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Makespan minimization for two-stage hybrid Flow shop with dedicated machines and additional constraints

Nacira CHIKHI and Moncef ABBAS

Faculty of Mathematics / USTHB
BP 32 El-Alia
Bab-Ezzouar, Algiers, Algeria
nacira.chikhi@gmail.com, mabbas@usthb.dz

ABSTRACT: *This paper considers a two-stage hybrid flow shop scheduling problem. The objective is the minimizing of maximum completion time of all the jobs. There exist two dedicated machines at stage 1 and one machine at stage 2. Each job is defined by two operations processed on the two-stages in series. Depending on the job type, the job is processed on either of the two machines at stage1 and must be processed on the single machine at stage2. the jobs are transported between the stages by a robot or a conveyor. After the problem formulation, we present lower bounds for the objective function. We then discuss a few polynomially solvable cases of the problem and present the solution algorithms. Since a general case of the problem is strongly NP-hard, we propose two heuristics to find approximate solutions to the general case. Computational experiments are done on a number of randomly generated test problems, and the test results are reported.*

KEYWORDS: *sheduling, flexible flow shop, transport, two-stage makespan.*

1 Introduction

Our problem may be defined as follows. We are given a set N of n independent jobs distributed in two disjoint subsets which have to be scheduled on a two-stage flow shop in series; the first stage contains two dedicated machines and the second stage contains one common single machine with unlimited buffer spaces. We assume that all the jobs are available at time 0 and each job has exactly two operations to be executed on the two-stages. Preemption is not allowed and the machines can process only one job at a time. Additionally, transportation times are considered. They occur if a job changes from one machine to another between which the transport takes place. We assume that all these transport operations have to be done by a single transport robot (conveyor) which can carry up to c jobs in one shipment. The transportation time from one machine to the other machine is denoted by t (the round-trip requires 2t). We assume that loading and unloading times are included in the processing times of jobs and are not considered separately. Our goal is to schedule the jobs so as to minimize the makespan.

As applications of the dedicated machines cover a large class of real problems: service industry (H. C. Hwang et al., 2004), semiconductor manufacturing environment (G. Centeno and R. L. Armacost, 2004),..., similar examples can be found in process industry such as chemical and pharmaceutical industries. We can find another example at a global manufacturing firm where products are first fabricated in one location and transferred to factories in different countries for final assembly. In this case, each factory can be considered as a machine. The products areas sembled in different locations because they are sold in those countries and by doing so, the firm can minimize tariffs and reduce the delivery time to local customers (J. Yang, 2010a). Our work considers dedicated machines at stage 1 which is common in real-world situations. This problem may arise in a manufacturing environment, The products are initially processed on two different machines at first assembly depending of their specifications. After their transportation, they must go through a common machine in a final fabrication stage, such as an inspection and testing station.

Literature review

The scheduling of flow shops with multiple parallel machines per stage, usually referred to as the hybrid flow shop (HFS), is a complex combinatorial problem encountered in many real world applications. (R. Ruiz et al, 2010) studied the hybrid flow shop scheduling problem. They presented a literature review on exact, heuristic and metaheuristic methods that have been proposed for its solution. A great deal of research has been done also on two-stage flow shop pro-

duction scheduling without transportation considerations. The scheduling problems on two stage hybrid flow shop are known to be strongly NP-hard (J. N. D. Gupta, 1988). A particular hybrid flow shop scheduling problem with the objective of minimizing makespan is first considered in (F. Riane et al, 2002). They developed three heuristics which run in polynomial time and one dynamic programming algorithm which runs in exponential time. They empirically evaluated their solution procedures. (J. Yang, 2010b) established that the same problem is unary NP-complete. (J. Yang, 2010a) considered the problem of minimizing total completion time in a two-stage hybrid flow shop scheduling problem with dedicated machines at stage2; established the complexity of several variations of the problem and presented optimal solutions for a few special cases. (O.Ceyda and M.T.L. Bertrand, 1997) studied the problem of minimizing makespan in a two-stage flowshop scheduling problem with a common second-stage machine, they showed that the problem is NP-hard and presented an heuristic procedure for the solution of the problem. (W. Besbes et al, 2010) considered the 2-stage flow shop problem with parallel dedicated machines and developed two approximate methods. The first approximate method is based on the Johnson's and Fifo rules. The second one is a genetic algorithm based approach. (Sung and Min, 2001) studied a two-machine flow shop scheduling problem by considering different machine sequences, such as batch to discrete, batch to batch and discrete to batch. In (N. Dridi et al, 2001), the two-stage hybrid flow shop with dedicated machines was studied. Some basic properties, a set of lower bounds and two polynomial cases were provided and a new heuristic was developed to sequence jobs in such a way that the obtained makespan corresponds to the lower bound. A case study in a two-stage hybrid flow shop with setup time and dedicated machines in (H.T. Lina and C.J. Liao, 2003)is taken from a label sticker manufacturing company. The objective is to schedule one day's mix of label stickers through the shop such that the weighted maximal tardiness is minimized. An heuristic is proposed to find the near-optimal schedule for the problem.

