MULTI-STAGE OPTIMIZATION IN SUPPLY CHAIN: AN INDUSTRIAL CASE STUDY

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MULTI-STAGE OPTIMIZATION IN SUPPLY CHAIN: AN INDUSTRIAL CASE STUDY

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ABSTRACT: *Inspired from a real case study of an Algerian supply chain company, this paper proposes an original solution for the optimization problem in supply chains. The analysed supply chain includes the suppliers, manufacturers and distribution centers, considering that orders come from clients to distribution centers. A mathematical model that solves the optimization problem regarding durations and costs in the execution of client orders is proposed. Many original constraints are supported in this mathematical model such as suppliers and manufacturers capacities and balanced stocks. The aim of the proposed method is to find a near-optimal solution that minimizes criteria of cost and time along the whole supply chain process. The mathematical model was tested on an agro alimentary supply chain using CPLEX and although we obtained encouraging results compared to the ones provided by a heuristic used by the industrial managers, further analysis should consider integrating other constraints to obtain a more accurate model.*

KEYWORDS: *Supply chain, optimization, linear program, coordination, transportation.*

1 INTRODUCTION

The supply chain management is fundamental for the good functioning of a company. According to (Simchi-Levi et al., 1999), "Supply Chain Management (SCM) is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements". A similar definition of SCM can be found in (Lazaros and Papageorgiou, 2009), the author insists on the fact that the management of supply chains is a complex task mainly due to large size of the physical supply network and uncertainties.

In this paper, we focus on a multi objective formalization of a supply chain (SC) in agro-alimentary sector taking into account many particular constraints like suppliers and manufacturers capacities and balanced stocks; we propose an original mathematical model, described in a modular way to find the solution that meet customer demands and minimize costs and time, while maintaining products quality, along the agro alimentary SC using the sub models. In the following section we describe the SC that gave us inspiration to design our model.

1.1 Description of the case study

The application of our work will be done on a SC including seven enterprises of the Metidji Group, an Algerian company, in which one of them provides transportation services and the other enterprises produce various types' flours, semolina and snacks and also provides the corn processing.

Generally, a SC can be model in three distinct stages: manufacturer/supplier of product-specific materials (parts), producer where finished products are assembled according to customer orders and a set of customers who generate final demand for the products (Sawik, 2009). In our study, the SC is constituted by customers, distribution centers, suppliers and manufacturers (figure 1).

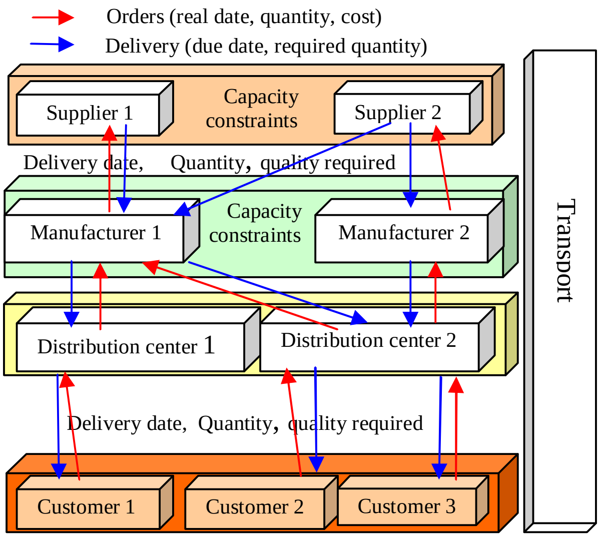

Figure 1: Representation of the SC case study

1.2 Current optimization mechanism

The current optimization mechanism used by industrials is based on the widespread MRP II (Manufacturing Resource Planning). Figure 2 shows the MRP II operating principle.

