New Integrated Approach For Solving Multi-level Lot Sizing and Scheduling problems

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NEW INTEGRATED APPROACH FOR SOLVING MULTI-LEVEL LOT SIZING AND SCHEDULING PROBLEMS

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ABSTRACT: *In this paper, a new procedure for solving the multi-level lot sizing and scheduling problem in an integrated manner is presented. A Lagrangian heuristic is used to generate feasible production plans for a fixed sequence of operations, by solving the dual problem of production planning and applying a smoothing procedure to repair production plans. Detailed capacity constraints are considered in the lot sizing mathematical model to guarantee consistency between planning and scheduling decisions; and in order to consider multi-level production requirements, nomenclature constraints are included, using echelon stock formulation.*

KEYWORDS: *lot sizing, scheduling, multi-level problems, integrated approach, Lagrangian heuristic*

1 INTRODUCTION

Production planning and scheduling are ones of the most important phases of production management. However, decisions concerning these two activities are generally taken in a sequential order. Thus, production planning is first performed at the tactical decision level and, the different jobs are then supposed to be scheduled at the operational decision level. The problem of this strategy is that at the tactical level, one has no a detailed view about system capacity in terms of scheduling. Therefore, the information about capacity becomes aggregated at the tactical level, thus not guaranteeing that scheduling constraints are respected. By consequence, the production plans may be unfeasible. For many years, this hierarchical approach was accepted and used. Thus, methods like the well-known MRP (Material Requirements Planning) approach (Vollman et al., 2005), which considers an infinite system capacity, were lagerly spread and included in several production systems. Then, the inconsistency of decisions was recognized and researchers put an effort on integrating production planning and scheduling. That was how the approach MRP-II (Manufacturing Resources Planning) was created. It improved the quality of solutions in terms of feasibility by considering aggregated capacity constraints. Nevertheless the production plans could still be unfeasible, because starting times of operations are not considered by this kind of constraint. In fact, to include precedence information between operations and starting times, the mathematical model must include detailed capacity constraints, as mentioned in (Lasserre, 1992). This is the only way that a lot sizing model guarantee feasible production plans in a complex manufacturing system.

In the last years, various works have been proposed for solving the production planning and scheduling problem in an integrated manner. The most part of them are focused on solving small bucket problems like: the discret lot sizing and scheduling problem (DLSP), the continuous setup lot sizing problem (CSLP) or the proportional lot sizing and scheduling problem (PLSP), where the number of setups per period is limited. For example, Gicquel et al. (2009) use a multi-attribute product structure to model production systems with the characteristics of a DLSP. With respect to big bucket problems, the problems usually treated are the capacitated lot sizing problem (CLSP) and the general lot sizing and scheduling problem (GLSP). The mathematical models used to solve these small and big bucket problems guarantee feasible solutions for manufacturing systems where the production of each item requires the completion of only one operation, or in case of multiple operations per product, each item can be treated on only one resource. Therefore, in these cases, detailed capacity constraints are not necessary to assure consistency between tactical and operational level. Nevertheless, some works include sequencing constraints in order to consider sequence-dependent setup costs and times (see for example Menezes et al., 2011).

On the other hand, when the production of an item

requires the completion of several operations and each item has to be produced on several resources (e.g. job-shop configuration), the use of capacitated lot sizing models does not guarantee feasible solutions at the operational level. That is why additional constraints concerning precedence between operations and starting times have to be considered. In the litterature, few works deal with this kind of problem. Dauzère-Pérès and Lasserre (1994) propose an iterative approach for job-shop problems, which solves the lot sizing problem for a given sequence of operations and then the scheduling problem for the computed production plan, Hasse and Kimms (2000) present a tailor-made enumeration method for single-stage single-machine production systems. Dauzère-Pérès and Lasserre (2002) study the impact of sequencing decisions in lot sizing and scheduling problems. Wolosewicz et al. (2006) present an integrated approach based on a Lagrangian heuristic for solving the single-level lot sizing and scheduling problem for a fixed sequence. Erdirik-Dogan and Grossmann (2008) study the integrated problem for single-stage multi-item continuous plants with parallel lines and propose a bilevel decomposition algorithm to solve the planning problem (upper level) and the scheduling problem (lower level). Li and Ierapetritou (2010) present an iterative rolling horizon method for chemichal industry environments, where at each iteration, the respect of detailed capacity constraints is required only for the current and previous planning periods. As these works consider single-level problems, products are independent between them from the point of view of material requirements.

