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A NASH EQUILIBRIUM SIMULATION MODEL OF THE AUCTION BASED DAY AHEAD ELECTRICITY GENERATION MARKET

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ABSTRACT: *We address the auction based day ahead electricity generation market and present a simulation model employing the Nash equilibrium notion. The model modifies a previous formalism proposed in the related literature and employs empirical data distributions of the market clearing price as registered by the market independent system operator. The model is effective when power suppliers with different generation capacities are considered, and can forecast the market competitiveness in different scenarios with respect to the number and capacity of participating bidders. The proposed framework is effectively applied to a data set regarding the Italian electricity market. The model can be employed as a basis for a decision support tool both for market participants and regulators.*

KEYWORDS: *electricity generation market, business intelligence, modeling, simulation, Nash equilibrium, market dynamics.*

1 INTRODUCTION

In the last decades numerous Western countries have witnessed a progressive deregulation of electricity. The energy market is gradually transitioning from being dominated by large producers to a market in which liberalization and privatization are encouraged in order to improve competition and efficiency. As a result, in today's energy market different alternative power suppliers are available and consumers may negotiate significant commercial elements, such as energy price, supplying conditions, etc.

This work focuses on energy generation, a production sector characterized by several peculiarities, e.g., the impossibility to store the product apart from minor quantities, and the fact that a permanent equilibrium between demand and supply has to be attained to preserve the energy network system stability. In such a complex scenario, business intelligence approaches can be a solution to address numerous emerging issues, such as (Argotte, 2009): trend prediction, analysis of supply and demand, modeling and simulation of market behavior.

This paper addresses the emerging topic of modeling and simulating the electricity generation market. In the new energy market context, electricity price is no longer determined by a power authority but rather by competitive bidding behaviors of the available suppliers. As a result, the actual operation of the generating units depends on the complex interactions between the available producers. The resulting risks to which electricity firms are exposed significantly increases their need for suitable models that may be employed as a starting point for decision support tools to forecast the market dynamics, by simulating the competitiveness of the various suppliers and deriving the possible scenarios of price and competetiveness variations.

A competitive electricity market is typically regulated by an Independent System Operator (ISO), whose task is to keep the physical integrity of the transmission system while providing nondiscriminatory access to all participants in the market. Two main models can be considered for such a regulated market structure: the bilateral market and the pool or auction (Nogueira *et al.*, 2003). In a bilateral market sellers and consumers transact directly with each other. On the contrary, the pool is a market structure in which suppliers and buyers transact based on some sort of auction under the supervision of the ISO. This paper considers the auction based day ahead electricity market. In the day ahead electricity market, before the actual opening of the market, consumers submit hourly demand offers while producers submit hourly supply bids for the next day: each bid consists of a sealed offer stating a quantity as well as a price at which the producer is willing to sell for the hourly trading period of the next day. Based on the received supply bids and demand offers, the ISO runs a market clearing algorithm that matches production with demand producing a series of hourly prices and accepted quantities. In particular, the ISO arranges the bids for each trading period in the increasing price order and determines the so-called Market Clearing Price (MCP) as the price of the bid of the last supplier needed to meet the announced demand (price clearing process). All suppliers whose bid are below or equal to the MCP supply power are admitted to produce and they are paid the MCP. A supplier is called (infra/extra) marginal if its bid equals (is below/above) the MCP.

This paper extends the work by You *et al.* (2010) that propose a method to analyze power suppliers bidding auctions based on order statistic. In particular, we present a novel Nash equilibrium-based model for simulating the auction based day ahead electricity market. The modeling framework enhances the recalled formalism in terms of flexibility and effectiveness, allowing to consider different production quantities in the auction. In addition, the model has the objective of investigating how the energy market competitiveness varies with the number of power suppliers and their generation capacity. The presented modeling framework is applied to the Italian day ahead electricity market, showing the effectiveness of the proposed technique by simulation and validation in the MATLAB software.

