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SEEKING STABILITY OF SUPPLY CHAIN MANAGEMENT DECISIONS UNDER UNCERTAIN CRITERIA

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ABSTRACT: *This paper tackles the question of the anticipation of the supply chain partner's decisional behaviour under uncertain criteria. In other words, we propose a model to support sequential decisions under uncertainty where the decision maker has to make hypothesis about the decision criteria. For example, Hurwicz criterion weights extreme optimism and pessimism positions and a classic criticism of this criterion consisting in the difficulty of the weight assessment and the involving decision instability. To achieve this, we present a method based on fuzzy representation of weight vision. Finally, the model allows sequential decision of a Decision Tree to be compute thanks a pignistic probabilities treatment of the fuzzy representation of the decision maker optimism-pessimism index. This approach is illustrated through an industrial case study.*

KEYWORDS: *sequential game, Hurwicz criteria, imprecision, stable decision making, decision tree, supply chain*

1 INTRODUCTION

1.1 Industrial problem statement

For an industrial Decision Maker (DM) in a supply chain, the anticipation of his partners' decisional behavior faced to uncertainty is a current complex real life situation. Knowing that his decision will be followed by a sequence of partners' decisions and other uncertain events, he has to anticipate and integrate the rational behavior of these partners during their own decision-making processes. In this paper, we more specifically consider a Supplier-Customer relationship from a dyadic supply chain. The customer is a worldwide dermo-cosmetic manufacturer and the supplier is a packaging product manufacturer. We adopt the point of view of an industrial manager of the customer who has to study the possibility to improve his supply chain performance in implementing new forms of collaboration. He has to choose between traditional and advanced ordering methods (decision also called here collaboration protocol choice). Furthermore, a second decision will concern the parameter setting of this protocol. But, the customer DM knows that the supplier will define his lot sizing strategy in response (third decision). This situation may be described as a multi-agent sequential decision problem. In addition to the "sequential" dimension, the DM is confronted to a multi-actor problem. So, decisions will not be made with the same performance objective. Each actor may seek to achieve his or her own performance criteria (inventory level, stock-out, order fulfillment…). Furthermore, these sequential and multi-actor decisions have to be taken on the basis of future uncertain events (scraps, breakdowns, delays…). Here, we will not focus on the detail of these events. We only consider a global event that influences the performance evaluation. In our example, the customer is a worldwide manufacturer who is able to build and to analyze a lot of historical data. He can therefore have a given behavior in front of uncertainty. The problem is the anticipation of the partner's behavior: optimistic, pessimistic…?

1.2 Research objective

A decision problem can be defined as a situation where a Decision Maker (DM) has to choose between several possibilities. This decision is referred to as a decision under or with uncertainty, when, at the decision time, the DM is not able to perfectly anticipate the results of his choices. Furthermore, in a real dynamic situation, the DM does not make a single decision, but a sequence thereof, characterized by a sequential arrival of relevant pieces of information. Consequently, the decision depends on the information available at the decision time such as a supplier who is waiting for the details of the contract with his customer to fix his inventory/production strategy according to his perception of the future possible market behavior. For all that, the first decision (protocol) has to take into account the future decisions (supplier's inventory strategy) and events (uncertain performance). This kind of problem is called uncertain dynamic (or sequential) decision and is supported by the use of Decision Trees (DT).

Whether individual or collective, a decision problem can be defined as a situation where a DM has to choose a decision d^* among a set of possibilities, $\overline{D}=\{d_1;...;d_p\}$, which have assessable consequences (Bouyssou *et al.* 2009). Let S be the set of possible states of the world that will be met after choosing $d^* \in \overline{D}$, and X the set of the potential consequences. The DM's choice of d^* could be defined as a function f_{d^*} from S to X that associates to each possibility $s \in S$ a precise consequence $f_{d^*}(s) \in X$.

$$\begin{aligned} f_{d^*}: \quad & S \rightarrow X \\ & s \mapsto f_{d^*}(s) \end{aligned} \tag{1}$$

The value (also called utility function) attached to each result can be represented as an application u from X to $\mathbb{R}$ that associates for each $f_d(s) \in X$ a value $u(f_d(s)) \in \mathbb{R}$ (Von Neumann and Morgenstern 1947).

$$\begin{aligned} u: \quad & X \rightarrow \mathrm{R} \\ & f_d(s) \mapsto u(f_d(s)) \end{aligned} \tag{2}$$

The expression "decision under uncertainty" is often used to describe a decision situation with a lack of knowledge about S. This lack of knowledge could be detailed in two points: (i) different states of the word could be met (S could be composed by more than a single state: $S=\{s_1;\ldots;s_S\}$) and (ii) the level of knowledge about the likelihood of each of these states may be poor. The terms "risk" and "uncertainty" are currently and differently used in the literature to describe this second dimension. According to Knight's distinction (Knight 1921), *decisions under risk* refer to decision situations where the DM is able to describe S by means of a known or a knowable probability distribution. Otherwise, he speaks about *decision under uncertainty*.

Confronted to the lack of knowledge, each possible choice of DM induces two potential consequences. The choice made by the DM depends on the assessment made to characterize the two possible situations. Different evaluation functions (V) have been proposed to characterize the DM's behavior in the face of risk or uncertainty. Faced with uncertainty, models based on likelihood assessment of the situation have been developed (Expected ($V \equiv EU$) and Subjective Utilities ($V \equiv SEU$), (Von Neumann et Morgenstern 1947; Savage 1954)). DT computation is based on these models. However, whether objective or subjective, the use of probability distributions is confronted to two main difficulties: (i) the DM's capacity to estimate the probability of each possible event (Moussa *et al.* 2006); (ii) the problem of the unique probability assumption. We refer to the Allais paradox (EU) (Allais 1953) or Ellsberg paradox (SEU) (Ellsberg 1961); they show that the risk perception depends on the context and the DM. Therefore, in some cases, EU or SEU cannot describe the DM's behavior and the decision theory proposes different criteria. If a uniform probability distribution on possible states is used, Laplace criterion $(V \equiv L)$ is a probabilistic way to model the DM behavior faced to the lack of information. The Wald criterion, also called maximin $(V \equiv W^-)$ and maximax $(V \equiv W^+)$ is an approach based on a qualitative representation of the DM's attraction to respectively the worst or the best situation (Wald 1950). Hurwitz (1951) proposes to weigh these two extreme behaviors with a parameter α in order to reflect the DM's pessimism degree (or optimism degree). This criterion $(V \equiv H)$ allows the DM's optimism degree to be more precisely described. The Savage regret-based decision model (1951) (also called minmax regret $(V \equiv S)$) proposes to make decisions based on the extent to which a decision-maker could have done better ex-post.

