Optimal maintenance and replacement decisions under technological change with consideration of spare parts inventories

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ABSTRACT: Classical spare parts inventory models assume that the same vintage of technology will be utilized throughout the planning horizon. However, replacement often occurs in the form of a new technology. A consequence of such replacements is the rendering of existing spare parts inventories obsolete and the necessity to purchase a new set. This paper aims to study the impact of spare parts inventory on maintenance and replacement decisions under technological change. We model this problem as a Markov decision process where the decisions are made as a function of the state of the asset itself, the technological environment, and the current spare parts inventory. The actions available to the decision maker are to do nothing, imperfectly or perfectly maintain the system, or replace the asset. The replacement decision is complex in that one must be decide with which technology available on the market to replace the current asset. Under technological change, the do nothing and repair options have significantly more value as they allow for the appearance of even better technologies in the future. Our model yields policies that demonstrate how replacement and maintenance strategies should be adjusted due to considerations of technological evolution spare parts inventory levels.

1 INTRODUCTION

Maintenance is key in ensuring the efficient use of equipment as well as an efficient production process. If assets often break down or operate in bad condition, this may cause lower product quality, increased energy consumption and reduced revenue. Therefore, the objective of maintenance is no longer to simply overcome failures, but the process to predict and prevent loss at the management level. Managers need to analyze all relevant information to assess the profitability of equipment, give sound investment decisions, and consider possible cost saving. In particular, high spare parts inventory is one important factor causing increased maintenance costs. Lack of inventory increases asset downtime due to waiting time when necessary spare parts are not available. Hence, consideration of the spare parts inventory problem is an essential task of managers.

There has been intensive research to study the different aspects of spare parts inventory problems such as management issues, multi-echelon problems, age-based replacement, repairable spare parts, problems involving obsolescence, etc. We can consider a spare parts inventory review in [13]. As the authors comment, spare parts inventories totally differ from other manufacturing inventories. Their function is to assist maintenance staff in keeping equipment in operating condition. The close relation between spare parts inventories and maintenance has been discussed in several articles. [11] study the joint-optimization of age-based replacement and spare parts inventory policy (S, s). Under a block replacement strategy, [24] utilize dynamic programming to solve the spare parts ordering problem while [3] and [22] present management policies for a manufacturing system, aiming to optimize the maintenance strategy with continuous review spare items inventory jointly. [4] and [23] extend the problem by also studying minimal repair for minor failures while [8] consider repair capacity of degraded/failed units after they were replaced by spare parts.

However, all the above models are constructed on the assumption that the same vintage of technology will be utilized throughout the planning horizon. They do not allow us to take into account the appearance of new technology with performance improvement. This information is very important for managers who often confront the technology investment decision. They must weigh the benefits of utilizing the available equipments with their stock of spare parts and the revenues of investment in new technology. Nevertheless, there are few articles that take into account the technological development in the spare parts inventory problem. Those that do are generally based on the introduction of an economical loss when new technology appears by a cost of obsolescence. [14] does not explicitly consider obsolete cost. They include it in the holding cost in a multi-echelon system. [7] develops models that can be seen as extensions of the EOQ formula for fast moving spare parts subject to sudden obsolescence risk. The authors examine the effects of obsolescence on costs under several different conditions: constant obsolescence risk, no shortages allowed; varying obsolescence risk, no shortages allowed and varying obsolescence risk, shortages allowed. However, they relax the interactive relation of spare parts inventory and maintenance strategy by considering a constant rate demand until the moment of obsolescence.

On the end of the spectrum, models devoted to maintenance optimization involving technological change do not consider the spare part inventory impact [2, 5, 6, 9, 10, 12, 15].

This gap in the literature motivates us to develop an appropriate model to meet management's requirements: optimization of maintenance cost while simultaneously updating information on the technological development and considering the impact of spare parts inventory levels to make sound investment decisions. With such a model, we can examine how spare parts inventory levels will influence the replacement decision and how much better a new technology must be in order to overcome the obsolescence of existing spare parts inventory.