The remainder of the paper is organized as follows. The next Section presents an overview of methods for modeling the energy market. Hence, Section 3 details the novel equilibrium based model for the simulation of the electricity generation market. The subsequent Section presents the case study, which refers to the Italian electricity market and enlightens the effectiveness of the model. The paper is concluded by a Section summarizing the proposed contribution and by an up-to-date reference Section.

2 ELECTRICITY MARKET: MODELING AND SIMULATION

A taxonomy of literature contributions regarding electricity market modeling is presented by Ventosa *et al.* (2005), who identify three major trends: optimization models for a single firm, market equilibrium models, and simulation models.

Models in the first class focus on the problem of choosing the bidding strategy of each competing power supplier to maximize its profit. The resulting formalisms rely on stating and solving optimization problems in which a single objective function has to be optimized subject to a set of technical and economical constraints. According to the manner in which the price clearing process is represented, these models can be grouped according to two basic categories: exogenous price models and models based on price as a function of the firm's decision. In particular, in the first subclass the price clearing process is represented as exogenous to the firm's optimization problem, i.e., the MCP is regarded as an input parameter to the optimization problem. On the contrary, the other subclass of models explicitly considers the influence of suppliers on price.

In addition, the second class collects the so-called equilibrium models that are approaches considering the simultaneous profit maximization program of each firm competing in the market. The market behavior is modeled based on the concept of Nash equilibrium: the energy market reaches an equilibrium when each bidder adopts a strategy which is optimal, given its opponents' strategies. Also this class of models can be viewed as grouping two categories. The most widespread formalisms are the so-called Cournot equilibrium models, in which firms compete based on quantity strategies, whereas the so-called supply function equilibrium models consider producers competing with respect to their offer curve strategies.

The third class of energy market modeling formalisms considers simulation models that provide a more flexible and less computationally intensive way to represent the market with respect to the two former classes. This class of modeling frameworks typically includes two subcategories: agent based simulation models and simulation models related to equilibrium models. Agent based simulation models employ agents to represent the distributed decision making dynamics of the energy market. Such autonomous decision-making entities are characterized by bounded rationality and are able to learn, adapt and reproduce. Hence, agents take decisions based on a set of heuristic rules. Moreover, simulation models related to equilibrium models are dynamical models that implement iteratively equilibrium models belonging to the second class of discussed formalisms to capture market dynamics including evolutions caused by strategic behaviors of bidders.

In the context of simulation models, market simulators based on the Nash equilibrium are emerging approaches since determining market equilibria is a desirable objective both for market regulators such as ISO and market participants (i.e., consumers and producers): on the one hand they allow ISO to monitor and detect market power and on the other hand they allow buyers and sellers to refine their bids considering long-term bidding strategies of their competitors (de la Torre *et al.*, 2004). Among these simulation models, we recall the work by Otero-Novas *et al.* (2000) who present a simulation model considering the profit maximization of each supplier and taking decisions by a two level decision process referring to the Nash Cournot equilibrium. In addition, Day and Bunn (2001) propose a simulation model that constructs optimal supply functions based on the so-called supply function equilibrium model. This approach has been used for some simulation applications: Abeygunawardana et al. (2009) studied the equilibrium of the Italian market in order to evaluate the effects of carbon price variations and CO_2 emissions price, while Ardakani et al. (2008) describe the behavior of bidders trying to find the Nash Cournot equilibrium according to a hierarchical agent-based learning. Another way of addressing the problem is based on conjectural models (Diaz et al., 2010): bidders offer to produce a certain power quantity at an equilibrium price. This can be obtained by equalling marginal revenues to marginal costs: bidders have to make conjectures on other bidders' behavior in order to determine the former term.