Faced to this diversity of criteria, a lot of authors have studied the capability of each of them to model the behavior of the DM facing a lack of knowledge. The present paper is focused on the complete ignorance situation (no information about a probability distribution). Seale et al. (1995) underline the inability of L, W^-, W^+, S "to account for individual differences" between DMs. "Aside from differences in the utility for outcomes, all the DMs are supposed to behave identically. The only exception is Hurwicz model". The differential subjective weighting factors allow variability in behavior across individuals to be represented (eq. 3).

$$H_u(f_d) = \alpha \times \min_{s\in S}(u(f_d(s))) + (1-\alpha) \times \max_{s \in S}(u(f_d(s))) \tag{3}$$

$$d^* = \arg\max_{d \in \overline{D}}(H_u(f_d)) \tag{4}$$

However, this coefficient (also called optimism-pessimism (o-p) index) is also the weak point of the criterion (Seale *et al.* 1995, Ballestero 2002) : (i) implicit is the assumption that each DM has a unique and stable o-p index; (ii) as shown by the multitude of questionnaires or other scales purporting to measure the o-p index, this evaluation is a hard task whose propensity to capture the actual o-p index is disputed; (iii) it is difficult to estimate the o-p index with precision from experimental protocols (an interval is easier); (iv) decision may be very unstable in the vicinity of particular value(s) of the o-p index (noted α^* in the Figure 1) where the evaluations of each decision consequence may be close.

Figure 1 illustrates the sensitivity of the evaluation function to the value of α where a DM has to choose between 2 decisions a_1 and a_2. In this example, a_1 has to be preferred if $\alpha < \alpha^*$ and a_2 if $\alpha > \alpha^*$.

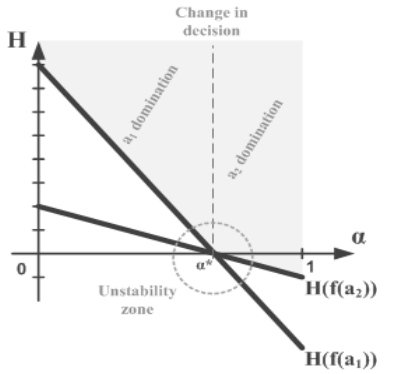

Figure 1 : Decision using Hurwicz coefficient

The purpose of this paper is to tackle these weaknesses through the proposition of a model allowing a stable decision to be extracted from a fuzzy knowledge of a partner's optimism-pessimism index.

We propose the use of both Hurwicz criterion and pignistic probabilities (Smets, 2005) to compute decision trees. To achieve this, we provide some background on possibility theory, pignistic probability and decision trees. Then, we present our proposition to model and support the decision-making process. Finally, we illustrate this proposition through our industrial case study before concluding and proposing future research works.

2 BACKGROUND

2.1 Representation of imprecision

In this section, we present a model to represent the imprecision on the information (possibility distribution) and a measure that evaluates the stability of the decision (pignistic probability).

2.1.1 Possibility distribution

Imprecise information is modeled by expressions of the form $v \in A$ where A is a subset of S that contains more than one element. Imprecision is always expressed by a disjunction of values (Dubois and Prade, 2009) defined by a possibility distribution on S. $v \in A$ means that all values from v outside A are supposed to be impossible.

A possibility distribution π_v attached to an ill-known quantity v quantifies the plausibility of values taken by v. π_v is a function of S into the scale of plausibility L ([0,1] for numerical possibility).

A numerical possibility distribution defines a random set $(m,F)_\pi$, having, for $i=1,\dots,M$, the following focal sets E_i with masses $m(E_i)$ (Dubois and Prade 1982) :

$$\begin{cases} E_i = \{x \in S \mid \pi(x) \geq \lambda_i\} \\ m(E_i) = \lambda_i - \lambda_{i-1} \end{cases} \quad (5)$$

2.1.2 Pignistic probability distribution

The pignistic probability is based on the Laplace principle, it consists in supposing an equal repartition of masses $m(E)$ over each element of focal set E for a random set (m,F) (eq.6). It has been proposed by (Smets, 2005) and is equivalent to the Shapley value (Shapley 1953) in game theory.

$$Pg^S(x) = \sum_{E \subseteq S, x \in F} \frac{m(E)}{|E|} \forall x \in S \, . \quad (6)$$

The pignistic probability distribution is used in simulation of "fuzzy variables" (Chanas and Nowakowski, 1988). It can be viewed as the subjective probability the decision-maker would provide, had his knowledge be faithfully represented by the possibility distribution π_v.

For example, we have possibility distribution over two possible criteria: $\Pi(c_1) = 1 \quad \Pi(c_2) = 0.8$. To compute the pignistic probability of each criterion

Let us first compute the masses $m(E_i)$ of the criterion.

In this case, the values of λ_i are linked to the possibility degree of the choice of the different criteria: thus they are discrete values: $\lambda_0 = 0$; $\lambda_1 = 0.8$; $\lambda_2 = 1$.

