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ABSTRACT: *This paper considers a selective maintenance policy for multi-component systems for which a minimum level of reliability is required for each mission. Such systems need to be maintained between consecutive missions. The proposed strategy aims at selecting the components to be maintained (renewed) after the completion of each mission such that a required reliability level is warranted up to the next stop with the minimum cost, taking into account the time period allotted for maintenance between missions. This strategy is applied to binary-state systems subject to propagated failures with global effect, and failure isolation phenomena. A numerical example based on such a system is presented to illustrate the modeling approach.*

KEYWORDS: *Selective Maintenance; stochastic dependence; Failure isolation; Propagated failures.*

1 INTRODUCTION

Maintenance strategies for systems made of one component or being possibly assimilated to single-component systems have been extensively treated in the literature for several decades. However, in the last years, an increasing interest has been focused on the development and optimization of maintenance policies for multi-component systems. This is related to the fact that in real life, industrial as well as transport systems and many other types of equipment are generally made of many components which may have one or more types of dependence (economic, stochastic or structural). Economic dependence means that jointly maintaining some components may be cheaper than maintaining them separately. Stochastic dependency implies that failure or degradation of one component can affect the state of one or more other components of the system. Structural dependency is the fact that maintaining one component may imply the maintenance or at least the disassembly of one or more other components. Cho and Parlar (1991), Van der Duyn Schouten (1996), Dekker et al.(1997) and more recently Nicolai and Dekker (2006) provide overviews of optimal maintenance policies of multi-component systems with and without dependency.

In the present work, we focus on multi-component systems for which a high reliability level is required for each mission of known duration to be accomplished. Such systems (like manufacturing equipment, aircrafts, ships, computer systems, military weapons, etc.) must be maintained between consecutive missions. The problem consists in selecting the components to be maintained after the completion of each mission such that a required reliability level is warranted up to the next stop with the minimum cost and taking into account the limitations on maintenance time and resources before the start of the next mission. This problem has been tackled by Cassady et al.(2001a) in the case of series-parallel systems and for more general structures displaying redundancy with stochastically independent components. They developed a method to decide which failed components should be repaired before the next mission and which components should be left in a failed condition. They also optimize these selective maintenance decisions in situations where the objective is to maximize the system's reliability under budget and time constraints, and also in the case where maintenance time is minimized under the constraints of cost and reliability. This work has been extended by the same authors Cassady et al.(2001b) considering not only renewal of failed components but also the possibility to perform minimal repairs on failed components and preventive replacement of functioning ones. They addressed the case of system reliability maximization under budget and time constraints considering time dependent failure rates for all components whose failures are stochastically independent and whose lifetimes follow a Weibull distribution. They also used simulation in combination with their analytical model to be able to deal with a succession of missions.

More recently, Rajagopalan and Cassady (2006) considered the problem of finding the number of failed

components which should be replaced in order to maximize the system reliability for next mission under a maintenance time constraint. The considered systems have a series-parallel structure with constant failure rate components and stochastically independent failures. The authors improved the original total enumeration method proposed by Rice et al.(1998). The need of speeding up the solving procedure becomes of first importance when large-size problems (systems with great number of components) are addressed. For such cases, Galante and Passannanti (2009) developed an exact algorithm for solving the same selective maintenance problem allowing a drastic reduction of the solution space for series-parallel systems.

As it can be noticed through the above mentioned papers and others in the literature, it is always supposed that the maintenance actions are performed one after the other, making the time constraint expressed in terms of the sum of the replacement durations of each component. In this paper, we consider situations, often encountered in practice where the maintenance actions on different components start at the same time and they are carried out simultaneously. Moreover, most if not all the works on selective maintenance consider series-parallel systems or more general system structures involving redundancy with components failures being local and stochastically independent. In this work, we model the selective maintenance concept considering, on one hand, economic dependency and on the other hand, we apply it to complex systems, with functional dependence, subject to failure propagation and isolation effects. Propagated failures are common cause-failures originated from a component of a system causing the failure of the entire system (global effect) or the failure of some of its sub-systems (selective effect). Propagated failures with global effect can be caused by imperfect fault coverage despite the presence of adequate redundancy and fault tolerant mechanism (see Amari and al.(1999)). They can also simply be due to a destructive effect of failures of some components of the system.