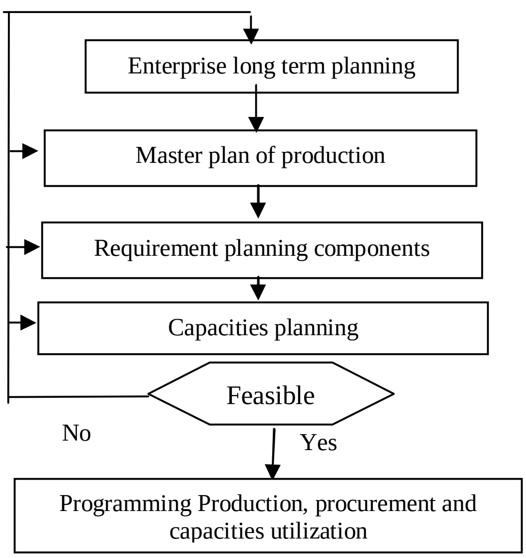

Figure 2: Operating principle of MRP II

Even though companies have implemented this model by using ERP (Enterprise Resource Planning) systems, integrating capacity constraints and other variables, this model suffers from several lacks. Indeed, the ERP applications are software suites that help organizations to integrate their information flows and business processes by using a single database that collects and stores data in real time. The ERP implements MRP II modules that allow adjustments in the desire charge and available capacity for each production center. The major limits of such systems are their weakness to make simulations or to present an explanation of the obtained solutions, and the lack of evidence in the optimality of the solutions found. The Metidji Group uses an ERP system and aside of the aforementioned limitations, the actual system does not take into account the Customer Relationship Management (CRM). For the companies constituting Metidji group, this constitutes the main drawback, making it hard to take care of customers' due dates and to select efficiently their supplier's.

Currently, the following heuristic is used to obtain solutions:

For each customer demand, DCs are first affected according to the availability of product requested and second, to the transport cost.

When the quantity requested is not available, distributors send manufacturing orders to a manufacturer. The choice of a manufacturer is made according to the product availability and the procurement costs from manufacturer to DCs.

If quantities of raw materials used in fabrication are not sufficient, procurement orders are send to the suppliers that are selected according to the availability and procurement cost.

The aim of this paper is to propose a solution that integrates different constraints of the whole supply chain and that respects also the customer due-dates, with the best prices.

The remaining of the paper is organized as follows; section 2 presents a literature review of papers on SC optimization. In section 3, we explain our proposition to model the SC of the industrial group and the methodology to solve it. Some experiments are carried out to validate the proposed approach in section 4. Finally, some conclusions and future works are presented in section 5.

2 LITERATURE REVIEW

2.1 Optimization in supply chain (SC)

In literature, there is a considerable number of works in SC optimization, particularly for the minimization of duration and costs in the global SC on one hand, and for coordination between stages in SC on the other hand. In recent years, mathematical research in SC has developed many approaches and solutions. In the following sub sections, we present different works established in the field of optimization to ensure good SC performances.

2.1.1 Inventory management

Replenishment and inventory management represents one of the issues in SC research. (Lim et al., 2006) considered various replenishment forms of raw materials, components and end-products in each node. In the same line of research, specific works have been proposed. For example, (Funaki, 2010) proposed an approach to solve the dynamic placement of inventory to meet challenges concerning compliance date of receipt in the machine industry. (Bilgen, 2010) proposed an application of fuzzy mathematical programming approach to solve the production allocation and distribution in SC network. The proposed models are illustrated through a case study in consumer goods industry.

2.1.2 Coordination in SC

Concerning coordination in SC, (Selçuk et al., 1999) proposed a set of decisions that need to be considered in jointly optimizing production/distribution planning. (Sawik, 2009) focused on coordination in SC customer driven. The author estimated that the manufacturing, supply and assembly schedules are determined simultaneously. (Arshinder et al., 2008) worked on SC customer driven. The majority of research on SC coordination aims to simultaneously minimize total cost of material ordering and holding, manufacturing setup and finished products holding. (Benjamin, 1989) proposed a simultaneous optimization of production, transport and inventory using a non linear programming model,. (Sousa et al., 2008) presented a two-level planning approach for the redesign and optimisation of production and distribution of an agrochemicals SC network.