- $E_1 = \{c_1 ; c_2\}$ with $m(E_1) = 0.8 - 0$
- $E_2 = \{c_1\}$ with $m(E_2) = 1 - 0.8$

From equation 6 we have $Pg(c_1) = \frac{m(E_1)}{2} = \frac{0.8}{2} = 0.4$

and $Pg(c_1) = \frac{m(E_1)}{2} + \frac{m(E_2)}{1} = \frac{0.8}{2} + 0.2 = 0.6$

Motivation: While in a finite case providing subjective probability degrees makes sense, it is too difficult for a DM to provide precise continuous subjective probability. In that case it is more user-friendly to ask for weak information (like support and mode), represent it faithfully in possibility theory, and extract the pignistic probability from it.

2.2 Sequential decision

In a real dynamic situation, the DM does not make a single decision, but a sequence of decisions characterized by a sequential arrival of relevant pieces information. This type of problem is called uncertain dynamic decision. The decision made at time t depends on the information available at t. By hypothesis, the information known at t is still known at $t+\Delta t$. The incoming information is currently presented as "events. They are the results of an external independent entity, for example nature. In such conditions, we can call $\beta_t = \{e_t^1;\dots;e_t^m\}$ and $\beta_{t+1} = \{e_{t+1}^1;\dots;e_{t+1}^n\}$ the sets of known events at time t and $t+1$. β_{t+1} defines a partition of the set β_t . We call $\overline{D}^T = \{D_1,\dots,D_T\}$ the set of decisions that have been made at times (D_1) < time(D_2) < … < time(D_T) respectively.

This kind of problem has induced many research works and specifically in the Artificial Intelligence literature. They are relevant in situations where a DM has a sequence of decisions (at prescribed times) to make. In this context, a strategy, called Δ, is defined as a particular sequence of choices among the decisions (one choice per decision). The set of all strategies is denoted by $\overline{\Delta}^T$. The target is therefore to support the DM who must choose the best strategy, $\Delta^* = \max_{\Delta \in \overline{\Delta}^T} \Delta$. All decisions are fixed when the strategy is applied.

A Decision Tree (DT) is often used to represent this kind of decisions. A DT may be defined as a directed graph $T = (\mathcal{N}, \mathcal{E})$ with $\mathcal{N}$ the set of nodes and $\mathcal{E}$ the set of arcs inside which there exists a unique node (root node), from which there is a single path between each node. The set of nodes is made of (Nielsen and Jaffray 2006):

- $\mathcal{N}_D$: the set of decision nodes (represented by squares). They characterize states where the DM has to decide and to choose one alternative among several ones. Each output arc of a decision node represents an alternative (some $d \in \overline{D}^i$) ;
- $\mathcal{N}_C$: the set of chance (or event) nodes (represented by circles). Event nodes represent the sources of uncertainty in the problem, i.e. nature states. Each output arc of an event node shows a possible state of the world after the event occurred (some $s \in \mathcal{S}$);
- $\mathcal{C}$: the set of terminal nodes (leaves). A leaf is defined as a node without children ($child(\mathcal{N})=\varnothing$, with $\mathcal{N} \in \mathcal{C}$) and represents a terminal state of the sequential decision problem (a final consequence). A utility value is associated to each node ($u(\mathcal{N})$, $\mathcal{N} \in \mathcal{C}$).

In a DT, a strategy Δ is therefore defined as a set of arcs: $\Delta = \{(N, N') : N \in \mathcal{N}_D^{\Delta}, N' \in \mathcal{N}^{\Delta}\} \subseteq \mathcal{E}$ where $\mathcal{N}_D^{\Delta} = \mathcal{N}_D \cap \mathcal{N}^{\Delta}$ and $\mathcal{N}^{\Delta} \subseteq \mathcal{N}$ is the set of nodes involved in the strategy Δ, i.e. the set of nodes made of :

- The root node **:** $\mathcal{N}_r$ (a decision by hypothesis);
- A unique child for each decision of the strategy, i.e. $\mathcal{N} \in \mathcal{N}_D^{\Delta}$;
- All the children of an event node met in the strategy, i.e. $\mathcal{N} \in \mathcal{N}_C^{\Delta} = \mathcal{N}_C \cap \mathcal{N}^{\Delta}$.

We call $\overline{\Delta}^T$, the set of strategies in a given DT, *T*. An example of DT is given on Figure 2. It represents a decision situation where a DM has to decide D_1, then the event E_1 will occur, after what a second decision D_2 will be made followed by a last event E_2. Formally:

- $\mathcal{N}_D = \{D_1; D_2\}$, with $\overline{D}^1 = \{d_1^1; d_1^2\}$ and $\overline{D}^2 = \{d_2^1; d_2^2\}$;
- $\mathcal{N}_C = \{E_1; E_2\}$, with $S_{E_1} = \{e_1^1; e_1^2\}$ and $S_{E_2} = \{e_2^1; e_2^2\}$;
- $\beta_1 = \{e_1^1; e_1^2\}$ and $\beta_2 = \{e_1^1 \cap e_2^1; e_1^1 \cap e_2^2; e_1^2 \cap e_2^1; e_1^2 \cap e_2^1\}$

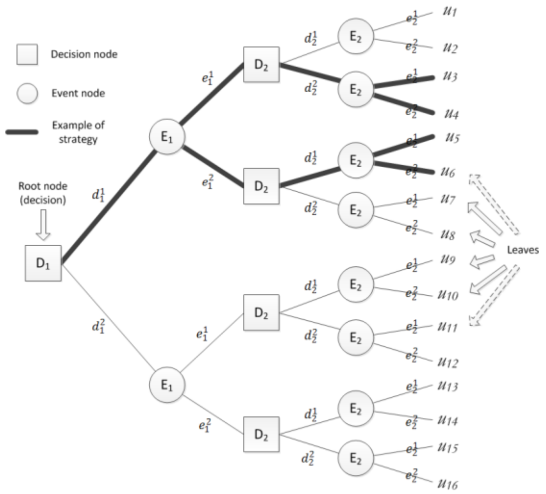

Figure 2 : Example of Decision Tree

Table 1 provides the list of strategies induced by this tree. The strategy illustrated on Figure 2 (in bold) appears in grey in the Table 1. Enumerating the strategies may become a very hard computational problem because of the complexity of the decision situation (the number of strategies increases exponentially). Different methods have been proposed to find the best strategy.