Very few works deal with multi-level problems including scheduling decisions. Generally, the capacity is respected by using the multi-level capacitated lot sizing problem (MLCLSP) formulation which is a variation of the CLSP. The difference is that inventory balance constraints take into account bill-of-material information. Similarly to capacitated single-level problems, when there are precedence contraints between operations belonging to the same product or sharing the same resource, the MLCLSP formulation does not guarantee feasible solutions. Buschkühl et al. (2010) presents a litterature review of solution approaches for capacitated lot sizing problems, with special emphasis on the MLCLSP. Stadtler (2011) studies the proportional lot sizing and scheduling problem (PLSP) with zero lead times in a multi-level single-machine manufacturing system. Wu et al. (2011) present two MIP models for solving capacitated multi-level lot sizing problems with backlogging. Sahling et al. (2009) propose a new algorithm for solving the MLCLSP with setup carry-overs. Integrated multi-level, multi-item, multi-resource problems have not been studied a lot. One of the few works is the one proposed by Fandel and Stammen-Hegene (2006), which study a multi-level general lot sizing and scheduling problem (MLGLSP) with multiple machines and consider sequence dependent setup times and costs.

In this paper, we present an integrated approach for solving multi-level big bucket problems with multiple resources, multiple products and multiple operations per product, considering a fixed sequence of operations. The configuration of the production system corresponds to a job-shop. The solution method is based on an extension of a previous work treating single-level problems. In that work, the difficulty was to generate production plans respecting detailed capacity constraints. In the current work, as there is a bill-of-material relationship between products, nomenclature constraints have to be also respected to guarantee a feasible solution. With respect to complexity, the CLSP is known to be $\mathcal{NP}$-hard (Florian et al., 1980). Then, the MLCLSP is also $\mathcal{NP}$-hard. Moreover, considering detailed capacity constraints entails a difficulty increase. Therefore, the integrated problem we consider is also $\mathcal{NP}$-hard.

This paper is organized as follows. In section 2, we describe the nature and context of the kind of problems we are considering and the mathematical model. In section 3, we present the integrated approach. Then, in section 4 some numerical tests are illustrated, and finally in section 5 we give the conclusions about this work.

2 PROBLEM DESCRIPTION

We consider manufacturing systems, where the production of several products has to be planed and scheduled over several periods, with the objective of satisfying the clients demand at a minimum cost. Performing the production planning corresponds to decide appropriately production targets, i.e. lot sizes of each item at each period, and solving the scheduling problem corresponds to determine the best sequence of operations for each resource and the starting time of each operation.

This paper is particularly focused on multi-level problems. It means that it exists a nomenclature or bill-of-material structure which defines the relationship (in terms of material requirements) between the different items to be produced. In other words, some items are components or successors of other products, and material requirements of successors entails also material requirements for predecessors. The most basic items (products without predecessors) of the nomenclature integrate the first or lowest level of the production structure, and the most sophisticated items (products without successors) form the last or uppest level. An assembly system is a typical exemple of a multi-level problem.

The difference between multi-level and single-level problems is that, in the second case there is no relationship of requirements between products. It allows the production targets to be determined using a dynamic lot sizing algorithm, where the lot sizing problem is decomposed into several subproblems (a subproblem per product). In multi-level cases, as not only the external demand influences decisons about production targets but also the internal demand, it is not easy to divide the problem into a given number of subproblems for independent products.

Therefore, the major difficulty of solving a lot sizing problem is the impossibility to effectively apply a dynamic lot sizing algorithm, as the one proposed by Wagner and Whitin (1958).

Furthermore, as we are integrating decisions concerning the scheduling problem, detailed capacity constraints have to be also included in the mathematical model. Then, in order to implement a dynamic lot sizing procedure for solving a problem per product, two constrainst (capacity and nomenclature) have to be relaxed or not considered. Therefore, the slack between the output of this kind of procedure and a feasible solution might be considerably large.

Formulation

We take some notations introduced by Clark and Armentano (1993) to define a mathematical model for solving a multi-level lot sizing problem, and we integrate it in the model we used for single-level problems (Wolosewicz et al., 2006).

Thus, the different variables and parameters of our model are defined using the following notation.

Decision variables:

- X_{il}: quantity of product i available at the end of period l (production target).
- Y_{il}: setup variable (= 1 if product i is produced at period l, 0 otherwise).

Other variables:

- I_{il}: inventory level of product i at the end of period l.
- E_{il}: echelon stock of product i at period l.