This paper proposes a simulation tool recalling a more recent approach to equilibrium-based simulation of the electric market, which has been introduced by You *et al.* (2010). In particular, they present an electricity market model coming from the sealed bid auction theory and based on the so-called order statistic method. Assuming that all suppliers have the same generation capacity and that they have no withholding strategy, the number of winners is fixed when the power demand is given and the market clearing price is an order statistic. Hence, the probability of a supplier winning/losing the bid is obtained by the order statistic method. Moreover, starting from the so-called Independent Private Values (IPV) assumption (McAfee and McMillan, 1987), all bidders are considered to have costs drawn from the same probability distribution. Thanks to the symmetry hypothesis, the resulting bidding function is common to all bidders. In addition, the bidding function is assumed to be increasing with the cost. As a consequence of these hypotheses, the probability for the bidder to be awarded is a function of its cost.

3 THE PROPOSED NASH EQUILIBRIUM-BASED SIMULATION MODEL

This paper presents a Nash equilibrium-based simulation model of the day ahead electricity market that enhances the recalled equilibrium model introduced by You *et al.* (2010), with some appropriate modifications allowing a more flexible use of the model by the decision maker.

3.1 Problem Statement

In this sub-section we formulate the assumptions underlying the presented model that are listed in the following.

A1. *Energy availability:* we assume that the total power supply quantity is always greater than the total demand;
A2. *Exogenous price*: we assume that no bidder is able to raise the MCP by itself (without loosing its customers), i.e., no one is a price-taker;
A3. *Independent private values*: according to the IPV assumption, we assume that every bidder has its own known production cost;
A4. *No withholding strategy:* we assume that every bidder is available to produce electricity until its production capacity, requiring the same unitary price for each produced electricity unit;
A5. *Empirical price distribution:* we assume that the MCP probability density function is empirical and may be derived from real data of the analyzed electricity market;
A6. *Nash equilibrium:* we assume that every bidder is rational and submits the bid maximizing its expected profit.

We call $\boldsymbol{S}$ the set of N available electricity suppliers participating to the day ahead electricity market. In the t-th considered trading period, denoting Q the total fixed power demand and announced by the ISO, each producer $S_i \in \boldsymbol{S}$ submits a bid $b_i=(p_i, q_i)$ where p_i is the required price per electricity unit (expressed in [€ MWh^{-1}]) and q_i is the generation capacity (expressed in [MWh]). Note that for the sake of simplicity we neglect representing the demand and the bids as a function of t.

Assuming that $\boldsymbol{S}=\{S_1,\ldots,S_N\}$ is ordered in an increasing unit cost order in the considered trading period, the Electricity Market Clearing Problem (EMCP) can thus be stated as follows.

EMCP. Given the demand Q in a trading period, determine a positive integer M such that $\sum_{i=1}^{M-1} q_i < Q \le \sum_{i=1}^{M} q_i$.

Hence, solving the EMCP leads to determining M (infra) marginal power suppliers.

Note that, because of assumption A1, the EMCP has always a solution, i.e., there always exists a positive integer $M \le N$ satisfying the *EMCP*.

3.2 Simulation Model Definition

By assumption A3, a widely used hypothesis in auction theory, each bidder does not know the other bidders' costs, but knows a common probabilistic distribution from which all costs are drawn (Vickrey, 1961). Hence, the i-th bidder's unitary producing cost c_i (expressed in [€ MWh^{-1}]) can be drawn from a uniform distribution $c_i \sim \mathrm{Uni}(\underline{c}_i, \overline{c}_i)$.

Accordingly, by assumption A6 each bidder $S_i \in \boldsymbol{S}$ submits a bid $b_i=(p_i, q_i)$ maximizing its expected profit, given its production cost and the probability to be awarded. According to such a definition, the offer may be written as:

$$p_i = \arg\max \pi_i = \\ = \arg\max \begin{pmatrix} (MCP - c_i) q_i \cdot \mathrm{P}(i \text{ inframarginal}) + \\ +(p_i - c_i)\cdot(Q - \sum_{j=1}^{m-1} q_j)\cdot \mathrm{P}(i \text{ marginal}) \end{pmatrix} \quad (1)$$

where π_i is the expected profit of the bidder (expressed in €) while P(i inframarginal) and P(i marginal) respectively indicate the probability that the submitted bidder is inframarginal or marginal.