A few related machine scheduling models with transportation considerations have been studied in the literature. The earliest scheduling papers that consider transportation are (P.L. Maggu et al, 1981) and (P.L. Maggu et al, 1982). (B.M.T. Lina et al, 2007) addressed a three-machine assembly-type flow shop scheduling problem, which frequently arises from manufacturing process management as well as from supply chain management. Machines one and two are arranged in parallel for producing component parts individually, and machine three is an assembly line arranged as the second stage of a flow shop for processing the component parts in batches. Whenever a batch is formed on the second-stage machine, a constant setup time is required. (L. X. Tang et al, 2000) studied a scheduling problem in a steelmaking shop. Two models arising from steelmaking and refining operations are considered. The first model assumes that there is a converter at the steelmaking operation and a refining furnace at the refining operation. A transporter with capacity one is available to carry out jobs from converter to a refining furnace. The second model considers a more practical situation in which jobs are processed in identical parallel converters first, and then the jobs coming from the same converter are transported by a dedicated trolley with capacity one to the next operations. (ZL. Chen and CY. Lee, 2001) studied scheduling problem with semi-finished jobs and finished jobs delivery in which capacity of transporters and transportation times are explicitly considered in flow shop or parallel machines environment. (N. Chikhi and M. Abbas, 2012) established complexity results on two stage hybrid flow shop with batch transfer between stages. (Y. Crama et al, 2000) reviews production planning and scheduling models in automated manufacturing systems that include material handling devices. (Y. Crama and J. Van de Klundert, 1999) established the validity of the conjecture that 1-unit cycles yield optimal production rates for 3-machine robotic flowshops. (N. Brauner and G. Finke, 1997) proved that 1-unit cycles do not necessarily yield optimal solutions for cells of size four or large and some complexity results of one-cycle robotic flow-shops are established in (N. Brauner et al, 2003). In the current literature, the allocation of the operations to each machine is assumed to be constant and for given processing times the optimum robot move cycle minimizing the cycle time is to be determined. In some manufacturing operations such as chemical electroplating this assumption is meaningful and these operations mostly require no-wait constraints (see A. Agnetis,2000). (A.Che et al, 2012) considered scheduling multiple robots in a no-wait re-entrant robotic flowshop. (C.B. Chu, 2006) addresses an improved algorithm for 2-cyclic identical part scheduling in a no-wait robotic flow shop where exactly two parts enter and two parts leave the production line during each cycle. (N. Brauner, 2008) considered the problem of identical part production in cyclic robotic cells. (S. Li, 1997) considers a two-stage hybrid flow shop with a single machine at stage 1 and multiple identical machines at stage 2. The flowshop is characterized by major and minor setups, part families and batch production allowing split and no split at stage 2. In (V. Suresh, 1997), the author considers a two-stage flow shop with multiple processors in each stage. The processors are of the unrelated type. A two-stage algorithm is developed to minimize the makespan. (L. Tang and H. Gong, 2008) considered the coordinated scheduling problem of hybrid batch production on a single batching machine and two-stage transportation con-

necting the production, where there is a crane available in the first-stage that transports jobs from the warehouse to the machine and there is a vehicle available in the second-stage to deliver jobs from the machine to the customer. They prove that this problem is strongly NP-hard and a polynomial time algorithm is proposed for a case where the job transportation times are identical on the crane or the vehicle. (J. Hurink and S. Knust, 2001) studied the m-machine flow shop scheduling problem with the consideration of transportation times. In this paper, we study a two-stage flow shop production problem with dedicated machines at the first stage and one single common machine at the second stage which is different from those machine environment studied in a simple two-stage productions problem. Furthermore, transportation time and conveyor capacity are considered.

Notation

The problem thus defined is denoted $TF3|\sigma = 2, \upsilon = 1, c \geq 1|Cmax(N)$ (TF to denote a flow shop problem with transportation, $\sigma = 2$ means that every job is constituted of two operations and $\upsilon = 1$ means that there is only one conveyor of capacity "c"). Let N1 and N2 be two disjoint subsets of N such that $N1 \cap N2 = \emptyset$. and $N1 \cup N2 = N$. Let $n1 = |N1|$ be the number of jobs belonging to the first job type and $n2 = |N2|$ be the number of jobs belonging to the second one. Cmax denotes the maximum completion time (makespan).