Strategy	Description	Consequences $(card(\sigma_{\Delta_i}) = card(\beta_2))$	Eval.
Δ_1	$=\{d_1^1; d_2^1 \text{ if } e_1^1; d_2^1 \text{ if } e_1^2\}$	$\sigma_{\Delta_1} = \{u_1; u_2; u_5; u_6\}$	$V(\sigma_{\Delta_1})$
Δ_2	$=\{d_1^1; d_2^2 \text{ if } e_1^1; d_2^2 \text{ if } e_1^2\}$	$\sigma_{\Delta_2} = \{u_3; u_4; u_7; u_8\}$	$V(\sigma_{\Delta_2})$
Δ_3	$=\{d_1^1; d_2^1 \text{ if } e_1^1; d_2^2 \text{ if } e_1^2\}$	$\sigma_{\Delta_3} = \{u_1; u_2; u_7; u_8\}$	$V(\sigma_{\Delta_3})$
Δ_4	$=\{d_1^1; d_2^2 \text{ if } e_1^1; d_2^1 \text{ if } e_1^2\}$	$\sigma_{\Delta_4} = \{u_3; u_4; u_5; u_6\}$	$V(\sigma_{\Delta_4})$
Δ_5	$=\{d_1^2; d_2^1 \text{ if } e_1^1; d_2^1 \text{ if } e_1^2\}$	$\sigma_{\Delta_5} = \{u_9; u_{10}; u_{13}; u_{14}\}$	$V(\sigma_{\Delta_5})$
Δ_6	$=\{d_1^2; d_2^2 \text{ if } e_1^1; d_2^2 \text{ if } e_1^2\}$	$\sigma_{\Delta_6} = \{u_{11}; u_{12}; u_{15}; u_{16}\}$	$V(\sigma_{\Delta_6})$
Δ_7	$=\{d_1^2; d_2^1 \text{ if } e_1^1; d_2^2 \text{ if } e_1^2\}$	$\sigma_{\Delta_7} = \{u_9; u_{10}; u_{15}; u_{16}\}$	$V(\sigma_{\Delta_7})$
Δ_8	$=\{d_1^2; d_2^2 \text{ if } e_1^1; d_2^1 \text{ if } e_1^2\}$	$\sigma_{\Delta_8} = \{u_{11}; u_{12}; u_{13}; u_{14}\}$	$V(\sigma_{\Delta_8})$

Table 1: Enumeration of strategies

The main method is based on the *backward induction* principle (or *dynamic programming*). The DT is visited from the leaves to the root by reasoning on subtrees. It is based on the *consequentialism* concept, proposed by Hammond (1988) then discussed by Machina (1989). To summarize, a consequentialist DM does not take into account past events and is focused only on the future choices and events. It is now demonstrated that this sophisticated behavior is efficient for probabilistic models. However, with non-probabilistic models, it may fail to extract the best strategy from $\overline{\Delta}^T$. Therefore, alternative approaches have been developed such as *Resolute Choice* by (McClennen (1990), or *Veto-Process* by Jaffray (1999) and Nielsen and Jaffray (2006). The first one enforces, by definition, the dynamic consistency of the DM. The second one allows dynamic programming advantages to be preserved by extending the spectrum of strategies considered by each node.

2.3 Game theory

The previous part has presented research work about dynamic decision from the Artificial Intelligence literature point of view. Associated methods are based on a common hypothesis: all decisions of the dynamic decision problem have to be made by the same DM. However, many real-life situations, such as met in SCM, involve multi-actor confrontation and collaboration levels. Consequently, the optimal choice for a DM depends on the others DM. DMs are described as in strategic interaction. It clearly defines a game theory context where each DM may be seen as a player that seeks to maximize his own profit. A game could be *cooperative* or *non cooperative*. In the first class of games, all players are linked with restrictive agreement(s). They define a coalition. In the second class of games, it is not possible to organize coalitions. This kind of game could be described in two different ways:

- *Strategic form* game: collection of strategies defining all the possible actions of each player in all possible situations with associated profits (also called payoffs).
- *Extensive form* game: tree that describes how the game is played. It is a dynamic description of the game because it specifies the sequence of decisions made by players. An event may be considered in a node, where "nature" will choose randomly a situation at different times of the game. Each decision node represents a player who has to decide and information available at a prescribed time. Payoffs (potential consequences for each player) associated to each scenario (a particular sequence of decisions and events) are represented by the leaves.

According to the interactions occurring between SC partners, SCM has become a natural application area for of game-theory. These game-theoretical applications in SCM have been differently surveyed: from a game-theoretical point of view (Cachon and Netessine 2006) or from SCM attributes point of view (Cachon 2003; Leng and Parlar 2005). These surveys show that a lot of models have been proposed in order to study the impact of given SCM decision levels (inventory-related decisions, decision in production/pricing, revenue sharing, quantity flexibility contract…). Here, we address the specific question of non-zero sum non-cooperative dynamic games with perfect (no simultaneous decision) symmetric (the same knowledge for all players) and complete (each player knows all strategies and associated payoffs) information. Furthermore, this game is not repeated. Therefore, algorithms based on the *Dynamic Programming* principle, i.e., backward induction, have to be preferred to search and find (if they exist) equilibria in this kind of game (Cachon and Netessine 2006).