2.1.3 Periods' consideration in SC

In the 90s, (McDonald and Karimi, 1997) proposed a multi-period linear programming model which takes into account the available processing time of all lines, transportation and shortage costs. Compared to our problem, this is an interesting case since demands with penalization and product families are considered. (Jin-Kwang et al., 2000) and (Kallrath, 2002) presented multi period optimisation models that concern operational decisions and strategic planning for multi-site production networks respectively. (Chen et al., 2003) proposed an approach based on multi-product, multi-period model for multi objective planning solved by applying a two phases fuzzy method. (Karimi et al., 2005) proposed a two-step procedure for the tank container management problem by combining an event based approach with a linear programming algorithm. (Huang and Karimi, 2006), (Al-Ameri et al., 2008) and (Dondo et al., 2008) used MILP (Mixed Integer Linear Programming) for transhipment operations of liquid chemical cargos, ship-scheduling problem and operational level of multiple vehicle pickup and delivery problem respectively. (Lazaros and Papageorgiou, 2009) focused on mathematical programming models for SC optimisation problems in the process industry at strategic and tactical levels.

2.1.4 Design of SC

In SC design aspect, (Naraharisetti et al., 2008) presented a MILP model for making efficient capacity management and SC re-design decisions for a corporation. (You and Grossmann, 2008) proposed a bi-criteria MINLP optimisation framework to consider simultaneously economics and responsiveness of multi-site, multi-echelon process in SC networks.. (Sousa et al., 2005) proposed a MILP model for the optimal product allocation–distribution structure of a pharmaceutical company SC. (Tsiakis et al., 2001) described a MILP to determine production capacity allocation among products, optimal layout and flow allocations of the distribution network by minimising an annualised network cost.

2.1.5 Uncertainty in SC

To take into account uncertainty, (Voudouris, 1996) developed a mathematical model where the objective function aims at representing the flexibility of the plant to absorb unexpected demands. (Lee and kim, 2000) considered the stochastic character of the production and distribution capacities. (Aliev et al., 2007) included a fuzzy-genetic approach to aggregate production and transport based on fuzzy mathematical programming including uncertainty demand and production capacities. (Mula et al., 2010) and (Ryu et al., 2004) proposed a solution which incorporates production and holding capacities into demand uncertainty models by reformulating them with multi-parametric linear programming. (Selim et al., 2008) proposed a Multi-objective mathematic model for decentralized and centralized collaborative production and transport planning in SCs. Fuzzy programming has been used to incorporate uncertainty. In (Roghanian et al., 2007), the authors proposed a probabilistic bi-level linear multi-objective programming model for SCs using randomness in demand with production capacities and available resources. Also, (Peidro et al., 2008) proposed a review for SC quantitative models under uncertainty.

2.1.6 Multi objective models for SC

This section shows relevant works related to multi objective programming models in SC. (Mula et al., 2010) present a recent review of the literature of mathematical programming in SC. (Chern and Hsieh, 2007) proposed a multi-objective linear programming model and a genetic algorithm for master production planning, taking into account lead time and many costs (production, transport, outsourcing, inventory …). In (Torabi and Hassini, 2008), authors used a possibilistic mixed multi objective integer linear programming (MOILP); it considers several conflicting objectives simultaneously and take into account different constraints. (Sabri and Beamon, 2000) developed a steady-state mathematical model for SC management by combining strategic and operational design and planning decisions using an iterative solution procedure. A multi-objective optimisation procedure is used to take into account multiple performance measures.

2.2 Optimization in agro alimentary SCs

To our knowledge, there are few works considering the specificity of the agro-alimentary sector that is addressed in this paper. (Bilgen and Ozkarahan, 2007) proposed a multi-period optimization model for bulk grain blending and shipping for a SC in the cereal sector. This work did not take into account inventory levels, service levels, backorders and replenishment capacities.

Expiration date of raw materials and products is one of the capital constraints in the food and agro-alimentary sector. Deteriorating the inventory was originally studied by (Ghare and Schrader, 1963) and (Ping-Hui et al., 2007). Many works were proposed in that direction, eg., (Ping-Hui et al., 2007) that developed deteriorating inventory replenishment. The product expiration date indicates the latest time at which the product may be used. The customers' demand always decreases as the product is closer to the expiration date; the authors insisted on the fact that the product delivery time is uncertain owing to capacity constraints, imperfect products, uncertain material supplies and production processes.