Let us denote $N1 = \{J_1, J_2, ..., J_{n_1}\}$ the set of jobs of Type "1" and $N2 = \{J_{n_1+1}, J_{n_1+2}, ..., J_{n_1+n_2}\}$ the set of jobs of Type "2".

If $J_j \in N1$, then the first operation of this job must be processed on the machine M_1 during one processing time P_{j1} j = 1,2... ,n1 and this job is transported toward M_3, the second operation of this job is processed on M_3 during one processing time $P_{j2}(j = 1, 2..., n1)$.

If $J_j \in N2$, the first operation of this job must be processed on the machine M_2 during one processing time P_{j1} j= $n_1 + 1$,... ,n and transported toward M_3. The second operation of this job will be processed on M_3 during one processing time P_{j2} $j = n_1 + 1$,... ,n.

The present article is organized as follows: the second section is devoted to the mathematical formulation, where a mixed integer linear programming model is proposed to determine the schedule with minimum makespan. This model has been tested using the Cplex solver in section 3. Section 4 is dedicated to the calculation of lower bounds . We establish complexity of some special cases in section 5. In section 6, a general case is studied and some particular cases that can be solved polynomially are presented. In section 7, two heuristics are presented for the solution of the general case and some numerical tests are carried out to show the performance and the efficiency of the different heuristics in the last section. Finally, we provide a conclusion at the end of this article.

2 Modeling

We present a mixed integer linear programming model for this problem in order to determine the sequence of jobs that minimizes the total makespan criterion.

For every job J_i, we define the following integer variables:

- p_{i1}: is the processing time of a job J_i on the first stage, i = $\overline{1, n}$
- p_{i2}: is the processing time of a job J_i on the second stage, i = $\overline{1, n}$
- t_{is}: is the starting time of the operation O_{is} of the job J_i on the stage s, i = $\overline{1, n}$ and s= $\overline{1, 2}$.
- d_{ip}: is the transportation time of job J_i, i = $\overline{1, n}$

For every pair (J_i, J_j) of jobs, we introduce the following binary variables:

- α_{ij} equal to 1 if $d_{is} < d_{js}$, and 0 otherwise.
- x_{ijs} equal to 1 if $t_{is} < t_{js}$, and 0 otherwise.
- C_{max} max completion time of all the jobs.

The mixed integer linear programming formulation for the general problem is:

Minimize C_{max};
subject to:

$t_{j2} \geq t_{i2} + p_{i2} - M * (1 - x_{ij1})$	$\forall\ i, j \in N(i \neq j)$...(1)
$t_{i2} \geq t_{j2} + p_{j2} - M * x_{ij1}$	$\forall\ i, j \in N(i \neq j)$...(2)
$t_{j1} \geq t_{i1} + p_{i1} - M * (1 - x_{ij2})$	$\forall\ i, j \in N_s(i \neq j)$...(3)
$t_{i1} \geq t_{j1} + p_{j1} - M * x_{ij2}$	$\forall\ i, j \in N_s(i \neq j)$...(4)
$x_{ij1} + x_{ji1} = 1$	$\forall\ i, j \in N(i \neq j)$...(5)
$x_{ij2} + x_{ji2} = 1$	$\forall\ i, j \in N_s(i \neq j)$...(6)
$\alpha_{ij} + \alpha_{ji} \leq 1$	$\forall\ i, j \in N(i \neq j)$...(7)
$d_{jp} - d_{ip} \geq 2t * \alpha_{ij} - \alpha_{ji} * M$	$\forall\ i, j \in N(i \neq j)$...(8)
$d_{ip} - d_{jp} \geq 2t * \alpha_{ji} - \alpha_{ij} * M$	$\forall\ i, j \in N(i \neq j)$...(9)
$\sum_{j=1, i \neq j}^{n} (1 - \alpha_{ij} - \alpha_{ji}) \leq c - 1$	$\forall\ i, j \in N$...(10)
$d_{is} \geq t_{i1} + p_{i1}$	$\forall\ i, j \in N$...(11)
$t_{i2} \geq d_{is} + t$	$\forall\ i, j \in N(i \neq j)$...(12)
$t_{i2} + p_{i2} \leq C_{max}$	$\forall\ i, j \in N(i \neq j)$...(13)
$x_{ijs},\ \alpha_{ij} \in \{0, 1\}$	$\forall\ i, j \in N, s = \overline{1, 2}$...(14)
$t_{is},\ \ d_{is} \in IN$	$\forall\ i, j \in N, s = \overline{1, 2}$...(15)
Where M is a very large value	