Parameters:

- d_{il}: external demand of product i at the end of period l.
- D_{il}: internal demand of product i at the end of period l.
- $\mathcal{DS}(i)$: set of direct successors of product i in the gozinto tree.
- $\mathcal{AS}(i)$: set of all successors of product i.
- $\mathcal{DP}(i)$: set of direct predecessors of product i in the gozinto tree.
- g_{ij}: gozinto factor, i.e. number of units of product i required to produce one unit of product j ($g_{ij} = 0$ if $j \notin \mathcal{DS}(i)$).
- n_{ij}: number of paths relating products i and j in the nomenclature structure.
- p_{ij}^n: total requirement of product i to produce one unit of product j through path n
- $P^n(i,j)$: n^{th} path connecting product i and its successor j.
- L_i: lead time of product i.
- $K^n(i,j)$ sum of lead times to obtain product j from i by following the path n in the nomenclature.
- $M(i)$: maximum sum of lead times to obtain product i.
- c_i^p: manufacturing cost per unit of product i.
- c_i^{inv}: inventory cost per unit of product i.
- e_i: echelon stock cost per unit of product i
- c_i^s: setup cost per unit of product i.
- p_o^u: processing time of operation o per unit of product $i(o)$.
- s_i^t: setup time of operation o per unit of product $i(o)$.
- c_l: length of period l (available capacity).
- $\mathcal{O}$: set of operations
- $\mathcal{L}$: set of last operations in the routing.
- $\mathcal{F}$: set of first operations in the routing.
- $i(o)$: product associated to operation o.
- $l(o)$: period associated to operation o.
- $S(y)$: sequence of operations associated to sequence y. $((o,o') \in S(y)$ means that operation o precedes operation o' in the sequence of a resource).
- N: number of products.
- T: number of periods of the planning horizon.

Let us explain some concepts before presenting the mathematical model. In multi-level lot sizing problems, the perception about demand and inventory takes another dimension, because of the internal relationship between products. Thus, balance material equation is not represented in terms of inventory, but in terms of echelon stock, and the demand acquires internal needs. In what follows, these new characteristics are introduced.

Echelon stock

The echelon stock of a given product at given moment, is defined in the litterature as the existing quantity of material of this item in the system, as finished product or as component, to satisfy the external and internal demands. Contrary, the inventory level of a given product at a given moment is its real available quantity of material, i.e. it changes as a function of production targets of successor products and not as a function of the internal demand. In our model, the echelon stock is considered at the end of the period.

There are two forms to define the echelon stock of a product i at period l (Afentakis and Gavish, 1986). Either as a function of the inventory level of all its successors, or in a recursive manner as function of the echelon stock of only its direct successors. The first definition can be represented by equation (1).

$$E_{il} = I_{il} + \sum_{j \in \mathcal{AS}(i)} \sum_{n=1}^{n_{ij}} p_{ij}^{n} I_{j,l+K^{n}(i,j)} \tag{1}$$

In the above equation, the sum of lead times to obtain product j from i by following the path n is calculated using equation (2)

$$p_{ij}^{n} = \prod_{k \in P^{n}(i,j)-j} g_{k,s^{n}(k)} \tag{2}$$

where $s^{n}(k)$ is a successor product of item k.

The recursive definition of the echelon stock is represented by equation (3).

$$E_{il} = I_{il} + \sum_{j \in \mathcal{DS}(i)} g_{ij} E_{j,l+L(i)} \tag{3}$$

On the other hand, the inventory level may be modeled by equation (4).

$$I_{il} = I_{i,l-1} + X_{il} - d_{il} - \sum_{j \in \mathcal{DS}(i)} g_{ij} X_{j,l+L(i)} \tag{4}$$

The echelon stock cost per unit of product i can be calculated through equation (5).

$$e_i = c_i^{inv} - \sum_{j \in \mathcal{DP}(i)} c_j^{inv} g_{ji} \tag{5}$$

Demand

The demand of product i at period l is equal to the sum of its external demand (client demand) and its internal demand (internal requirements). This definition can be represented by the recursive equation (6)

$$D_{il} = d_{il} + \sum_{j \in \mathcal{DS}(i)} g_{ij} D_{j,l+L(i)} \tag{6}$$

A non-recursive expression of the demand correspond to equation (7).

$$D_{il} = d_{il} + \sum_{j \in \mathcal{AS}(i)} \sum_{n=1}^{n_{ij}} p_{ij}^{n} D_{j,l+K^{n}(i,j)} \tag{7}$$

In what follows, we present the mathematical model for multi-level lot sizing and scheduling problems. It is an extension of the one proposed for single-level problems, with the particularity that bill-of-material constraints are taken into account and inventory variables are replaced by echelon stock variables.