2.3 Limits and challenges in an agro-alimentary SC optimization

The review highlighted some disadvantages and limitations of the above-described contributions regarding an agro-alimentary SC optimization. First of all, most of these approaches do not take into account all the SC actors at the same time. It is important to consider all of them and to include all the parameters and constraints. On one hand, a good representation of information and materials flows will be obtained, and on the other hand that lead to optimal solutions by generating coherent and complete models.
Also, in the agro-alimentary sector, many authors proposed models concerning deteriorating inventories, but they do not take into account product categories in manufacturing level. This problem arises in our case study: the actual system does not consider the setup cost and time in some cases: in production (more precisely, in a manufacturer called Menut's that produces snacks and others cereals), the transition from one product to another requires tool changing when category, aroma or weight products are different. In that case, a setup time and setup cost are supported. In our model those constraints must be considered. The optimization of these variables affects the expiration date as well as the production costs.
In addition, other lacks exist in the actual system used in the case study. First, simulations are not possible and explanation of planning and scheduling results is a serious problem for experts working in Metidji enterprises. Another aspect is that stock balancing is a major issue for the company. In fact, the system must always keep balance in inventories so that no inventory shortage is allowed.
To respond to the cited limits, we propose a new mathematical model based on a multi-objective solution. In the next section we describe our contribution.

3 PROPOSITION

In this paper, we propose an approach that can provide near optimal solutions minimizing duration and cost along a generic agro-alimentary SC (we will apply this model to the case study in the last part of this paper). We have chosen mathematical programming to obtain a formal representation that could lead to the proof of optimality.

Our model takes into account many aspects that are not yet considered in the current system (view sections 2.2 and 2.3). However, we specify that we obtain an off-line solution, neither uncertainly nor perturbations are considered.
Section 3.1 shows the proposed mathematical model.

3.1 Mathematical formulation of the problem

The studied agro-alimentary SC network presented in figure 2 can be represented by a multi-echelon graph G (N, A), where the set of nodes N are corresponding to the SC actors (Suppliers, Manufacturers, Depositors and customers) and the set of arcs A are the different flows (material or information). Note that in order to simplify the problem, the retailers are neglected in this work but will be considered for future works.
The global functioning of the agro-alimentary SC is as follow:
When the costumers make an order to the distribution center (DC) of some products, this order is placed on a period t (t=1..T) corresponding to its due-date. A DC d is chosen to respond the order in the limited due-date and by optimizing different costs (inventory, handling inventory and transportation). If no DC is able to respond to the order (not enough in stock), orders are send from the DC to manufacturers. The later tries in turn to respond by looking to the stock or by producing. The delivery and production lead time are taken into account to meet the customer orders.
The manufacturers, depending on their needs, order the raw materials from suppliers that optimize costs (procurement and handling cost) and respect the due dates.
This global problem is modelled by a subdivision into three sub-problems according to the different echelons (customer-DC, manufacturer and supplier). For each level, the objective function and the considered constraints are detailed in the following part. For comprehensive reason, we start by describing the lower level (Depositor-Customer).
For the remaining features of our model, we consider the following parameters and variables:

Indexes and Sets:
A planning horizon of T periods. t = 1..T
S: set of suppliers, s =1..S,
M: set of manufacturers, m = 1..M,
D; set of DCies, d=1..D,
C: set of customers, c = 1..C,
P: set of products, p=1..P,
E: set of raw materials, e=1..E,

Parameters:

$DC_{c,p,t}$	Demands of product p from customer c in period t
$hd_{d,p}$	Handling cost of product p at DC d
$hm_{m,p}$	Handling cost of product p at manufacturer m