We formulate a discrete-time, non-stationary Markov decision process (MDP) to determine the optimal action plan. To model the technological evolution, we combine the geometric model and uncertain apparition model of technology. The geometric technological evolution model is presented by [1, 2, 12, 21, 18, 19, 20]. They utilize the geometric model to form the cost functions in vintage equipment or in time. Unlike these articles, we present technology change by the improvement of the expected deterioration rate and the expected profit function within a period. In addition, we also consider non-stationary likelihood of new technology's apparition over time. Thereby, we overcome the disadvantages of the geometric model proposed by [2]. Recall [16, 17] also consider the non-stationary probability of the appearance of new technologies. But in their model, Nair focuses on the problem of capital investment decisions due to technological change rather than physical deterioration of equipment. To simplify its exposition, he also does not consider salvage values while we establish a reasonable salvage value function which is proportional to the purchase price of technology at that time, decreasing in the remaining lifetime and incurs an even greater reduction when it becomes obsolete due to new technology appearances.

The remainder of this paper is structured as follows: In Section 2, we present our mathematical formulation model and its assumptions. Next, in Section 3, the performance of our model is discussed through numerical examples. Finally, conclusions and future work are discussed in Section 4.

2 MODEL AND ASSUMPTIONS

2.1 *Problem Statement*

Consider a repairable asset which is accompanied by a cargo of n spare parts. They are utilized for the maintenance of the asset and are not sold separately in the market, i.e., we cannot replenish the spare parts store when it is empty. This is a common assumption for special spare parts because it can be difficult and costly to find original and compatible spare units. In making this assumption we de not consider the optimization of the inventory policy. Our primary goal is to study the impact of the spare parts inventory level on maintenance and replacement decisions under technological change rather than determining what should be the optimal order level/order quantity for the spare parts.

The asset operates continuously from the new state $X = 0$ and is characterized by its expected deterioration rate. In the failure state, denoted m, the asset continues to operate but unprofitably. To reveal the deterioration and the spare parts inventory level, periodic inspections are performed. The inter-inspection interval, τ, defines the decision epochs.

We assume that only one new technology can appear in a decision interval, τ. We introduce p_{i+1}^{k+1}, the non-stationary probability that technology k+1 appears in the interval τ given the latest available technology at decision epoch i is k. The difference in the technological generations is modeled by an improvement factor on the expected instantaneous deterioration rates and the accrued profit within a decision period.

Let (x, k, j, s) be the system state at the beginning of the i^{th} decision epoch with s spare parts in stock for maintenance of the asset having deterioration level x. The asset in use belongs among the technological generation j while the latest available technology in the market is k, $k \geq j$. In each decision epoch the possible actions are:

1) Do nothing (*DN*): The asset continues to deteriorate until the next decision epoch and generates a profit $q_i(x)$. Note that $q_i(x)$ is the expected profit within a period when the deterioration state at the beginning of that period is x and the utilized technology is j. The spare

parts inventory level is not changed, so the holding cost within this period is sc_s where c_s is the holding cost per spare part unit in a decision period.

2) Imperfect maintenance (*IM*) restores the asset to a given deterioration level, $\max(0, x - d)$ where d models the maintenance efficiency. An imperfect maintenance cost, c_1 is incurred and θ spare parts are utilized to replace the degraded units, so the spare part inventory level is reduced by: $s - \theta$. As we assume that the maintenance time is negligible, in the next decision interval the asset deteriorates from the level $\max(0, x - d)$ and generates a profit $g(\max(0, x - d))$.

3) Perfect maintenance (*PM*) restores the asset to the initial deterioration level $X = 0$ (as-good-as-new) and the expected profit within next decision interval is $g(0)$. This action requires $\eta(x)$ spare parts that depends on the deterioration state of the asset. Simultaneously, a perfect maintenance cost c_2 is incurred ($c_2 > c_1$).

Note that as we assume the spare parts are only supplied when we buy a new asset, we can execute maintenance actions if and only if there are sufficient numbers of spare parts in stock.

