point of the optional public goods game becomes an attractor for decreasing returns and a repeller for increasing returns.

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In practice, the rich dynamics afforded by scale would provide many options for the proper design of sanctioning systems to support the evolution of cooperation.

Appendix

A.1. The strongly altruistic rewarding

We here turn to a strongly altruistic variant of pool-rewarding, in which the rewards resulting from an R-player will be shared among *other* contributors only. We assume that if there exists no other contributor, the investment to the incentive from a single R-player will be exactly refunded to her. The expected reward for a C-player turns into

$$P_C^2 = r_2 c_2 \left(\frac{x}{1-z} \right) \left(1 - z^{N-1} \right),$$

and that for an R-player is reduced from P_C^2 by the expected incentive cost $c_2 \left(1 - z^{N-1} \right)$. Eqs. (7) and (8) turn into

$$\dot{f} = -c_2 f \left(1 - f \right) \left(1 - z^{N-1} \right),$$

$$\dot{z} = -z(1-z)[c_2 \left(r_2 - 1 \right) f \left(1 - z^{N-1} \right) - \sigma].$$

Since $\dot{f}$ is negative in the interior of the state space S_3, $int\, S_3$, there is no interior fixed point. If $c_2 \left(r_2 - 1 \right) - \sigma \le 0$, then $\dot{z} > 0$ holds in $int\, S_3$, and thus, all interior orbits converge to the vertex D.

If $c_2 \left(r_2 - 1 \right) - \sigma > 0$, the system has a new equilibrium at $\left(f, z \right) = \left(1, \left(1 - \frac{\sigma}{c_2 \left(r_2 - 1 \right)} \right)^{\frac{1}{N-1}} \right)$ on the edge D-R, which is a source. The vertex D is a sink, while the vertex R still remains a saddle. We consider the z-isocline that is the set where $\dot{z} = 0$: in $int\, S_3$, this is the set where $f = \frac{\sigma}{c_2 \left(r_2 - 1 \right) \left(1 - z^{N-1} \right)}$. The interior component forms a curve that connects the new fixed point and the point $\left(f, z \right) = \left(\frac{\sigma}{c_2 \left(r_2 - 1 \right)}, 0 \right)$ on the edge R-C, and divides $int\, S_3$

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to two regions: one region where $\dot{z} < 0$ and the other where $\dot{z} > 0$. The last one includes the vicinity of the edge C-D given by $x = 0$. Since $\dot{f} < 0$ holds in $int\, S_3$, any interior orbit which starts in the state with $\dot{z} < 0$ has to travel to the region where $\dot{z} > 0$. Hence, all interior orbits converge to the vertex D.

A.2. The Hamiltonian System

Divide the right-hand side of Eqs. (7) and (8) by the function $f(1-f)z(1-z)$, which is positive for any (f,z) in the interior of the unit square $[0,1]^2$. Hence,

$$\dot{f} = \frac{-F(z)}{z(1-z)} =: -g(z), \quad \dot{z} = \frac{\sigma - c_2(r_2-1)f}{f(1-f)} =: -l(f).$$

This transformation corresponds to a change in velocity and does not affect orbit. Let us introduce $H(f,z) := G(z) + L(f)$, where $G(z)$ and $L(f)$ are primitives of $g(z)$ and $l(f)$, respectively:

$$G(z) = c_2\left(1 - \frac{r_2}{N}\right)\log z + c_2(r_2-1)\log(1-z) + R(z),$$

$$L(f) = \sigma \log f + [c_2(r_2-1) - \sigma]\log(1-f).$$

with $R(z)$ bounded on $[0,1]$. Thus, we obtain the Hamiltonian system

$$\dot{f} = -\frac{\partial H}{\partial z}, \quad \dot{z} = \frac{\partial H}{\partial f}.$$

Because the system is conservative and the Hamiltonian attains a strict global maximum at $(\hat{f},\hat{z})$ if $c_2(r_2-1) - \sigma > 0$ and $1 < r_2 < N$, the interior equilibrium Q is a stable point surrounded by closed orbits. Indeed, all interior orbits are closed: $G(z) \to -\infty$ as $z \to 0,1$ if $1 < r_2 < N$ and $L(f) \to -\infty$ as $f \to 0,1$ if $0 < \sigma < c_2(r_2-1)$. Hence, $H \to -\infty$ uniformly near the boundary of $[0,1]^2$ and thus all constant level sets of H are closed curves around $(\hat{f},\hat{z})$. The solutions have to remain on the constant level sets and thus return to their starting points.