$hm_{m,e}$	Handling cost of raw material e at manufacturer m
$TC_{i,j}$	Transportation cost from node I to node j in the network G.
$PCM_{m,d,p}$	Procurement cost of product p from manufacturer m to DC d
$PCS_{s,m,e}$	Procurement cost of raw material e from supplier s to manufacturer m
$PrC_{m,p}$	Production cost of product p at manufacturer m
$LC_{d,c,p}$	Delivery Lead time for product p from DC d to customer c
$LD_{m,d,p}$	Lead time for product p from manufacturer m to DC d
$LD_{s,m,e}$	Delivery lead time for raw material e from supplier s to manufacturer m
$CD_{d,t}$	Storage capacity of DC d in period t
$CM_{m,t}$	Storage capacity of manufacturer m in period t for finish product
$CP_{m,t}$	Production capacity of manufacturer m in period t
$CME_{m,t}$	Storage capacity of manufacturer m in period t for raw material
$CS_{s,e,t}$	Capacity of supplier s in period t for raw material e
$SC_{m,p}$	Setup cost of product p at manufacturer m
$\mu_{m,p}$	Utilization rate of capacity in manufacturer m when producing one unit of product p
$\sigma_{e,p}$	Rate of raw material e to producing one unit of product p
$\delta_{m,p}$	Setup time of product p at manufacturer m

Variables

$ID_{d,p,t}$	Stock level of product p at DC d and period t
$IM_{m,p,t}$	Stock level of product p at manufacturer m and period t
$IE_{m,e,t}$	Stock level of raw material e at manufacturer m and period t
$QD_{m,d,p,t}$	Quantity of product p procured from manufacturer m to DC d in period t
$QM_{m,p,t}$	Quantity of product p produced at manufacturer m in period t
$QE_{s,m,e,t}$	Quantity of raw material e procured from supplier s to manufacturer m in period t
$DM_{m,p,t}$	Demands of product p at manufacturer m in period t
$DD_{m,p,t}$	Demands of product p at DC d in period t
$DM_{m,e,t}$	Demands of product p at manufacturer m in period t
$DE_{m,e,t}$	Demands of material raw e for manufacturer m in period t
$\lambda D_{d,c,p,t}$	Binary variable set to 1 if DC d is chosen for customer c needs of product p in period t;0 otherwise
$\lambda M_{m,d,p,t}$	Binary variable set to 1 if manufacturer m is chosen for DC d needs of product p in period t;0 otherwise
$SE_{m,p,t}$	Binary variable set to 1 there is a setup of product p in t; 0 otherwise

3.1.1 Depositor-Customer Model

In this stage, the customers make orders to DCs. The DC aims to minimize the costs related to handling stock, transportation and procurement from manufacturers. Let f_1 be the objective function of this level:

$$f_1 = \sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{t=1}^{T}((hd_{d,p} * ID_{d,p,t}) + \sum_{m\in M}(PCM_{m,d,p} * QD_{m,d,p,t}) + \sum_{c\in C}\lambda D_{d,c,p,t} * TC_{d,c}) \quad (1.1)$$

The minimization of *f1* should respect the inventory and capacity constraints.

The customer demands placed in period *t* should be shifted to the period $t\text{-}LC_{p,d,c}$ in the DC consideration to meet the customer due-date.

The DC demands are equal to the assigned customers' demands.

$$DD_{d,p,t} \geq \sum_{c\in C} DC_{c,p,(t+LC_{dcp})} * \lambda D_{d,c,p,t}, \forall d, \forall p, \forall t \quad (1.2)$$

$$ID_{d,p,t} = ID_{d,p,(t-1)} + \sum_{m\in M} QD_{m,d,p,t} - DD_{d,p,t}, \forall d, \forall p, \forall t \quad (1.3)$$

$$\sum_{p\in P} ID_{d,p,t} \leq CD_{d,t}, \forall d, \forall t \quad (1.4)$$

The customer demand in a period t should be covered by one DC.

$$\sum_{d\in D} \lambda D_{d,c,p,t} = 1, \forall p \in P, \forall c \in C, \forall t = 1..T \quad (1.5)$$

Constraints (1.2) compute the demand at DCs level, taking into account the assignment of customer to DC.

The balancing stock is respected by imposing the constraints (1.3). The inventory is limited by the available capacity (constraints (1.4)).

The following constraints define the nature of each variable.