Moreover, in practice, many systems experience what is called failure isolation. Xing and Levitin (2010) define this phenomenon as follows "the failure of one component (referred to as a trigger component) can cause other components (referred to as dependent components) within the same system to become isolated from the system, which on one hand, makes the isolated dependent components unusable; and on the other hand, prevents the propagation of the failures originated from those dependent components". I/O controllers of peripheral devices in a computer system are part of many real-world systems with isolation effect. Indeed, when the I/O controller fails, the connected peripheral devices become unusable and at the same time the computer becomes insensitive to any failure originated from those peripheral devices (Xing et al., 2009). Another example displaying both propagated failures with global effect and isolation is a computer network (Xing and Levitin (2010)). Indeed, computers communicate through Network Interface Cards (NIC). In case a NIC fails (considered here as the trigger), a connected computer (considered as a dependent component) becomes inaccessible but at the same time it prevents the propagation of failures like viruses for example from this computer to the network. This happens only if the propagated failure (the virus) originated in the computer occurs after the NIC failure, otherwise the virus could go into the entire network and crash it completely. This example illustrates the competition in time between the failure of the trigger components and failures initiated from the dependent components. Xing and Levitin (2010) established an analytical and combinatorial method to assess the system reliability in this case of competing propagated failures and failure isolation effect due to the functional dependence. Systems of this kind are considered in this paper within the context of selective maintenance with no limitation on the type of component's time to failure (local or propagated) distributions.

The remainder of the paper is organized as follows. The mathematical model corresponding to the considered selective maintenance problem is presented in next section along with the working assumptions and used notation. Section 3 reports the method developed by Xing and Levitin (2010) to assess the reliability of the considered type of systems subject to failure isolation and propagation effects. A numerical example is presented in section 4 to illustrate the developed model. Finally, the paper is concluded in section 5 with indications on current and future related research.

2 THE MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a multi-component system required to perform a series of successive missions and whose components can be maintained during scheduled downtime periods between successive missions. The problem consists in selecting the components to be maintained (renewed) after the completion of any mission (k) and before the start of the next mission (k+1), such that a required reliability level is warranted up to the next stop after D_{k+1} time units, with the minimum cost and without exceeding the time Δ_k scheduled for maintenance between missions k and k+1. This should be done considering that it is possible to pay a penalty cost for extending this maintenance period between missions to a certain extent.

The following assumptions are made:

- At the end of a given mission period, each component (as well as the system) is either functioning or failed.
- All maintenance actions consist in components renewals. They could be preventive renewals of working components or renewals of failed ones.
- The replacement actions of the selected components start at the same time. Hence, the required duration to complete all replacements

is equal to the largest required period per selected component.

The following notations are used:

f_{il} : Probability density function associated with time to local failure of component i. i=(1,2,…,n) , n being the number of components of the system.
R_{il} : The reliability function associated with time to local failure of component i.
f_{ip} : Probability density function associated with time to propagated failure of component *i.*
R_{ip}: The reliability function associated with time to propagated failure of component i.
F_s: The unreliability function associated with time to failure of the system.
t_i: The time required to replace the component i between missions.
D_k: The duration of mission k.
Δ_k : The downtime period between missions k and k+1 allotted to perform maintenance actions.
β_k: the extension coefficient of maintenance periods Δ_k ($\beta_k \geq 0$), i.e. a maintenance period Δ_k can be extended by a maximum of ($\Delta_k . \beta_k$).
C_p: The penalty cost per time unit due to the extension of a maintenance period Δ_k.
C_{ri}: The replacement cost of component i.
C_w: The maintenance labor cost per time unit.
C_f : A fixed cost incurred for dismantling and reassembling the system in case at least one component is to be replaced. This cost is incurred only one time in case more than one component is replaced.
C(k): The total maintenance cost incurred to maintain the system between mission k and mission k+1.
$E_i(k)$: The age of component i at the end of mission k.
$A_i(k+1)$: the age of component i at the beginning of mission k+1.
M(k): The maintenance decision vector made of n elements, each one is either equal to 1 (replace the corresponding element) or equal to 0 (do not replace the corresponding element). $M(k) = \left\{m_1^k, m_2^k ..., m_n^k\right\}$.