Mathematical model

$$\min \sum_{i=1}^{N} \sum_{l=1}^{T} (c_i^{s} Y_{il} + c_i^{p} X_{il} + e_i E_{il}) \tag{8}$$

Subject to

$$E_{i,l-1} + X_{il} - E_{i,l} = \sum_{j \in \mathcal{DS}(i)} g_{ij} D_{j,l+L(i)} \quad \forall i = 1,...,N; \forall l = 1,...,T \tag{9}$$

$$\sum_{j \in \mathcal{DS}(i)} g_{ij} E_{j,l+L(i)} - E_{il} \leq 0 \quad \forall i = 1,...,N; \forall l = 1,...,T \tag{10}$$

$$r\left(o_c^f\right) + \sum_{o \in c} \left(p_o^u X_{i(o)l(o)} + s_o^t Y_{i(o)l(o)}\right) \leq \sum_{l=1}^{l(o_c^l)} c_l \quad \forall c \in C(y) \tag{11}$$

$$X_{il} \leq \left(\sum_{k=l}^{T} D_{ik}\right) Y_{il} \quad \forall i = 1, ..., N; \forall l = 1, ..., T \tag{12}$$

$$X_{il} \geq 0, E_{il} \geq 0 \quad \forall i = 1, ..., N; \forall l = 1, ..., T \tag{13}$$

$$Y_{il} = \{0; 1\} \quad \forall i = 1, ..., N; \forall l = 1, ..., T \tag{14}$$

$$X_{il} = 0 \quad \forall i = 1, ..., N; \forall l \leq M(i) \tag{15}$$

The objective function of the model is represented by equation (8), which corresponds to the minimization of the total cost, i.e. the sum of manufacturing, echelon stock and setup costs. Inventory balance constraints are modeled by equation (9). Equation (10) represents nomenclature or bill-of-material constraints. It forces the production of components to be enough to satisfy requirements of successors. Equation (11) corresponds to detailed capacity constraints, where variable $r(o)$ can take different values according to equations (16)–(18), which guarantee that first operations can start at a period between 0 and $l(o)$ if $L_{i(o)} = 0$ or between $l(o) - L_{i(o)}$ and $l(o)$ if $L_{i(o)} > 0$. On the other hand, last operations start and finish during their associated production period $l(o)$. Constraints (12) relate the continuous production variable X to the binary variable Y. Non-negative constraints are represented by equation (13). Constraints (14) correspond to the definition of the binary variable for setup decision, and, by considering constraints (15), the production of item i at period l is not possible if l is not greater than the maximum sum of lead times required to produce this item. This is true considering that $E_{i0} = 0 \; \forall i$.

$$r(o) = \begin{cases} \sum_{l=1}^{l(o)-L_{i(o)}} c_l, & L_{i(o)} < 0 \\ 0, & otherwise \end{cases} \quad \forall o \in \mathcal{F} \tag{16}$$

$$r(o) = \sum_{l=1}^{l(o)-1} c_l, \quad \forall o \in \mathcal{L} \tag{17}$$

$$r(o) = \begin{cases} \max\left(\sum_{l=1}^{l(o)-L_{i(o)}} c_l, \sum_{l=1}^{l(o)-1} c_l\right), & L_{i(o)} < 0 \\ \sum_{l=1}^{l(o)-1} c_l, & otherwise \end{cases} \quad \forall o \in \mathcal{F} \cap \mathcal{L} \tag{18}$$

3 INTEGRATED APPROACH

In Section 2, we mentioned that using a dynamic lot sizing algorithm for solving multi-level problems was difficult because this type of procedures considers uncapacitated systems and independent products. In single level cases, the challenge of solving in an integrated manner the lot sizing and scheduling problem is to satisfy capacity constraints, taking into account that in dynamic lot sizing algorithms the capacity is not respected. In multi-level cases, a new challenge is incorporated: statisfying nomenclature constraints.

To solve the lot sizing problem we apply the Lagrangian relaxion (Lemaréchal, 2001) over capacity and nomenclature constraints, as in (Tempelmeier and Derstroff, 1996), and we use the subgradient multipliers method (Parker and Rardin, 1988) to update Lagrangian multipliers. We combines the Lagrangian relaxation with the dynamic lot sizing algorithm proposed by Wagelmans et al. (1992). It allows the results to vary as a function of Lagrangian multipliers values. However, as two constraints of the model are relaxed, the proposed production plans do not correspond to feasible solutions. The cost associated to a production plan is in reality a lower bound.

