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AN OPTIMAL MAINTENANCE/PRODUCTION PLANNING FOR A MANUFACTURING SYSTEM UNDER THE MACHINE AVAILABILITY AND SUBCONTRACTING CONSTRAINT

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ABSTRACT: *This paper deals with a production and maintenance problem of manufacturing system under the availability of the machine and subcontracting constraints. We have developed an integrated production/maintenance policy for a manufacturing system satisfying a random demand by using a subcontracting machine.*
In order to ensure a simultaneous economic production planning with an optimal maintenance strategy, a joint optimization is made in order to minimize the total cost of production, holding, subcontracting and maintenance costs. An analytical study and a numerical example are presented to prove the developed approach.

KEYWORDS*: Manufacturing system, random demand, availability of machine, Failure rate, subcontracting, simultaneous optimization.*

1- INTRODUCTION

In an economic context, the internal and external environment of enterprises is characterized by markets subject to strong competition, and customer expectations and requirements are becoming higher in quality, cost and delivery times. One of the first actions in a hierarchical decision chain is the elaboration of an aggregated production plan, (Tsubone, et al., 1991) and (see Hax and Candea, 1985).

To remain competitive, companies need to better manage their operating costs and optimize their production systems. It is therefore necessary to develop industrial strategies (production, maintenance) and define a methodology for task scheduling production and maintenance.

Maintenance strategies and Production control of manufacturing systems subject to uncertainties such as demand fluctuations, system availability and variation machine failures. In order to limits these uncertainties; many companies have recourse to the industrial subcontracting. A number of approaches have been studied in the literature and most of them concern the determination of the economic manufacturing quantity for different products on a single or multiple manufacturing systems.

An integrated approach of maintenance policies and production planning and control has recently become an important research area. In this context, (O.S.Silva Filho, 2005) deals with a chance-constrained stochastic production-planning problem under hypotheses of imperfect information of inventory variables. The optimal production plan obtained by minimizing the expected cost. More then it's interested to develop an optimal maintenance strategy with considering the manufacturing system degradation according to the production rate. (N. Rezg, S. Dellagi. and A.Chelbi, 2008) presented a mathematical model and a numerical procedure which allows determining a joint optimal inventory control and age based preventive maintenance policy for a randomly failing production system. (Rezg, N.,Xie, X.,and Mati,Y, 2004) presented a common optimization of the preventive maintenance and stock control in a production line made up of N machines. (Van der Duyn Schouen, F.A., and Vanneste, 1995) addressed a production line of two machines separated by a buffer and proposed a preventive maintenance policy based not only on the age of the machine but also on the size of the buffer, both of which are used to determine when to perform a preventive maintenance action. The best time at which a preventive maintenance action in a manufacturing system must be carried out is very important for minimizing the total cost of maintenance and production.

In this paper, we will study a problem of an integrated maintenance policy for a manufacturing system calling upon subcontractor. Many studies address subcontracting in different areas, such as industry (Lehtinen, 1999), (Cagliano, 2002) and (Bertrand, 2001), aerospace (Amesse, 2001), construction (Tserng, 2002), project management (Gutierrez, 2000), trade and supply chain

(Andersen, 1999), (Andersen, 2000). In manufacturing various studies treating subcontracting in the literature. New maintenance/production strategies by taking into account the context of subcontractor are studied by (S. Dellagi, N. Rezg, X. Xie, 2007), developed and optimized a new maintenance policy with taking into account machine subcontractor constraints. A case study, which proves the influence of the subcontractor constraints on the optimal maintenance strategy adopted, has been presented in (S. Dellagi, Rezg, N, 2007b). Dealing with this frame, two cases of maintenance and production strategies, which are subcontractor and contractor constraints, have been treated in (Dahane et al, 2008). (Hajej, Z., S. Dellagi and N. Rezg. 2009) presented a new production and maintenance policies. These policies take into account the influence of the production rate on the material failure rate in order to establish the optimal maintenance strategy.
The paper is organized as follows. In section 2, we present the problem formulation. The analytical studies are developed for evaluating maintenance and production strategies in section 3. A simple numerical example is presented in chapter 5. Finally, the conclusion in Section 6.

