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A CONDITION-BASED MAINTENANCE MODEL FOR AVAILABILITY OPTIMIZATION FOR STOCHASTIC DEGRADING SYSTEMS

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ABSTRACT: *This paper proposes a condition-based maintenance approach for availability optimization problem. The system to be maintained is subject to stochastic degradations and assumed to be continuously monitored. Imperfect preventive maintenance actions are made on the basis of the hybrid hazard model and the condition to perform a preventive maintenance corresponds to a system reliability threshold. After a number of preventive maintenance cycles, the system is replaced by a new one. The maintenance optimization problem to be solved consists on finding the optimal reliability threshold together with the optimal number of preventive maintenance cycles to maximize the average system availability. A mathematical model is proposed and numerically solved. A numerical example is provided for illustration.*

KEYWORDS: *Reliability, Condition-based preventive maintenance, Optimization, Hazard rate model.*

1 INTRODUCTION

The framework of preventive maintenance for reparable systems was initiated by Barlow and Hunter in their seminal paper published in 1960 (Barlow & Hunter 1960) (see also the book by Barlow and Proshan (Barlow & Proshan 1996)). Since, a variety of mathematical models appeared in the literature for the design of optimal maintenance policies for reparable systems. For a survey, the reader may refer, for example, to (Cho & Parlar 1991, Dekker 1996) and (Jardine & Tsang 2006), and the references therein. Further generalizations of Barlow and Hunter work are proposed by Nakagawa and his coauthors (see the recent reference (Nakagawa & Mizutani 2009)). In (Nakagawa & Mizutani 2009), Nakagawa and Mizutani extended the well known results of periodic replacement with minimal repair policy as well as that of block replacement policy from infinite into finite time horizon setting. For more details about preventive maintenance policies made within finite time horizon, one may refer to the book of Nakagawa (Nakagawa 2008). Lugtigheid and his coauthors gave also an interesting review of a number of maintenance models (Lugtighei, Jardine & Jiang 2007).

In the last decade, an increasing effort is made to

encourage the usage of a specific type of preventive maintenance called either condition-based maintenance or predictive maintenance. Condition based maintenance (CBM) consists to monitor, either continuously or according to a regular time intervals, a particular health condition of a system so that to prevent its failure and to determine appropriate preventive maintenance to be performed. Within the CBM approach, system health conditions is obtained via observations, collections and analysis of data such as temperature, gas emission, vibration, and so on. According to a level of system health condition, maintenance actions are then scheduled to reduce system failures while reducing total operation costs. In the existing literature, a large variety of mathematical models, methods as well as techniques have been developed to deal with CBM of systems (to cite a few, see for example (Wang 2000) (Grall, Bérenguer & Dieulle 2002) (A.Ghasemi, Yacout & Ouali 2007) (Tian & Liao 2011) and the references therein). In the work by Wang (Wang 2000), a mathematical model is proposed to determine the optimal critical system health threshold together with monitoring intervals in condition based maintenance. The proposed model is derived on the basis of the regression growth model whose coefficients are assumed to be of known distri-

butions. In (Grall et al. 2002), Grall and his coauthors gave an analytical model for inspection and replacement of a single-unit system in the CBM setting. In (Grall et al. 2002), replacement threshold as well as inspection times are considered as decision variables for the maintenance policy whose objective function consists to minimize the expected total maintenance cost per time unit. Ghasemi *et al.* in their work (A.Ghasemi et al. 2007) established a CBM optimization model on the basis of partially observed Markov decision process. In their paper, the system degradation is represented by using the proportional hazards model. The optimization problem formulated is solved by using dynamic programming technique. More recently Tian and Liao propose, on the basis of proportional hazards model, a CBM model for a system with more than one component (Tian & Liao 2011).

The present paper studies a condition-based maintenance optimization problem. The system considered is assumed to be continuously monitored and degrades according to a given stochastic process. Imperfect preventive maintenance actions are made on the basis of hybrid hazard model and the condition to perform a preventive maintenance corresponds to a system reliability threshold. Therefore, the system undergoes preventive maintenance when its reliability reaches a threshold level R_{th} . In the case where the system fails before reaching threshold R_{th} , a corrective maintenance is then carried out. After a number N of preventive maintenance actions, the system is replaced by a new one. Both preventive and corrective maintenance actions are made identically on the basis of the hybrid hazards model which take into account two factors, namely the age reduction factor and hazards increase factor (Nakagawa & Mizutani 2009). As pointed out by Wang in his paper (Wang 2000), the choice of reliability threshold R_{th} and number N of preventive maintenance actions to be done, before replacing the system by a new one, will obviously have an economic impact on the performance of the maintenance policy. Indeed, on one hand, a low value of the threshold R_{th} implies a long and an uninterrupted use of the system but with, however, an increased risk of system failures. On the other hand, if the reliability threshold is set to a high value, the number of system failures is then reduced but with an increasing preventive maintenance cost and unavailability. Therefore, the maintenance optimization problem to be solved consists on finding the optimal reliability threshold R_{th}^* together with the the optimal number N^* of preventive maintenance actions to maximizes the average availability of the system.

