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A FREEWAY TRAFFIC MODEL IN A FIRST ORDER HYBRID PETRI NET FRAMEWORK

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ABSTRACT: *The paper presents a model for traffic state estimation and control of the freeways. The model is based on a First-Order Hybrid Petri Net (FOHPN) framework, a hybrid Petri net formalism including continuous places holding fluid, discrete places containing a non-negative integer number of tokens and transitions, which are either discrete or continuous. In order to suitably describe the dynamics of the freeway traffic flow, we allow updating the transition firing speed as a function of the markings modeling the freeway traffic. Moreover, we propose an online optimal control coordination of speed limits with the objective of maximizing the flow density. The use of FOHPNs offers several significant advantages with respect to the model existing in the related literature: the graphical feature enables an easy modular modeling approach and the mathematical aspects efficiently allow simulating and optimizing the system. The effectiveness of the FOHPN formalism is shown by applying the proposed modeling technique to a stretch of a freeway in the North-East of Italy.*

KEYWORDS: *Freeway traffic, First-Order hybrid Petri nets, Modeling, Optimization, Simulation.*

1 INTRODUCTION

Several management and control methods are proposed to improve performance of freeway networks. Among them, control strategies such as ramp metering, speed limits, and route recommendation are recognized as the most effective ways to relieve the freeway traffic congestion (Ghods *et al.*, 2010, Carlson *et al.* 2010, Hegyi *et al.* 2005).

Traffic control strategies can be classified in three main approaches (Ghods *et al.*, 2010). The first category consists of offline or open loop strategies that are based on historical data for the ramp metering (Kotsialos *et al.*, 2002). Such control approaches are inaccurate in the predictive traffic demands and accidents.

The second approach is based on closed-loop methods, which derive the control decisions on real-time data from traffic sensors. These controls do not provide any optimization procedures and are heuristics in nature (see Papageorgiou and Kotsialos, 2002 for a extensive review).

The third approach includes predictive control strategies that use online and offline information to predict the future state and manage the system by variable speed limits or ramp metering control (Hegyi, 2005, Di Febbraro *et al.* 2001). Among them, the Model Predictive Control (MPC) is a control strategy in which the control action at each sampling instant is obtained by solving on line a finite-horizon open loop optimization problem,

using the current state of the system as the initial state. However, the MPC show the drawback of a high computational complexity that adversely affects the online applications.

In this paper we propose a model predictive control strategy to face the congestion problem of freeway networks equipped with variable speed limits.

In order to evaluate the performance of the considered control schemes, some simulation models are formulated in the related literature. A popular model is the well-known second order macroscopic dynamic model of traffic flow proposed in (Payne 1971) and applied in many real cases by Papageorgiou *et al.* (1990a) and Papageorgiou *et al.* (1990b). Moreover, the traffic flow variables are estimated in real-time for a considered freeway stretch (Wang *et al.* 2007) with adequate time resolution and spatial resolution. In a recent paper, Wang *et al.* (2009) propose a stochastic version of the non-linear second-order macroscopic traffic flow model and a simple traffic measurement model, based on which the traffic state estimator is designed by extended Kalman filtering.

A MPC problem devoted to optimize the variable speed limit is proposed by Sacone *et al.* (2011) that adopt a first order dynamic model. The authors show that using the first-order model for the prediction in the model predictive control scheme involves an advantage in computation times in comparison with the second order model due to its simpler form.

The contribution of this paper is twofold. First, we model the freeway system in a First-Order Hybrid Petri Net (FOHPN) framework, a hybrid Petri net formalism including continuous places holding fluid, discrete places containing a non-negative integer number of tokens and transitions, which are either discrete or continuous (Balduzzi *et al.* 2000). Hybrid Petri nets are used to model urban transportation networks (Di Febbraro *et al.* 2004, Zhang *et al.* 2008, Dotoli *et al.* 2008) since they allow taking advantage of modeling the traffic flows as fluids and the traffic lights as the event-driven dynamics.

In (Dotoli *et al.* 2011), FOHPNs are used to model freeway traffic but the event-driven dynamics is determined by the traffic lights of the ramp metering. The present paper modifies the dynamics of the FOHPN in order to allow updating the transition firing speed as a marking function modeling the freeway traffic flow. The resulting model is a first order macroscopic, time-varying state model of traffic flow that is based on the space discretization and able to combine both time-driven and event-driven dynamics. In particular, the hybrid dynamics of the freeways is described by modeling the traffic flow as fluids (the continuous dynamics) and unpredictable events (i.e., accidents and lane interruptions) as event-driven dynamics.

Second, we propose an online optimal control coordination of speed limits with the objective of maximizing the flow density. The obtained controller can be used on line when the traffic is congested to predict the future state and manage the system by variable speed limits control.

The use of FOHPNs offers significant advantages with respect to the models existing in the related literature: the graphical feature enables an easy modeling approach and the mathematical aspects efficiently allow simulating and optimizing the system. Moreover, the model is able to describe also particular situations such as the lane changes and the unpredictable accidents and lane blockings.

The effectiveness of the FOHPN formalism is shown by applying the modeling technique to a stretch of a freeway in the North-East of Italy. Some simulation studies illustrate how the proposed model is able to provide a support to analyze the strategies to solve the congestions due to accidents and lane blockings.