It is very complex to define a Nash equilibrium according to the recalled tendering mechanism. You *et al.* (2010) solve this problem under the very restrictive and poorly realistic hypothesis that all bidders sell the same capacity. In this paper, instead, in order to assess how the availability of different bidders production capacities affects the competitiveness of the electricity market, a simplified Nash equilibrium (assumption A6) is adopted (McAfee and McMillan, 1987). In particular, we assume that each awarder bidder is paid not the MCP but the asked amount. According to this equilibrium, it holds :

$$p_i = \arg\max \pi_i = \arg\max \left((p_i - c_i) q_i \cdot \mathrm{P}(MCP \geq p_i) \right), \quad (2)$$

where $\mathrm{P}(MCP \geq p_i)$ is the probability that the submitted bid price p_i is not above the MCP and the auction is therefore awarded to the bidder.

The presented model is based on the empirical approach which allows considering more flexibility in the market scenario conditions than those in You *et al.* (2010), like different production capacities for different bidders. Based on assumptions A2 and A5, MCP can therefore be defined as a stochastic variable, distributed according to a empirical distribution. Let us suppose that L observation of MCP occurrences (MCP_1, MCP_2,…, MCP_L) are available. Let us define ($MCP_{(1)}$, $MCP_{(2)}$,…, $MCP_{(L)}$) the set of MCP occurrences in an increasing order. The empirical cumulative probability function is thus defined as follows:

$$F_{MCP}(x) = \begin{cases} 0 & \text{for } x < MCP_{(1)} \\ 1/L & \text{for } MCP_{(1)} < x \leq MCP_{(2)} \\ \ldots & \ldots \\ (L-1)/L & \text{for } MCP_{(L-1)} < x \leq MCP_{(L)} \\ 1 & \text{for } x \geq MCP_{(L)} \end{cases} \quad (3)$$

Finally, we remark that both the hypotheses of absence of price-taker bidders and empirical price distribution works optimally for a large number of bidders. Moreover, the described model neglects aspects of the electricity market management regarding congestion on transportation lines. Consequently, procedures for congestion management, like zonal pricing, are disregarded.

4 CASE STUDY

The model described in Section 3 is employed for the simulation and analysis of the Italian day ahead electricity market. The scope of this simulation is to show the effectiveness of the model and investigate how the number of participating bidders and their production capacity affect the market competitiveness. In particular, competitiveness is measured in the considered trading period by means of a suitable indicator, the so-called Hirschman Herfindahl Index (HHI), defined as the sum of squares of market shares of all the market participants as follows (Hirschman, 1964):

$$HHI = \sum_{i=1}^{N} \left[Q_i \cdot 100 \right]^2 , \quad (4)$$

where Q_i is the market percentage share of the demand associated to the i-th bidder in the considered trading period. Clearly, the value of HHI can range between 100/N (perfect competition) and 10000 (absolute market power, no competition): larger values define a decreasing competition level. In particular, a market is respectively considered competitive when HHI is below 1000 (diluted oligopoly) and moderately concentrated if HHI ranges between 1000 and 1800 (normal oligopoly); on the contrary, HHI values above 1800 mean uncompetitive markets (Leeprechanon *et al.*, 2002).

The model is tested using data of the day ahead market from the Italian electricity market, available at the GME (*Gestore Mercati Energetici*, which is the Italian ISO) website (GME, 2012). The data for the empirical MCP probability distribution and for determining the mean market demand have been collected from the GME database. In particular, the considered data are the national MCP values in the trading periods ranging from 12 am to 2 pm of all the days in the March-April period in the years from 2004 to 2011. This choice is made in order to neglect seasonal effects on the MCP data to the highest possible extent.