3 DEFINITION OF PROBLEM WITH PERFECT KNOWLEDGE ON CRITERIA

In this paper we address a particular problem of sequential decision involving two decision makers (DM1 and DM2). Moreover, we consider that DM1 takes his/her decision before DM2 ignoring the behavior of DM2 and of nature that plays just once. Both DMs can use different criteria to make the decision. This problem can be modeled by a decision tree (Figure 3).

If we consider that DM1 perfectly knows the criterion of DM2 (and his/her own), the problem can easily be solved by dynamic programming:

- For each node j of decision of DM2, choice of the optimal decision d_j^{2*} using the criteria of DM2.
- Then, choice of the optimal decision d^{*1} of DM1 using the criteria of DM1 and taking into account the decision of DM2.

In real context, DM1 and DM2 have limited knowledge of their own decision criterion (their precise degree of optimism for example) with precision. Furthermore DM1 does not know with precision the decision criterion of DM2. In this context, a stable decision has to be made in front of uncertainty regarding these criteria.

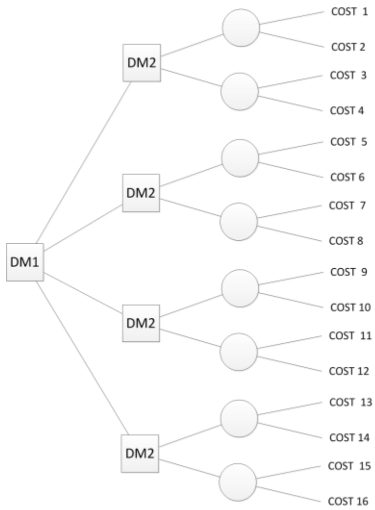

Figure 3 : Decision tree of problem considered

Thus, in the next section, we propose to solve the previous problem in the context of imperfect knowledge on criteria considering that this imperfect knowledge is modeled with possibility distributions on o-p indices.

4 RESOLUTION OF PROBLEM UNDER IMPRECISION ON CRITERIA

In this section, we first introduce the main principles of our method of choosing a stable decision in a game with two players when (i) DM1 makes his decision before DM2 and (ii) DM1 knows with imperfection both DM1 and DM2 criteria. Then, we detail some steps of this method when (i) both DM1 and DM2 criteria are

Hurwicz criteria and (ii) the optimism degree of this criterion is known with imprecision.

4.1 Approach overview

To evaluate the stability of decisions in front of the possible criteria, we choose to use the concept of pignistic probabilities (i.e. §2.1.2). Indeed the decision that has the maximal pignistic probability to be optimal is the one that is optimal for most criteria, taking into account the uncertainty on the criterion.
In order to compute the pignistic probability of each decision (stability degree), we have to know for which criteria this decision is optimal and then to sum the pignistic probabilities of these criteria.

In the considered problem the decision of DM1 depends on the criteria of DM1 and DM2. To solve this problem we have to solve a decision tree for each combination of criteria ($|C^1| \times |C^2|$). In the context of imprecise on o-p indices for Hurwicz criterion, an efficient method is proposed to compute the set of criteria for which a given decision is optimal (§4.2.2).

4.1.1 Method

We adopt the following notations:

- C^i : set of criteria of c^i of DM*i* with i=1 to 2
- D^1 : set of decisions d^1 of DM1
- D_j^2 : set of decisions d_j^2 of DM2
- j : index of decision node of DM2 with *j*=1 to $|D^1|$
- $D = D_1^2 \times \ldots \times D_J^2$: set of decision vectors $\vec{d}^2 = (d_1^2, \ldots, d_J^2)$ of DM2
- $C_2(\vec{d}_2) = \{c^2 | \vec{d}^2 \text{ is optimal}\}$: set of criteria $c^2 \in C^2$ for which decision vector $\vec{d}^2$ is optimal.
- $C_{12}(d_1, \vec{d}_2) = \{(c_1, c_2) | d_1 \,\&\, \vec{d}_2 \text{ are optimal}\}$: set of pairs of criteria (c_1, c_2) for which d^1 is optimal for $c^1 \in C^1$ and $\vec{d}^2$ is optimal for $c^2 \in C^2$.
- $C_1(d_1) = \{(c_1, c_2) | d_1 \text{ is optimal}\}$: set of pairs of criteria (c_1, c_2) for which decision d^1 is optimal $C_1(d_1) = \bigcup_{\vec{d}_2 \in D} C_{12}(d^1, \vec{d}^2)$
- $\mathrm{Pg}(c^i)$: Pignistic probability of criteria c^i
- $\mathrm{Pg}(d^1)$ is the pignistic probability that d^1 is an optimal decision. The optimality of the decision depends on the criterion c^1 of DM1 and the criteria c^2 of DM2. It depends on the probability to have c^1 and c^2 . So $\mathrm{Pg}(d^1)$ is the sum of the probabilities of pairs (c_1, c_2) for which d^1 is optimal.

Thus, the problem of stability maximization can be written as follows (eq. 7):

$$\begin{aligned} &\max_{d^1 \in D^1} \mathrm{Pg}(d^1) \\ &= \max_{d^1 \in D^1} \sum_{C_{12}(d_1, \vec{d}_2)} \sum_{\vec{d}_2 \in D} \mathrm{Pg}(c^1) \times \mathrm{Pg}(c^2) \\ &= \max_{d^1 \in D^1} \sum_{C(d_1)} \mathrm{Pg}(c^1) \times \mathrm{Pg}(c^2) \end{aligned} \quad (7)$$

Method to choose the most opti-stable decision $d^1 \in D^1$:

- **Step 1.** Computation of $C_2(\vec{d}^2)$ for each vector $\vec{d}^2 \in D$ (cf §4.3)
- **Step 2.** Computation of $C_{12}(d^1, \vec{d}^2)$ for each vector $\vec{d}^2$ such that $C_2(\vec{d}^2) \neq \phi$ and each $d^1 \in D^1$ (cf §4.4)
- **Step 3.** Computation of $C_1(d^1) = \sum_{\vec{d}_2 \in D} C_{12}(d^1, \vec{d}^2)$ for each $d^1 \in D^1$
- **Step 4.** Selection of the decision $d^1 \in D^1$ such that $\sum_{C(d_1)} \mathrm{Pg}(c^1) \times \mathrm{Pg}(c^2)$ is maximal