The remainder of the paper is organized as follows. The next section recalls the basics of FOHPN. Afterwards, Section 3 describes the freeway modular model and Section 4 defines its dynamics. In addition, Section 5 presents the control strategy used to optimize the traffic flow. Finally, Section 6 illustrates the considered case study and Section 7 draws the conclusions.

2 BASIC OF FIRST ORDER HYBRID PETRI NETS

This section recalls the basics of FOHPN.

2.1 The FOHPN Structure and Marking

A FOHPN is a bipartite digraph described by the six-tuple $PN=(P, T, \mathbf{Pre}, \mathbf{Post}, \Delta, F)$. The set of places $P=P_d \cup P_c$ is partitioned into a set of discrete places P_d (represented by circles) and a set of continuous places P_c (represented by double circles). The set of transitions $T=T_d \cup T_c$ is partitioned into a set of discrete transitions T_d and a set of continuous transitions T_c (represented by double boxes). The set of discrete transitions $T_d=T_I \cup T_S \cup T_D$ is further partitioned into a set of immediate transitions T_I (represented by bars), a set of stochastic transitions T_S (represented by boxes) and a set of deterministic timed transitions T_D (represented by black boxes). We denote $T_t=T_S \cup T_D$, indicating the set of timed transitions.

Matrices \mathbf{Pre} and \mathbf{Post} are respectively the $|P| \times |T|$ pre- and post-incidence matrices, where $|A|$ denotes the cardinality of set A . Such matrices specify the net digraph arcs and are defined as follows:

$$\mathbf{Pre}, \mathbf{Post}: \begin{cases} P_c \times T \rightarrow \mathbf{R}^+ \\ P_d \times T \rightarrow \mathbf{R}^+ \end{cases}$$

We require that $\forall t \in T_c$ and $\forall p \in P_d$ $\mathbf{Pre}(p,t) = \mathbf{Post}(p,t)$ (well-formed nets).

Function $\Delta: T_t \rightarrow \mathbf{R}^+$ specifies the timing of timed transitions. In particular, each $t \in T_S$ is associated to the average firing delay $\Delta(t_j) = 1/\lambda_j$, where λ_j is the average transition firing rate. Each $t \in T_D$ is associated to the constant firing delay $\Delta(t_j) = FT_j$. Moreover, $F: T_c \rightarrow \mathbf{R}^+ \times \mathbf{R}_\infty^+$ specifies the firing speeds of continuous transitions, where $\mathbf{R}_\infty^+ = \mathbf{R}^+ \cup \{+\infty\}$. For any $t \in T_c$ we let $F(t_j) = (V_{mj}, V_{Mj})$, with $V_{mj} \leq V_{Mj}$, where V_{mj} (V_{Mj}) is the minimum (maximum) firing speed of the transition.

Given a FOHPN and a transition $t \in T$, we define $\bullet t = \{p \in P: \mathbf{Pre}(p,t) > 0\}$ and $t \bullet = \{p \in P: \mathbf{Post}(p,t) > 0\}$ (pre- and post-set of t , respectively). The corresponding restrictions to continuous or discrete places are ${}^{(d)}t = \bullet t \cap P_d$ or ${}^{(c)}t = \bullet t \cap P_c$. Similar notations may be used for pre- and post-sets of places. The net incidence matrix is $C(p,t) = \mathbf{Post}(p,t) - \mathbf{Pre}(p,t)$. The restriction of C to P_X and

$$T_X \text{ (with } X, Y \in \{c, d\}) \text{ is } C_{XY}. \text{ A marking } \mathbf{m}: \begin{cases} P_d \rightarrow \mathbf{R}^+ \\ P_c \rightarrow \mathbf{R}^+ \end{cases}$$

is a function assigning each discrete place a non-negative number of tokens (represented by black dots) and each continuous place a fluid volume; m_i denotes the marking of place p_i . The value of a marking at time τ is $\mathbf{m}(\tau)$.

The restrictions of \mathbf{m} to P_d and P_c are \mathbf{m}^d and \mathbf{m}^c . A FOHPN system $\langle PN, \mathbf{m}(\tau_0) \rangle$ is a FOHPN with initial marking $\mathbf{m}(\tau_0)$. Continuous and discrete transitions fire

as follows: 1) a discrete transition $t \in T_d$ is enabled at \mathbf{m} if for all $p_i \in \bullet t$, $m_i \geq \text{Pre}(p_i, t)$; 2) a continuous transition $t \in T_c$ is enabled at \mathbf{m} if for all $p_i \in {}^{(d)}t$, $m_i \geq \text{Pre}(p_i, t)$. Moreover, an enabled transition $t \in T_c$ is said strongly enabled at \mathbf{m} if for all $p_i \in {}^{(c)}t$, $m_i > 0$; $t \in T_c$ is weakly enabled at \mathbf{m} if for some $p_i \in {}^{(c)}t$, $m_i = 0$. In addition, if $\langle PN, \mathbf{m} \rangle$ is a FOHPN system and $t_j \in T_c$, then its Instantaneous Firing Speed (IFS) is denoted v_j and: 1) if t_j is not enabled then $v_j = 0$; 2) if t_j is strongly enabled, it may fire with any IFS $v_j \in [V_{mj}, V_{Mj}]$; 3) if t_j is weakly enabled, it may fire with any $v_j \in [V_{mj}, V_j]$, where $V_j \leq V_{Mj}$ depends on the amount of fluid entering the empty input continuous place of t_j . We call $\mathbf{v}(\tau) = [v_1(\tau) \ v_2(\tau) \ \dots \ v_{|T_c|}(\tau)]^T$ the IFS vector at time τ . Any admissible IFS vector \mathbf{v} at \mathbf{m} is a feasible solution of:

$$\begin{aligned} V_{Mj} - v_j &\geq 0 & \forall t_j \in T_\varepsilon(\mathbf{m}) \\ v_j - V_{mj} &\geq 0 & \forall t_j \in T_\varepsilon(\mathbf{m}) \\ v_j &= 0 & \forall t_j \in T_v(\mathbf{m}) \\ \sum_{t_j \in T_\varepsilon} C(p, t_j) v_j &\geq 0 & \forall p \in P_\varepsilon(\mathbf{m}) \end{aligned} \quad (1)$$

where $T_\varepsilon(\mathbf{m}) \subset T_c$ ($T_v(\mathbf{m}) \subset T_c$) is the subset of continuous enabled (not enabled) transitions at \mathbf{m} and $P_\varepsilon(\mathbf{m}) = \{p_i \in P_c \mid m_i = 0\}$ is the subset of empty continuous places. The set of all solutions of (1) is denoted $S(PN, \mathbf{m})$.

2.2 The net dynamics

The net dynamics combines time-driven and event-driven dynamics.

Macro events occur in two cases: i) a discrete transition fires or the enabling/disabling of a continuous transition takes place; ii) a continuous place becomes empty.

The time-driven evolution of the marking of a place $p_i \in P_c$ is:

$$\dot{m}_i(\tau) = \sum_{t_j \in T_c} C(p_i, t_j) v_j(\tau). \quad (2)$$

If τ_k and τ_{k+1} are the occurrence times of two subsequent macro-events, within time interval $[\tau_k, \tau_{k+1}[$ (macro period) the IFS vector $\mathbf{v}(\tau_k)$ is assumed constant. Then the continuous behaviour of a FOHPN for $\tau \in [\tau_k, \tau_{k+1}[$ is as follows:

$$\begin{aligned} \mathbf{m}^c(\tau) &= \mathbf{m}^c(\tau_k) + \mathbf{C}_{cc} \mathbf{v}(\tau_k) (\tau - \tau_k) \\ \mathbf{m}^d(\tau) &= \mathbf{m}^d(\tau_k). \end{aligned} \quad (3)$$

The net evolution at the occurrence of a macro-event is:

$$\begin{aligned} \mathbf{m}^c(\sigma_k) &= \mathbf{m}^c(\tau_k^-) + \mathbf{C}_{cd}(\tau_k) \\ \mathbf{m}^d(\sigma_k) &= \mathbf{m}^d(\tau_k^-) + \mathbf{C}_{dd}(\tau_k) \end{aligned} \quad (4)$$

with $\sigma(\tau_k)$ the firing count vector of discrete transition t_j at τ_k .

We associate to each $t_j \in T_i$ a timer η_j and $\boldsymbol{\eta}(\tau_k)$ is the timers vector associated to timed transitions at τ_k . Hence, the timer evolution within period $[\tau_k, \tau_{k+1}[$ $\forall t_j \in T_i$ is: $\eta_j(\tau_{k+1}) = \eta_j(\tau_k) = 0$ if t_j is not enabled; $\eta_j(\tau_{k+1}) = \eta_j(\tau_k) + (\tau - \tau_k)$ if t_j is enabled. When t_j is disabled or fires, its timer is reset to zero.

The system state at time τ_k , given by the marking and timers, is $\mathbf{x}(\tau_k) = [\mathbf{m}^c(\tau_k)^T \ \mathbf{m}^d(\tau_k)^T \ \boldsymbol{\eta}(\tau_k)^T]^T$. The system input is $\mathbf{u}(\tau_k) = [\tau_{k+1} - \tau_k \ \sigma(\tau_{k+1})^T]^T$, collecting the current macro-period length and the transition that will fire at the end of such macro-period. Hence, the behavior of the system can be described within the macro-period $[\tau_k, \tau_{k+1}[$ as follows:

$$\begin{aligned} \begin{bmatrix} \mathbf{m}^c(\tau_{k+1}) \\ \mathbf{m}^d(\tau_{k+1}) \\ \boldsymbol{\eta}(\tau_{k+1}) \end{bmatrix} &= \begin{bmatrix} \mathbf{I} & 0 & 0 \\ 0 & \mathbf{I} & 0 \\ 0 & 0 & \mathbf{D}(\tau_k) \end{bmatrix} \begin{bmatrix} \mathbf{m}^c(\tau_k) \\ \mathbf{m}^d(\tau_k) \\ \boldsymbol{\eta}(\tau_k) \end{bmatrix} + \\ &+ \begin{bmatrix} \mathbf{C}_{cc} \mathbf{v}(\tau_k) & \mathbf{C}_{dc} \\ 0 & \mathbf{C}_{dd} \sigma(\tau_k) \\ \mathbf{f}(\tau_k) & 0 \end{bmatrix} \begin{bmatrix} \tau_{k+1} - \tau_k \\ \sigma(\tau_{k+1}) \end{bmatrix}. \end{aligned} \quad (5)$$

The elements of matrix $\mathbf{D}(\tau_k)$ and vector $\mathbf{f}(\tau_k)$ are elements equal to 0 or 1 and depend on the macro-event occurring at the sampling instant τ_k (Balduzzi *et al.* 2000).