The rest of the paper is organized as follows. The next section addresses the hybrid hazard rate. It also shows how this model can be exploited to predict system reliability. In Section 3, the condition-based

maintenance is introduced and formally modeled on the basis of the results obtained in Section 2. A numerical example is provided in Section 4. The conclusion and future works are drawn in Section 5.

2 HYBRID HAZARD MODEL AND SYSTEM RELIABILITY PREDICTION

Assume that imperfect preventive maintenance actions are carried out at the end of N cycles (intervals) each of which the duration is T_k ($k = 1, 2, \dots, N$). According to the hybrid hazards model, the failure rate of the system after the $(k-1)^{th}$ preventive maintenance is denoted by $h_k(t)$ and defined recursively as:

$$\begin{aligned} h_k(t) &= b_{k-1} h_{k-1}(t + a_{k-1} T_{k-1}) \\ t &\in [0, T_k], k = 2, \dots, N \end{aligned} \quad (1)$$

where a_{k-1} and b_{k-1} states, respectively, for the age reduction coefficient and the hazard rate increase coefficient such that $0 \leq a_{k-1} \leq 1$ and $b_{k-1} \geq 1$. the hazard rate $h_1(t)$, for $t \in [0, T_1]$, is the hazard rate of the system assumed initially new. By taking particular values for coefficients a_k and b_k , the hybrid hazard model becomes pure proportional hazard model or an age reduction model (Nakagawa & Mizutani 2009). From the above equation, it is easy to write the hazard rate function $h_k(t)$ in the following form:

$$h_k(t) = \left(\prod_{i=1}^{k-1} b_i \right) h_1 \left(t + \sum_{i=1}^{k-1} a_i T_i \right). \quad (2)$$

By exploiting the recursion rule given by Equation (1), it is possible to predict the process degradation of the system, i.e. to predict the system health condition given by the reliability threshold R_{th} in the case of the present paper. Indeed, the system reliability $R_k(t)$ in the k^{th} cycle is given by:

$$R_k(t) = \exp \left(- \int_0^t h_k(x) dx \right), \quad (3)$$

By using Equation (2), the reliability $R_k(t)$ can be rewritten as:

$$R_k(t) = \exp \left(- B_k \int_{A_k}^{t+A_k} h_1(x) dx \right), \quad (4)$$

where $A_k = \sum_{i=1}^{k-1} a_i T_i$ and $B_k = \prod_{i=1}^{k-1} b_i$.

Preventive maintenance times T_k ($k = 1, \dots, N$) corresponds to the time the system reliability reaches the threshold level R_{th} . it follows that:

$$R_{th} = \exp \left(- \int_0^{T_k} h_k(t) dt \right). \quad (5)$$

By using basic algebraic operations, the time instant T_k can easily be obtained such that:

$$T_k = \mathcal{H}_1^{-1} \left(\mathcal{H}_1(A_k) - \frac{\log(R_{th})}{B_k} \right), \quad (6)$$

where the function $\mathcal{H}_1(t) = \int_0^t h_1(x)dx$ is the cumulative hazard rate and $\mathcal{H}_1^{-1}(t)$ is the inverse function of $\mathcal{H}_1(t)$. In the particular case where the system lifetime is Weibull distributed with the shape and scale parameter denoted, respectively, by β and η , the above equation becomes:

$$T_k = \eta \left(\left(\frac{A_k}{\eta} \right)^\beta - \frac{\log(R_{th})}{B_k} \right)^{\frac{1}{\beta}} - A_k. \quad (7)$$

according to the above case, the reliability $R_k(t)$ at the k^{th} cycle is then written as:

$$R_k(t) = \exp \left[B_k \left(\frac{A_k}{\eta} \right)^\beta - \left(\frac{t + A_k}{\eta} \right)^\beta \right].$$