A.3. The second-order sanctioning

We examine an extensive model in which rewards for cooperators (second-order freeloaders) will be reduced by $100\alpha\%$ ($0 \le \alpha \le 1$), under the assumptions that

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$c_2(r_2-1)>\sigma$ and $1<r_2<N$. In the extension, the expected payoff for a cooperator is given by

$$P_C^2=(1-\alpha)r_2c_2\left(1-\frac{1-z^N}{N(1-z)}\right)\left(\frac{x}{1-z}\right),$$

and, Eqs. (7) and (8) turn to

$$\dot{f}=-f(1-f)[F(z)-\alpha(c_2(r_2-1)+F(z))f],$$

$$\dot{z}=-z(1-z)[c_2(r_2-1)f-\sigma-\alpha(c_2(r_2-1)+F(z))f(1-f)].$$

In the interior of S_3, there exists at most one fixed point $Q=(f_Q,z_Q)$ such that

$$f_Q=\frac{\sigma}{\alpha\sigma+(1-\alpha)c_2(r_2-1)} \quad \text{and} \quad F(z_Q)=c_2(r_2-1)\ \frac{\alpha f_Q}{1-\alpha f_Q}.$$

The fact that $F(z)$ is monotonically decreasing and $F(z_Q)\geq 0$ yields that $0<z_Q\leq\hat{z}$, where $\hat{z}$ is the unique solution of $F(z)=0$. f_Q increases and z_Q decreases, with increasing α. This implies that as α increases, Q moves towards the edge R-C. As α crosses a threshold α_P given by $\frac{F(0)}{c_2(r_2-1)+F(0)}$, a new equilibrium with $(f,z)=\left(\frac{N-r_2}{\alpha r_2(N-1)},0\right)$ enters the edge R-C through the vertex R, which then turns into a sink. The boundary equilibrium, P, is a saddle point, unstable within the edge and stable to invasion of defectors. As α further increases, P moves towards the vertex C, and when α crosses another threshold α_Q given by $\frac{F(0)}{\sigma+F(0)}$, Q exits S_3 through P, which then turns into a source. For larger values of α, S_3 has no interior equilibrium but P still remains within the edge. Preliminary numerical simulations imply that Q is a source for $\alpha>0$, and all interior orbits converge, if $0<\alpha<\alpha_P$, to a heteroclinic cycle on the boundary of S_3, and if $\alpha_P<\alpha\leq 1$, to the vertex R.

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Figure Captions

Figure 1. Defectors (first-order freeloaders) prevail. Oscillations do not occur and the interior state space has no fixed point. (a) All interior states evolve towards the vertex D. (b) In the boundary case that $c_2\left(r_2-1\right)-\sigma=0$, the edge D-R is a line of fixed points. All interior orbits converge to a stable (lower) segment of the edge. Random drift and occasional invasion of the missing C-player will eventually send the state to the vertex D. Parameters: $N=5$; $r_1=3$; $c_2=1$; $r_2=1.2$ (a) or 1.4 (b); $\sigma=0.4$; and $c_1=1$ (in the case of **WA**), $c_1=0.4$ (in the case of **SA**).

Figure 2. Rewarders prevail. Oscillations do not occur and all interior states evolve towards the vertex R. The interior state space has no fixed point. Parameters: $N=5$; $r_1=3$; $c_2=1$; $r_2=5.5$; $\sigma=0.4$; and $c_1=1$ (in the case of **WA**), $c_1=0.4$ (in the case of **SA**).