2- PROBLEM DESCRIPTION

In this study, we are concerned with the problem of the jointly optimal production and maintenance planning problem formulation of a manufacturing system composed of one machine *M1* which produces a single product, working at a rate *u*, in order to meet a random demand characterized by a Normal distribution. The Normal mean and standard deviation parameters are respectively denoted by $\hat{d}$ and σ_d^2.

To make up the rest of the unmet demand by the machine *M1*, the system use a subcontractor, composed of a machine *M* 2 which produces the same kind of product. The principal machine availability depends on the number of partitions of preventive maintenance actions and the production rate.
Points of view reliability, the system is prone to random failure. The probability degradation law of machine *M* is described by the probability density function of time to failure *f(t)* and for which the failure rate $\lambda(t)$ increases with time and according to the production rate. Failure of machine *M1* can be prevented by preventive maintenance actions.
Our objective is to establish simultaneously an economical production plan and an optimal preventive maintenance period satisfying the randomly demand, with taking account the machine availability and the subcontractor constraint.

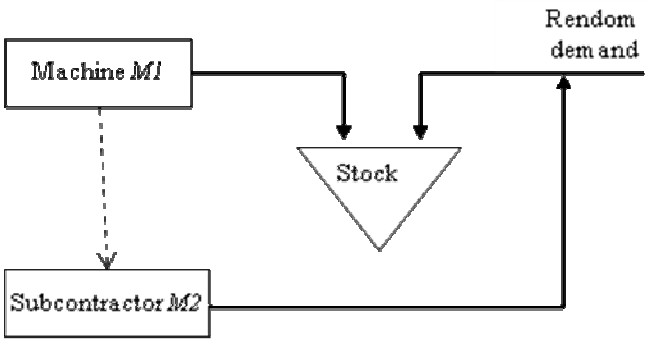

Figure 1: Problem description

2.1 NOTATION

H	finite production horizon
Δt	period length of production
$S(k)$	inventory level at the end of the period k ($k=1,\ldots\ldots,H/\Delta t$)
$u(k)$	production rate at period k
$u_S(k)$	The subcontractor production rate for period k
U_{max}	maximal production rate
$\beta(N)$	availability of system
θ	probabilistic index (related to customer satisfaction)
N	Number of partitions of preventive maintenance actions
C_p	unit production cost
C_{ps}	unit production cost of subcontractor machine
C_s	holding cost of a product unit during the period k
Mp	preventive maintenance action cost
Mc	corrective maintenance action cost
μp	preventive maintenance action delay
μc	corrective maintenance action delay
$\Gamma(.)$	maintenance cost
$\varphi_p(.)$	the average number of failure
$\lambda_i(t)$	failure rate fonction
$F(.)$	total expected cost of production and inventory over the finite horizon H

2.2 PROBLEM FORMULATION

To establish an economical production plan and optimal maintenance strategy, we define a stochastic model that minimizes the total costs over a finite horizon. The goal of the production/maintenance plan is to determine the greatest combination of production rate, inventory level and failure rate that minimizes the total costs over a planning horizon. In our model the customer satisfaction is made at the end of each period. The maintenance policy is taking into account the production rate in

determining the optimal number N of partitions of preventive maintenance actions to be carried out.

Formally, the problem is defined as follows:

$$\underset{(u,N)}{Min}\{F(u)+\Gamma(N)\}$$

With
$F(u)$: The total cost including production and inventory.
$\Gamma(N)$: The maintenance cost.

Under the following constraints:

$$S(k+1)=S(k)+\beta(N).u(k)-d(k) \quad k=0,1,.....,H-1 \qquad (1)$$

$$0 \le u_k \le U_{max} \quad k=0,1,.....,H-1 \qquad (2)$$

$$\text{Prob}\left[S(k+1)\ge 0\right]\ge\theta \quad k=0,1,.....,H-1 \qquad (3)$$

Where the first constraint denotes the inventory balance equation for each time period. The equation (2) defines the maximal rate of machine *M1*. The constraint (3) imposes the service level requirement for each period and denotes the lower physical limit of inventory variable. The probabilistic constraint of inventory is taken as a chance-constraint in order to ensure that the inventory level is greater than zero with conditional probability of at least θ at each time period *k*.

We seek to optimize the cost model associated with the preventive maintenance with minimal repair policy derived above. Note that the production rate over the horizon H has an impact on the failure rate $\lambda(t)$. Consequently, the objective here is to take into account the production rate in determining the optimal number of partitions N^* of preventive maintenance actions to be carried out, which in turn means that the preventive maintenance action takes place at $T^* = H/N^*$ tu (time unit).