4.1.2 Example

We illustrate the method in a general context, where DM1 does not know if DM2 will use the minmax criteria (with probability 0.6) or Laplace (with probability 0.4) and DM1 doesn't know if he/she will use the indicator $g(d_1, d_2, n)$ (with probability 0.7) $h(d_1, d_2, n)$ (with probability 0.3) within the criteria minmax :

- $C^2 = \left\{ \max_n f(d_2, n); \sum_n \frac{f(d_2, n)}{|N|} \right\}$
- $C^1 = \left\{ \max_n g(d_1, d_2, n); \max_n h(d_1, d_2, n) \right\}$

DM1 has 2 possible decisions {1;2} and DM2 has two possible decisions {one, two} and the nature three possible realisations {a, b, c}. The evaluation of decision strategies is represented on Table 2 and Figure 4.

		f(*d*,*n*)		*g*(*d*,*n*)	*h*(*d*,*n*)
DM1	DM2	max	Laplace	max	max
1	one	10	8	10	12
	two	14	7	11	10
2	one	20	10	14	11
	two	15	12	9	15

Table 2: Evaluation of the decision strategies

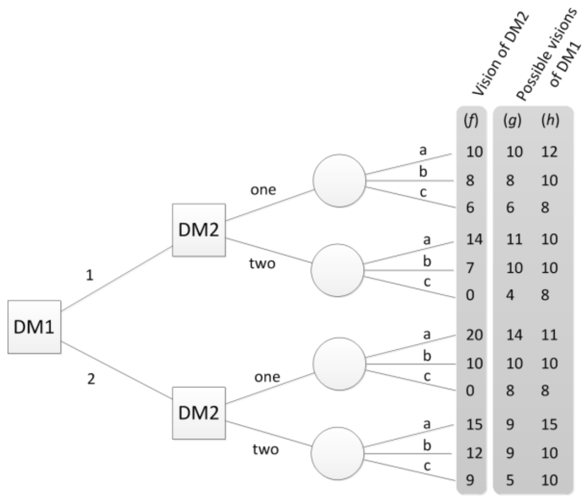

Figure 4 : DT of the example

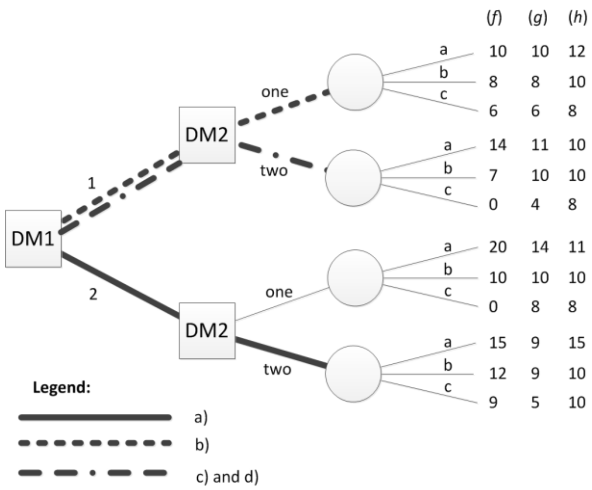

Figure 5 : Result of the DT computation

To solve this problem, we have to compute the possible optimal solution for the 4 combinations of the criteria (Table 3) in order to compute the stability degree of each decision.

To identify the decision that has the highest degree of stability, we compute the optimal solutions and they probabilities and the stability degrees of the decision of DM1. Results are detailed in Table 3 and illustrated on Figure 5 (optimal decisions in each case appear in boldface).

Combination		Optimal decision : DM1	Optimal decision : DM2	*Pg*
a)	$\max f(d,n)$ $\max g(d,n)$	2	one if 1 two if 2	0.6*0.7=0.42
b)	$\max f(d,n)$ $\max h(d,n)$	1	one if 1 two if 2	0.6*0.3=0.18
c)	$Laplace\ f(d,n)$ $\max g(d,n)$	1	two if 1 one if 2	0.4*0.7=0.28
d)	$Laplace\ f(d,n)$ $\max h(d,n)$	1	two if 1 one if 2	0.4*0.3=0.12

Table 3: Results of problem

The stability degrees of decision 1 and 2 can thus be computed $C_1(1) = 0.18+0.28+0.12=\mathbf{0.58}$; $C_1(2)=0.42$.

It can be concluded that decision 1 of DM1 is the most stable with a degree of stability = 0.58.

4.2 Problem with imprecise optimism degree

In the considered problem, the possible criteria are the Hurwicz criterion with imprecise value of optimism degree *α*. In this section, we describe how to compute the sets $C_2(\vec{d}^2)$ and $C_{12}(d^1,\vec{d}^2)$, in this imprecise optimism degree context.

4.2.1 Model of imprecise degree of optimism

The model is based on the hypothesis that DM1 is able to give two possibility distributions on the value of *α*: possibility distribution $\tilde{\alpha}^1$ on his/her degree of optimism and possibility distribution $\tilde{\alpha}_1^2$ on the possible degree of optimism of DM2. To evaluate the stability of decision (pignistic probability to be optimal), we must know the pignistic probability of each criteria. So, first we build pignistic probability distribution from possibility distribution (cf §2.1.2).