Figure 3. Rock-scissors-paper cycles. All three corners of the simplex S_3 are saddle points and the boundary of S_3 represents a heteroclinic cycle. The interior of S_3 has a unique fixed point Q, which is a center surrounded by closed orbits. Parameters: $N=5$; $r_1=3$; $c_2=1$; $r_2=3$; $\sigma=0.4$; and $c_1=1$ (in the case of **WA**), $c_1=0.4$ (in the case of **SA**).

Figure 4. The effects of second-order sanctions. (a) The existing interior fixed point Q turns into a repeller by cutting off $a\%$ rewards for cooperators. The population converges to a heteroclinic cycle on the boundary of S_3. (b) For a sufficiently high a, the vertex R can be a global attractor. At the same time, S_3 has a boundary fixed point P, which divides the basins of attraction of rewarders and cooperators on the edge R-C and is stable if there is an invasion of defectors. Parameters: $N=5$; $r_1=3$; $c_2=1$; $r_2=3$; $\sigma=0.4$; and $c_1=1$ (in the case of **WA**), $c_1=0.4$ (in the case of **SA**). The rewards are cut by the following percentages (a) $a=10$ and (b) $a=20$.

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Fig 1

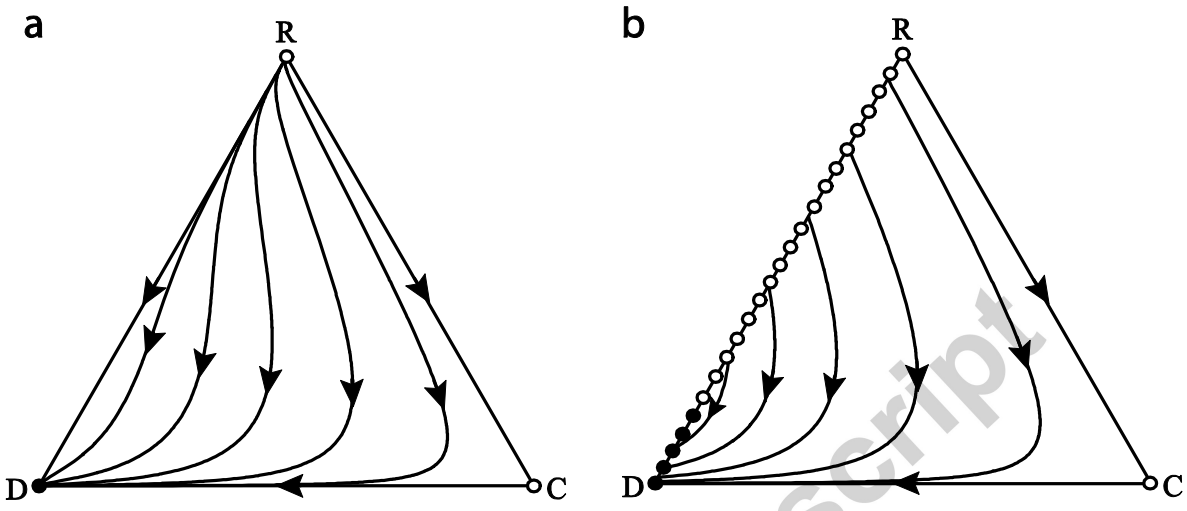

Fig 2

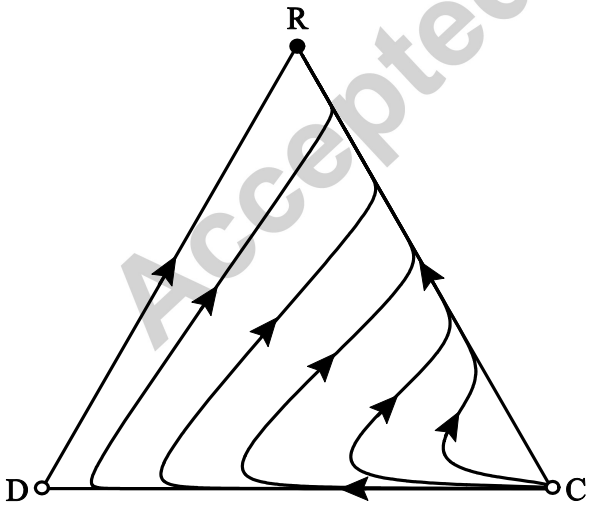

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Fig 3

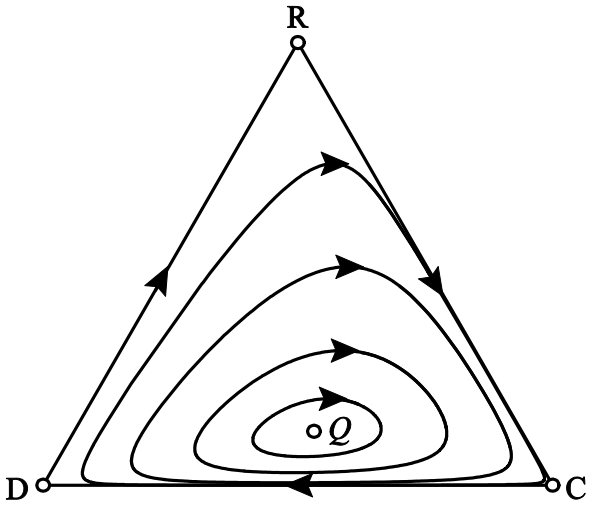
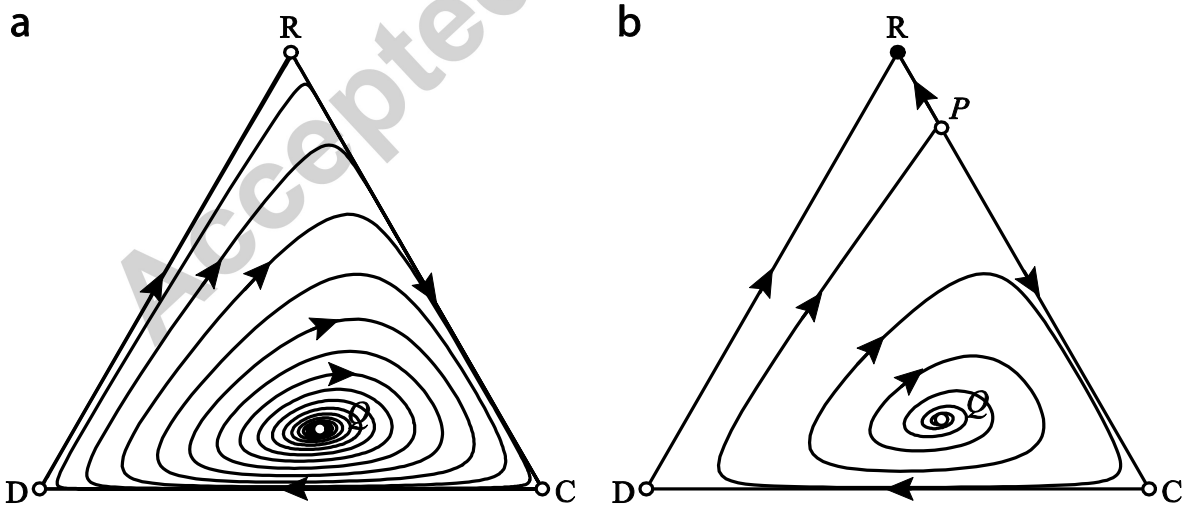


Fig 4



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Highlights

> We investigate how a reward fund affects the evolution of cooperation.
> The reward fund can evolve in the presence of second-order free riders.
> Rewarders, second-, and first-order free riders can form a rock-scissors-paper cycle.
> The replicator dynamics can fully be analyzed despite the nonlinear payoff functions.
> The model does not require iterated game, reputation, or optional participation.

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