The binary variables:

m_i (k): The replacement decision of component i at the end of mission k.

$$m_i(k) = \begin{cases} 1 & \text{if component i is replaced} \\ & \text{between missions k and k+1.} \\ 0 & \text{if component i is not replaced} \\ & \text{between missions k and k+1.} \end{cases}$$

Y_i (k): The component i state at the end of mission k.

$$Y_i(k) = \begin{cases} 1 & \text{if component i is functioning at} \\ & \text{the end of mission k.} \\ 0 & \text{if component i is in failed state at} \\ & \text{the end of mission k.} \end{cases}$$

X_i (k+1) : The state of component i just before the start of mission k+1.

$$X_i(k+1) = \begin{cases} 1 & \text{if component i is functioning} \\ & \text{at the start of mission k+1.} \\ 0 & \text{if component i is in failed} \\ & \text{state at the start of mission k+1.} \end{cases}$$

$$z(k) = \begin{cases} 1 & \text{if } \Delta_k \text{ is lower than the largest required} \\ & \text{replacement period per component.} \\ 0 & \text{otherwise.} \end{cases}$$

$$\Phi(k) = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} m_i(k) \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

$\Phi(k) = 1$ corresponds to the situation of replacing at least one component. In this case, the setup cost is incurred only one time (i.e. economic dependency).

At the end of mission k, the decision maker should consider both possible states (working or failed) and the age of each component. Table 1 shows the effects of performing or not performing a replacement of a component on its state and age. Suppose 'a' being the age of the component at the end of mission k.

	Component i state and age	
State and age at the end of mission k	$Y_i(k)=0$ $E_i(k)=a$	$Y_i(k)=1$ $E_i(k)=a$
State and age in case replacement is performed	$X_i(k+1)=1$, $A_i(k+1)=0$	$X_i(k+1)=1$, $A_i(k+1)=0$
State and age in case replacement is not performed	$X_i(k+1)=0$, $A_i(k+1)=a$	$X_i(k+1)=1$, $A_i(k+1)=a$

Table 1. Effect of replacing or not replacing a component.

The total cost incurred by the replacement of a component i is expressed as follows:

$$C_f + \left(C_{ri} + C_w t_i\right) \quad (1)$$

Hence, the problem can be formulated mathematically as follows:

Min C (k) =

$$\sum_{i=1}^{n}\left[C_{ri}+C_w t_i\right]\times m_i(k)+\left[\left(\mathrm{Max}\left(t_i\times m_i(k)\right)-\Delta_k\right)\times C_p\times z(k)\right]+\left[C_f\times\Phi(k)\right] \quad (2)$$

Subject to.

$$F_S(D_{k+1})\le F_S^* \quad (3)$$

$$\mathrm{Max}(t_i\times m_i(k))\le \Delta_k+\beta_k\Delta_k \quad (i=1,...,n) \quad (4)$$

Equation (3) expresses the reliability constraint. F_s^* being the probability of failure of the system not to be exceeded for mission (k+1). Equation (4) expresses the fact that for a given maintenance option (vector M(k)), the longest maintenance period among those of the components to be maintained should be smaller than the maintenance scheduled period Δ_k plus its maximum extension period.