$QD_{m,p,d,t} \in N,\ \forall\, m, p, d, t.$
$ID_{d,p,t} \in N,\ \forall\ p, d, t.$
$\lambda D_{d,c,p,t} \in \{0,1\},\ \forall\, d, p, c, t.$

3.1.2 Manufacturer Model

The objective at the manufacturing level is to minimize production costs, transportation costs, inventory handling costs and to responding to all order of lower level. The objective function is noted f_2.

$$f_2 = \sum_{p=1}^{P}\sum_{m=1}^{M}\sum_{t=1}^{T}((hm_{m,p} * IM_{m,p,t}) + (PrC_{m,p} * QM_{m,p,t}) + SC_{m,p} * SE_{m,p,t} + \sum_{d\in D}\lambda M_{m,d,p,t} * TC_{m,d})$$

The constraints of capacity and stock handling should be respected. Note that in this level two type of capacity are considered: storage capacity and production capacity.

$$DM_{m,p,t} \geq \sum_{d \in D} DD_{d,p,(t-LD_{mdp})} * \lambda M_{m,d,p,t} \quad (2.1)$$

$$IM_{m,p,t} = IM_{m,p,(t-1)} + QM_{m,p,t} - DM_{m,p,t}, \forall m, \forall p, \forall t \quad (2.2)$$

$$\sum_{p \in P} IM_{m,p,t} \leq CM_{m,t}, \forall m, \forall t \quad (2.3)$$

$$\sum_{p \in P} (\mu_{m,p} * QM_{m,p,t} + \delta_{m,p} * SE_{m,p,t}) \leq CP_{m,t},, \forall m, \forall t \quad (2.4)$$

$$QM_{m,p,t} \leq BM * SE_{m,p,t}, \ \forall m, \forall p, \forall t \quad (2.5)$$

The DC demand in a period t should be covered by one manufacturer.

$$\sum_{m \in M} \lambda M_{m,d,p,t} = 1, \forall p \in P, \forall d \in D, \forall t = 1..T \quad (2.6)$$

The DCs demands are computed according to the customer selection to each DC (the constraints (2.1)).
The stock in a period should be balanced by the demand and the production at the same period and the stock of the previous period (the constraints (2.2)).
The production capacities consider the processing time and the setup time (the constraints (2.4)).
The constraints (2.5) impose that if there is no setup, then no production is launched. Otherwise, the production is limited by a big number BM.

The following constraints define the nature of each variable.

$QM_{m,p,t} \in N, \ \forall m, p, t.$
$IM_{m,p,t} \in N, \ \forall m, p, t.$
$\lambda M_{m,d,p,t} \in \{0,1\}, \ \forall m, d, p, t.$

3.1.3 Procurement Model

At the Supplier-Manufacturer level, the objective is to supply manufacturers with the raw materials necessary for production.
The composition of products is provided by the bill of materials (BOM). Based on the BOM, the raw material requirement is determined. The objective, for which the function denoted f_3, is to minimize procurement and handling costs.

$$f_3 = \sum_{e=1}^{E} \sum_{m=1}^{M} \sum_{t=1}^{T} ((he_{m,e} * IE_{m,e,t}) + \sum_{s=1}^{S} (PCS_{s,m,e} * QE_{s,m,e,t}))$$

As for the previous models, this model is subjected to the capacity and handling stock constraints.
The raw material demand is at least equal to the demand of product at manufacturer that may be not honored by inventory or production.

$$DE_{m,e,t} \geq \sum_{p \in P} QM_{m,p,(t-LM_{s,m,e})} * \sigma_{e,p}, \forall m, \forall e, \forall t \quad (3.1)$$

$$IE_{m,e,t} = IE_{m,e,(t-1)} + \sum_{s=1}^{S} QE_{s,m,e,t} - DE_{m,e,t}, \forall m, \forall e, \forall t \quad (3.2)$$

$$\sum_{e \in E} \left(IE_{m,e,t} \right) \leq CME_{m,t}, \qquad \forall m, \forall t \quad (3.3)$$

For a supplier, the total order for a material raw should not exceed its capacity.

$$\sum_{m \in M} QE_{s,m,e,t} \leq CS_{s,e,t}, \quad \forall s, \forall e, \forall t \quad (3.4)$$

Constraints (3.1) imposes that the material raw demand should be at least equal to product demand multiply by the rate σ.
The inventory of each period is limited by the manufacturer capacity (constraints (3.3)).
The nature of each variable is defined by the following constraints.