Constraints (1) and (2) (respectively (3) and (4)) are the disjunctive constraints at each stage. Constraints (5) and (6) require that each machine executes only one job at a time. Constraints (7),(8), (9) and (10) are constraints on the conveyor (vehicle) and on the jobs to be transported: Constraints (7) express that all jobs must be transported between

the stages. Constraints (8) and (9) indicate that any job J_i is transported between the stages either before or after another job J_j, or at the same time and show that the transportation time of a round-trip of the vehicle requires 2t. Constraints (10) express the fact that the number of transported jobs at any time must be smaller than the vehicle capacity. Constraints (11) ensure that a job cannot be transported from the first to the second stage, only once the first operation of this job is finished. Constraints (12)and (13) concern the second stage. Constraints (12) induce that the processing time of the second operation of a job can only begin once the job has arrived to the second stage.Constraints (13) imply that the end of processing of any job is lower or equal to the makespan. Constraints (14) and (15) indicate the type of variables.

From this formulation, we can derive a lower bound by relaxing the constraints (14) and (15). The relaxed problem can be solved using a linear programming solver (Cplex for example).

3 Testing of the model with CPLEX

The linear models with integer and binary variables can be solved by efficient solvers such as LINGO, CPLEX, etc. Our mathematical model has been tested on a Personal Computer(Intel core 2 duo 2 GHz)using Cplex Solver. The processing times p_{i1} and p_{i2} are generated by a uniform law in $[1, 30]$. We fixed the number of jobs and let the vehicle capacity and the transportation time vary. For every case, the average execution time (for which the optimal or the feasible solution is obtained) is computed in seconds. We have gotten good results for instances of reduced size. The results are given in the Table 1.

(a) $n = 6$, $avr - time(optimal)$

n	c : capacity	t:trans-timet	avr-time
6	2	1	3,64 sec
6	2	5	5,71 sec
6	3	1	3,62 sec
6	3	5	5,63 sec

(b) $n = 10, 50$, $avr - time(feasible)$

n	c : capacity	t:trans-time	avr-time
10	2	1	0,33 sec
10	2	5	0,28 sec
10	3	1	0,31 sec
10	3	5	0,28 sec
50	20	1	23,73 sec
50	20	5	23,09 sec
50	30	1	23,15 sec
50	30	5	22,71 sec

Table 1: Results obtained by the Cplex solver.

4 Study of bounds

The workload of a machine k corresponds to the sum of job processing times that the machine must carry and it is denoted: $W_k(Q)$ with Q a subset of N.

$$W_1(N1) = \sum_{1 \le j \le n1} p_{j1} \ , \ W_2(N2) = \sum_{(n_1+1) \le j \le n} p_{j1}(1)$$

$$W_3(N) = \sum_{1 \le j \le n} p_{j2} \tag{2}$$

Lemma 1 $Cmax \ge W_3(N) + \min_{1 \le j \le n} \{p_{j1}\} + t$

Proof 1 *The makespan of the workshop is larger or equal to the total workload of any machine, that is $C_{max} \ge W_3(N)$. Equality may occur only in the case where transportation are equal to zero. We assume that all jobs are processed on all the machines without idle time.*

All jobs must be transported from the first stage to M_3 in order to be processed. If J_1 is the first job processed on the first stage, it will be the first to be processed on M_3 during time p_{12}. Therefore, the completion time is $C_{max} = p_{11} + t + W_3(N)$. Thus $Cmax \ge W_3(N) + \min_{1 \le j \le n} \{p_{j1}\} + t$.

Lemma 2 $Cmax \ge \max\{W_1(N1), W_2(N2)\} + \min_{1 \le j \le n} \{p_{j2}\} + t$

Proof 2 *The makespan of the workshop is larger or equal to the workload of any machine, that is $C_{max} \ge \max\{W_1(N1), W_2(N2)\}$. We first assume that all jobs are processed on the two-stages without idle time. All jobs must be transported from the stage1 to stage2 in order to be processed on M_3. Let $J_{(s)}$ be the last job processed on stage1, requiring a transportation time t. Assuming that it is going to be processed on M_3 right after its arrival during one time p_{s2}, then the completion time of $J_{(s)}$ on M_3 is $C_{s2} = C + p_{s1} + t + p_{s2}$, where C is the time completion on the stage1 before the job $J_{(s)}$. Then $C_{s2} = \max\{W_1(N1), W_2(N2)\} + t + p_{s2}$, therefore $C_{s2} \ge \max\{W_1(N1), W_2(N2)\} + t + \min_{1 \le i \le n} \{p_{i2}\}$. Thus $C_{max} \ge \max\{W_1(N1), W_2(N2)\} + t + \min_{1 \le i \le n} \{p_{i2}\}$.*

Lemma 3 $Cmax \ge \{(\lceil \frac{n}{c} \rceil - 1) * 2t + t + \min_{1 \le j \le n} \{p_{j2}\} + \min_{1 \le j \le n} \{p_{j1}\}\}$.