3 THE FREEWAY MODEL

Based on the idea of the bottom-up approach, this section proposes a modular FOHPN model to describe a freeway traffic system. Such a method can be summarized into two steps: decomposition and composition. Decomposition consists in partitioning a system into several subsystems. In freeway systems this division can be based on the determination of the structural entities, i.e., freeway stretches, on-ramp and off-ramp links, and control entities, i.e., vehicle flow interruptions and restoration procedures. All these subsystems are modeled by FOHPN modules. On the other hand, composition involves the interconnections of these sub-models into a complete model, representing the whole system.

3.1 The Generic Freeway Stretch Model

We assume that a generic freeway stretch is divided into links with length L and, consequently, a specified vehicle capacity C . We consider the set of continuous places P_c partitioned into two sets: the set P_L of the places modelling the link and the set P_{Ca} of the places modelling the capacities. Figure 1 depicts the model of a generic link constituted by two adjacent lanes i and i' . Each lane is modeled by two continuous places: $p_{Li} \in P_L$ and $p_{Ci} \in P_{Ca}$ model lane i , $p'_{Li} \in P_L$ and $p'_{Ci} \in P_{Ca}$ model lane i' . In particular let us consider lane i in Fig. 1: marking m_{Li} repre-

sents the vehicles in the link lane, while m_{C_i} describes its available space. Hence, if the link lane i can accommodate C_i vehicles and is initially empty, then the initial markings are $m_{L_i}(0)=0$ and $m_{C_i}(0)=C_i$.

With reference to Fig. 1, vehicles that enter (exit from) link i are modeled by transitions t_{i-1} and t_{i-1}^* (t_i and t_i^*). More precisely, transition t_{i-1} models the vehicle flow entering the link i , while t_{i-1}^* models the vehicle flow that changes lane and moves from link i' to link i . On the other hand, t_i represents the vehicle flow that continues to travel in the same lane of the next link, while t_i^* represents the vehicle flow that changes lane in the next link.

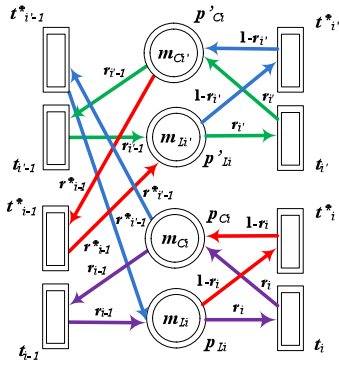


Figure 1: The FOHPN model of a two lane link.

The IFS of each transition $t_i \in T_c$ is $v_i \in [V_{mi}, V_{Mi}]$ where $V_{mi}=0$ and V_{Mi} represent respectively the minimum and maximum vehicle flow in the link. Moreover, when transition $t_i \in T_c$ is weakly enabled, it may fire with any $v_i \in [V_{mi}, V_i]$ with $V_i = V_{Mi}$. Weight r_i represents the fraction of vehicle flow remaining in the lane and $(1-r_i)$ is the changing lane fraction flow. Moreover, the weights r_{i-1} , r_{i-1}^* , r_{i-1} and r_{i-1}^* depends by the previous link $i-1$. For the sake of simplicity, in the next figures we do not report the weights of the arcs.

3.2 The On-ramp and Off-ramp Models

The FOHPN module modelling the on-ramp is shown in Fig. 2. In particular, transitions t_{lk} and t_{ok} respectively model the input and output vehicle flow from the ramp. Moreover, m_{L_k} represents the vehicle number occupying the ramp and m_{C_k} the available space in term of vehicles. Finally, the vehicles are routed from the ramp to the freeway link i by the arc from transition t_{ok} to place p_{Li} .

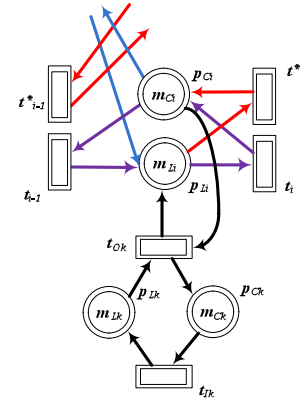


Figure 2: The FOHPN model of the on-ramp.

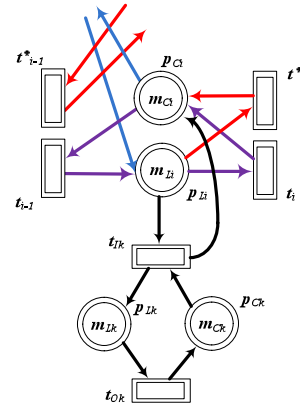


Figure 3: The FOHPN model of the off-ramp.

The off-ramp model is very similar to the on-ramp one and is shown in Fig. 3. In this case the vehicles flow from the freeway lane to the ramp. Hence, transitions t_{lk} and t_{ok} respectively model the input and output vehicle flow from the ramp and the weight r_k is the vehicle fraction that leaves the highway by the off-ramp.

3.3 The Vehicle Flow Interruption

The vehicle flow can be interrupted by unpredictable events, such as accidents. We describe a model to represent the interruption that is shown in Fig. 4. The accident event occurrence is modelled by transition $t_1 \in T_S$. When the transition t_1 occurs, the vehicle flow is interrupted, because the transitions t_i , t_{i-1}^* , t_i and t_{i-1}^* are disabled. Moreover, transition $t_2 \in T_S$ is associated with the time delay necessary to the freeway operators for starting the restoration procedures. When the traffic condition are restored, t_3 fires and place p_3 , if marked, enables the recovery procedures.