$QE_{s,m,p,t} \in N, \ \forall s, m, p, t.$
$IE_{m,e,t} \in N, \ \forall m, e, t.$

3.2 Solution methodology

To solve the global optimization problem of an agro-alimentary SC, the global model including the three objective functions (f_1, f_2 and f_3) and the constraints of the three sub-models must be optimized. However, this global model is very large in number of variables and constraints.
Our strategy is to solve the different problems separately and coordinate between the levels by exchanging outputs and inputs.
As shown in figure 3, the procedure starts by considering customer orders to compute the DC requirements. This is done using the Costumer-Depositor model. The output of this model allows the computation of demands at the manufacture level.
Knowing the manufacturers requirement, it is then possible to compute production quantities and the raw materials requirement for each one of them. In this stage, the manufacturer model is used.
Finally, at the last level, the procurement model is used to compute the raw material that should be procured from suppliers to each manufacturer.

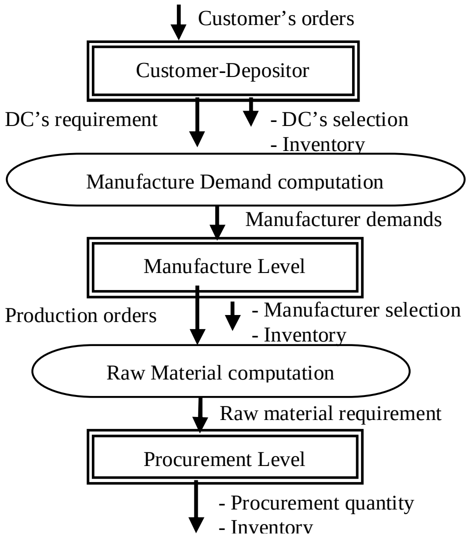

Figure 3: Solution methodology

4 EXPERIMENTATION

To illustrate the proposed solution, an example is developed in this section to validate our methodology using the previously introduced case study.
The solution approach is coded on C++ using the Concert Technologies of IBM Cplex (IBM, 2010).
The comparison of our methodology to the current system is complicated due to the confidentiality of the company data. More, it is difficult to find in literature benchmarks that correspond to our SC scheme. For this purpose, we compare the obtained solutions with a heuristic developed regarding the current functioning process of the company SC. According to our knowledge, this heuristic describes with a good level of adequacy the behaviour of the studied agro-alimentary SC's managers. This heuristic is given in the following sub-section.

4.1 Proposed planning heuristic for comparison study

As it was mentioned in section 2.1, in the system, there is a lack of evidence in the optimality of the solutions found and the respect of customers' due-dates, which generates high costs along the SC.
We have implemented the heuristic in order to compare its results with those of the mathematical model.
The steps of the heuristic are detailed by the sequence diagram presented at the appendix.

4.2 Illustrative example

At this stage, we do not have complete data sets from the studied company. Thus, to fix the input data used, the same strategies presented by (Sabri and Beamon, 2000) and (Wolosewicz et al. 2006) to generate randomly instances have been used.
The following parameters are considered:

- Number of customers = 2,
- Number of DCs = 2,
- Number of plants = 2,
- Number of suppliers = 2,
- Number of raw material = 2,
- Number of period = 4,

Delivery time between different levels is set to one period. The initial inventory is set 0 at all levels. The customer demand is given by the following matrix (for each product p, and each period t, see table1):

Customer 1

	t1	t2	t3	t4
p1	8	6	1	4
p2	4	7	0	4

Customer 2

	t1	t2	t3	t4
p1	2	9	3	3
p2	9	3	4	7

Table 1: Customers' demands

The different parameters and input data are uniformly generated in the intervals as shown in table 2.