To develop the analytical model, we assume that:

- The cost of storage, production and subcontracting production, respectively Cs, Cpr and Cps are known and constant
- The standard deviation of demand $\sigma_d(k)$ and the average demand $\hat{d}(k)$ for each period k are known and constant.
- Mp and Mc costs incurred by the actions of preventive and corrective maintenance are known and constant, with Mc >> Mp.

We recall that our objective is to determine the jointly optimal production and maintenance planning over a time horizon H.

Our problem is formally presented as follows:

$$(U^*, N^*) = \min\left(F(u), \Gamma(N)\right)$$
$$\text{with} \quad U=\left(u(1),u(2),.....u(k)....u(H-1)\right) \text{ and } N=(1,2,...)$$

The system model is defined by an equation of state with continuous components. This equation is called the stock level, is given by equation (1), with $S(0)=S_0$, where S_0 is the initial stock level.
The average total cost of production, subcontracting and storage over a time horizon H is given by:

$$F(u)=\sum_{k=0}^{H}\left[C_s.E\left\{(S(k)^2)\right\}\right]+\sum_{k=0}^{H-1}\left[C_{pm}.\beta(N).u(k)^2+C_{ps}.u_s(k)^2\right] \qquad (4)$$

With: $u_s(k)^2=\max\left\{0, E\left\{\left(d(k)-(S(k)+\beta(N).u(k)\right)^2\right\}\right\}$

Remark:

$u(H)^2$ is not included in the cost formulation because we don't consider the production command at the end of the horizon H.

The total cost of maintenance until time H is:

$$\Gamma(N) = N.M_p + M_c.\varphi(U,N) \qquad (5)$$

With

$\varphi_{(U,N)}$: the average number of failure

$U=(u(1),u(2),\ldots,u(H-1))$: the production rates vector during the horizon $H.\Delta t$

So our problem is defined as follows:

$$\underset{(U,N)}{Min}\left\{\begin{array}{l}\sum_{k=0}^{H}\left[C_s.E\left\{(S(k)^2)\right\}\right] \\ +\sum_{k=0}^{H-1}\left[\begin{array}{l}C_{pm}.\beta(N).u(k)^2+ \\ C_{ps}.\underbrace{\max\left\{0,E\left\{\left(d(k)-(S(k)+\beta(N).u(k)\right)^2\right\}\right\}}_{u_s(k)^2}\end{array}\right] \\ +N.M_p+M_c.\varphi_{(U,N)}\end{array}\right\}$$

Under the following constraints :

$$S(k+1)=S(k)+\beta(N).u(k)-d(k) \quad k=0,1,.....,H-1$$
$$\text{Prob}\left[S(k+1)\ge 0\right]\ge\theta \quad k=0,1,.....,H-1$$
$$0 \le u(k) \le U_{max} \quad k=0,1,.....,H-1$$

$$\beta(N) = \frac{H - (\mu_c.\varphi(U, N) + N.\mu_p)}{H} \qquad (6)$$

Where the equation (6) define the availability of machine which depends on the production rate and the number of partitions of preventive maintenance actions.

3- ANALYTICAL STUDIES

After given an idea of the two policies (production and maintenance) in previous section, we would like to show the jointly optimization of production and maintenance strategy by the analytical study of two policies and establish the deterministic equivalent problem.

3.1 PRODUCTION POLICY

- ***Production, subcontracting and holding cost***

This section focuses on transforming the total cost into an analytical expression deterministic which will then be easier to solve. Thus the production, subcontracting and holding costs simplified as:

Lemma1:

$$F(u) = \sum_{k=0}^{H}\left[C_s.\left(k.(\sigma_d)^2 + \hat{S}(k)^2\right)\right] + \sum_{k=0}^{H-1}\left[\begin{array}{l} C_{pm}.\beta(N).\hat{u}(k)^2 + \\ C_{ps}.\max\left\{0, \left(\hat{d}(k) - (\hat{S}(k) + \beta(N).\hat{u}(k)\right)^2 + k.(\sigma_d)^2\right\}\end{array}\right] \qquad (7)$$

where

$\hat{S}(k)$: means stock level at the end of the period *k*

Proof ;