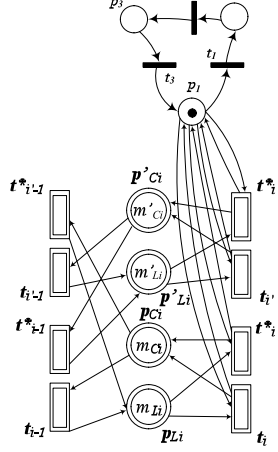


Figure 4: The FOHPN model of the vehicle flow interruption.

4 THE FREEWAY DYNAMICS

The model adopted in this work is based on the macroscopic traffic theory and, in particular, on the first order dynamic model of traffic flow. However, the FOHPN dynamics recalled in Section 2 is not sufficient to describe the complex relationships of a generic freeway. Indeed, let us consider a simple example regarding two consecutive links constituted by only one lane (see Figure 2). We assume that the system is in the initial state $m_{L_i}(0)=m_{L_{i+1}}(0)=0$, $m_{C_i}(0)=m_{C_{i+1}}(0)=C$ and the IFS of each transitions $t_j \in T_c$ is $v_j \in [0, V_{M_j}]$ with $j=i-1, i, i+1$. In this state, the transitions t_i and t_{i+1} are weakly enabled and they may fire with $v_j \in [0, V_j]$ where $V_j = V_{M_{i-1}}$ and $j=i, i+1$.

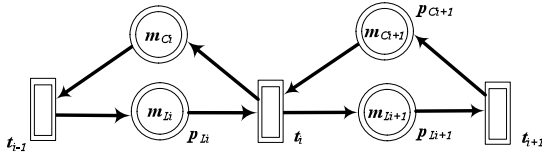


Figure 5: The FOHPN model of two links.

We consider two possible cases:

- a) $v_{i-1}=v_i=v_{i+1}$
- b) $v_{i-1}>v_i>v_{i+1}$.

In case a), by (3) the behaviour of the system is the following:

$$\begin{cases} m_{L_i}(\tau) = m_{L_i}(0) + (v_{i-1} - v_i)(\tau - \tau_k) \\ m_{L_{i+1}}(\tau) = m_{L_{i+1}}(0) + (v_i - v_{i+1})(\tau - \tau_k) \end{cases}$$

Hence, the markings $m_{L_i}(\tau)=m_{L_{i+1}}(\tau)=0$, $m_{C_i}(\tau)=m_{C_{i+1}}(\tau)=C \forall \tau > \tau_k$. In this scenario it is not possible to determine the vehicle occupation of the lane that results empty.

On the contrary, in case b) the markings m_{L_i} and $m_{L_{i+1}}$ tend to increase linearly to the limit value of their capacities C , when a new macro-event occurs. Even in this case, the freeway occupation linearly increases without taking into account the traffic law.

4.1 The Traffic Model in the FHPN framework

In order to provide a more general model that can describe a generic freeway, we introduce some changes in the FOHPN dynamics.

Consider link i in Fig. 1, the IFS v_i (vehicle/min) of $t_i \in T_c$ modeling the output of vehicles from link i is determined by the following stationary flow-density relationship (Wang *et al.* 2009) at time τ_k :

$$v_i(\tau_k) = \rho_i(\tau_k) \cdot V_{f_i} \cdot \exp \left[-\frac{1}{a_i} \left(\frac{\rho_i(\tau_k)}{\rho_{cr_i}} \right)^{a_i} \right] \quad (6)$$

where:

$\rho_i(\tau_k)$ (in veh/km/lane) is the traffic density, i.e., the number of vehicles in the link i , at the time τ_k divided by the link length L_i ;

V_{f_i} (in km/min) is the free flow speed of vehicles in link i ;

ρ_{cr_i} (in veh/km/lane) is the critical density.

The values of V_{f_i} , ρ_{cr_i} and a_i are usually not precisely known beforehand and may be different from site to site and may vary with environmental and further external conditions. However, as mentioned in (Papageorgiou *et al.*, 1990a) such parameters can be determined by off-line model calibration. In the presented FOHPN model, (6) can be written as follows:

$$v_i(\tau_k) = \frac{m_{L_i}(\tau_k)}{L_i} \cdot V_{f_i} \cdot \exp \left[-\frac{1}{a_i} \left(\frac{m_{L_i}(\tau_k)}{m_{cr_i}} \right)^{a_i} \right] \quad (7)$$

where $m_{cr_i} = \rho_{cr_i} \cdot L_i$ is the marking corresponding to the critical density.

Moreover, the minimum and maximum vehicle flows in the link are obtained from (7) as follows:

$$V_{mi}=0, V_{Mi} = V_{f_i} \cdot \frac{m_{cr_i}}{L_i} \cdot e^{-\frac{1}{a_i}}, \forall t_j, t_j^* \in T_c. \quad (8)$$

As explained in section 2.1, the IFS vector $\mathbf{v}(\tau_k)$ is constant in the macro period $[\tau_k, \tau_{k+1}[$. Hence, at each macro-event occurrence the model has to assign to each transition $t_i \in T_c$ the IFS determined by (7). Since the macro-event occurrences are asynchronous, the dynamics of the

FOHPN does not allow complying with (7). Therefore, we introduce new types of macro-events that determine the proper updating of the IFS vector in function of the continuous place markings.

