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SUPER FACILITY VERSUS CHAINING IN DEMAND ALLOCATION PROBLEMS UNDER DISRUPTIONS

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ABSTRACT:

In this paper we study a capacity planning and demand allocation problem where we aim to assign the demand for a set of products to a set of potential facilities that are subject to disruptions. Our objective is to find the optimal solution that minimizes the total cost while maximizing the demand satisfaction. We consider the process flexibility as a strategy to mitigate disruptions impacts and compare, in terms of cost, two specific flexible configurations: the single chain configuration and the super facility configuration. The single chain configuration refers to a specific facility configuration in which each facility is configured to fulfill only two products and each product can be assigned to only two facilities. All the facilities and products are connected so as to form a single chain. In the super facility configuration each facility is dedicated to a product and the super facility is configured to handle all products. We propose an algorithm that can be used to find which configuration is better in terms of cost and facing disruptions under symmetry and balance conditions.

KEYWORDS: demand allocation; flexibility; chaining; disruptions; super facility

1 INTRODUCTION

We consider a capacity planning and demand allocation problem where we aim to assign the demand of a set of products to a set of facilities. We consider settings where facilities are subject to disruptions. In order to mitigate the disruptions impacts we consider flexible configurations. The literature on flexibility is very rich. Chou et al. (2008) and Graves (2008) present literature reviews of this growing research area. In this paper we focus on two particular flexible configurations: chaining and super facility configurations. In the chaining configuration introduced by Jordan and Graves (1995), each facility is configured to fulfill demand from exactly two products and each product can be fulfilled from exactly two facilities and the facilities are configured in such a manner that they form a single connected chain with an overlap between each pair. The super facility configuration is composed of a set of facilities dedicated to a set of products and one fully flexible facility called the super facility that can handle the demand of all products. Many researchers have been interested in studying the chaining configuration and have compared its performance to that of the fully flexible configuration in the case of uncertain demand (Gurumurthi and Benjaafar (2004); Jordan et al.

(2004); Iravani et al. (2005); Aksin and Karaesmen (2007); Bassambo et al. (2009); Chou et al. (2010); Chou et al. (2011); Hopp et al. (2010); Iravani et al. (2011); Lim et al. (2011)).

However, few works were interested in the super facility configuration as a more practical configuration than chaining. We can refer for example to Balasubramian et al. (2011) who applied the super facility configuration to the medical field. Another example is the work of Askin et al. (2011) that presents a comparison between the chaining and the super facility configurations and proposes conditions under which each configuration may be preferred to the other.

Most of the work on flexibility addressed the case of uncertain demand. Literature related to uncertain capacity often dealt with the issue of disruptions while studying the strategic aspect of the facility location problem and proposed various models for designing robust supply chains (Snyder and Daskin (2005), Snyder et al. (2006), Lim et al. (2010), Qi et al. (2010), Cui et al. (2010)).

The results that evaluate chaining and super facility configurations are based on numerical results, simulation experiments, or approximations. In this paper, we provide exact analytical results that characterize conditions

for comparing the super facility and the single chain configurations and determining the optimal configuration to adopt in the face of disruptions. The objective is to minimize the sum of capacity investment cost, demand allocation cost and lost sales cost due to facilities disruptions while maximizing demand satisfaction. The remainder of this paper is organized as follows. In section 2, we provide a general model for the capacity planning and demand allocation problem. In Section 3, we present a series of results that allow us to compare between the super facility and the single chain configurations. In Section 4, we present a conclusion and future research directions.

2 PROBLEM FORMULATION

We consider a capacity planning and demand allocation problem where facilities are subject to failures. Our objective is to minimize total cost whether facilities are operational or under failure. Besides solving the problem, our objective is to study the flexibility that is needed to mitigate failures. The following is the notation we use to formulate the problem:

- a_{ij} : the unit assignment cost of product i to a facility j ,
- v_{ij} : the amount of capacity (number of units of capacity) that a product i consumes if assigned to facility j ,
- d_i : the expected demand for product i ,
- u_j : the capacity of facility j ,
- q_j : the probability of failure of each facility,

We will give a particular attention to the super facility and single chain configurations. In the single chain configuration, each facility can handle exactly two products and each product demand can be fulfilled by exactly two facilities in a way that the whole setting forms a closed chain. For the super facility configuration, we consider that each facility is dedicated to a product, i.e. each facility is configured to produce only one product. In addition, there is a particular facility, referred to as the "super facility" and denoted by B , that is configured to produce all products. This facility can also fail with a probability of failure q_B . A unit demand of product i fulfilled from facility B incurs a unit fulfillment cost a_{iB} and consumes a capacity amount v_{iB} . The unit capacity cost for the super facility is denoted by f_B .