We have

$$F(u) = \sum_{k=0}^{H}\left[C_s.E\left\{(S(k)^2)\right\}\right] + \sum_{k=0}^{H-1}\left[\begin{array}{l} C_{pm}.\beta(N).u(k)^2 + \\ C_{ps}.\underbrace{\max\left\{0, E\left\{\left(d(k) - (S(k) + \beta(N).u(k)\right)^2\right\}\right\}}_{u_s(k)^2}\end{array}\right]$$

And $E\left\{S(k)^2\right\} = k.(\sigma_d)^2 + \hat{S}(k)^2$

$$\Rightarrow E\left\{\left(d(k) - (S(k) + \beta(N)u(k)\right)^2\right\} = E\left\{\begin{array}{l} d(k)^2 \\ -2d(k)\left(S(k) + \beta(N)u(k)\right) \\ +\left(S(k) + \beta(N)u(k)\right)^2\end{array}\right\}$$

$$= E\left\{\begin{array}{l} d(k)^2 - 2d(k)S(k) - 2d(k)\beta(N)u(k) \\ +S(k)^2 + 2S(k)\beta(N)u(k) + \beta(N)^2u(k)^2\end{array}\right\}$$

$$= E\left\{d(k)^2\right\} - 2.E\left\{d(k)\right\}.E\left\{S(k)\right\} - 2\beta(N)E\left\{d(k)\right\}.E\left\{u(k)\right\} + E\left\{S(k)^2\right\} + 2.\beta(N)E\left\{S(k)\right\}.E\left\{u(k)\right\} + \beta(N)^2.E\left\{u(k)^2\right\}$$

$$= \hat{d}(k)^2 - 2\hat{d}(k).\hat{S}(k) - 2\beta(N).\hat{d}(k).u(k) + k.(\sigma_d)^2 + \hat{S}(k)^2 + 2\beta(N).u(k).\hat{S}(k) + \beta(N)^2.u(k)^2$$

$$= \left(\hat{d}(k) - \left(\hat{S}(k) + \beta(N).u(k)\right)\right)^2 + k.(\sigma)^2$$

So: $u_s(k)^2 = \max\left\{0, \left(\hat{d}(k) - \left(\hat{S}(k) + \beta(N).u(k)\right)\right)^2 + k.(\sigma)^2\right\}$

Therefore:

$$F(u) = \sum_{k=0}^{H}\left[C_s.\left(k.(\sigma_d)^2 + \hat{S}(k)^2\right)\right] + \sum_{k=0}^{H-1}\left[\begin{array}{l} C_{pm}.\beta(N).\hat{u}(k)^2 + \\ C_{ps}.\max\left\{0, \left(\hat{d}(k) - (\hat{S}(k) + \beta(N).\hat{u}(k)\right)^2 + k.(\sigma_d)^2\right\}\end{array}\right]$$

- The inventory balance equation

$$\hat{S}(k+1) = \hat{S}(k) + \beta(N).\hat{u}(k) - \hat{d}(k) \quad k = 0,1,.....,H-1 \qquad (8)$$

- The service level constraint:

For the probabilistic constraint, is transformed the service level constraint into a deterministic equivalent constraint by specifying certain minimum cumulative production quantities that depend on the service level requirements.

Lemma 2:

$$\Pr ob\left(S(k+1) \geq 0\right) \geq \theta \Rightarrow \left(u(k) \geq U_\theta\left(S(k), \theta\right)\right) \quad k = 0,1,.....,H-1$$

With:

$U_\theta(\,)$: Minimum cumulative production quantity.

$$U_\theta\left(S(k), \theta\right) = \frac{V_{d,k} \times \varphi_{d,k}^{-1} + \hat{d}(k) - \hat{S}(k)}{\beta(N)}$$

V_{d_k} : Variance of demand *d* at period *k*

φ: Cumulative Gaussian distribution function with mean $\hat{d}$ and finite variance V_{d_k}