4.2.2 Determination of $C_2(\vec{d}^2)$

In this section we give the framework of the algorithm to compute $C_2(\vec{d}^2)$:

- **Step 1.** Computation, of the value of α^2 for which decision d_j^2 changes, denoted by α^2_{change}, for each node of decision of DM2, (cf: Figure 1)
- **Step 2.** Computation of the set of α^2 such that vector $\vec{d}^2$ is optimal for DM 2: $C_2(\vec{d}^2)$

The maximal cardinality of $C_2(\vec{d}^2)$ appears when all decisions are optimal for a given α^2 and each α^2_{change} are different for each decision nodes of DM 2. Thus, in the worst case, we have $|D_1|\times|D_2|$ set $C_2(\vec{d}^2)$.

4.2.3 Determination of $C_{12}(d^1,\vec{d}^2)$

After determining all $C_2(\vec{d}^2)$, we compute the set $C_{12}(d^1,\vec{d}^2)$ for each $d^1 \in D^1$. The framework of the algorithm is:

- **Step 1.** Compute the value of α^1 for which decision d_1^1 changes, denoted by α_{change}^1, for each $\vec{d}_2$ such that $C_2(\vec{d}^2) \neq \phi$, (cf Figure 1)
- **Step 2.** Build the set of α^1 such that $\vec{d}_2$ is optimal vector of DM 2 and d_1^1 is optimal d: $C_{12}(d^1, \vec{d}^2)$

In the worst case we must compute $|D_1| \times \alpha_{change}^1$ for each $C_2(\vec{d}^2)$ so at most $|D_1|^2 \times |D_2|$ $C_{12}(d^1, \vec{d}^2)$.

5 APPLICATION OF THE METHOD

In this section, we apply the method on the case study context that has been described in the introduction: a dyadic supply chain where the customer, a worldwide dermo-cosmetic maker, has to choose a collaboration protocol (2 possibilities) with its packaging product supplier. According to the traditional collaboration protocol the customer has to release orders (a product, a quantity) and the supplier responds. A DM decision variable is the order lead time (here 12, 8 or 6 weeks). With the advanced collaboration protocol the customer commits on purchases associated to a family of products 8 weeks in advance (family aggregation is related to supplier's set up considerations). Then, the customer releases delivery needs expressed in product 1 week in advance. A DM decision lever is the minimal volume associated to the family engagement (here 50000, 100000 or 150000 products).

5.1 Problem modeling

According to the notation defined in previous parts, we denote by DM1 the customer and by DM2 the supplier. Three sequential decisions have to be made.

- DM1 has to define the decision protocol (2 possibilities),
- Then, its parameter (3 possibilities).
- Then, DM 2 will define his lot sizing strategy (3 possibilities).

In addition, the performance of the supply chain will be subject to a global uncertain event that models the uncertainty of the performance due to different risk sources (scrap, production/transport delay, breakdowns…) (7 possible situations).

DM1 has to choose one decision: the decision protocol with its parameter (Table 4) before DM2 chooses the lot sizing strategy.

Notation	Protocol decision	Parameter decision
1	Advanced collaboration	Low volume (50000)
2	Advanced collaboration	Medium volume (100000)
3	Advanced collaboration	High volume (150000)
4	Basic order	Little order lead time (6w)
5	Basic order	Medium order lead time (8w)
6	Basic order	Big order lead time (12w)

Table 4 : Notations for DM1's decisions

The Table 5 summarizes the problem according to the notations introduced in the previous part:

Description	Notation	Observation
Protocol decision within its parameter (DM 1)	d^1	$d^1 \in D^1$
Lot sizing strategy (DM 2)	d^2	$d^2 \in D^2$
Uncertain Event	n	$n \in N$
DM 1's Hurwicz coefficient	α^1	$\alpha^1 \in \tilde{\alpha}^1$
DM 2's Hurwicz coefficient	α^2	$\alpha^2 \in \tilde{\alpha}^2$

Table 5 : Global notations used

According to the quantity of scenarios that have to be evaluated, we use a simulation tool called LogiRisk for the evaluation of each scenario (each leaf of the tree). Developed in Perl language, it is dedicated to tactic and mostly strategic SC planning processes. This simulator is based on a discrete event simulation modeling approach. Authors have established a generic representation of the different planning processes for each SC actor based on the MRPII (Manufacturing Resource Planning) processes. An upstream planning process is used between partners: plans are made by the customer and passed to its suppliers. The procedure is repeated all over the chain in the upstream direction. No information circulates downstream (Lamothe *et al*, 2007, Marques *et al*. 2009).
The customer's cost function is 2/3 average customer's stock-out 1/3 average customer's stock and supplier's cost function is 1/2 average supplier's stock-out 1/2 average supplier's inventory level.

From those simulations we build the decision tree (Figure 6).

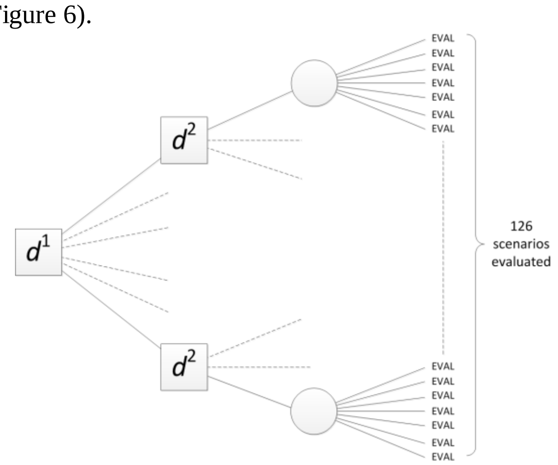

Figure 6 : DT of the study

5.2 Problem Solving

The customer gives the two possibility distributions on the optimism degree of himself/herself and on the supplier. The optimism degrees are represented in Figure 7. The DM1 is pessimistic (black line) and the DM2 is known as optimistic (grey line) by DM1.