Parameter	**Interval**
The inventory costs at all levels	[1, 4],
The procurement costs from manufacturer to DC	[5,16]
The procurement costs from supplier to manufacturer	[50,150]
The transportation costs from DCs to customer	[10, 30]
The transportation costs from plants to DCs	[20, 40],
The storage capacities of DCs, manufacturers and suppliers	Generated according to demands at each level.
The production cost at plants	[5,16]
The setup time	[1,30]
The setup cost	According to setup time
The consumption rate of product	Rate [10%, 60%] of capacity
Rate of raw material in product	[0,10]

Table 2: Parameters and input data

Once applied, our solution methodology gave the following results:
The demands at DCs (d1 and d2) are given in table 3.

Results obtained in level 1:

For d1

	t1	t2	t3	t4
p1	0	0	0	0
p2	0	4	11	0

For d2

	t1	t2	t3	t4
p1	15	4	7	0
p2	10	0	0	0

Table 3: Calculated demands at DCs

As we can see in those tables, the customer demands are aggregated at DCs level. All customer demands of the first period are ignored since the delivery time is one

period. The demand of product p1 is affected to DC d2. The DC demand in the first period equals to 15 including the two customer's demand at period 2 (6 and 9).
The total cost at this level, calculated using the function f1, according to the generated data is equal to 471.
In the manufacturer level the demand are given in table 4 (for manufacturer m1 and m2).

Results obtained in level 2:

Demands of m1

	t1	t2	t3	t4
p1	0	0	0	0
p2	0	11	0	0

Demands of m2

	t1	t2	t3	t4
p1	4	7	0	0
p2	4	0	0	0

Table 4: Calculated demands of manufacturers

The decision taken by the model at this stage is to affect the highest quantity of demand of products to m2.
The total cost is equal to 642 including production cost, setup cost and transportation cost (calculated using the function f2). Finally, to produce the manufacturer demands, the plant m1 receives the material raw from first supplier s1 (22 and 66 for each raw material e1 and e2). The plant m2 receives the material raw from first supplier s1 (49 and 35 for each raw material).

Results obtained in level 3:
The total costs at supplier level, calculated using the function f3, are equal to 12438.
For comparison purpose, the execution of the planning heuristic has been made. Another solution more expensive than the one obtained by our model has been found. This solution is equal to 19631 (796 at the first level, 718 at the second and 18144 at the last one).

Table 5 shows global costs obtained by using the heuristic (H) and the mathematical model (MM), according to configurations of sets presented in section 3.1.

Configuration C,P,D,M,S,E,T	Heuristics results (HR)	Mathematical model results (MMR)	GAP %
2,2, 3,3,3,3,4	57215	38940	49
4,3,4,2,3,3,5	168900	153070	10
10,12,5,5,6 ,10	5592136	5486871	19
10,5,5,5,5,5,15	5095984	5033481	1
15,5,5,5,5,5,15	6807883	6605339	3
15,5,5, 5,5,5,10	3094634	2874916	7
15,10,10,10,10,10,10	17558430	16357206	7
20,10,10,5,10,5,10	9556325	9428271	1

Table 5: Costs obtained by H and MM

To analyse the performance of the mathematical model, we use the GAP given by the following formulas:

$$GAP = \frac{HR - MMR}{MMR} * 100$$

The proposed model improves the actual solution by 12.12 % in average.

5 CONCLUSION AND FUTURE WORKS

In this paper we proposed a mathematical model to optimize SC taking into consideration some innovative constraints. Its objective was to guarantee the satisfaction of customers' demands in terms of costs and real delivery dates.
The proposed solution methodology was compared to a heuristic modelling the behaviour of the industrial managers of the agro-alimentary SC. The results obtained by executing the heuristic approach and the mathematical model show that the last one is more efficient than the heuristic.
Nonetheless, many perspectives seem interesting. First, the proposed model can be extended by including wholesalers and consideration of details in the raw materials acquisition. Considering different production types for each manufacturer can be a good future contribution because of product diversity, product categories, weight and aroma. Expiration dates must also be integrated. Finally, improving the proposed model to handle reactivity will be considered. For this purpose, the coupling with a multi-agent system, widely used to handle reactivity in supply chains, will be considered.

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APPENDIX The sequence diagram of the heuristic.