φ^{-1} : Inverse distribution function

Proof: *See Hajej-Dellagi-Rezg,IJPR 2011*

3.2 MAINTENANCE POLICY

For the maintenance policy, we seek to minimize the cost associated with a schedule of future preventive

maintenance and replacement activities. Our maintenance policy adopted in the problem is a periodic preventive maintenance policy with minimal repair. More precisely, the machine will operate over a given horizon *H.Δt*. the maintenance policy adopted is as follows: the H.Δt production periods is divided equally into *N* parts of duration *T*. Perfect preventive maintenance actions are is performed periodically at times *i.T*, *i*=0,1,…, with $N \in \{1,2,3\ldots\}$ and *N.T=H* following which the unit is as good as new. Whenever a failure occurs between preventive maintenance actions, the system undergoes a minimal repair. It is assumed that the repair and replacement times are negligible.
The analytic expression of the total maintenance cost is as follows:

$$\Gamma(N) = N.M_p + M_c.\varphi(U,N)$$

If we assume that $\lambda(t)$ represents the linear failure rate function at production period t is expressed as following

$$\lambda_k(t) = \lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{max}} \cdot \lambda_n(t)$$
$$\forall t \in [0, \Delta t]$$

We noticed that the maintenance policy is tightly related to the system degradation. That is why we adopted the production rate in order to take into account the influence of the production rate on the failure rate λ(t). Letting $L(T) = \int_0^T \lambda_s(t)\,dt$ denotes the expected failure number incurred over the interval [0,*T*] , the average failure number over the horizon H is:

$$\varphi(U,N) = \sum_{j=0}^{N-1} \left[\sum_{i=In(j\times\frac{T}{\Delta t})+1}^{In\left((j+1)\times\frac{T}{\Delta t}\right)} \int_0^{\Delta t} \lambda_i(t) + \int_0^{(j+1)\times T - In\left((j+1)\times\frac{T}{\Delta t}\right)\times\Delta t} \lambda_{In\left((j+1)\times\frac{T}{\Delta t}\right)+1}(t)\,dt + \int_{(j+1)\times T}^{\left(In\left((j+1)\times\frac{T}{\Delta t}\right)+1\right)\times\Delta t} \frac{\left(\left(In\left((j+1)\times\frac{T}{\Delta t}\right)+1\right)\right)}{U_{max}} \times \lambda_n(t)\,dt \right]$$

With :

$$\lambda_i(t) = \lambda_{i-1}(\Delta t) + \frac{u(i)}{U_{max}} \cdot \lambda_n(t) \quad \forall t \in [0, \Delta t]$$
$$\Rightarrow \lambda_i(t) = \lambda_0(t_0) + B_i + \frac{u(i)}{U_{max}} \cdot \lambda_n(t)$$

With $B_i = \sum_{l=In\left[j\times\frac{T}{\Delta t}\right]+1}^{i-1} \frac{u(l)}{U_{max}} \cdot \lambda_n(\Delta t)$; $B_1 = 0$ and $\lambda_0(t_0) = \lambda_0$

Proof: see *thesis Hajej2010*

4- OPTIMIZATION

The objective of the jointly optimization of production and maintenance strategy is to determine simultaneously the optimal number of partition *N** and the optimal production plan U* over the horizon *H*.
Thus, the equivalent deterministic model can be now formulated as follows :

$$\underset{(U,N)}{Min} \sum_{k=0}^{H} \left[C_s.\left(k.(\sigma_d)^2 + \hat{S}(k)^2\right) \right]$$
$$+ \sum_{k=0}^{H-1} \left[\begin{array}{l} C_{pm}.\beta(N).\hat{u}(k)^2 + \\ C_{ps}.\max\left\{0, \left(\hat{d}(k) - (\hat{S}(k) + \beta(N).\hat{u}(k))\right)^2 + k.(\sigma_d)^2\right\} \end{array} \right]$$
$$+ N \times M_p + M_c \times \sum_{j=0}^{N-1} \left[\sum_{i=In(j\times\frac{T}{\Delta t})+1}^{In\left((j+1)\times\frac{T}{\Delta t}\right)} \int_0^{\Delta t} \lambda_i(t) + \int_0^{(j+1)\times T - In\left((j+1)\times\frac{T}{\Delta t}\right)\times\Delta t} \lambda_{In\left((j+1)\times\frac{T}{\Delta t}\right)+1}(t)\,dt + \int_{(j+1)\times T}^{\left(In\left((j+1)\times\frac{T}{\Delta t}\right)+1\right)\times\Delta t} \frac{\left(\left(In\left((j+1)\times\frac{T}{\Delta t}\right)+1\right)\right)}{U_{max}} \times \lambda_n(t)\,dt \right]$$

