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## OPPORTUNITY STUDY ON THE ADOPTION OF AN EXTENDED TWO-DIMENSIONAL WARRANTY FOR A RANDOMLY FAILING PRODUCT

S. BOUGUERRA, N. REZG

LGIPM/ ENIM  
1 route d'Ars Laquenexy CS65820,  
57078 Metz Cedex 3  
bouguer@univ-metz.fr, rezg@univ-metz.fr

A. CHELBI

CEREP / ESSTT  
5 Avenue Taha Hussein, BP 56  
Bab Menara, 1008 Tunis - Tunisie  
Anis.chelbi@planet.tn

**ABSTRACT:** *In this paper a decision model is developed to study the opportunity provided by offering an extended two-dimensional warranty for a randomly failing product. The proposed mathematical model considers different maintenance policies to be adopted during the product's life cycle as well as the usage rate of the product by the consumer. The total expected costs incurred by the manufacturer and by the consumer during the product's life cycle are expressed in order to determine the maximum extra cost the consumer should pay and the minimum price at which the manufacturer should sell the extended warranty.*

**KEYWORDS :** *two-dimensional warranty, extended warranty, preventive maintenance.*

### 1 INTRODUCTION

With increasing competition manufacturer have more incentive to improve services and conditions of sale in order to attract a customer seeking elements that reassure the proper functioning of the system for a maximum time.

Among the elements that have been revisited in recent decades to provide consumers with a longer duration of use of their property and to have more confidence to the customer relationship and supplier warranty.

Several review papers are available on the general subject of warranty policies; they deal with different aspects related to product warranty (warranty vs. product design, warranty vs. maintenance, warranty vs. costs analysis over the product lifecycle, etc.). We mention the following: (Murthy, 1990), (Murthy and Blischke, 1992), (Blischke and Murthy, 1994), and (Murthy and Djamaludin, 2002).

In many practical situations such as in the automotive and aeronautic industry, warranty contracts are based on a two-dimensional warranty; such a warranty is generally characterized by a two-dimensional region with one axis representing the product's age and the other representing its usage.

This region is usually a rectangle  $[0, K_w] \times [0, L_w]$ , the warranty ceases when the sold item reaches the age  $K_w$ , or earlier if the usage exceeds  $L_w$ . (Singpurwalla, 1987) and (Moskowitz, 1988) dealt with the cost analysis of two-dimensional warranties characterized by a rectangular region. (Murthy and al, 1995) developed three two-dimensional policies with warranty regions that are different from the basic rectangular warranty region. They studied two-dimensional failure-free warranty policies for non-repairable items. They derived expressions for the expected warranty cost per item

sold and for the expected life cycle cost for the different warranty regions. (Iskander and Murthy, 2003) dealt with repair-replace strategies for products sold with two-dimensional failure-free warranty. (Baik et al, 2004) discussed two-dimensional failure modeling for a system where degradation is due to age and usage. They extended the concept of minimal repair from the one-dimensional warranty case to the two-dimensional case and characterized the failures over a two-dimensional region under minimal repair. An application of this important result to a manufacturer's servicing costs for a two-dimensional warranty policy has been given and they compared the minimal repair strategy with the strategy of replacement at failure.

(Iskander and al, 2005) looked at a warranty servicing strategy for items sold with two-dimensional warranty where the failed item is replaced by a new one when it fails for the first time in a specific region of the warranty and all other failures are repaired minimally. The region is characterized by two parameters. They derived the optimal values for these parameters which minimize the total expected warranty servicing cost. (Jung and Bai, 2007) proposed a method of estimating lifetime distribution for products under two-dimensional warranty for which age and usage are used simultaneously to determine the eligibility of warranty claims.

(Chukova and Johnston, 2006) considered a rectangular warranty region and divided it into three disjoint sub-regions, so that the associated expected warranty servicing cost per item sold are minimized. [Manna and al, 2006] proposed a methodology to determine the optimal warranty region for fixed as well as for flexible policies from among several types of regions. On the other hand, they attempted to find the warranty region that suits the customers best. (Vintr Z. and Vintr M.,

2007) dealt with an estimation of costs when two-dimensional warranty is granted. The paper includes a procedure of estimation of costs of the warranty based on a research on consumers' behavior. (Manna and al, 2008) studied warranty cost under a rectangular two-dimensional warranty policy for both repairable as well as non-repairable products. (Jack and al, 2009) discussed a repair-replace strategy for the manufacturer of a product sold with a two-dimensional warranty. The strategy is based on a specified region of the warranty defined in terms of age and usage with the first failure in the region rectified by replacement and all other failures being minimally repaired.