For this reason, we integrate a smoothing procedure to the solution method, in order to repair the violated constraints, thus providing upper bounds, which correspond to feasible solutions. In what follows, the global method (Lagrangian relaxation + Smoothing procedure) is called *Lagrangian heuristic*.

The Lagrangian heuristic algorithm is illustrated in figure 1.

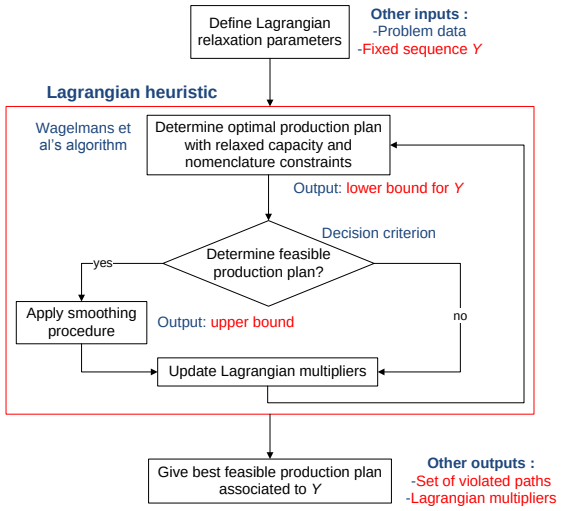

Figure 1: Lagrangian heuristic algorithm

In the rest of this section, the Lagrangian relaxation and the smoothing procedure are explained in detail.

3.1 Lagrangian relaxation

As mentioned above, the Lagrangian relaxation is used to decompose the optimization problem into a number of easy-to-solve subproblems dualizing capacity and nomenclature constraints. The objective is to find an optimal production plan without considering these two contraints, using the algorithm proposed in (Wagelmans et al., 1992). Capacity and nomenclature constraints are relaxed because of the difficulty to integrate them in a lot sizing optimization algorithm. Nevertheless, for the case of capacity constraints, despite the fact that the difficulty of the problem decreases (because it becomes uncapacitated), relaxing all capacity constraints is time-consuming.

Therefore, the strategy is to relax only the most violated capacity constraints, and to update only the Lagrangian multipliers associated to them. It allows the Lagrangian heuristic to reduce the necessary time to complete an iteration, without deteriorating the lower bound quality.

Lagrangian multipliers associated to capacity constraints are updated according to the following steps.

First of all, let $\mathcal{CH}$ be the set of violated paths. Then,

- Set all the Lagrangian multipliers to 0 and $\mathcal{CH} = \emptyset$.
- At each iteration:
  1. Find the most violated path in the graph, i.e. the path cv which maximizes the expression: $r(o_c^f) + \sum_{o \in c} (p_o^u X_{i(o)l(o)} + s_o^t Y_{i(o)l(o)}) - \sum_{l=1}^{l(o_c^l)} c_l$, where $c = cv$.
  2. If $cv \notin \mathcal{CH}$ then $\mathcal{CH} = \mathcal{CH} \cup cv$.
  3. Update all the multipliers associated to the paths of $\mathcal{CH}$.

For the case of nomenclature constraints, a Lagrangian multiplier is defined for each product at each period (set (i, l)), and the number of relaxed constraints is always the same. It corresponds to the number of sets (i, l).

The cost associated to the production plan obtained at the first iteration corresponds to the *absolute lower bound* (ALB). As at the first iteration all Lagrangian multipliers are set to zero, the proposed production plan is optimal for the correspondent uncapacitated single-level lot sizing problem. The greatest lower bound obtained represents the *lower bound* (LB) of the problem solved for a fixed sequence y. This lower bound is supposed to be close to a feasible solution.

By relaxing capacity and nomenclature constraints, the objective function of the dual problems is:

$$
\begin{aligned}
F = \min & \sum_{i=1}^{N} \sum_{l=1}^{T} \left\{ \left(c_i^s + \sum_{c \in C(y)} \beta_c \sum_{o \in c} s_o^t \right) Y_{il} \right. \\
& \left. + \left[c_i^p + \sum_{k=l}^{T} (e_i + \gamma_{ik}) + \sum_{c \in C(y)} \beta_c \sum_{o \in c} p_o^u \right] X_{il} \right\} \\
& - \sum_{i=1}^{N} \sum_{l=1}^{T} \sum_{k=l}^{T} (e_i - \gamma_{ik}) D_{ik} \\
& + \sum_{c \in C(y)} \beta_c \left[r(o_c^f) - \sum_{l=1}^{l(o_c^l)} c_l \right] \\
& + \sum_{i=1}^{N} \sum_{l=1}^{T} \sum_{j \in DS(i)} \sum_{k=l}^{T} \gamma_{ik} g_{ij} X_{j,l+L(i)}
\end{aligned} \tag{19}
$$