In the chosen simulation period, each bidder is considered to have unitary cost drawn from a uniform probability distribution whose bounds are respectively 30 and 70 €/MWh, according to an estimation made considering the Italian energy production mix and the cost of different energy sources. Four scenarios are addressed, each characterized by a fixed demand (equal for all scenarios), by varying two parameters in each simulation: the number of participating bidders (from N=10 to N=50) and their production capacities in the considered trading period, as a share of the market demand (whose range varies in the different scenarios). More precisely, under Scenario 1 in each simulation all participating bidders are assumed to have the same production capacity, ranging from 10% to 50% of the overall demand in different iteration (with a 2% pace). On the contrary, in Scenario 2 we consider three categories of equal capacity bidders, whose capacities are in the 1:2:3 proportion: bidders of the first group can produce from 10% to 24% of the market demand in different iterations (pace 1%), while second and third group bidders can generate respectively twice and three times as the competitors of the first group. In addition, the latter two scenarios consider the presence of dominant bidders in the market, i.e., suppliers characterized by larger capacity than other competitors so that they can satisfy a significant share of the market demand by themselves. In particular, in Scenario 3 we consider two dominant bidders, each able to provide up to 50% of the overall market demand, while in Scenario 4 we assume there is one dominant bidder that can provide 100% of the demand. In both the latter scenarios, non dominant bidders can produce from 5% to 19% of the total market demand in different iterations (pace 1%)

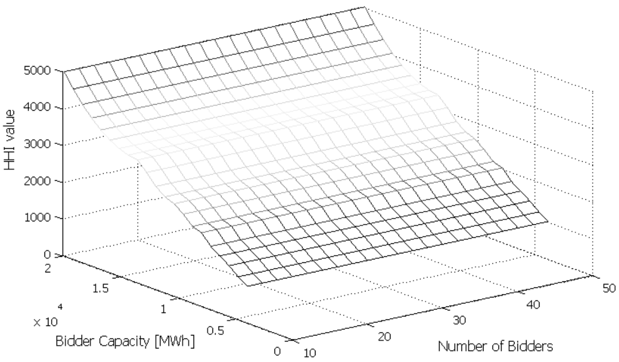

Figure 1: HHI index versus bidder number and capacity in Scenario 1.

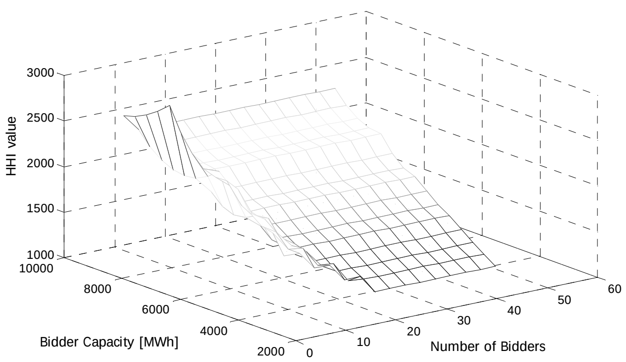

Figure 2: HHI index versus bidder number and capacity in Scenario 2.

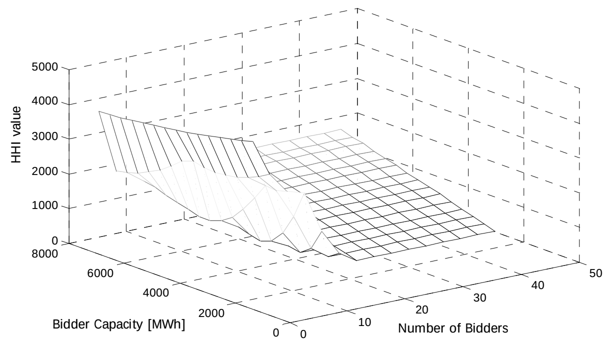

Figure 3: HHI index versus bidder number and capacity in Scenario 3.