The great majority of the works on warranty consider a fixed warranty period. In practice many products are sold with the possibility to extend the warranty period either at purchasing time or sometimes at the end of the warranty period. Extended warranty helps manufacturers keeping in touch with consumers after the expiration of the warranty period. This issue related to extended warranty has not received much attention so far. The academic research on this specific subject is relatively recent. From an economic perspective, Padmanabhan V. (1995) developed a mathematical model considering consumer heterogeneity in usage habits. The article argues that variation in consumer usage habits for the product creates variation in the consumer's demand for an extended warranty.

(Lam Y. and Lam P.K.W., 2001) developed an extended warranty model that includes a free repair period and an extended warranty period. Consumers have the choice to renew or not the warranty at the end of the free perfect repair period. No preventive maintenance is considered. Different choices with different cost implications for consumers and manufacturers were presented. The exact expressions of the total expected discounted cost, and the long-run average cost per time unit for the consumer and the manufacturer were derived to obtain the optimal policies for the consumer.

(Gi Mun Jung and Dong Ho Park, 2003) developed an optimal periodic preventive maintenance policy following the expiration of warranty. They considered two types of warranty policies: renewing warranty and non-renewing warranty. The product is maintained free of charge or with prorated cost on failure on failure to the consumer during the warranty period. And the user will have to repair or replace the failed product at their own expenses during the post warranty period. The authors derived the expressions for the expected maintenance costs for the periodic preventive maintenance following the expiration of warranty when applying two types of warranty policies and obtain the optimal number and the optimal period for such post warranty maintenance policies by minimizing the expected long-run maintenance cost per unit time.

(Chen J.A. and Chien Y.H., 2007) considered the situation of repairable products sold under a free-replacement renewing warranty. Two types of failures are considered: minor failures rectified by minimal

repair and catastrophic failures that can be removed only by replacement. For the free-replacement renewing warranty policy, if the product fails due to a catastrophic failure it is replaced by a new product and a new warranty, at no cost to the buyer. The contractual obligation of the warranty per unit sold incurred by the manufacturer expires at the instant the product passes the end of the warranty period without any catastrophic failure. The authors derived the expression of the expected total cost from the manufacturer and the buyer perspectives.

(Shaomin W. and Phil L., 2011) analyzed the lifecycle cost of a product protected by both standard and extended warranty policies from a consumer's perspective. They assumed that the product has two types of failures: minor and catastrophic which can be corrected respectively with minimal repair and replacement. They also assumed that the length of the extended warranty can be selected among available choices. They derived optimal values of the opportunity-based age replacement and the duration of the extended warranty which minimize the expected life cycle cost per time unit.

(Bouguerra et al., 2011) considered the problem related to the adoption of an extended warranty period for randomly failing products sold with a one-dimensional warranty. They developed a mathematical model to study the opportunity provided by the extended warranty for the buyer as well as for the manufacturer.

In this paper we extend the model proposed by (Bouguerra et al., 2011) to the case of an extended two dimensional warranty. We develop a mathematical model to study the opportunity provided by the extended two-dimensional warranty from the consumer and the manufacturer perspectives. We will express the total expected cost incurred by each side during the product's life cycle in order to determine, for any given situation, the maximum extra cost the consumer should pay for the extended warranty, and the minimum price at which the manufacturer should sell it. The model allows looking if there is a zone of possible compromise yielding a win-win relationship with respect to the extended warranty.

Obviously, performing preventive maintenance (PM) on the product over the warranty and the extended warranty periods has an impact on the warranty servicing cost. For this reason, we will tackle the problem considering different options in terms of maintenance strategies adopted during the product's lifecycle.

In the next section, we present the working assumptions and the corresponding mathematical model considering different maintenance policies. Section 3 is dedicated to an illustrative numerical example with a discussion of the obtained results. We conclude in section 4 with some comments and indications about further research on the same topic.

## 2 MATHEMATICAL MODEL

### 2.1 Notation and working assumptions

The following notation will be used:

$r_0(t_h)$ : rate of occurrence of failures with no PM

$r_m(t_h)$ : rate of occurrence of failure with PM

$C_r$ : cost of each minimal repair action

$C_m$ : cost of each imperfect PM action

$C_w$ : cost of the extended warranty paid by the consumer

$re$ : nominal usage per time unit

$A_i$ : usage rate of the product (varies from user to user).