Therefore, by using the lot sizing optimization algorithm mentioned above, the decision of production setup varies as a function of the following term:

$$
c_i^s + \sum_{c \in C(y)} \beta_c \sum_{o \in c} s_o^t \tag{20}
$$

and if we consider that each product is independent of each other, optimal lot sizes varies as a function of:

$$
c_i^p + \sum_{k=l}^{T} (e_i + \gamma_{ik}) + \sum_{c \in C(y)} \beta_c \sum_{o \in c} p_o^u \tag{21}
$$

Nevertheless, as it exists nomenclature constraints relating the products, decisions about lot sizes of a product cannot be taken in isolation. As shown in equation (19), the production target of successors products has an influency on the objective function of the dual problem, so this information must not be neglected. The problem is that lot sizing optimization algorithm mentioned above, as well as the other ones, is designed to optimize production targets of independent products. For this reason, a transformation of equation (19) is realized, considering the information about successor products requirements. To perform this idea, direct successors requirements have to be included in expression (21), by adding the last variable part of equation (19) in terms of direct predecessors, as follows:

$$\left[c_i^p + \sum_{k=l}^{T} (e_i + \gamma_{ik}) + \sum_{c \in C(y)} \beta_c \sum_{o \in c} p_o^u + \sum_{j \in DP(i)} \sum_{k=l-L(i)}^{T-L(i)} \gamma_{jk} g_{ji} \right] X_{il} \qquad (22)$$

Thus, the relaxed objective function allows the lot sizing optimization algorithm to provide different production plans according to the evolution of Lagrangian multipliers. However, the feasibility is not guaranteed. To satisfy the remaining violated capacity and nomenclature constraints, a smoothing procedure is applied.

3.2 Smoothing procedure

This procedure aims at performing the necessary production moves to repair violated capacity and nomenclature constraints. Thus, production targets are modified, by moving production portions to previous and later periods, untill the production plan becomes feasible. The cost associated to the new production plan corresponds to an upper bound (UB) for the fixed sequence y. Two smoothing procedures have been created, changing the priority order of constraints to be satisfied. Figures 2 and 3 illustrate the general algorithm of these two procedures.

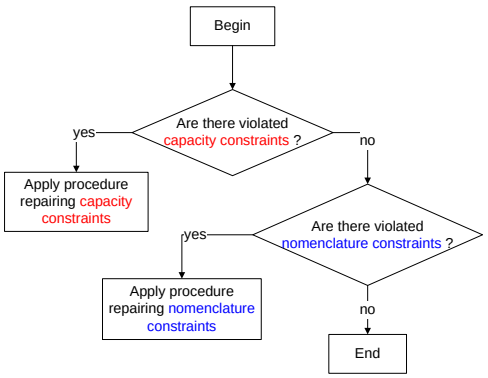

Figure 2: Procedure "Capacity-Nomenclature"

The first smoothing procedure, named "Capacity-Nomenclature" aims at satisfying first capacity constraints and then nomenclature constraints. Contrary, by considering procedure "Nomenclature-Capacity", nomenclature constraints are first satisfied.

In the first setp of the algorithm, the idea is to satisfy all constraints of one type (capacity or nomenclature) without increassing the violation of the other type, whereas in the second step, violated constraints of the

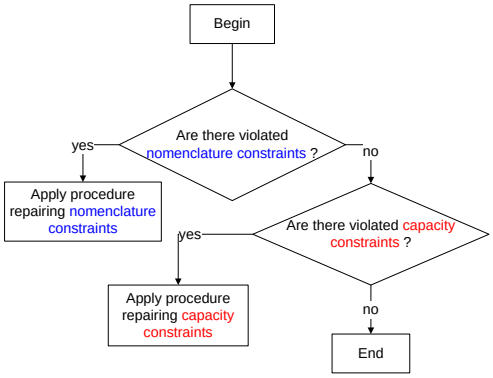

Figure 3: Procedure "Nomenclature-Capacity"

second type are repaired without violating constraints of the first type.

To complete the subprocedure that repaires capacity constraints, the following principal steps are performed:

- Select product to be moved and source period.
- Select target period and quantity to transfer.
- Realize production move.