In order to describe the configurations under study we let n denote the number of dedicated facilities in the super facility configuration and the number of facilities in the single chain configuration. Besides, we let y_{ij} denote the parameter associated with facility configuration, where $y_{ij} = 1$ if facility j is configured for product i and $y_{ij} = 0$ otherwise.

In the super facility configuration we have:

$$y_{i0} = 1 \quad \forall i \in I, \quad (1)$$

$$y_{iB} = 1 \quad \forall i \in I, \quad (2)$$

$$y_{ii} = 1 \quad \forall i \in I, \quad (3)$$

$$y_{ij} = 0 \quad \forall i \in I, \forall j \in J - \{0, B\} \text{ and } i \neq j \quad (4)$$

In the single chain configuration, we have:

$$y_{i0} = 1 \quad \forall i \in I, \quad (5)$$

$$y_{ii} = 1 \quad \forall i \in I, \quad (6)$$

$$y_{i+1} = 1 \quad \forall i \in I, \quad (7)$$

$$y_{n1} = 1, \quad (8)$$

$$y_{ij} = 0 \quad \forall i \in I, \forall j \in J - \{0\} \text{ and } i \neq j \text{ and } i+1 \neq j \quad (9)$$

We consider a problem where failures are independent i.e., a facility can fail or operate independently of the status of the other facilities. To handle the uncertainty due to facility disruptions, we adopt a scenario-based approach and associate a scenario s with each possible combination of failed and non-failed facilities. We let S denote the set of all such scenarios. Each scenario, s , is characterized by a probability of occurrence, which we denote by p_s . We use u_{js} to refer to the capacity of facility j under scenario s , where $u_{js} = u_j$ if facility j is operational under scenario s and $u_{js} = 0$ otherwise. Under any scenario, a demand from product i that is not fulfilled from any facility incurs a lost sale cost denoted by a_{i0} . Note that facility 0 refers to a dummy facility with index 0 that is introduced for modeling convenience. Demand allocated to this facility, x_{i0s} , corresponds to demand from product i that is not fulfilled (lost sale) under scenario s , and incurs a demand fulfillment cost that is equal to the lost sales cost a_{i0} . We assume that $a_{i0} > a_{ij}$ for all i and j , that facility 0 is configured for all products, has unlimited capacity ($u_0 = \infty$), and never fails.

We will use two sets of decision variables:

- Assignment variables x_{ijs} is the amount of products i assigned to facility j under scenario s .
- Capacity level of the super facility u_B .

We let J denote the set of facilities (including facilities 0 and B), I the set of products, and S the set of scenarios. The joint capacity planning and demand allocation problem can be formulated as follows:

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} \sum_{s \in S} a_{ij} x_{ijs} + f_B u_B, \quad (10)$$

$$\text{Subject to} \quad \sum_{j \in J} x_{ijs} = d_i \quad \forall i \in I, \forall s \in S, \quad (11)$$

$$\sum_{i \in I} v_{ij} x_{ijs} \leq u_{js} \quad \forall j \in J, \forall s \in S, \quad (12)$$

$$v_{ij} x_{ijs} \leq u_{js} y_{ij} \quad \forall i \in I, \forall j \in J, \forall s \in S, \quad (13)$$

$$x_{ijs} \geq 0 \quad \forall i \in I, \forall j \in J, \forall s \in S, \quad (14)$$

$$u_B \geq 0. \quad (15)$$

The objective function (10) is the sum of two costs: the demand assignment cost over all possible scenarios and the capacity planning cost for the super facility. Constraints (11) ensure that the demand of each product is fully assigned under each scenario; constraints (12) guarantee that the total amount of demand assigned to a facility does not exceed its capacity; constraints (13) prevent demand from a product being assigned to a facility if that facility is not configured for that product; and constraints (14) and (15) are standard integrality constraints.