3 MAINTENANCE OPTIMIZATION MODEL

According to the CBM strategy adopted in the present paper, the system may experience N maintenance cycles. Within each cycle, the system is maintained either when it fails or its reliability reaches the threshold R_{th} . Corrective maintenance is carried out whenever the system fails while preventive maintenance is performed in the case where the system survives until the required reliability threshold is reached. Expected durations of corrective and preventive maintenance are, respectively, denoted by T_c and T_p . At the end of the N^{th} cycle, the system is replaced by a new one. The expected duration of system replacement is denoted by T_r . The optimal maintenance policy consists then on finding optimal values of both the number N of cycles and the critical system reliability threshold R_{th} that maximizes the average system availability. In the rest of this paper, the average system availability is denoted by $\mathcal{A}(R_{th}, N)$ as a function of the two parameters N and R_{th} . We define such availability as:

$$\mathcal{A}(N, R_{th}) = \frac{\sum_{k=1}^N UpTime(k)}{\sum_{k=1}^N (UpTime(k) + DownTime(k))} \quad (8)$$

where $UpTime(k)$ and $DownTime(k)$, for $k \in \{1, \dots, N\}$ represents the average time, during the k^{th} maintenance cycle, where the system is in its up state and down state, respectively. The following proposition gives $UpTime(k)$ and $DownTime(k)$.

Proposition 1 During the k^{th} maintenance cycle, $DownTime(k)$ and $UpTime(k)$ are evaluated such that:

$$DownTime(k) = \begin{cases} T_c(1 - R_{th}) + T_p R_{th} & \text{if } k = 1, \dots, N-1, \\ T_r & \text{if } k = N. \end{cases} \quad (9)$$

$$UpTime(k) = \int_0^{T_k} R_k(t) dt. \quad (10)$$

Proof. Let us denote by the random variable X_k the residual lifetime of the system before the k^{th} maintenance (preventive or corrective) is performed. Let $F_k(t)$ be the cumulative distribution function of X_k and $R_k(t) = 1 - F_k(t)$ be its reliability function. Since the system undergoes maintenance whenever its reliability reaches the threshold value R_{th} or fails before. It follows that probabilities to perform preventive and corrective maintenance are, respectively, given by $P(X_k > T_k) = R_k(T_k)$, which is equal to R_{th} , and $P(X_k \leq T_k) = F_k(T_k)$, for $k = 1, \dots, N-1$. Dealing with the N^{th} cycle, the system is replaced either at failure or at the time where the reliability threshold is reached. This induces a duration T_r with probability 1. On the other hand, the system remains in its operating state for a random time given by the random variable $\min(X_k, T_k)$ which also corresponds to the up time. It follows that the expected system operating time is given by:

$$E(\min(X_k, T_k)) = \int_0^{T_k} R_k(t) dt,$$

this ends the proof. ■

From the results of the above proposition, it follows that the average system availability $\mathcal{A}(N, R_{th})$:

$$\mathcal{A}(N, R_{th}) = \frac{\mathcal{N}(N, R_{th})}{\mathcal{D}(N, R_{th})}, \quad (11)$$

where:

$$\begin{aligned} \mathcal{N}(N, R_{th}) &= \sum_{k=1}^N \left(\int_0^{T_k} R_k(t) dt \right), & (12) \\ \mathcal{D}(N, R_{th}) &= (N-1)(T_c(1 - R_{th}) + T_p R_{th}) + T_r + \sum_{k=1}^N \left(\int_0^{T_k} R_k(t) dt \right). & (13) \end{aligned}$$

By maximizing the system availability $\mathcal{A}(N, R_{th})$, the optimal maintenance policy can be determined according to the optimal values obtained for both the reliability threshold and the number of maintenance cycles. From the value of the optimal reliability threshold together with the initial system hazard rate, time

instants where preventive maintenance actions should be carried out are derived straightforward from Equation (7). In the following section, a numerical example is provided to illustrate the proposed CBM optimization model.

4 NUMERICAL EXAMPLE

In this section a numerical example is conducted for a system whose lifetime follows a Weibull distribution with shape and scale parameters are, respectively, $\beta = 2.8$ and $\eta = 100$ unit of time. Age reduction coefficients a_k as well as hazard rate increasing coefficients b_k ($k = 1, \dots, N$) are obtained from formula initially used in (Zhou, Xi & Lee 2007):

$$a_k = \frac{k}{3k+7} \text{ and } b_k = \frac{12k+1}{11k+1}. \quad (14)$$

Corrective and preventive maintenance as well as replacement durations (unit of time) are set to values such that:

$$T_c = 40, T_p = 4, T_r = 100.$$

By varying simultaneously the reliability threshold R_{th} and the number N of cycles, Figure (1)) shows that the optimum value of the average system availability is obtained to be nearly equal to $\mathcal{A}(N^*, R_{th}^*) = 60.57\%$. For this availability value, the optimal reliability threshold is equal to $R_{th}^* = 0.47$, while the optimal number of scheduled preventive maintenance in addition to replacement is obtained to be $N^* = 5$. It follows that the system is replaced by a new one after the 5th preventive maintenance. The system replacement is performed whenever the system fails or its reliability reaches the threshold value $R_{th}^* = 0.47$. Optimal time intervals for scheduled preventive maintenance and replacement are given in Table 1. From this table, it can be seen that such time intervals decrease by the increase of cycles number. This is mainly due to both the system degradation and the imperfect maintenance actions carried out on the system.