Proof 3 *Let J_1 and J_n be the first and the last job (respectively) to execute on the workshop. All jobs*

must be transported to the second stage in order to be processed on machine M_3. As the vehicle capacity is equal to c and a round-trip takes 2t, then the necessary time to transport all jobs is greater or equal than $(\lceil \frac{n}{c} \rceil - 1) * 2t + t$. We assume that all jobs are processed on the two-stages without idle time and the conveyor transports the jobs without wait time.

The transportation of jobs takes place only if the processing of J_1 is finished on stage1. Also, at the end of the transportation of jobs, J_n must be executed on M_3. Therefore $C_{max} = p_{11} + (\lceil \frac{n}{c} \rceil - 1) * 2t + t + p_{n2}$, then $C_{max} \geq (\lceil \frac{n}{c} \rceil - 1) * 2t + t + \min_{1 \leq i \leq n} \{p_{i1}\} + \min_{1 \leq i \leq n} \{p_{i2}\}$.

We give three lower bounds $LB^{(1)}$, $LB^{(2)}$ and $LB^{(3)}$ for the objective function:

$$LB^{(1)} = W_3(N) + \min_{1 \leq j \leq n} \{p_{j1}\} + t. \quad (3)$$

$$LB^{(2)} = \max\{W_1(N1), W_2(N2)\} + \min_{1 \leq j \leq n} \{p_{j2}\} + t. \quad (4)$$

$$LB^{(3)} = \{(\lceil \frac{n}{c} \rceil - 1) * 2t + t + \min_{1 \leq j \leq n} \{p_{j2}\} + \min_{1 \leq j \leq n} \{p_{j1}\}\}. \quad (5)$$

Proposition 1 $LB = \max\{LB^{(1)}, LB^{(2)}, LB^{(3)}\}$ *is also a lower bound for the makespan.*

5 Complexity and special case

The problem $TF3|\sigma = 2, \upsilon = 1, c \geq 1|Cmax(N)$ is strongly NP-hard because one particular problem without transportation times denoted $F3|\sigma = 2|Cmax(N)$ was shown to be strongly NP-hard in (M.T.L. Bertrand,1999).

We show that problem $TF2|\upsilon = 1, c \geq 1|Cmax(N)$ studied in (C. Y. Lee and Z. L. Chen, 2001) is a special case of the problem $TF3|\sigma = 2, \upsilon = 1, c \geq 1|Cmax(N)$:

Remark 1 *If $N2 = \emptyset$ then problem $TF3|\sigma = 2, \upsilon = 1, c \geq 1|Cmax(N)$ is identical to problem $TF2|\upsilon = 1, c \geq 1|Cmax(N)$.*

Proof 4 *Since all jobs are processed on M_1 and no job is processed on M_2, the problem $TF3|\sigma = 2, \upsilon = 1, c \geq 1|Cmax(N)$ is identical to problem $TF2|\upsilon = 1, c \geq 1|Cmax(N)$.*

Theorem 1 *The decision version of problems $TF2|\upsilon = 1, c \geq 1|Cmax(N)$,$TF3|\sigma = 2, p_{j1} = p1, j \in N1, \upsilon = 1, c \geq 1|Cmax(N)$ and $TF3|\sigma = 2, p_{j1} = p1, j \in N2\upsilon = 1, c \geq 1|Cmax(N)$ are NP-complete.*

Proof 5 *The decision version of problem $TF2|\upsilon = 1, c \neq 2|Cmax(N)$ is NP- complete (ZL. Chen and CY. Lee, 2001). From Remark1, $TF2|\upsilon = 1, c \geq 1|Cmax(N)$ is a special case of $TF3|\sigma = 2, \upsilon = 1, c \geq 1|Cmax(N)$. Hence,the result with problem $TF3|\sigma = 2, \upsilon = 1, c \neq 2|Cmax(N)$ holds. Similarly,the decision version of problem $TF3|\sigma = 2, p_{j1} = p1, j \in N1, \upsilon = 1, c \neq 2|Cmax(N)$ is NP-complete since the decision version of problem $TF2|\upsilon = 1, c \neq 2|Cmax(N)$ is NP-complete. A similar argument can be applied to the other case.*

6 General case: c=1

Although this general case $TF3|\sigma = 2, \upsilon = 1, c = 1|Cmax(N)$ is NP-hard, there are some particular cases which can be solved in polynomial time:

- (1)If $\min p_{j2} \geq \max\{p_{j1}, 2t\}$ then an optimal schedule is obtained in $\theta(n)$.
- (2) If $\min p_{j1} \geq 2 \max\{max p_{j2}, 2t\}$ then an optimal schedule is obtained in $\theta(n)$.