For a given set of problem parameters, the above model can be solved for the optimal capacity amount in the super facility B . The optimal capacity level would clearly be affected by the unit capacity cost and the probability of facility failures. However, characterizing analytically the optimal capacity level is difficult due to the multiplicity of factors that may favor more or less capacity such as the differences in the product demands, facility capacity levels and probability of failures, and product-facility fulfillment and configuration costs. For these reasons, we focus in this paper on a particular super facility configuration where products and facilities have similar characteristics (identical costs, capacities, and probabilities of failures) as illustrated in figure 1, and we state the following "symmetry assumptions":

- Assignment cost: $a_{ij}=a \forall i \in I, \forall j \in J$.
- Probability of failure: $q_j = q \forall j \in J$.
- Lost sales cost: $a_{iL}=a_L \forall i \in I$.
- Capacity: $u_j = u \forall j \in J$.
- Demand: $d_i = d \forall i \in I$.

In the chaining setting, we consider that each facility has a capacity u_c and is subject to disruptions with a probability of failure q and each product is characterized by a demand d as illustrated in figure 2. In addition, we focus on cases where demand and capacity are roughly *balanced* so that each facility would have to be allocated some demand in order to fulfill total demand. In particular, we require that: $u > d$ and that $nd > (n-1)u_c$. This assumption ensures that the failure of one or more facilities always degrades performance and leads to some lost sales. In order to compare systems with the same amount of total capacity, we suppose that the total capacity in the super facility configuration is equal to the total capacity in the single chain configuration.

In a previous work (Benaïcha et al. 2012) we proved that under the stated symmetry and balance assumptions, when the following condition is fulfilled:

$$\sum_{k=m+1}^n \binom{n}{k} q^k (1-q)^{n-k} \leq \frac{f_B}{(1-q_B)(a_L - a)},$$

the optimal capacity of the super facility is $u_B=md$ where m is an integer such that $1 \leq m \leq (n-1)$. We also proposed an algorithm that allows us to determine the optimal capacity of the super facility.

Consequently, the total capacity in the super facility configuration is $nu+md$. Therefore, in order to have a fair comparison between the chaining and the super facility

configurations, we consider that $nu_c=nu+md$ which is equivalent to considering that each facility in the single chain configuration has a capacity of $u_c = u + \frac{m}{n}d$.

In the next section, we will compare the performance of the super facility configuration to that of the single chain configuration under disruptions.

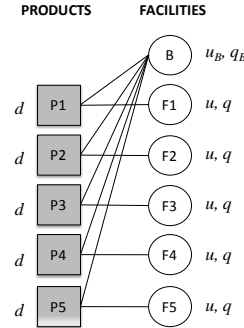


Figure 1: Example of a super facility configuration with 5 products, 5 facilities and the super facility.

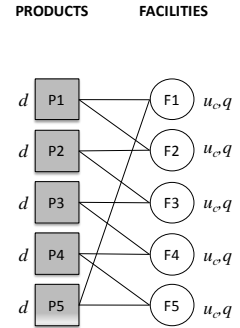


Figure 2: Example of a single chain configuration in a symmetric setting with 5 products and 5 facilities

3 ANALYTICAL RESULTS

In this section we present results in which we state conditions that allow us to compare between the super facility and the single chain configurations and to determine the optimal configuration to adopt in order to mitigate disruptions impacts.

3.1 Comparisons between the super facility and the single chain configurations

Result 1 For m , integer such that $1 \leq m \leq (n-1)$, $(u + \frac{m}{n}d)((1-q) - (1-q)^n) \geq d(1 - (1-q)^n)$ is a sufficient condition for having $Z_m^0 > Z_c$ where Z_m^0 denotes the total cost associated with the super facility configuration where the super facility capacity is equal to md and the probability of failure of the super facility $q_B=0$. Z_c denotes the total cost associated with the single chain configuration.

Proof: Let us suppose that by using the super facility capacity algorithm we determined the optimal super facility capacity $u_B=md$ where m integer verifies $1 \leq m \leq (n-1)$. Note that the total cost in the case of the single chain configuration is equal to the assignment cost since there is no fixed cost or operating a super facility. If there are no failures, all the demand can be assigned and the assignment cost is nda . If k out of the n dedicated facilities fail, then the $(n-k)$ remaining operational facilities can satisfy $(n-k)d$ products demand. If the super facility is under disruption then the system can only

satisfy $(n-k)d$ products demand. If the super facility is operating then we have two cases:

- If $k \leq m$ then all the demand can be satisfied and the assignment cost is nda .
- If $k > m$ then the amount of demand that can be satisfied is $(n-k)d + md$ and we will have $(k-m)d$ lost demand.