Expressed in units of usage/time unit

$K$ : length of the product life cycle

$K_w$ : the basic warranty period

$K_{we}$ : the warranty period including the basic period  $W$  and the extension

$L$ : nominal usage of the system at the end of the life-cycle

$L_w$ : nominal usage of the system at the end of the basic warranty period

$L_e$ : nominal usage of the system at the end of the extended warranty period

$\Delta h$ : période entre deux actions de maintenance préventive

$\Delta u$ : usage entre deux actions de maintenance préventive  $\Delta u = \Delta h \cdot re$

We consider randomly failing and repairable products sold with a basic two-dimensional warranty period  $[K_w, L_w]$  with the possibility to be extended to  $[K_e, L_e]$  for an additional cost  $C_w$  paid by the consumer when purchasing the product. The consumer has to decide whether to buy or not the extended warranty. This is generally a decision difficult to take for the consumer because he does not know if the extra cost  $C_w$  would exceed the potential repairs cost that would be borne by him in case he does not take the extended warranty. On the other hand, from the manufacturer's side, the extended warranty is worthwhile only when its price exceeds the cost of claims servicing borne by him during the additional warranty period. Hence, the price  $C_w$  of the extended warranty is a crucial variable of interest

Table 1 presents the expression of the number of PM actions observed during each period of the system's lifecycle.

Table 1: Number of PM actions over each interval

Number	$N_1$	$N_3$	$N_5$	$N_7$	$N_9$	$N_2$	$N_4$	$N_6$	$N_8$	$N_{10}$
Interval	$[K_w, K]$	$[K_e, K]$	$[0, K_w]$	$[0, K_e]$	$[0, K]$	$[L_w, L]$	$[L_e, L]$	$[0, L_w]$	$[0, L_e]$	$[0, L]$
Value	$\left\lfloor \frac{K - K_w}{\Delta_h} \right\rfloor$	$\left\lfloor \frac{K - K_e}{\Delta_h} \right\rfloor$	$\left\lfloor \frac{K_w}{\Delta_h} \right\rfloor$	$\left\lfloor \frac{K_e}{\Delta_h} \right\rfloor$	$\left\lfloor \frac{K}{\Delta_h} \right\rfloor$	$\left\lfloor \frac{L - L_w}{\Delta_u} \right\rfloor$	$\left\lfloor \frac{L - L_e}{\Delta_u} \right\rfloor$	$\left\lfloor \frac{L_w}{\Delta_u} \right\rfloor$	$\left\lfloor \frac{L_e}{\Delta_u} \right\rfloor$	$\left\lfloor \frac{L}{\Delta_u} \right\rfloor$

With:

$0 < \rho < 1$

for both the buyer and the vendor considered from opposite perspectives.

We restrict our analysis to the case where the basic and extended warranty periods are similar in shape; As shown in figure 1, this implies:

$$re = \frac{Lw}{Kw} = \frac{Le}{Ke} = \frac{L}{K} \quad (1)$$

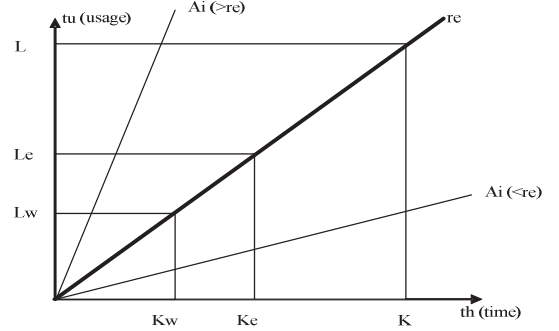


Figure 1: Product lifecycle with two-dimensional basic and extended warranties region

All failures during the basic and extended warranty periods are repaired by the manufacturer at no cost to the buyer. The repair actions are considered as minimal repairs with negligible duration keeping the system failure rate at nearly the same level as before failure. The costs of repairs occurring during the post-warranty period are borne by the buyer.