4) Replace (*R*) the asset by an available technology h in the market ($j \leq h \leq k$). The replacement time is also negligible. A cargo of n spare parts is supplied with the new asset. We assume the spares are only compatible with the same generation asset, hence, after replacement, the spare parts inventory level is n if we decide to replace by a newer generation asset and equals $n + s$ in the case of replacement by same-generation asset. The cost of such a replacement is given by the difference between the purchase price of the new asset $c_{i,h}$ and the salvage value $b_{i,j,k}(x)$. Note that the purchase price of a new technology asset can be estimated as well as the deterioration rates. This is realistic in case where the technical parameters and specifications of future designs may be assumed reasonably well beforehand. After replacement, the new asset generates a profit $g_h(0)$.

2.2 *Model formulation*

In this paper, we use a non-stationary MDP formulation to find the optimal maintenance-replacement policy to maximize the expected discounted value-to-go over the finite horizon time denoted by $V^{\pi}(x, k, j, s)$.

Let $V_i(x, k, j, s)$ denote the maximum expected discounted value from the decision epoch i, ($k \leq i$) to the last epoch N. Then, $V_1(x, k, j, s) = V^{\pi}(x, k, j, s)$.

$$V_i(x,k,j,s) = \max\left\{\begin{matrix} DN_i(x,k,j,s),\ IM_i(x,k,j,s)1_{\{s>\theta\}}, \\ PM_i(x,k,j,s)1_{\{s>\eta(x)\}}, R_i^h(x,k,j,s) \end{matrix}\right\} \tag{1}$$

where DN_i, IM_i, PM_i, R_i are the respective possible actions to do nothing, imperfectly maintain, perfectly maintain and replace at decision epoch i. Therefore, the complete MDP formulation is given by:

$$DN_i(x,k,j,s) = g_j(x) - sc_s + \lambda[\sum_{\forall k' \in \{k,k+1)} \sum_{\forall x' \in [x,m]} p_i^{k'} p_j(x'|x) V_{i+1}(x',k',j,s)] \tag{2}$$

$$IM_i(x,k,j,s) = -c_1 + DN_i(\max\{0, x-d\}, k, j, s-\theta) \tag{3}$$

$$PM_i(x,k,j,s) = -c_2 + DN_i(0,k,j,s-\eta(x)) \tag{4}$$

$$R_i^h(x,k,j,s) = b_{i,j,k}(x) - c_{i,h} + DN_i(0,k,j,s+n)1_{\{h=j\}} + DN_i(0,k,h,n)1_{\{h\neq j\}} \tag{5}$$

Where λ is a discount factor, $\lambda \in [0, 1]$.

3 NUMERICAL EXAMPLES

3.1 *Input parameters*

3.1.1 *The appearance probability of new technology*

We define the appearance probability of new technology k+1 at decision epoch i+1, given the latest available technology at decision epoch i is k, as a time increasing function:

$$p_{i+1}^{k+1} = (1 - \delta\varepsilon^{i-k}) \tag{6}$$

δ is the factor that reflects the non-appearance probability of next generation at next decision epoch when $k \equiv i$. And the factor ε characterizes the increasing rate of the appearance probability of new technology over time. We have $\delta, \varepsilon \in [0, 1]$.

3.1.2 *Deterioration process*

We consider a k-out-of-n system, specifically 2-out-of-5, where identical independent components have the time to failure T that follows the exponential distribution with parameter α_j (j is the technological generation of system). The improvement of α_j is discussed in the next paragraph. We choose $\tau = 2$ weeks.

For a new system, all components are functional and the deterioration state of system is $X = 0$. When one component fails, the system degrades one unit, and it breaks down when at least 4 items fail, system failure state is denoted by m (m = 4, 5). To perform IM, we utilize one spare part ($\theta = 1$) to replace the failed component and restore the system to the previous deterioration state (d = 1). With perfect maintenance, we replace all failed components $\eta(x) = x$ to restore the system to its initial state. The transition probability of deterioration state of system:

$$p_j(x'|x) = \binom{N-x}{x'-x}\left(1-\exp(-\alpha_j\tau)\right)^{x'-x}[\exp(-\alpha_j\tau)]^{N-x'}$$

$$\forall x, x' \in X:\{0, 1, 2, 3, 4, 5\} \tag{7}$$

As the components have an exponential life distribution, their conditional survivor function at age y is equal to the survivor function of a new component. If we do not consider the repair capacity of the system, the survivor function at time y, denoted by $\psi(x, t)$ depends only on the current deterioration state. (We have $\psi(m, t) = 0$).