Since failures are independent, the probability of a scenario s with k failures is given by:

$$p_{s/k} = q^k (1-q)^{n-k}. \quad (16)$$

Consequently, the expected total cost is given by:

$$\begin{aligned} Z_m = & mdf_B + q_B \left(\sum_{k=0}^n \binom{n}{k} p_{s/k} [(n-k)da + kda_L] \right) \\ & + (1-q_B) \left(\sum_{k=0}^m \binom{n}{k} p_{s/k} nda \right) \\ & + (1-q_B) \left(\sum_{k=m+1}^n \binom{n}{k} p_{s/k} [(n-k)da + mda + (k-m)da_L] \right). \end{aligned} \quad (17)$$

Since $q_B=0$, the total cost of the super facility configuration can be simplified as follows:

$$\begin{aligned} Z_m^0 = & mdf_B + \sum_{k=0}^m \binom{n}{k} p_{s/k} nda \\ & + \sum_{k=m+1}^n \binom{n}{k} p_{s/k} [(n-k)da + mda + (k-m)da_L] \end{aligned} \quad (18)$$

In the case of the single chain configuration, if k facilities fail, then the available capacity is $(n-k)u_c$. Because of chaining, when k facilities fail, at least $(n-k+1)$ products are still linked to the remaining facilities and because $nd > (n-1)u_c$ and $d < u < u_c$, we have $(n-k+1)d > (n-k)u_c$. Thus the demand that has to be assigned exceeds the available capacity, implying that the total assignment cost to facilities (under a scenario with k failures) is $(n-k)u_c a$ and the lost sales cost is $(nd - (n-k)u_c)a_L$, leading to a total expected assignment cost:

$$\begin{aligned} Z_c = & nda(1-q)^n \\ & + \sum_{k=1}^n \binom{n}{k} p_{s/k} [(n-k)u_c a + (nd - (n-k)u_c)a_L]. \end{aligned} \quad (19)$$

Let us compute the difference between the two costs:

$$\begin{aligned} Z_m^0 - Z_c = & mdf_B + \sum_{k=1}^m \binom{n}{k} p_{s/k} (nda - (n-k)u_c a) \\ & + \sum_{k=1}^m \binom{n}{k} p_{s/k} (nd - (n-k)u_c)a_L \\ & + \sum_{k=m+1}^n \binom{n}{k} p_{s/k} [(n-k)da + mda + (k-m)da_L] \\ & - \sum_{k=m+1}^n \binom{n}{k} p_{s/k} [(n-k)u_c a + (nd - (n-k)u_c)a_L]. \end{aligned} \quad (20)$$

The above expression can be simplified as follows:

$$\begin{aligned} Z_m^0 - Z_c = & mdf_B + \sum_{k=1}^m \binom{n}{k} p_{s/k} (n(u_c - d)(a_L - a) - ku_c(a_L - a)) \\ & + \sum_{k=m+1}^n \binom{n}{k} p_{s/k} (n(u_c - d)(a_L - a) - ku_c(a_L - a)) \\ & + \sum_{k=m+1}^n \binom{n}{k} p_{s/k} (kd(a_L - a) - md(a_L - a)), \end{aligned} \quad (21)$$

and then leads to the following:

$$\begin{aligned} Z_m^0 - Z_c = & mdf_B + \sum_{k=1}^n \binom{n}{k} q^k (1-q)^{n-k} (n(u_c - d)(a_L - a)) \\ & - \sum_{k=1}^n \binom{n}{k} q^k (1-q)^{n-k} (ku_c(a_L - a)) \\ & + \sum_{k=m+1}^n \binom{n}{k} p_{s/k} (k-m)d(a_L - a). \end{aligned} \quad (22)$$

In the above expression we recognize the expression of Bernstein polynomials which are defined as follows:

$B_{r,n}(x) = \binom{n}{r} x^r (1-x)^{n-r}$. Since Bernstein polynomials have the property of forming a partition of unity, i.e. $\sum_{r=0}^n B_{r,n}(x) = 1$. Using the following combinatorial property: $r \binom{n}{r} = n \binom{n-1}{r-1}$, we can prove that Bernstein polynomials also satisfy the following property:

$$\sum_{r=0}^n r \binom{n}{r} x^r (1-x)^{n-r} = nx. \quad (23)$$

Using these properties and the fact that $u_c = u + \frac{m}{n}d$, we can express the difference between costs as follows:

$$\begin{aligned} Z_m^0 - Z_c = & mdf_B + n(a_L - a) \left(\left(u + \frac{m}{n}d \right) ((1-q) - (1-q)^n) \right) \\ & - n(a_L - a) \left(d(1 - (1-q)^n) \right) \\ & + \sum_{k=m+1}^n \binom{n}{k} p_{s/k} (k-m)d(a_L - a). \end{aligned} \quad (24)$$

In the above expression we already know that $mdf_B > 0$ and $\sum_{k=m+1}^n \binom{n}{k} p_{s/k} (k-m)d(a_L-a) > 0$. Therefore, if $(u + \frac{m}{n}d)((1-q) - (1-q)^n) \geq d(1 - (1-q)^n)$ then $Z_m^0 > Z_c$. \square

With this result we stated a sufficient condition under which the single chain configuration with n facilities leads to a lower total cost than the super facility configuration with $n+1$ facilities where the super facility never fails. Also note that in this case we compared the symmetric system of the single chain configuration to a system presenting some asymmetry.

Result 2 For m , an integer such that $1 \leq m \leq (n-1)$, $(nu - (n-m)d)((1-q) - (1-q)^n) \geq \sum_{k=0}^m \binom{n}{k} p_{s/k} kd$ is a sufficient condition for having $Z_m^0 > Z_c$ where Z_m^0 denotes the total cost associated with the super facility configuration where the super facility capacity is equal to md and its probability of failure $q_B=0$. Z_c denotes the total cost associated with the single chain configuration.

Proof: Let us suppose that by using the super facility capacity algorithm we determined the optimal super facility capacity $u_B=md$ where m integer verifies $1 \leq m \leq (n-1)$. As in the *Result 1* we can express the difference between the two costs as follows:

$$\begin{aligned} Z_m^0 - Z_c &= mdf_B + \sum_{k=1}^n \binom{n}{k} q^k (1-q)^{n-k} (nu_c - d)(a_L - a) \\ &\quad - \sum_{k=1}^n \binom{n}{k} q^k (1-q)^{n-k} (ku_c(a_L - a)) \\ &\quad + \sum_{k=m+1}^n \binom{n}{k} p_{s/k} (k-m)d(a_L - a). \end{aligned} \quad (25)$$

By adding and subtracting the term $\sum_{k=0}^m \binom{n}{k} p_{s/k} kd(a_L - a)$ we obtain the following expression:

$$\begin{aligned} Z_m^0 - Z_c &= mdf_B - md(a_L - a) \sum_{k=m+1}^n \binom{n}{k} p_{s/k} \\ &\quad + (a_L - a) \left(\sum_{k=1}^n \binom{n}{k} q^k (1-q)^{n-k} (n-k)(u_c - d) \right) \\ &\quad - (a_L - a) \left(\sum_{k=0}^m \binom{n}{k} p_{s/k} kd \right). \end{aligned} \quad (26)$$

Using Bernstein polynomials properties presented in *Result 1* and the expression $u_c = u + \frac{m}{n}d$, we can express the difference between costs as follows:

$$\begin{aligned} Z_m^0 - Z_c &= mdf_B - md(a_L - a) \sum_{k=m+1}^n \binom{n}{k} p_{s/k} \\ &\quad + (a_L - a) \left((nu - (n-m)d)((1-q) - (1-q)^n) \right) \\ &\quad - (a_L - a) \left(\sum_{k=0}^m \binom{n}{k} p_{s/k} kd \right). \end{aligned} \quad (27)$$

Recall from *Result 1* that since $u_B=md$ is the optimal capacity of the super facility then we have

$$\frac{f_B}{(a_L - a)} \geq \sum_{k=m+1}^n \binom{n}{k} p_{s/k}, \quad q_B=0 \text{ in this case.}$$