For each component i with age $A_i(k+1)$ and state X_i (k+1) just before initiating mission (k+1), the reliability function related to local failures is expressed as follows:

$$\hat{R}_{il}(D_{k+1})=X_i(k+1)\times\frac{R_{il}(D_{k+1}+A_i(k+1))}{R_{il}(A_i(k+1))} \quad (5)$$

For the components which may also cause propagated failures, the corresponding reliability function is given by:

$$\hat{R}_{ip}(D_{k+1})=X_i(k+1)\times\frac{R_{ip}(D_{k+1}+A_i(k+1))}{R_{ip}(A_i(k+1))} \quad (6)$$

The age $A_i(k+1)$ is obtained as follows:

$$A_i(k+1)=E_i(k)-E_i(k)\times m_i(k) \quad (7)$$

And the state $X_i(k+1)$ is given by:

$$X_i(k+1)=Y_i(k)+m_i(k)\times(1-Y_i(k)) \quad (8)$$

Hence, given the state and age of each component observed at the end of mission k, for each possible maintenance decision vector M(k) among the theoretical number of 2^n, the decision maker can evaluate the cost C(k) and check the reliability constraint using the proposed mathematical model presented above. The best decision will be the one allowing the minimum cost while satisfying both constraints.

The proposed formulation holds for any type of system structure with any given number of components with any type of time to failure distributions. In next sections, we apply it to a particular type of complex binary-state systems subject to propagated failures with global effect, and failure isolation phenomena.

3 RELIABILITY ASSESSMENT OF SYSTEMS SUBJECT TO COMPETING PROPAGATED FAILURES AND FAILURE ISOLATION EFFECT.

We consider binary-state multi-component systems displaying functional dependence with competing propagated failures and failure isolation effect. Failure isolation happens when the failure of one component (the trigger component) causes other components (dependent components) to become unusable. Isolation prevents the propagation of failures initiated from those dependent components. All components can experience local failures and some of them may also be at the origin of propagated failures. The local failure and the propagated failure of the same component are mutually exclusive.

The evaluation approach proposed by Xing and Levitin (2010) to assess the reliability of such systems or sub-systems is summarized below in 3 steps. It is based on the total probability theorem.

Step1: Define the three different events (Ri) representing the order of failures in the trigger component and its corresponding dependent components. R1: 'the isolating/trigger element does not fail at all'; R2: 'at least one dependent element fails globally before the failure of the isolating element' and R3: 'the trigger element fails before any global failure originated in the dependent components'. Then, evaluate the occurrence probabilities for R1 and R2, and calculate Pr(R3) =1- Pr(R1) - Pr(R2). Step2: using conventional dynamic fault tree analysis methods (Dugan and Doyle, 1997), determine the conditional probabilities of system failure given the occurrence of Ri: $\Pr(\text{system fails}\,|\,R1)$, $\Pr(\text{system fails}\,|\,R2)$, $\Pr(\text{system fails}\,|\,R3)$. Step3: Compute the unreliability of the system as follows:

$$F_S(t)=\sum_{i=1}^{3}\Pr(\text{system fails}\,|\,R_i\rangle)*\Pr(R_i) \quad (9)$$

This reliability assessment method is used in this paper to evaluate the system reliability for mission k+1 $F_s(D_{k+1})$ to check the constraint expressed by equation (3).

4 NUMERICAL EXAMPLE

In order to illustrate our approach, let us consider a relatively simple four-component system with competing failures subject to failure isolation and propagation effects. The fault tree model of this example system is shown in figure 1 below. It is composed of four components A, B, C and D. The same system has been used by Xing and Levitin (2010) as an illustrative example considering all components as new (Age = 0).

The special gate in figure 1 is called functional dependence gate (FDEP) (Dugan and Doyle, 1997) is used to model the functional dependence. In this example, when the trigger component A fails the dependent components (B and C) become unusable. All the components can experience local failures and only the dependent components (B and C) can also cause propagated failures with global effect.