We assume that during the product lifecycle, PM are performed at instants  $\tau_j$ ,  $j=1, 2, \dots$ ; with  $\tau_0 = 0$ . The costs of PM are always paid by the consumer. PM is modeled using the arithmetic intensity reduction (ARI) well known concepts. We use the same modeling framework considered by (Chan and Shaw, 1993) to model the product intensity reduction. The authors state that the PM action reduces the failure intensity by an amount proportional to the current failure intensity. The corresponding model is given by:

$$r_m(t) = r_0(t) - \rho \sum_{j=0}^{N_t-1} (1 - \rho) r_0(\tau_{N_t-j}) \quad (2)$$

$\tau_j$ : Successive PM instants,  $\tau_0 = 0$

$N_t$ : Number of PM observed up to time  $t$

## 2.2 Failure rate expression

Users with usage rate ( $A_i < r_e$ ) will be called light users and those having a usage rate ( $A_i > r_e$ ) will be called hard users. The warranty servicing cost will obviously be impacted by the adopted maintenance policy for the product during its life cycle. That is why we develop the model considering the following maintenance options:

- **Option I:** no PM action over the product lifecycle. Only minimal repair actions at failures are performed.
  - **Option II:** periodic PM actions over  $[0, L]$
  - **Option III:** periodic PM actions within the post warranty region. For this option we consider two different possibilities:
    - **Option III-1:** PM actions carried out from the end of the basic warranty region,
    - **Option III-2:** PM actions carried out from the end of the extended warranty region,
  - **Option IV:** periodic PM actions during the warranty region, For this option we also consider two different possibilities:
    - **Option IV-1:** PM actions are carried out only within the basic warranty region,
    - **Option IV-2:** PM actions are carried out within both the basic and the extended warranties
- The expressions of the failure rate  $r_m(t)$  corresponding to each of the above listed options is given as follows:

- **Option I :**

For light users (3)

$$r_m(t_h) = r_0(t_h) \quad (4)$$

$$r_m(t_u) = r_0(t_u)$$

- **Option II :**

For light users ( $A_i \leq r_e$ ) (5)

$$r_m(t) = \begin{cases} r_0(t_h) & \text{if } t_h \leq \tau \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } \tau_1 < t_h \leq \tau_{N_i} \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_h > \tau_{N_i} \end{cases}$$

For hard users ( $A_i > r_e$ ) (6)

$$r_m(t) = \begin{cases} r_0(t_u) & \text{if } t_u \leq \tau_1 \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } \tau_1 < t_u \leq \tau_{N_i} \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_u > \tau_{N_i} \end{cases}$$

- **Option III-1:**

For light users ( $A_i \leq r_e$ ) (7)

$$r_m(t) = \begin{cases} r_0(t_h) & \text{if } t_h \leq \tau_1 \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } \tau_1 < t_h \leq \tau_{N_i} \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_h > \tau_{N_i} \end{cases} \quad (8)$$

For hard users ( $A_i > r_e$ )

$$r_m(t) = \begin{cases} r_0(t_u) & \text{if } t_u \leq \tau_1 \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } \tau_1 < t_u \leq \tau_{N_i} \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_u > \tau_{N_i} \end{cases} \quad (9)$$

- **Option III-2:**

For light users ( $A_i \leq r_e$ ) (9)

$$r_m(t) = \begin{cases} r_0(t_h) & \text{if } t_h \leq \tau_1 \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } \tau_1 < t_h \leq \tau_{N_i} \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_h > \tau_{N_i} \end{cases} \quad (10)$$

For hard users ( $A_i > r_e$ )

$$r_m(t) = \begin{cases} r_0(t_u) & \text{if } t_u \leq \tau_1 \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } \tau_1 < t_u \leq \tau_{N_i} \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_u > \tau_{N_i} \end{cases} \quad (11)$$

- **Option IV -1:**

For light users ( $A_i \leq r_e$ ) (11)

$$r_m(t) = \begin{cases} r_0(t_h) & \text{if } t_h \leq \tau_1 \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } \tau_1 \leq t_h \leq \tau_{N_i} \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_h \geq \tau_{N_i} \end{cases} \quad (12)$$

For hard users ( $A_i > r_e$ )

$$r_m(t) = \begin{cases} r_0(t_u) & \text{if } t_u \leq \tau_1 \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_u \leq \tau_1 \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_u \geq \tau_{N_i} \end{cases} \quad (13)$$

- **Option IV -2:**

For light users ( $A_i \leq r_e$ )

$$r_m(t) = \begin{cases} r_0(t_h) & \text{if } t_h \leq \tau_1 \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } \tau_1 \leq t_h \leq \tau_{N_i} \\ r_0(t_h) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_h \geq \tau_{N_i} \end{cases}$$