If, in addition, we have

$$(nu - (n-m)d)((1-q) - (1-q)^n) \geq \sum_{k=0}^m \binom{n}{k} p_{s/k} kd \text{ then we}$$

can conclude that $Z_m^0 > Z_c$. \square

With this result we stated another sufficient condition under which the single chain configuration with n facilities leads to a lower total cost than the super facility configuration with $n+1$ facilities where the super facility never fails. Now what can we conclude if

$$(nu - (n-m)d)((1-q) - (1-q)^n) < \sum_{k=0}^m \binom{n}{k} p_{s/k} kd ?$$

In fact, in this case and from the cost difference expression stated in *Result 2*, we can state other sufficient conditions based on the relationship between:

$$\begin{aligned} &\left(mdf_B + (nu - (n-m)d)(a_L - a)((1-q) - (1-q)^n) \right) \quad \text{and} \\ &\left(\sum_{k=m+1}^n \binom{n}{k} p_{s/k} md(a_L - a) + \sum_{k=0}^m \binom{n}{k} p_{s/k} kd(a_L - a) \right). \end{aligned}$$

This leads to the following result.

Result 3 For m , integer such as $1 \leq m \leq (n-1)$, a necessary and sufficient condition for having $Z_m^0 \geq Z_c$ is:

$$\left(mdf_B + (nu - (n-m)d)(a_L - a)((1-q) - (1-q)^n) \geq \sum_{k=m+1}^n \binom{n}{k} p_{s/k} md(a_L - a) + \sum_{k=0}^m \binom{n}{k} p_{s/k} kd(a_L - a) \right)$$

Where Z_m^0 denotes the total cost associated with the super facility configuration where the super facility capacity is equal to pd and its probability of failure is $q_B=0$. Z_c denotes the total cost associated with the single chain configuration.

Proof: Let us suppose that by using the super facility capacity algorithm we determined the optimal super facility capacity $u_B=md$ where m integer verifies $1 \leq m \leq (n-1)$. As in the *Result 2*, we can express the difference between both costs as follows:

$$Z_m^0 - Z_c = mdf_B - md(a_L - a) \sum_{k=m+1}^n \binom{n}{k} p_{s/k} + (a_L - a) \left(nu - (n-m)d \right) \left((1-q) - (1-q)^n \right) - (a_L - a) \left(\sum_{k=0}^m \binom{n}{k} p_{s/k} kd \right). \quad (28)$$

The difference between costs can also be expressed as follows:

$$Z_m^0 - Z_c = \left(mdf_B + nu - (n-m)d \right) (a_L - a) \left((1-q) - (1-q)^n \right) - \left(\sum_{k=m+1}^n \binom{n}{k} p_{s/k} md(a_L - a) \right) - \left(\sum_{k=0}^m \binom{n}{k} p_{s/k} kd(a_L - a) \right). \quad (29)$$

In this expression we identify two terms. The first term, $\left(mdf_B + nu - (n-m)d \right) (a_L - a) \left((1-q) - (1-q)^n \right)$, is a positive term that can be easily computed using the problem data. The second term,

$\left(\sum_{k=m+1}^n \binom{n}{k} p_{s/k} md(a_L - a) + \sum_{k=0}^m \binom{n}{k} p_{s/k} kd(a_L - a) \right)$, is also a positive term related to the probability of failure of the facilities and can be computed using a simple algorithm. The comparison between these two terms allows us to determine configuration is better according to the problem data. Therefore,

$$\left(mdf_B + nu - (n-m)d \right) (a_L - a) \left((1-q) - (1-q)^n \right) \geq \left(\sum_{k=m+1}^n \binom{n}{k} p_{s/k} md(a_L - a) + \sum_{k=0}^m \binom{n}{k} p_{s/k} kd(a_L - a) \right) \text{ is a}$$

necessary and sufficient condition for having $Z_m^0 \geq Z_c$. □

In fact, *Result 3* is a more general result since it provides a necessary and sufficient condition for concluding whether the single chain or the super facility is better in terms of cost. However, the complexity of computing the condition increases considerably compared to *Results 1* and *2*. In fact, computing complexity increases progressively from *Result 1* to *Result 3*. The idea here is to use an algorithm based on the three results in order to assist the decision maker in choosing the better configuration. In the next paragraph we propose this algorithm.