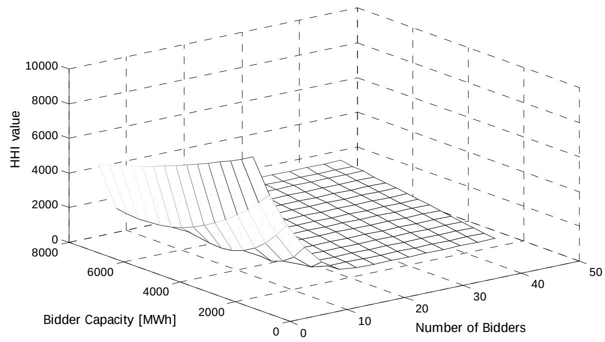

Figure 4: HHI index versus bidder number and capacity in Scenario 4.

As regards the number of bidders, in Scenario 1, the overall number of participating bidders is varied in each simulation by one unit from a minimum of 10 to a maximum of 50, while in Scenario 2, at each step a bidder for each of the three recalled categories is added. Moreover, in Scenarios 3 and 4, in each simulation the number of non dominant bidders is varied according to a pace of 3 units. As regards the production capacity of the bidders, this is defined as a function of the fixed demand supply. We recall that in Scenario 1 all bidders have the same capacity, while in Scenario 2 the capacity of low-capable bidders is varied and the other capacities are defined according to the cited 1:2:3 ratios. Finally, in Scenarios 3 and 4, the dominant bidders' capacities are fixed in all simulations, while the non-dominated bidders' capacity, assumed equal for all suppliers, varies in each simulation.

Figures 1 to 4 show how the considered variables affect the competitiveness in the different scenarios. In Scenario 1, the HHI is not affected by the number of bidding competitors, because the number of awarded bidders is constant for equal production capacities. On the contrary, the increase in their capacity determines an increase of the HHI index and, consequently, a reduction of the competition in the market: this is easy to foresee, because less bidders can fully satisfy a constant demand.

Scenario 2 and the corresponding figure 2 show a trend similar to Scenario 1, for a sufficient number of bidder (not inferior to 25), but with lower values of the HHI index. A peculiar trend can be observed for a small number of participating bidders: the HHI value increases more than linearly with the bidders' capacity, but after a peak it starts decreasing, due to the increased capacity of small bidders to erode the market shares of large bidders.

Scenarios 3 and 4 display similar HHI trends in figures 3 and 4, respectively. For a large number of small bidders, the oligopolistic effect of the dominant bidders is not relevant, while they strongly affect the competitiveness of the market when there are few bidders in the market.

In conclusion, the presented model can effectively simulate based on empirical data the dynamics of electricity markets both in the case of suppliers with the same generation capacity (as in the model by You *et al.*, 2010) and of bidders with different power market (which the recalled model cannot address). In the first case, the market competitiveness does not change when production costs are equal even when the bidder number increases. Naturally, when the production capacity of bidders increases competitiveness is decreased. Moreover, in the second case the market competitiveness increases by increasing the number of participating bidders. When the production capacity of bidders increases, competitiveness varies depending on the considered scenario.

5 CONCLUSION

Since the deregulation process started in many Western countries after the 1990s, the electric market has been widely studied in the related literature. One of the most used approaches has been the simulation of the operating of such markets.

In this paper we present a simulation model for representing auction based day ahead electricity generation markets. The model is based on a Nash equilibrium approach previously presented in the literature on sealed bid auctions and employs empirical distributions of the market clearing price to represent, under suitable assumptions, the behavior of the bidders participating in the pool. The proposed simulation model may be employed as a decision support tool by participants and regulators of the electricity market to analyze dynamics and foresee players behaviors and decide actions to increase market power or improve competitiveness.

The paper contributes with an early step of a research that still needs to consider several improvement opportunities. Firstly, the adopted approach needs validation by comparison with some more mature solutions, like the Cournot equilibrium, the supply function equilibrium and the conjectural variation approaches. Moreover, in order to better assess the effect of different production capacities among the generating companies, it is necessary to differentiate production costs with the bidder market power and increase the differentiation of production capacities of the power suppliers participating to the auction. Finally, a further development can be the application of the model to studying the effect of network congestions on the market efficiency, e.g., by comparing zonal pricing with other tools for MCP determination in case of congestion.

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