These two special cases are equivalent to those considered in (O. Ceyda and M.T.L. Bertrand, 1997) without transportation time, therefore, we omit the proof.

- (3) If $p_{j1} = p, p_{j2} = q$: An optimal ordering schedule is given by any sequence of jobs such that the jobs are processed alternatively.

Proof 6 *we have four cases:*

- *A) $p \leq 2t$($q \leq 2t \vee q > 2t$).*
- *B) $p > 2t$($q \leq 2t \vee q > 2t$).*

Let us denote J_{1b} and J_{nb} be the first and the last job (respectively) to be processed. First, we take the first job J_{1b} whose execution time on the first machine is p. Second, all jobs must be transported in order to be processed. As the vehicle capacity is equal to 1 and a round-trip takes 2t, then the necessary time to transport all jobs is equal to $(n-1) * 2t + t$ (for the case A) . The transportation of jobs takes place only if the processing of J_{1b} is finished on stage1. Also, at the end of the transportation of jobs, J_{nb} must be processed on M_3. Therefore for the case (A1) $C_{max} = p + (n-1) * 2t + t + q$, however $LB^{(3)} = p_{min1} + (n-1) * 2t + t + p_{min2}$ then $Cmax(N) = LB^{(3)}$.For the case (A2),there is no idle time on M_3,then $C_{max} = p + t + W_3(N)$, then $Cmax(N) = LB^{(1)}$, we deduce that the case (A)is a polynomial case. A similar argument can be applied to the case (B).

These three special cases are identified for theoretical interest only because it may be hard to find a real life problem satisfying these conditions.

6.1 Case where p_{i1} is identical for all $i \in N$

Algorithm 1 *1: Arrange the jobs in N in LPT rule relative to processing times on the common machine p_{i2} and denote the resulting list as ψ_N.*
2: Decompose the set ψ_N of n jobs into two mutually exclusive families N_1, N_2, depending of the job type.
3: Process the job of type 1 and type 2 on the corresponding machine at the first stage according to the order in ψ_N and transport them in the same order in ψ_N .
4: Process these jobs on the common machine at the second stage according to the order of their transportation.

Theorem 2 *The inverse order in ψ_N gives an optimal solution for the problem $TF3|\sigma = 2, p_{j2} = p_1, \forall i \in N, c = 1|Cmax$ with one common machine at the first stage and two dedicated machines at the second stage.*

Proof 7 *There is no idle time on the common machine at stage1.The processing times on the Stage2 are identical. Then in this situation, to minimize the makespan, the transportation time must be minimized and since the conveyor capacity is one, a simple pairwise argument can prove that among jobs, it is optimal to process jobs in SPT order.*

Theorem 3 *The algorithm 1 gives an optimal solution for the problem $TF3|\sigma = 2, p_{j1} = p_1, \forall i \in N, c = 1|Cmax$.*

Proof 8 *The problem $TF3|\sigma = 2, p_{j1} = p_1, \forall i \in N, c = 1|Cmax$ is symmetric to the problem $TF3|\sigma = 2, p_{j2} = p_1, \forall i \in N, c = 1|Cmax$ with one common machine at the first stage and two dedicated machines at the second stage. Therefore, from the preceding theorem, the inverse order in ψ_N is an optimal sequence and the precedent algorithm gives an optimal solution for the problem $TF3|\sigma = 2, p_{j1} = p_1, \forall i \in N, c = 1|Cmax$.*

6.2 Case where p_{i2} is identical for all $i \in N$

Algorithm 2 *1: Depending of the job type, decompose the set of n jobs into two mutually exclusive families N_1, N_2.*
2: Arrange the jobs in each family in SPT rule relative to the processing times p_{j1} and denote the resulting list as π_{N_1}, π_{N_2}.
3: Process the job of type 1 on M_1 according to π_{N_1} and process jobs of type 2 on M_2 according to π_{N_2}.
4: Let $C[j]$ be the completion time of the job J_j on the first stage.
5: Arrange the jobs in the increasing order of $C[j]$ and denote the resulting list π_N.
6: Transport the jobs to the order in π_N.
7: Process these jobs on the second stage in the order of their transportation.