3.2 Super facility and single chain algorithm

Based on results 1, 2 and 3, we propose the following algorithm that can be used in order to determine which is the better configurations in terms of cost that should be used : the single chain configuration or the super facility where the super facility is always operational ($q_B=0$). Recall that the single chain configuration is composed of n facilities that are subject to random failure with probability q , and that can satisfy the demand of n products. Each facility is configured to handle exactly 2 products and each product demand can be satisfied by exactly 2

facilities in a way that the whole configuration forms a closed chain with $2n$ links. The super facility configuration that we focus on in this paragraph is a setting with n facilities dedicated to n products and one super facility that is configured with all products and where all facilities are subject to random failure except the super facility ($q_B=0$). The super facility configuration presents $2n$ links.

Algorithm

Data:

- System size n : integer+
- Demand d : real+
- Capacity u : real+
- Assignment cost a : real+
- Lost sales cost a_L : real+
- Unit capacity cost f_B : real+
- Probability of failure q : real+ such as $0 \leq q < 1$
- Capacity level integer m : integer+

Variables:

- Summing integer: k : integer+
- Summing integer: t : integer+
- Sum A_1 : real+
- Sum A_2 : real+
- Expression α : real+
- Expression β : real+

Steps:

- (1) Initialization:
 - $\alpha = \left(u + \frac{m}{n} d \right) \left((1-q) - (1-q)^n \right) - d \left(1 - (1-q)^n \right)$
 - $\beta = \left(nu - (n-m)d \right) (a_L - a) \left((1-q) - (1-q)^n \right)$
- (2) if $\alpha \geq 0$ then $Z_m^0 > Z_c$ else go to (3)
- (3) Sum A_1 computation:
 - (i) $k=0$
 - (ii) $A_1=0$
 - (iii) while $k \leq m$ do:
 - $A_1 \leftarrow A_1 + \binom{n}{k} q^k (1-q)^{n-k} kd(a_L - a)$
 - $k \leftarrow k+1$
- (4) Comparison: If $A_1 \leq \beta$ then $Z_m^0 \geq Z_c$ else go to (5)
- (5) Sum A_2 computation:
 - (i) $t=m+1$
 - (ii) $A_2=0$
 - (iii) while $t \leq n$ do:
 - $A_2 \leftarrow A_2 + \binom{n}{t} q^t (1-q)^{n-t} md(a_L - a)$
 - $t \leftarrow t+1$
- (6) Comparison: If $mdf_B + \beta \geq A_1 + A_2$ then $Z_m^0 \geq Z_c$ else $Z_m^0 < Z_c$

End.

The above algorithm allows us to choose between two flexible configurations based on the problem data in the case of facility disruptions. Note that if $q=0$ then $\beta \geq 1$

and we will always have the inequality $A_1 \leq \beta$ verified which allows us to conclude that the single chain configuration leads to a lower total cost in this case. This result is expected since in the super facility configuration the system incurs the additional cost of operating the super facility. From the expressions of α and β , we can derive some conclusions according to the evolution of problem data. In fact, everything being equal, if the utilization d/u increases, one can note that α and β decrease which tends to favor the super facility configuration. On the other hand, if the number of facilities and products n increases, α and β increase making the single chain configuration the better one. We observe the same system behavior when the cost ratio $\frac{f_B}{(a_L - a)}$ increases. In fact,

this tends to favor the single chain configuration.

Note that the above algorithm and observations concern the case where the super facility is always operational. Now, one can be interested in the case where the super facility is subject to failure ($q_B \neq 0$). Deriving analytical results in this case is not easy.

4 CONCLUSION

In this work, we address a capacity planning and demand allocation problem where the facilities are subject to disruptions. Our objective is to minimize the sum of the capacity investment cost and the demand allocation cost while maximizing demand satisfaction. In order to mitigate disruptions impacts we focus on particular flexible configurations: the super facility and the single chain configurations. The super facility configuration is composed of a set of facilities dedicated to a set of products and one fully flexible facility called the super facility that can handle the demand of all products. In the single chain configuration, each facility is configured to fulfill demand from exactly two products and each product can be fulfilled from exactly two facilities and the facilities are configured in such a manner so that they form a single connected chain with an overlap between each pair. In this work we provide a comparative study of the two configurations based on analytical results and we propose an algorithm that allows for determining the optimal configuration to adopt based on the problem characteristics. In this paper we considered the case where the super facility is failure free. Evidently, results that show the superiority of the single chain configuration must still hold when the super facility is subject to failure. However, in this case, more effective sufficient conditions and other general results should be the subject of future research directions.

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