For hard users ( $A_i > re$ ) (14)

$$rm(t) = \begin{cases} r_0(t_u) & \text{if } t_u \leq \tau_1 \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } \tau_1 \leq t_u \leq \tau_{N_i} \\ r_0(t_u) - \rho \cdot \sum_{j=0}^{N_i-1} (1-\rho)^j r_0(\tau_{N_i-j}) & \text{if } t_u \geq \tau_{N_i} \end{cases}$$

For each option, the successive PM instants can be expressed as follows (table 2)

Table 2: PM instants for each option

Option	$\tau_i$	
II IV-1 IV-2	$\tau_i = N_i \cdot \Delta_h$	If $A_i < re$
	$\tau_i = N_i \cdot \Delta_u$	If $A_i > re$
III-1	$\tau_i = (N_i \cdot \Delta_h) + Kw$	If $A_i < re$
	$\tau_i = (N_i \cdot \Delta_u) + Le$	If $A_i > re$
III-2	$\tau_i = (N_i \cdot \Delta_h) + Ke$	If $A_i < re$
	$\tau_i = (N_i \cdot \Delta_u) + Le$	If $A_i > re$

### 2.3 Determination of the maximum extra cost the consumer should pay for the extended warranty

In what follows we state, the condition which should be satisfied so that buying the extended warranty region would be advantageous for the consumer. This condition is based on the fact that buying the extended warranty would be beneficial for the consumer if the total cost incurred to him, during the product lifecycle, would be lower than what it would cost him in case he does not take it. Let  $Cw$  be the maximum cost the consumer should pay for the extended warranty region. For each maintenance option considered, the extended warranty cost ( $Cw$ ) can be expressed as follows:

#### • Option I, II, III-1, IV-1: (15)

$$\begin{cases} \rightarrow \text{if } A_i \leq re \\ Cw \leq Cr \left[ \int_{Kw}^K r_m(t_h) dt_h - \int_{Ke}^K r_m(t_h) dt_h \right] \\ \rightarrow \text{if } A_i > re \\ Cw \leq Cr \left[ \int_{Lw}^L r_m(t_u) dt_u - \int_{Le}^L r_m(t_u) dt_u \right] \end{cases}$$

#### • Option III-2: (16)

$$\begin{cases} \rightarrow \text{if } A_i \leq re \\ Cw \leq Cr \left[ \int_{Kw}^K r_m(t_h) dt_h - \int_{Ke}^K r_m(t_h) dt_h \right] + Cm[N_1 - N_3] \\ \rightarrow \text{if } A_i > re \\ Cw \leq Cr \left[ \int_{Lw}^L r_m(t_u) dt_u - \int_{Le}^L r_m(t_u) dt_u \right] + Cm[N_2 - N_4] \end{cases}$$

#### • Option IV-2: (17)

$$\begin{cases} \rightarrow \text{if } A_i \leq re \\ Cw \leq Cr \left[ \int_{Kw}^K r_m(t_h) dt_h - \int_{Ke}^K r_m(t_h) dt_h \right] + Cm[N_7 - N_5] \\ \rightarrow \text{if } A_i > re \\ Cw \leq Cr \left[ \int_{Lw}^L r_m(t_u) dt_u - \int_{Le}^L r_m(t_u) dt_u \right] + Cm[N_8 - N_6] \end{cases}$$

### 2.4 Determination of the minimum price at which the manufacturer should sell the extended warranty

We establish below, the condition which should be satisfied so that selling the extended warranty region would be profitable for the manufacturer. This condition is based on the fact that selling the extended warranty would be beneficial for the manufacturer if the total cost incurred to him, during the product lifecycle, would be lower than what it would cost him in case he does not sell it. Let  $Cw$  be the minimum cost for which the manufacturer can sell the extended warranty region. For each maintenance option considered, the extended warranty cost ( $Cw$ ) can be expressed as follows:

$$\begin{cases} \rightarrow \text{if } A_i \leq re \\ Cw \geq Cr \left[ \int_0^{Ke} r_m(t_h) dt_h - \int_0^{Kw} r_m(t_h) dt_h \right] \\ \rightarrow \text{if } A_i > re \\ Cw \geq Cr \left[ \int_0^{Le} r_m(t_u) dt_u - \int_0^{Lw} r_m(t_u) dt_u \right] \end{cases} \quad (18)$$