Theorem 4 *The algorithm 2 gives an optimal solution for the problem $TF3|\sigma = 2, p_{j2} = p_2, \forall i \in N, c = 1|Cmax$.*

Proof 9 *There is no idle time on the two machines in the first stage. The processing times on the machine 3 are identical. Then in this situation, to minimize the makespan, the transportation time must be minimized and since the conveyor capacity is one, a simple pairwise argument can prove that among jobs in the same set such as N_1 or N_2, it is optimal to process jobs in SPT order.*

Theorem 5 *The inverse order in π_N gives an optimal solution for the problem $TF3|\sigma = 2, p_{j1} = p_1, \forall i \in N, c = 1|Cmax$ with one common machine at the first stage and two dedicated machines at the second stage.*

Proof 10 *The problem $TF3|\sigma = 2, p_{j2} = p_2, \forall i \in N, c = 1|Cmax$ is symmetric to the problem $TF3|\sigma = 2, p_{j1} = p_1, \forall i \in N, c = 1|Cmax$ with one common machine at the first stage and two dedicated machines at the second stage. Therefore, the inverse order in π_N is an optimal sequence and gives an optimal solution for the second problem.*

7 Heuristics

Recall that the general case $TF3|\sigma = 2, \forall i \in N, c = 1|Cmax$ is NP-hard, we propose two heuristics for its solution. These heuristics are based on the algorithm of Johnson (S.M. Johnson, 1954). We improved it in order to take into account the transportation times. Finally, we use the two rules SPT (Shortest Processing Time) and LPT (Longest Processing Time) that are based on the arranging of jobs.

Heuristic 1 *1: Depending of the jobs type , decompose the set of n jobs into two mutually exclusive families N_1, N_2.*
2: Construct an artificial F2//Cmax problem in relative to jobs in N with processing times on the first machine $p\prime_{i1} = \max\{p_{i1}, 4t\}$ and $p\prime_{i2} = p_{i2} + p_{i1}$ on the second machine.
3: Apply the modified Johnson's algorithm to this pseudo problem to solve this artificial problem with LPT(1)LPT(2) in relative to $p\prime_{i2}$ and denote the resulting list as ψ_N

4: *Process the jobs of type 1 and type 2 on the corresponding machine at the first stage according to the order in ψ_N and transport them according to the same order.*
5: *Process these jobs on the second stage in the order of their transportation.*

Heuristic 2 1: *Depending of the jobs type , decompose the set of n jobs into two mutually exclusive families N_1, N_2.*
2: *Construct an artificial F2//Cmax problem in relative to jobs in N with processing times on the first machine $p\prime_{i1} = p_{i1} + p_{i2}$ and $p\prime_{i2} = \max\{p_{i2}, 4t\}$ on the second machine.*
3: *Apply the modified Johnson's algorithm to this pseudo problem to solve this artificial problem with SPT(1)SPT(2) in relative to $p\prime_{i1}$ and denote the resulting list as π_N*
4: *Process the jobs of type 1 and type 2 on the corresponding machine at the first stage according to the order in π_N and transport them according to the same order.*
5: *Process these jobs on the second stage in the order of their transportation.*

8 Experimentation

To study the practical value of the methods that we develop, the heuristics H_1 and H_2 are empirically evaluated. Note that these two heuristics are indeed heuristics for the general case.
To generate large size test problems, the following parameters are considered:

- Number of jobs (n): 30, 100, 500, 1000.
- Processing times of jobs on the two-stage P_{i1} and P_{i2}: being generated from discrete Uniform distribution with range [1, 30], [1, 50] and [1, 100].
- Capacity of the transporter (c).
- Transportation time of transporter (t): being generated from discrete Uniform distribution with range [1, 10].

For each combination of number of jobs, we randomly generate 100 problem instances for the performance test of the heuristic algorithms. As there are 12 combinations for each heuristic, the total 2400 instances are generated. For each problem instance, the heuristic makespan, denoted by C(H), and lower bound of makespan, denoted by LB, are computed. The relative error ratio can be defined as:
Error Ratio (ER)=$\frac{C(H)-LB}{LB}$.

Table 2 and Table 3 summarize the results of computational experiments. For each combination, the average and maximum values of error ratios denoted by Avg.ER and Max.ER, respectively are used to evaluate the performance of the proposed heuristics. The average and maximum required time for applying each heuristic are computed (in milliseconds), respectively are denoted by Avg.Tim and Max.Tim.