To determine the product to transfer and the source period, we consider all products containing operations with no margin at each period and we take the job with largest processing time. The margin of an operation is calculated using equation (23).

$$margin_o = \begin{cases} \dfrac{d_{l_o} - d_{e_o}}{D_o}, & Y_{i(o)l(o)} = 1 \\ \dfrac{d_{l_o} - d_{e_o} - s_o}{D_o}, & Y_{i(o)l(o)} = 0 \end{cases} \qquad (23)$$

In the above equation, d_{l_o} and d_{e_o} are the latest and earliest starting times of operation o, respectively, and D_o is the total processing time of operation o.

The selection of the target period and the quantity to transfer is performed at the same time. We consider all periods without critical operations and we calculate the maximum quantity that can be moved from the source period, without creating backlogs. For each possible move, we compute the unitary transfer cost, and we select as target period the one with smallest unitary transfer cost. This steps are repeated until all capacity constraints are satisfied or until there are not more possible moves.

On the other hand, the procedure repairing nomenclature constraints follows the algorithm presented in figure 4.

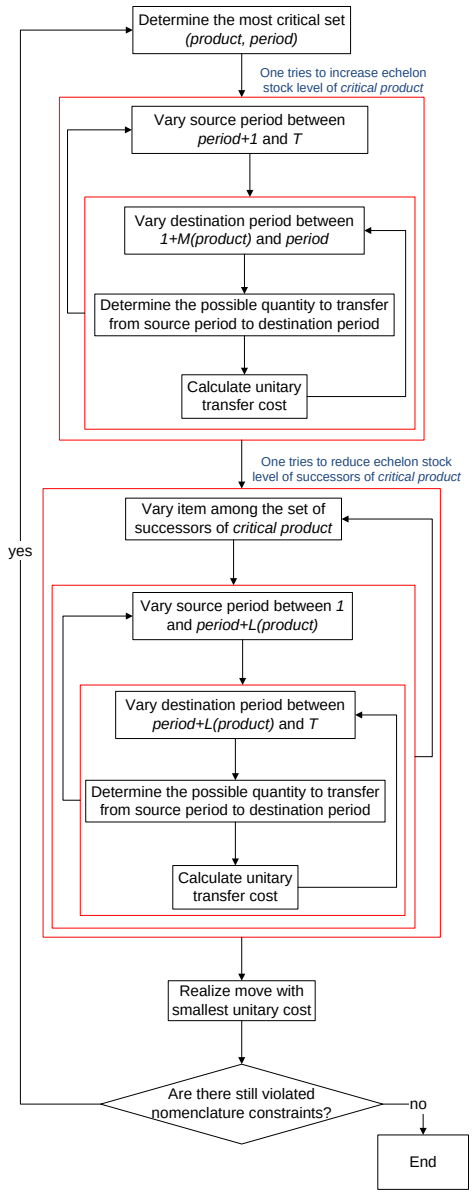

Figure 4: Procedure repairing nomenclature constraints

In this subprocedure, two kind of moves are possible. In fact, regarding equation (10), to satisfy material requirements of a set (i, l) we can increase the echelon stock level of product i at period l, by transfering production quantities from later periods $(l+1, ...T)$ to earlier periods $(1 + M(i), ..., l)$, or we can reduce the echelon stock of successor products of i, by moving production quantities from earlier periods $(1, ..., l + L(i))$ to later periods $(l + L(i), ..., T)$. In both cases, the maximum quantity of transfer is calculated. This quantity is restricted by the fact that capacity of periods must not become violated, and the violation degree of other nomenclature constraints must not increase. The move chosen to be performed is the one wich generates the smallest unitary transfer cost.

4 NUMERICAL TESTS

In this section, the performance of the approach is analyzed by solving an example of a multi-level problem. Our approach was run on Microsoft Visual C++ 2010 Express, and exact solutions were obtained using CPLEX 12.3. The machine used to compute results has an operating system of 64 bits (Microsoft Windows 7), a processor Intel core i7 4Quad @2.0GHz and 8 GB of RAM memory. The number of threads was not limited for the execution. Then CPLEX was able to perform parallel computation, thus reducing a lot execution times. On the other hand, our program uses only one thread.

The tested problem is presented in figure 5, where each circle represents a product and each arc corresponds to the material relationship between two products. The number writen on each arc is the gozinto factor. Thus, we are considering a relative small structure of 6 products. Additionaly, to complete the production of a product, 6 operations have to be performed on six different resources. The sequence is fixed in advance, so the problem consists in finding the best feasible production plan associated to the given sequence, respecting all constraints.