Such that :

$$\hat{S}(k+1) = \hat{S}(k) + \beta(N).\hat{u}(k) - \hat{d}(k) \qquad k = 0,1,\ldots..,H-1$$
$$u(k) \geq \frac{V_{d,k} \times \varphi_{d,k}^{-1} + \hat{d}(k) - \hat{S}(k)}{\beta(N)}$$
$$0 \leq u(k) \leq U_{max} \qquad k = 0,1,\ldots..,H-1$$
$$\beta(N) = \frac{H - \left(\mu_c.\varphi(U,N) + N.\mu_p\right)}{H}$$

5- NUMERICAL EXAMPLE

An example of a multi-period, single product, an aggregated production/maintenance planning problem is formulated by our model, which minimizes total costs over a finite planning horizon: *H*=120 *tu* (time unit). The information required to run this model is given in sequence.

- the monthly mean demand $\hat{d}_k$:

{d_1=*15*, d_2= *17*, d_3= *15*, d_4= *15*, d_5=*15*, d_6=*14*, d_7= *16*, d_8=*14*, d_9=*16*, d_{10}= *15*, d_{11}= *15*, d_{12}= *15*, d_{13}= *15*, d_{14}= *15*, d_{15}= *15*, d_{16}= *13*, d_{17}= *15*, d_{18}= *15*, d_{19}= *16*, d_{20}= *13*, d_{21}= *15*, d_{22}= *15*, d_{23}= ***14*, d_{24}= *16*, d_{25}= *16*, d_{26}= *16*, d_{27}= *14*, d_{28}= *15*, d_{29}= *15*, d_{30}= *14*, d_{31}= *15*, d_{32}= *16*, d_{33}= *14*, d_{34}= *16*, d_{35}=*14*, d_{36}= *14*, d_{37}= *17*, d_{38}= *16*, d_{39}= *14*, d_{40}= *14*, d_{41}=*15*, d_{42}=*15*, d_{43}=*15*, d_{44}=*14*, d_{45}= *15*, d_{46}=*14*, d_{47}=*14*, d_{48}=*15* , d_{49}=*14*, d_{50}=*14*, d_{51}=*15*, d_{52}= *13*, d_{53}=*15*, d_{54}= *15*, d_{55}=*15*, d_{56}=*17*, d_{57}= *14*, d_{58}= *16*, d_{59}= *16*, d_{60}=*15*, d_{61}=*14*, d_{62}= *13*, d_{63}= *15*, d_{64}= *14*, d_{65}=*13*, d_{66}=*13*, d_{67}= *16*, d_{68}=*15*, d_{69}=*15*, d_{70}= *14*, d_{71}= *14*, d_{72}= *15*, d_{73}= *15*, d_{74}= *14*, d_{75}= *14*, d_{76}= *13*, d_{77}= *12*, d_{78}= *16*, d_{79}= *16*, d_{80}= *16*, d_{81}= *15*, d_{82}= *15*, d_{83}= ***15***, d_{84}= *16*, d_{85}= *14*, d_{86}= *17*, d_{87}= *16*, d_{88}= *16*, d_{89}= *15*,**

d_{90}= *16*, d_{91}= *13*, d_{92}= *14*, d_{93}= *16*, d_{94}= *14*, d_{95}=*14*, d_{96}= *16*, d_{97}= *16*, d_{98}= ***13*, d_{99}= *17*, d_{100}= *14*, d_{101}=*17*, d_{102}=*14*, d_{103}=*16*, d_{104}=*14*, d_{105}= *16*, d_{106}=*16*, d_{107}=*14*, d_{108}=*15* , d_{109}=*14*, d_{100}=*14*, d_{111}=*15*, d_{112}= *15*, d_{113}=*16*, d_{114}= *14*, d_{115}=*16*, d_{116}=*14*, d_{117}= *15*, d_{118}= *15*, d_{119}= *14*, d_{120}=*14* }**

- the other data are presented as following :

C_{pr} =10 *mu*, C_s =0.5 *mu/k*, C_{ps} =30 *mu*, Mp = 3000 *mu*, Mc = 300 *mu*, u_{max}=17 *up* , S_0 = 10 *up*, μp = 0.1 *tu*, μc = 0.02 *tu.*

The demand is assumed Gaussian with the standard deviation is σ_d=*1.2*

The customer satisfaction degree, associated with the stock constraint, is equal to 90% (θ=0.9).