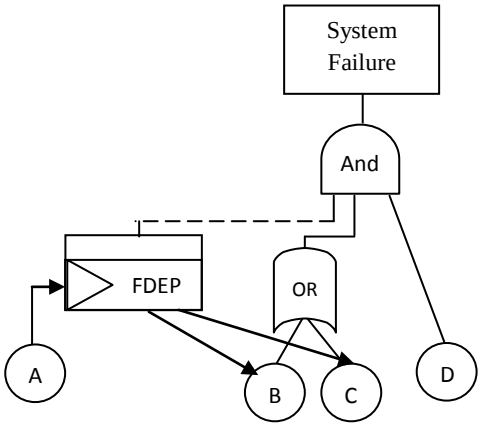

Figure 1: Fault tree model of the considered system.

For any given situation before the start of mission k+1 regarding the state $X_i(k+1)$ and the age $A_i(k+1)$ of each component (i=A,B,C,D), applying the reliability assessment method presented in section 3 in combination with equations (5) and (6), the probability of failure of the considered system over the duration D_{k+1} is expressed as follows :

$$F_s(D_{k+1}) = \hat{R}_{Al}(D_{k+1}) \times \left[1 - (\hat{R}_{Bp}(D_{k+1}) \times \hat{R}_{Cp}(D_{k+1})) + (\hat{R}_{Cp}(D_{k+1}) \times (1 - \hat{R}_{Dl}(D_{k+1})) \times (1 - \hat{R}_{Bl}(D_{k+1}))) + \left[\begin{array}{l} (1 - \hat{R}_{Dl}(D_{k+1})) \times (\hat{R}_{Bp}(D_{k+1}) - (1 - \hat{R}_{Bl}(D_{k+1})) \\ \times (1 - \hat{R}_{Cl}(D_{k+1}))) \end{array} \right] \right]$$
$$+ \left[\frac{1}{R_{Bp}(A_B(k+1)) \times R_{Cp}(A_C(k+1)) \times R_{Al}(A_A(k+1))} \times \int_0^{D_{k+1}} \int_0^{D_{k+1}} \int_{\min\{\tau_1,\tau_2\}}^{D_{k+1}} f_{Bp}(\tau_1 + A_B(k+1)) f_{Cp}(\tau_2 + A_C(k+1)) f_{Al}(\tau_3 + A_A(k+1)) d\tau_3 d\tau_2 d\tau_1 \right]$$
$$+ ((1 - \hat{R}_{Dl}(D_{k+1})) \times \left[1 - \hat{R}_{Al}(D_{k+1}) - \left[\begin{array}{l} \frac{1}{R_{Bp}(A_B(k+1)) \times R_{Cp}(A_C(k+1)) \times R_{Al}(A_A(k+1))} \times \int_0^{D_{k+1}} \int_0^{D_{k+1}} \int_{\min\{\tau_1,\tau_2\}}^{D_{k+1}} f_{Bp}(\tau_1 + A_B(k+1)) f_{Cp}(\tau_2 + A_C(k+1)) \\ f_{Al}(\tau_3 + A_A(k+1)) d\tau_3 d\tau_2 d\tau_1 \end{array} \right] \right] \quad (10)$$

The details of the development of this expression are given in (Maaroufi et al. 2011).

We suppose the time to local and propagated failures follow Weibull distributions with scale parameter λ and shape parameter α as shown below. When $\alpha = 1$ it reduces to an exponential distribution.

$\lambda_{Al} = 0.01$ and $\alpha_{Al} = 1$;
$\lambda_{Bl} = \lambda_{Cl} = 0.012$ and $\alpha_{Bl} = \alpha_{Cl} = 3$;
$\lambda_{Bp} = \lambda_{Cp} = 0.001$ and $\alpha_{Bp} = \alpha_{Cp} = 1$;
$\lambda_{Dl} = 0.02$ and $\alpha_{Dl} = 3$;

The following arbitrarily chosen input data are considered.