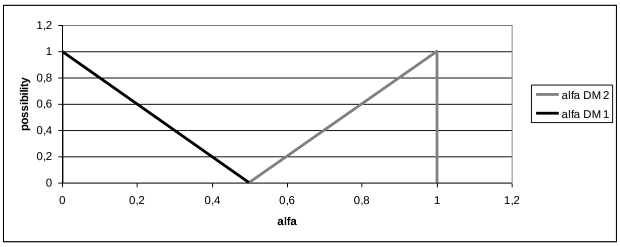

Figure 7 : optimism degree (alfa) of DM 1 and DM 2

From the simulation we build the decision tree (table 3) with 6 decisions for DM 1 and 3 decisions for DM 2 and the cost function for each DM (DM1: customer's cost and DM2: supplier's cost).
DM1 has six possible decisions {1;2;3;4;5;6} and DM2 three possible decision {1;2;3}. Table 6 represents the DT with the cost value of the study problem.

		Supplier's cost		Customer's cost	
DM1	DM2	min	max	min	max
1	1	7.175	7.696	0.471*	0.537*
	2	14.516	17.563	0.415	0.475
	3	20.436	25.396	0.411	0.453
2	1	6.022	6.907	0.422	0.462
	2	13.078	14.34	0.380	0.425
	3	18.92	21.57	0.375	0.414
3	1	5.905	6.956	0.414	0.468
	2	12.975	14.734	0.382	0.420
	3	18.267	21.257	0.374	0.412
4	1	6.177	7.272	0.547	0.656
	2	11.862	14.444	0.505	0.605
	3	17.268	20.824	0.478*	0.554*
5	1	6.427	6.946	0.571	0.622
	2	12.131	13.985	0.567	0.624
	3	17.540	20.445	0.542	0.639
6	1	7.307	7.549	0.765	1.009
	2	13.010	14.628	0.763	1.009
	3	18.968	21.294	0.765	1.008

Table 6: Data of problem

Decision 1 of DM2 is Pareto-optimal for all decisions of DM1. In other worlds, decision 1 has the minimal "min" and minimal "max" for each decision of DM1. So, whatever the optimism degree of DM2, DM2 chooses decision 1 for each node.

$$C_2((1,1,1,1,1,1)) = [0.5;1] \quad \forall \vec{d}^2 \neq (1,1,1,1,1,1) \quad C_2(\vec{d}^2) = \phi$$

Then we compute the set $C_{12}(d_1,(1,1,1,1,1,1))$ for each $d^1 \in D^1$. Whatever the optimism degree of DM1 decision 1,4,5,6 can be chosen:

$$\forall d^1 \neq 2{,}3 \quad C_{12}(d^1,(1,1,1,1,1,1)) = \phi$$

DM1 has two possible optimal solutions: solution 2 and 3. To compute $C_{12}(d^1,(1,1,1,1,1,1))$ we compute the α^1_{change} (Figure 8): $C_{12}(2,(1,1,1,1,1,1)) = [0;0.429]$ and $C_{12}(3,(1,1,1,1,1,1)) = [0.429;0.5]$

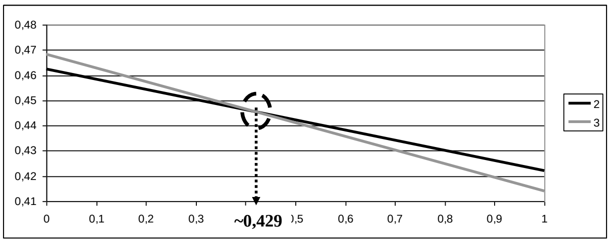

Figure 8 : Analysis of decision 2 and 3

In this example $C_1(d^1) = C_{12}(d^1,(1,1,1,1,1,1))$:

$$C_1(2) = C_{12}(2,(1,1,1,1,1,1)) = [0;0.429]$$
$$C_1(3) = C_{12}(3,(1,1,1,1,1,1)) = [0.429;0.5]$$

To choose between decisions 2 and 3 we compute the pignistic probability that decision 2 is optimal: $Pg(\alpha^1 \in [0;0.429]) \cong 0.992$ and the pignistic probability that decision 3 is optimal: $Pg(\alpha^1 \in [0.429;0.5]) \cong 0.008$. So, DM1 chooses decision 2.

From an industrial point of view, the approach presented in this paper could be used with two main objectives. For an "optimality" seeking objective, this approach allows imprecise information about the optimism-pessimism index to be used to identify the most plausible decision, in other words the most stable decision if the latter will be made numerous times. In the example presented in the last case study, the customer (DM1) is able to conclude that, according to the context defined in the problem, he has to prefer the advanced form of collaboration with a medium volume of engagement (family).

However, the model proposed may be applied to emphasize and identify "risky" situations. In the example, a customer (DM1) confronted to supplier (DM2) characterized by a poor capacity to deal with high family volume engagement (few possibilities of family aggregation for example) has to give priority to the improvement of the basic ordering form (through order lead time decreasing, i.e. DM1 decision 4) compared to imposing an advanced ordering form with a low volume of engagement (DM1 decision 1). This situation (distinguished with * in Table 6) illustrates the necessity for the DM to be supported in order to rank improvement schemes. In the example the advanced ordering form may not be "the" best solution according to the context.

6 CONCLUSION

In this paper we focused on a decision problem in a dyadic collaborative supply chain. More precisely we addressed the problem of decision making for a customer, taking into account the future decision of his supplier under imprecise information on the criteria of the two SC partners. We proposed a decision method for the criterion ensuring optimal stability. In other words we focus on the decision that has the best chance to be optimal under an imprecise criterion.

Industrial DMs are daily confronted to the problematics of exploiting their empirical knowledge of their partners' decisional behavior. This knowledge is rarely precise and quantified. Being able to exploit this knowledge may be a strategic advantage in term of value creation and conservation. The model presented in this paper and the associated case study illustrates the advantage to identify the most stable decision under imprecise knowledge, i.e. the most probable decision, even if research efforts have to be made to improve the robustness of the results (sensitivity analysis) and to use real life collaboration experience in order to express imprecise vision of partners' decisional behaviors.

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