k	1	2	3	4	5
T_k	90, 45	78, 91	64, 75	51, 71	41, 18

Table 1: Time intervals for scheduled preventive maintenance and replacement

5 CONCLUSION

This paper investigated the optimization problem of a condition-based maintenance. The system considered is assumed to be continuously monitored and degrades according to a given stochastic process. Imperfect preventive maintenance actions are made on the basis of hybrid hazard model and the condition

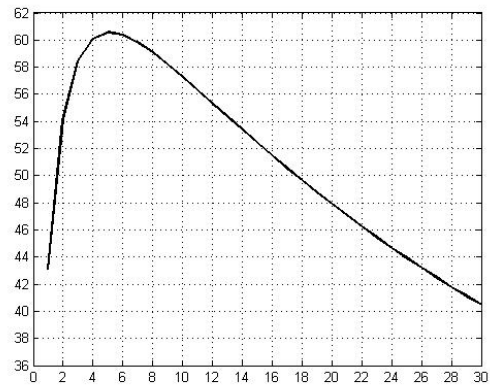


Figure 1: Average system availability versus the number of PM cycles: case of optimal reliability threshold

to perform a preventive maintenance corresponds to a system reliability threshold. The hybrid hazard model offers the advantage to combine the effect of two coefficients, namely the age reduction coefficient and the hazard rate increase coefficient. Furthermore, it allows to represent the system degradation and the imperfectness of the preventive maintenance. A mathematical model is then proposed and numerically solved. The optimal maintenance policy allows to derive the critical reliability threshold together with the number of preventive maintenance actions that maximizes the average system availability. As a possible improvement of the present work, it is of great interest to investigate on how both coefficients of the hybrid hazard model can be obtained. Another issue consist to deal with the problem of simultaneous optimization of such maintenance strategy combined with production constraints.

References

- A.Ghasemi, Yacout, S. & Ouali, M. (2007). Optimal condition based maintenance with imperfect information and the proportional hazards model, *International Journal of Production Research* **45**(4): 989–1012.
- Barlow, R. & Hunter, L. (1960). Optimum preventive maintenance policies, *Operations Research* **8**(1): 90–100.
- Barlow, R. & Proschan, F. (1996). *Mathematical theory of reliability*, reprint (31 december) edn, Society for Industrial and Applied Mathematics, US.
- Cho, D. I. & Parlar, M. (1991). A survey of maintenance models for multi-unit systems, *European Journal of Operational Research* **51**: 1–23.
- Dekker, R. (1996). Application of maintenance optimization models: a review and analysis, *Relia-*

bility Engineering and system safety **51**(3): 229–240.

- Grall, A., Bérenguer, C. & Dieulle, L. (2002). A condition-based maintenance policy for stochastically deteriorating systems, *Reliability Engineering and system safety* **76**: 167–180.
- Jardine, A. & Tsang, A. (2006). *Maintenance, replacement and reliability, theory and applications*, Taylor and Francis.
- Lugtighei, D., Jardine, A. & Jiang, X. (2007). Optimizing the performance of a repairable system under a maintenance and repair contract, *Quality and Reliability Engineering International* **23**: 943–960.
- Nakagawa, T. (2008). *Advanced reliability models and maintenance policies*, Springer.
- Nakagawa, T. & Mizutani, S. (2009). A summary of maintenance policies for a finite interval, *Reliability Engineering and system safety* **94**(1): 89–96.
- Tian, Z. & Liao, H. (2011). Condition based maintenance optimization for multi component systems using proportional hazards model, *Reliability Engineering and system safety* **96**: 581–589.
- Wang, W. (2000). A model to determine the optimal critical level and monitoring intervals in condition-based maintenance, *International Journal of Production Research* **38**(6): 1425–1436.
- Zhou, X., Xi, L. & Lee, J. (2007). Reliability-centered predictive maintenance scheduling for continuously monitored system subject to degradation, *Reliability Engineering and system safety* **92**(4): 530–534.