A macro event occurs at time τ_{k+1} when the following condition is verified by some continuous place markings:

$$|m_i(\tau_{k+1}) - m_i(\tau_k)| = K_i \quad (9)$$

where K_i is a threshold value defined for each continuous place $p_i \in P_c$.

In the considered model we define the following threshold values:

$$K_i = \frac{C_i}{N_i} \text{ for each } p_{Li} \in P_L \quad (10)$$

$$K_i = C_i \text{ for each } p_{Ci} \in P_{Ca}. \quad (11)$$

where N_i is the segment number of equal length through which the stationary flow density relationship is approximated.

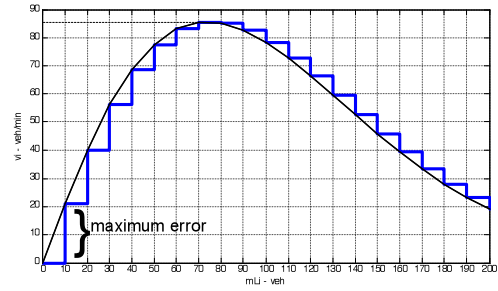
The resulting dynamics is a piecewise linear model for the traffic flow: when the variation of the number of vehicles in a place $p_{Li} \in P_L$ overcomes the threshold, a macro-event occurs and the IFS of each $t \in p_{Li}^{(c)}$ is updated according to (7). Since the threshold of the marking associated with the capacity places is equal to their capacity (see equation (11)), they do not determine any macro event. Introducing such new type of macro-event means to approximate the relation v_i of (7) by N_i segments of equal length K_i .

Since the slope of the function representing (7) is maximum in the interval $[0, K_i]$ (see for example Fig. 6a), the maximum error $E_{v_i}^{\max}$ in the determination of the flow v_i is the following:

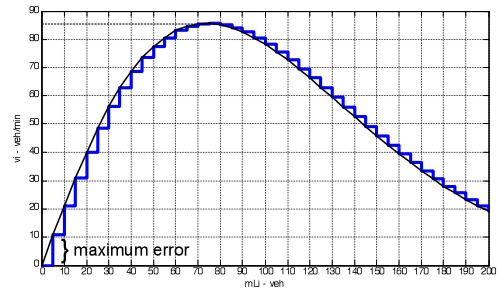
$$E_{v_i}^{\max} = \frac{K_i}{L_i} \cdot V_{f_i} \cdot \exp \left[-\frac{1}{a_i} \left(\frac{K_i}{m_{cr_i}} \right)^{a_i} \right]. \quad (12)$$

In Fig. 6 the shape of (7) is reported with the following values: $V_{f_i} = 2.16$ km/min, $L_i = 1$ km, $a_i = 1.62$, $m_{cr_i} = 73.58$ vehicles. Moreover, in Fig. 6a we use the threshold $K_i = 10$ vehicles, while in Fig. 6b the threshold $K_i = 5$ vehicles. Considering the data of Fig. 6a, the maximum error results $E_{v_i}^{\max} = 21.08$ veh/min, while in

the case of Fig. 6b it is $E_{v_i}^{\max} = 10.71$ veh/min. However, we have to design a controller that can be used on line when the traffic is congested (i.e., $m_{Li} > m_{cr_i}$) to predict the future state and manage the system by variable speed limits control. Hence, the error in the most interest zone of the flow density relationship is very low: for example for $m_{Li} = 130$ vehicles, the error in the flow evaluation is equal to 6.90 (3.47) veh/min considering $K_i = 10$ (5).



(a)



(b)

Figure 6. Stationary flow density relationship with (a) $K_i = 10$ vehicles, (b) $K_i = 5$ vehicles.

5 FLOW OPTIMIZATION

The linear time-varying system model (5) combines both time-driven and event driven system dynamics. The control approach adopted in this paper for dealing with the problem of regulating traffic behaviour on freeway stretches is a kind of MPC. In a MPC scheme a finite-horizon optimization is solved over a prediction horizon for optimizing a suitable objective function subject to constraints on control variables and state variables. In the presented approach, the control strategy selects the vector \mathbf{v} in each macro-period on the basis of the knowledge of the system state and in order to optimize a particular objective function. To this aim, the IFS vector \mathbf{v} to be applied to the successive macro-period can be selected by a controller that minimizes an objective function subject to the set of linear constraints (1) and the stationary flow-density relationship (7). The controller intends to maximize the traffic flow in the congested lanes, i.e., it

imposes that the vehicles density in each congested link is as close as possible to the critical density.

Moreover, we point out that at each macro-event the critical links (places $p_{Li} \in P_L$) are the ones that exhibit a marking $m_{Li} > m_{cri}$. More formally, we consider the following set of places:

$$P_{cr}(\tau_k) = \left\{ p_i \in P_L \mid m_i(\tau_k) > m_{cri} \right\}.$$

In addition, the controller has to select the IFS of transitions that are in input of places $p_i \in P_{cr}(\tau_k)$ in order to decrease the corresponding marking. On the contrary, the IFS of the remaining transitions can remain unchanged. Hence, the following transition set of the IFS components is defined:

$$V_{cr}(\tau_k) = \left\{ v_j \mid t_j \in {}^{(c)}P_{cr}(\tau_k) \right\}$$

The controller chooses the vector \mathbf{v}^* that minimizes the following performance index:

$$J = \sum_{p_{Li} \in P_L} \left[m_{cri} - (m_{Li}(\tau_k) + \left(\sum_{t_j \in T_c} C(p_{Li}, t_j) \cdot v_j \right) \cdot N \Delta \bar{\tau}) \right]^2.$$

where $\Delta \bar{\tau}$ is the average time delay between two consecutive macro-events and N is the number of the macro-events that we consider for the optimization.