	P_{i1}, P_{i2}	[1, 30]	[1, 50]	[1, 100]
n = 30	Avg.ER	0.060	0.058	0.059
	Max.ER	0.130	0.113	0.114
	Avg.Tim	35.74	32.7	32.38
	Max.Tim	47	47	47
n = 100	Avg.ER	0.020	0.018	0.0189
	Max.ER	0.057	0.028	0.029
	Avg.Tim	86.26	115.58	110.3
	Max.Tim	109	141	125
n = 500	Avg.ER	0.004	0.003	0.003
	Max.ER	0.019	0.004	0.004
	Avg.Tim	528.81	449.31	526.44
	Max.Tim	577	546	983
n = 1000	Avg.ER	0.002	0.001	0.001
	Max.ER	0.009	0.002	0.002
	Avg.Tim	884.52	675.08	914.17
	Max.Tim	999	999	999

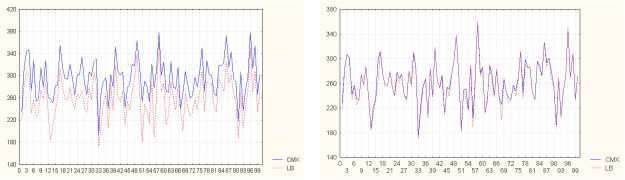
Table 2: Computational results of error ratios of the heuristic H_1(c = 1)

	P_{i1}, P_{i2}	[1, 30]	[1, 50]	[1, 100]
n = 30	Avg.ER	0.014	0.001	0.0003
	Max.ER	0.118	0.029	0.010
	Avg.Tim	38.53	41.05	40.58
	Max.Tim	47	63	47
n = 100	Avg.ER	0.006	0.0008	0.0001
	Max.ER	0.059	0.010	0.003
	Avg.Tim	83.48	130.50	123.58
	Max.Tim	110	297	172
n = 500	Avg.ER	0.001	0.0001	0.00001
	Max.ER	0.014	0.0023	0.0002
	Avg.Tim	504.38	492.97	467.52
	Max.Tim	780	515	530
n = 1000	Avg.ER	0.0008	0.0001	0.00002
	Max.ER	0.0083	0.002	0.0007
	Avg.Tim	447.76	888.9	771.86
	Max.Tim	999	967	999

Table 3: Computational results of error ratios of the heuristic H_2(c = 1).

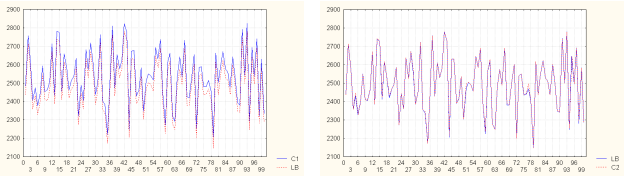
For the two heuristics, we compared the lower bound LB and the results of the makespan C_i obtained by the heuristic H_i.

From the computational results, we can observe that very small difference exists between the lower bound and the heuristic makespan. So, it reveals that both of them are close to the optimal solution and the heuristic algorithms can find near-optimal solution efficiently. We also observe that the ratio errors follow



(a) ***H1:***$pi1, pi2 \in \overline{1,50}$**,** $t \in \overline{1,10}$ **(b)** ***H2***$pi1, pi2 \in \overline{1,50}$**,** $t \in \overline{1,10}$

Figure 1: Comparison between C_i and the lower bound LB (Case:n=10)



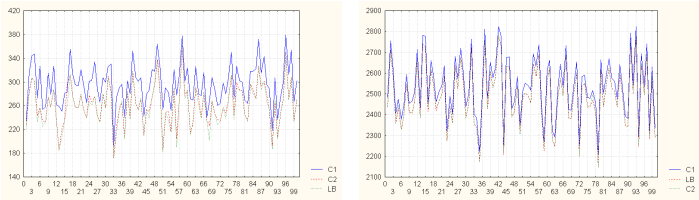
(a) ***H1:***$pi1, pi2 \in \overline{1,50}$**,** $t \in \overline{1,10}$ **(b)** ***H2:***$pi1, pi2 \in \overline{1,50}$**,** $t \in \overline{1,10}$

Figure 2: Comparison between C_i and the lower bound LB (Case:n=100)

a decreasing trend as the number of jobs increases. In general, results obtained for the different tests reveal that the heuristics give very good results but the heuristic H_2 is better than the heuristic H_1.

9 Conclusion

We studied a two-stage hybrid flow shop scheduling problem with two dedicated machines at stage 1 and one machine at stage 2. The stages are connected by a conveyor. The performance criteria chosen is the total execution time of all the jobs (makespan). We presented and modeled our problem as a linear program in integer and binary variables. We also proposed lower bounds for the objective function, that may be taken as references to assess the quality of solutions obtained by the developed methods. Some polynomially solvable cases of the problem are analyzed and solved in polynomial time. We developed two heuristics to find approximate solutions to the general case with computational experiments.



(a) ***Case (n=10)*** **(b)** ***Case (n=100)***

Figure 3: Comparison between C(i) and the lower bound LB ($pi1, pi2 \in \overline{1,50}$, $t \in \overline{1,10}$)

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