**Table 3: Compromise intervals for the extended warranty cost for different usage rates**

	Option I	Option II	Option III-1	Option III-2	Option IV-1	Option IV-2
Light users	[1000,1000]	[213.57, 224.92]	Ø	Ø	Ø	[341.69, 3003.11]
Hard users	[4000,4000]	[854.28, 899.69]	Ø	Ø	Ø	[1155.81, 11112.44]

### 3 NUMERICAL EXAMPLE

To illustrate our approach, we consider a situation with the following input data which have been arbitrarily chosen considering nevertheless realistic settings, the units considered for age and usage are years and 1000 km, respectively. We consider a system with a basic warranty period  $K_w=2$  years which can be extended for one year until  $K_e=3$  years. The system's lifecycle  $K=9$  years. The corresponding usage of the system is  $L_w=12$ ,  $L_e=18$ ,  $L=54$  allowing a nominal usage per time unit  $r_e=6$ . Periodic PM actions are performed over different periods of the system's lifecycle every  $\Delta=0.33$  year for a cost  $C_m=100$  € paid by the consumer and a minimal repair cost  $C_r=200$  €.

The system's time to failure has a Weibull distribution with the following parameters: shape parameter  $\alpha=2.0$  and scale parameter  $\lambda=1.0$ . Finally, the ARI parameter for PM actions is  $\rho=0.5$ .

We applied the model with the input data mentioned above. We have looked, for each maintenance option and for different consumer usage rates, for the existence of a compromise (win-win) interval  $[a,b]$  where 'a' stands for the minimum price at which the manufacturer should sell the extended warranty, and 'b' represents the maximum extra cost the consumer should pay for the extended warranty when buying the product. Table 3 shows the obtained results.

According to the obtained results illustrated in table 3, one can notice that, for example, for options II and IV-2 the win-win interval for the extended warranty exists for both hard and light users. For option II and for light users the minimum extended warranty cost that the manufacturer could accept to sell the extended warranty is 341.69 monetary unit and the maximum extra cost the consumer can pay for the extended warranty is 3003.11 monetary unit, the optimal case from the two perspectives is two pay 1672.40 monetary unit.

On the other hand, for the two options (II, IV-2), the compromise interval of the extended two-dimensional warranty region exists for light and hard users. The minimum cost for which the manufacturer should sell the extended warranty and the maximum extra cost the consumer should pay for the extended warranty is higher when consumer behavior is harder. This is obviously due to the fact that with a hard user, the failure rate will increase and consequently the total expected cost incurred from maintenance actions (repairs) will increase.

We can also note, that for option I there's no zone of compromise for the extended two-dimensional war-

ty cost. For this option one can notice that the win-win interval is reduced in this case to a unique identical value for both of them. If the extended warranty cost is higher than 1000 monetary units, taking the extended warranty would not be advantageous for the consumer; this conclusion is also valid for the manufacturer if the extended warranty cost is lower than 1000 monetary units. The threshold values obtained for both the consumer and the manufacturer is the same due to the fact that they both correspond to the minimal repair costs incurred during the interval  $[W, W_e]$ . Since there is no PM, the expected number of minimal repairs during this period remains obviously the same with or without the extended warranty.

For options III-1, III-2 and IV-1, the PM actions are performed during a short period over the system's life cycle duration. Therefore, these actions have almost no effect on the failure rate of the system. This yields higher costs incurred by the manufacturer compared to the costs incurred by the consumer during the extended two-dimensional warranty period. Consequently, one can notice the nonexistence of a win-win zone for these options.

### 4 CONCLUSION

In this paper we developed a mathematical model to study the opportunity provided by offering and adopting an extended two-dimensional warranty for a randomly failing product.

Different maintenance options were considered during the product's life cycle. For each of them, the total expected cost incurred by the consumer and the manufacturer has been expressed.

We looked for the possible existence of a compromise zone for the extended warranty cost. We expressed the maximum extra cost the consumer should pay for the extended warranty, and the minimum price at which the manufacturer should sell it.

According to the obtained numerical results, such a compromise is not always possible. It depends mainly on the consumer usage rate, the adopted maintenance policy, the repair cost as well as on the PM frequency.

To extend this work, we are currently studying the same problem considering imperfect repair following failure. Another perspective is to consider a variable deterministic or stochastic usage rate.

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