Hence, the objective of the controller is to minimize the difference between the vehicles in the link at time τ_k ($m_{Li}(\tau_k)$) and a reference represented by the marking corresponding to the critical density m_{cri} , in order to maximize the freeway vehicle flow.

Referring to the generic link shown in Fig. 1, the following Programming Problem is solved at each macro-event occurrence:

$$\min_{\mathbf{v} \in V_{cr}(\tau_k)} J \quad (13)$$

s.t.

$$\mathbf{v} \in S(PN, \mathbf{m}) \quad (13a)$$

$$0 < v_j + v^*_{t_j} \leq v_j(\tau_k) \quad \forall t_j, t_j^* \in \bullet P_{cr}(\tau_k) \quad (13b)$$

$$0 < v_j + v^*_{t_j} = v_j(\tau_k) \quad \forall t_j, t_j^* \notin \bullet P_{cr}(\tau_k) \quad (13c)$$

$$v_j = v^*_{t_j} \quad \forall t_j, t_j^* \in T_c \quad (13d)$$

The constraints (13a) impose that the solution $\bar{\mathbf{v}}$ is an admissible IFS vector. Moreover, the constraints (13b) force the input transitions of each place of *critical marking* to have an IFS vector minor or equal of the flow den-

sity (7). On the other hand, the IFS of transitions that are not in input of a place of critical marking can be equal of the flow density by (5). In addition, the constraints (13d) impose that the flow density in output of the same place are equal. Obviously, if the traffic is not congested, then the critical occupancy cannot be reached and the value of the objective function J can not decrease.

Note that the controller defined by the least square problem (13) selects the IFS of transitions, i.e., the average vehicle speed in each lane stretch to guarantee the selected marking of the continuous places. More precisely, the recommended speed of the vehicles for the i -th link can be determined by the optimal solution $\bar{\mathbf{v}}$ as follows:

$$vel_i = 60 \cdot \bar{v}_i \cdot \frac{L_i}{m_{cri}} e^{\frac{1}{a_i}} \text{ km/h.} \quad (14)$$

6 THE SYSTEM DESCRIPTION AND RESULTS

The case study considers a stretch of the A4 freeway of the North-East of Italy between the tollbooths of Portogruaro and San Stino. The data used in the simulation model (i.e., inter-arrival times and path choices) are provided by the Autovie Venete s.p.a. that is the company managing the considered freeway.

	Places	length (km)	Capacities
Link 1	p_1, p_7	2.0	400
Link 2	p_2, p_8	2.0	400
Link 3	p_3, p_9	2.0	400
Link 4	p_4, p_{10}	2.0	400
Link 5	p_5, p_{11}	2.0	400
Link 6	p_6, p_{12}	2.0	400
On ramp 1	P_{25}	0.5	100

Table 1: The freeway stretch description

6.1 Simulation Description

Figure 7 describes the FOHPN modular model of the considered freeway: the section Portogruaro-San Stino is 12 km long and is composed of two lanes for each carriageway. We divide the freeway stretch into 6 links. Table 1 shows the places that model the links, the on-ramp and the corresponding values of the capacities. Moreover, we assume that an accident occurs after 8 minutes and that one lane is in such a case interrupted.

The IFS v_j associated with the generic continuous transition t_j modelling the traffic flow is determined at each macro period by (7). Furthermore, we estimate the parameter of (7) by the data provided by Autovie Venete and we obtain the following values:

$$V_{f_i} = 2.16 \text{ km/min, } \rho_{cri} = 73.58 \text{ veh/km/lane, } a_i = 1.62 \text{ for}$$

the freeway links $i=1,\dots,6$;

$V_{f_i}=0.83$ km/min, $\rho_{cr_i}=73.58$ veh/km/lane, $a_i=1.62$ for the on ramp.

In any case, we chose the threshold (10) equal to $K_i=20$ vehicles.

The described system is simulated in three scenarios characterized by different values of the IFS v_j associated with the continuous transitions modeling the freeway input flows, i.e., the input transitions t_1, t_2, t_3 and t_4 , and the on-ramp input transitions t_{29} . Table 2 shows the IFS (in veh/min) that are associated with the system input and on-ramp transitions in the three scenarios S0, S1 and S2.

	v_1	v_2	v_3	v_4	V_{23}
S0	13.2	1.5	13.2	1.5	7.4
S1	39.6	4.5	39.6	4.5	14.8
S2	52.8	6	52.8	6	29.6