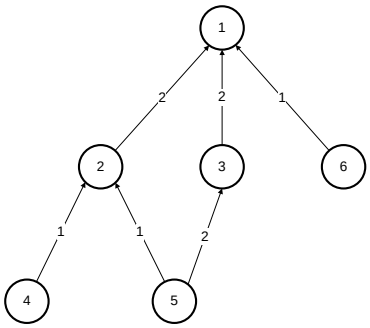

Figure 5: Multi-level problem

Unitary costs and lead times are presented in table 1. External demands vary between 5 and 15 units, and echelon stock costs are calculated using equation (5).

Item	c_i^p	c_i^s	c_i^h	$L(i)$
1	4	60	15	1
2	4	60	4	1
3	4	60	2	1
4	4	60	1	1
5	4	60	1	1
6	4	60	2	1

Table 1: Problem data

Several instances were tested using planning horizon lengths of 5, 10 and 20 periods. Additionaly, different period capacities were considered, by modyfing the capacity ratio *capa* and calculating the period capacity c_l as follows:

$$c_l = capa \sum_{o \in l} \left(p_o^u X_{i(o)l(o)} D_{i(o)l(o)} + s_o^t Y_{i(o)l(o)} \right) \quad \forall l \quad (24)$$

Exact solutions and absolute lower bounds are presented in table 2. As we can observe, the computational time increases when the number of periods increases and even more when the capacity ratio is tight. Instance 10 shows that time may increase considerably when increassing T and reducing *capa*. Nevertheless, in this case, as the nomenclature structure is relatively small and the sequence of jobs is fixed, the computational time remains short.

Instance	T	*capa*	*ALB*	Optimum	time (s)
1	5	0.60	4474	4726	0.59
2	5	0.65	4474	4715	1.03
3	5	0.70	4474	4690	1.03
4	5	0.75	4474	4690	0.70
5	5	0.80	4474	4690	1.17
6	10	0.65	9965	10560	2.57
7	10	0.70	9965	10442	2.23
8	10	0.75	9965	10438	1.67
9	10	0.80	9965	10418	2.07
10	20	0.65	22796	24140	20.73
11	20	0.70	22796	23961	4.41
12	20	0.75	22796	23908	3.84
13	20	0.80	22796	23908	4.85

Table 2: Exact solutions and absolute lower bounds

Solutions provided by our approach are presented in table 3.

Instance	*LB*	Cap-Nom		Nom-Cap	
		UB	time(s)	*UB*	time(s)
1	4474.0	4747	0.98	4747	0.62
2	4661.2	4715	0.95	4715	0.83
3	4622.6	4692	0.72	4692	0.72
4	4604.2	4726	0.42	4726	0.44
5	4595.5	–	0.33	4732	0.36
6	10227.5	–	4.24	10798	4.09
7	9965.0	10547	2.00	10524	1.84
8	9965.0	10550	1.58	10550	1.33
9	9965.0	10533	1.96	10549	1.28
10	23280.6	24920	5.85	24885	2.73
11	23059.6	24398	3.37	24454	2.54
12	23777.6	24073	5.63	24073	1.70
13	23773.6	24114	1.82	24114	2.21

Table 3: Solutions provided by our approach

We note that using smoothing procedure "Nomenclature-Capacity" is more effective than using "Capacity-Nomenclature", because the first one always guarantees feasible solutions and gives globaly better solutions in a shorter time. With respect to the lack between upper bounds and optimal solutions, the largest gap is of 3.09% and it corresponds to instance 10. Even if gaps and computational times using "Nomenclature-Capacity" are considerably short, there is still a margin of improvement in terms of solution quality. One possibility to improve results consists in reducing the restriction of smoothing procedure moves, by allowing partial violation of one type of constraints when searching for satisfying the other one.

5 CONCLUSIONS

A new method to solve the integrated problem of multi-level production planning and scheduling in multi-item multi-resource systems was presented. In order to reduce the complexity of the problem and facilitate the computation of lower bounds, the Lagrangian relaxation technique has been used relaxing detailed capacity constraints and nomenclature constraints. This technique was combined with a lot sizing optimization algorithm designed for uncapacitated independent products, by including a mathematical relationship between components and successors in the relaxed objective function. A smoothing procedure has been also created, generating upper bounds which represent costs of feasible solutions. The performance of this integrated approach was tested, and results prove that it is able to provide feasible solutions.

As perspectives, we are considering adding more constraints to take into account more situations of common manufacturing problems in supply chains.

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