Finally, we suppose that the failure time of the principal machine *M1* has a degradation law characterized by a Weibull distribution. The Weibull scale and shape parameters are respectively β=100 and α=2.

Using the Nelder-mead method with MATHEMATICA, we obtained the production plan the most economical, which is set out in the table below.

u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
16	14	17	16	17	15	16	14	17	17
u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}	u_{17}	u_{18}	u_{19}	u_{20}
13	17	17	16	17	16	15	17	13	17
u_{21}	u_{22}	u_{23}	u_{24}	u_{25}	u_{26}	u_{27}	u_{28}	u_{29}	u_{30}
17	17	13	14	17	16	17	17	15	11
u_{31}	u_{32}	u_{33}	u_{34}	u_{35}	u_{36}	u_{37}	u_{38}	u_{39}	u_{40}
15	15	13	17	13	15	13	12	17	17
u_{41}	u_{42}	u_{43}	u_{44}	u_{45}	u_{46}	u_{47}	u_{48}	u_{49}	u_{50}
17	15	16	17	12	13	14	17	17	17
u_{51}	u_{52}	u_{53}	u_{54}	u_{55}	u_{56}	u_{57}	u_{58}	u_{59}	u_{60}
15	17	10	13	17	17	16	13	11	14
u_{61}	u_{62}	u_{63}	u_{64}	u_{65}	u_{66}	u_{67}	u_{68}	u_{69}	u_{70}
17	13	15	15	12	16	13	17	13	15
u_{71}	u_{72}	u_{73}	u_{74}	u_{75}	u_{76}	u_{77}	u_{78}	u_{79}	u_{80}
17	17	12	15	14	15	16	15	12	16
u_{81}	u_{82}	u_{83}	u_{84}	u_{85}	u_{86}	u_{87}	u_{88}	u_{89}	u_{90}
15	12	17	17	16	17	15	17	17	11
u_{91}	u_{92}	u_{93}	u_{94}	u_{95}	u_{96}	u_{97}	u_{98}	u_{99}	u_{100}
13	16	16	12	16	17	16	15	17	17
u_{101}	u_{102}	u_{103}	u_{104}	u_{105}	u_{106}	u_{107}	u_{108}	u_{109}	u_{110}
8	17	16	14	17	17	17	15	6	17
u_{111}	u_{112}	u_{113}	u_{114}	u_{115}	u_{116}	u_{117}	u_{118}	u_{119}	u_{120}
8	17	17	17	17	16	9	12	17	10

Table 1: The economical production plan

Figure 2 shows the curve of total production and maintenance cost according to N. We conclude that the optimal preventive maintenance period number obtained is *N**=3.

Since that applying the preventive maintenance action at this optimal period *T**= 20 Δt, we obtained a minimal total cost including production and maintenance costs C*= 100570 *mu* (monitory unit).

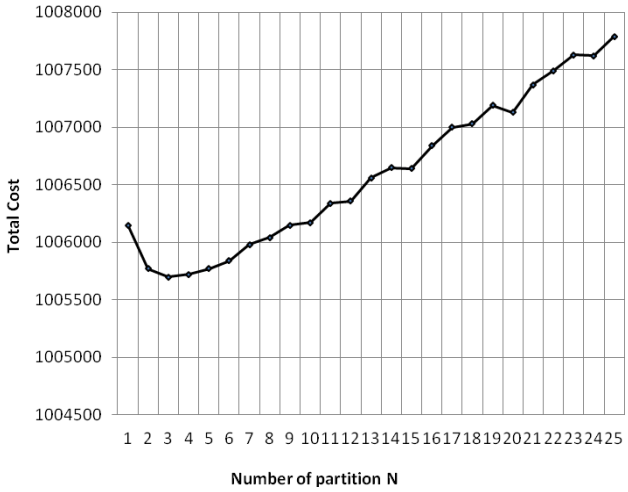

Figure 2: Total cost depending to *N*

6- CONCLUSION

In this work, we are interested in a manufacturing system which calling upon a subcontractor machine in order to satisfy economically a random demand under some constraints such as the random demand, a subcontracting constraint and the availability of machine.

In order to obtain a simultaneous optimal production and maintenance scheduling, we have transformed our problem from a stochastic one to a deterministic one.

In the numeric example, we were able to determine a simultaneous optimal production plan U* which is described in table 1and the optimal number of partition of the preventive maintenance plan N*=3.

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