Maintenance labor cost per time unit $\mathbf{C_w}$	400 $
Replacement cost for each component i, $\mathbf{C_{ri}}$	1200 $
Penalty cost per time unit, $\mathbf{C_p}$	200 ($ / time unit)
The setup cost, $\mathbf{C_f}$	50 $
Time $\mathbf{t_i}$ required to replace the component i, in time units.	(t_A=0.5 , t_B=1.2 ,t_C=0.9, t_D=0.8)
Time allotted for maintaining the system before the start of mission k+1, Δ_k	1 time unit
Expected duration of mission k+1, $\mathbf{D_{k+1}}$	25 time units
The extension coefficient of the maintenance period $\boldsymbol{\beta_k}$	1

Table 2. Input parameters.

Let's suppose that at the end of mission k, the state and the age of each component are given by:

$\boldsymbol{Y_i(k)}$	$Y_A(k) = 0$	$Y_B(k) = 1$	$Y_C(k) = 0$	$Y_D(k) = 1$
$\boldsymbol{E_i(k)}$	$E_A(k) = 10$	$E_B(k) = 16$	$E_C(k) = 11$	$E_D(k) = 15$

Table 3. Input parameters regarding the state and age of each component at the end of mission k.

It is interesting to notice that this state of the system is possible in case the failure of component C has been a local failure (if it was a propagated one, all components would have been in a failed state) and it certainly happened before the failure of component A because if the latter failed before, it would have isolated component C.

The decision maker must select the components to be replaced such that the probability of failure of the system during next mission k+1 does not exceed the maximum allowed failure probability F_s^* = 0.15 (reliability requirement of 85%) with the minimum cost.

Using the mathematical model combined with equation (10), and considering all feasible maintenance options vectors M(k), we obtain the optimal solution M(k)* = (m_A^*= 0, m_B^*= 0, m_C*= 0, m_D* =1) which corresponds to a replacement of component D only. This solution yields the minimum cost C(k)*= 1570$ with system failure probability during next mission $F_s(D_{k+1})$ = 12.22 % (lower than 15%). As shown in table 4, the other components remain in the same state found at the end of mission k with the same age (only D is preventively renewed).

$X_i(k+1)$	$X_A(k+1)=0$	$X_B(k+1)=1$	$X_C(k+1)=0$	$X_D(k+1)=1$
$A_i(k+1)$	$A_A(k+1)=10$	$A_B(k+1)=16$	$A_C(k+1)=11$	$A_D(k+1)=0$

Table 4. Components State and age before starting mission k+1.

Note that in order to find out if there is at least one feasible solution satisfying the reliability constraint (equation 3), one has first to check if the vector M(k) = (1,1,1,1) corresponding to a renewal of all components allows the satisfaction of the constraint. In case it does not, there is no need to test any other maintenance option. The operations manager should find a way to shorten the next mission duration such that the reliability requirement can be met.

5 CONCLUSION

This paper proposed a formulation of the problem of selective maintenance for multi-component systems to help the decision maker selecting the components to be maintained between consecutive missions such that a required reliability level is guaranteed up to the next stop with the minimum cost and taking into account limitations on available time for maintenance. In this respect related to the time constraint, we considered situations where all the components replacements start at the same time. Also, the proposed approach takes into account economic dependence on one hand, and on the other hand, it has been applied to the case of complex multi-component systems with stochastic dependence subject to failure isolation and propagation effects. Though, the mathematical model can be applied to any kind of system structure with any kind of components failure time distribution, provided one can assess the system's reliability function with new or aged components.

This work is currently being extended in some respects including the consideration of a planning horizon with several successive planned missions, the possibility of performing imperfect maintenance actions between missions and the development a procedure which minimizes the number of maintenance decision vectors M(k) to be explored given any combination of state and age for each component, hence minimizing the computation time of the optimal solution.

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