Table 2: The IFS of the simulations in each scenario

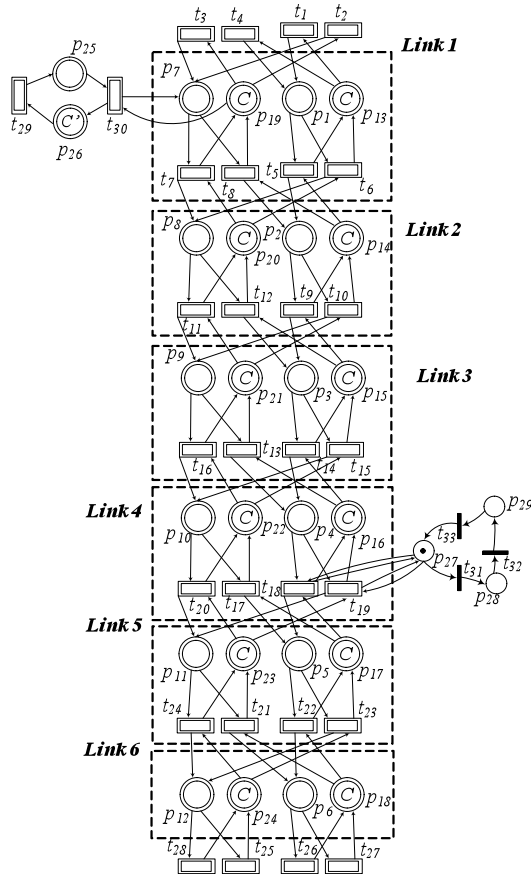


Figure 7. The FOHPN model of the case study

The scenario S0 uses the real data provided by the Autovie Venete and the scenarios S1 and S2 are more congested systems. In each case, we show the efficiency of the model to predict the behaviour of the freeway. In particular, we evaluate by the simulations three performance indices:

N_{av} , i.e., the average number of vehicles in the freeway;

N_{ll} , i.e., the average number of vehicles in the last link (places p_6 and p_{12});

N_{cong} , i.e., the number of congested links (links where the average marking is about equal to the capacity of the link).

6.2 Simulation Results

The FOHPN shown in Fig.7 and the Programming Problem (13a-d) are respectively simulated and implemented in MATLAB environment. The value of the performance indices are determined by a simulation run of 900 minutes, considering a transient period of 120 minutes and the initial markings corresponding to the empty system.

First of all, the model is validated by comparing the performance index N_{av} (see scenario S0 in Table 1) with the annual average value N_{real} provided by Autovie Venete. Two cases are considered: case 1) no accident occurs; case 2) the accident occurs after 500 minutes ($FT_{31}=500$ min). In particular, by the data provided by Autovie Venete, the freeway operators start the recovery procedures 30 min after the accident ($FT_{32}=30$ min) and complete the restore operations in 120 min ($FT_{33}=120$ min). The simulation results are the following: $N_{av}=215.8$ veh and $N_{real}=215$ in case 1), and $N_{av}=985.3$ veh and $N_{real}=1015$ veh in case 2). Considering that the error performed by the simulation is about 0.37% in case 1 and 2.9% in the more complex case 2), the results prove that the simulation closely represents the actual system.

	S0	S1	S2	S2C
N_{av} (veh)	215.8	727.9	1046.9	934.7
N_{ll} (veh)	35.7	113.6	131.2	145.8
N_{cong} (link)	0	0	2 (p_7, p_{25})	0

Table 3: The simulation results

Table 3 summarizes the results of the simulations for the described scenarios S0, S1 and S2. The last scenario named S2C considers the scenario S2 under the system control scheme (13)-(13d).

In the scenarios S0 and S1 the links are not congested ($N_{cong}=0$) and in S1 the number of vehicles in the freeway is increased, as expected. In the scenario S2 the places p_7 and p_{25} are congested. In particular, the congestion of the place p_{25} (on-ramp) depends from the congestion of the place p_7 (a lane of the link 1). In this case, the proposed control scheme is applied with $P_{cr}(\tau_k) = \{p_7, p_{25}\}$ and $V_{cr}(\tau_k) = \{v_2, v_3, v_{29}\}$. The

obtained optimal solution is the following: $\bar{v}_2 = 50.4$, $\bar{v}_3 = 50.6$ and $\bar{v}_{29} = 29.6$, i.e. the recommended speed limit for the first link is about 80 km/h and for the on ramp is 50 km/h. The simulation results show that under the proposed control technique the average number of vehicles in the freeway (N_{av}) decreases. Moreover, the average number of vehicles in the last link N_{ll} increases in scenarios S2C since the upstream congestion is solved.

Summing up, the application of the proposed control action drives the system state to a regular traffic condition by imposing the speed limits. Moreover, the computational effort is very important to apply on line the control scheme. The optimization problems resulting from the considered case study are generally solved in few seconds and this proves the applicability of the controller in the real cases.

7 CONCLUSION

The paper develops a model for traffic state estimation and control of the freeways. The model is based on the First-Order Hybrid Petri Nets (FOHPNs), a hybrid Petri net formalism able to describe the traffic flow by continuous dynamics and the unpredictable events by event driven dynamics. Moreover, in order to suitably describe the freeway traffic flow, the dynamics of the FOHPN is modified by adding new macro-events. The resulting model is a first order macroscopic, time-varying state model of traffic flow that is based on the space discretization and is able to combine both time-driven and event system dynamics. Moreover, the FOHPN model allows us to define an online optimal control coordination of speed limits with the objective of maximize the flow density.

The modeling and control techniques are applied to a stretch of a freeway in the North-East of Italy. The simulation results validate the model and show the effectiveness of the proposed control scheme based on speed limits.

Future research will consider a larger simulation campaign of more complex real systems in order to refine the model and the control schemes. In particular, the freeway input flows will be considered variable during the seasons and week days. Furthermore, the proposed management system will be applied to solve accidents and huge lane blockings.

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