$$\Psi(x,t) = \sum_{z=K}^{N-x}\binom{N-x}{K}\left(\exp(-\alpha_j t)\right)^z(1-\exp(-\alpha_j t))^{N-x-z}$$

$$\forall x \in \{0, 1, 2, 3\}; \tag{8}$$

Therefore, the mean residual lifetime, $MRL(x)$ of system is:

$$MRL(x) = \int_0^\infty R(x,t)dt \tag{9}$$

3.1.3 *Impact of technological evolution*

We model the impact of the technological evolution on the expected failure rate α_i of items with the following decreasing geometric function:

$$\alpha_j = ab^{-(j-1)} \tag{10}$$

where a, b are constants.

We choose arbitrarily values for a, b in Table 1.

The profit function is also improved by technological generations. Moreover, we know that the asset will operate less efficiently when its deterioration state is greater. Therefore, the expected profit function within a decision interval τ is decreasing by deterioration state and the greater the deterioration state is, the faster the decrease of the profit function. To reflect these characteristics, we use the following function to characterize the accrued profit.

$$g_{i,j} = \left(g_0 - a_1\exp(x)\right)\left(a_2 - \exp\left[r_1(1-j)\right]\right) \tag{11}$$

where a_1, a_2, g_0, r_1 are constants, chosen arbitrarily in Table 1. In case of failure, the do nothing action is still allowed but the profit in the next decision epoch $g(m)$ is assumed to be zero.

Additionally, under technological evolution, the purchase price of a new asset is assumed to be decreasing over time after appearance and normally increasing over technological generation:

$$c_{i,k} = c_{1,1}v^{i-1}u^{k-1} \tag{12}$$

$c_{1,1}$ is the purchase price of the first of technological generation at the first decision epoch; v, u are constants characterizing the change of purchase price over time and over technological generation. We choose arbitrarily $c_{1,1}$, v, u in Table 1.

We assume the salvage value is a function of the current purchase price of the technology, the difference between technological generation and the Mean Residual Lifetime (MRL). Thus, we propose the following function for the salvage value, $\forall x \in [0, 5]$ h, r are constant, chosen arbitrarily in Table 1.

$$b_{i,j,k}(x) = hr^{k-j}c_{i,j}\frac{MRL(x)}{MRL(0)} \tag{13}$$

Table 1. The input parameters for the Example 1

Appearance probability, Discount factor	δ	ε	λ		
	0.99	0.999	0.98		
Profit	g_0	r_1	a_1	a_2	
	415	0.05	15	1.5	
Maintenance, Holding cost, Failure rate	c_1	c_2	c_s	a	b
	80	120	1	2	0.9
Salvage value & Purchase price	h	r	$c_{1,1}$	v	u
	0.7	0.8	750	0.998	1.2

3.2 *Analysis of numerical experiments*

For the numerical examples, we consider only five technological generations. We also assume the storage capacity for spare parts is limited. We assume that the maximal storage capacity is 20 units and the cargo of spare parts accompanied with new asset comprises 15 units. The optimal policy for the first ten decision epochs of Example 1 in cases where k = 1, 2, 3 is given in Table 2. Note that with time horizon $N \geq 190$, this policy does not change.

With the used technology j, given that generation k is the latest available technology, the optimal decision is defined according to the current deterioration x and stock level s. Each column in Table 2 represents a combination of k and j given by the vector (k, j) in the header. For each vector (k, j) the different combinations of stock level and system state are given in the left and right sides of the columns, respectively. There are different system states associated with each stock level and the optimal action is given for each system state at that stock level. We find that in the cases where there is no obsolescence problem i.e., (k, j) = (1,1); (2,2); (3,3), the replacement decision is made only at the low stock levels (s < 5). For example, with (k, j) = (1, 1) at stock level s = 0 when we cannot perform maintenance, thus the optimal policy prescribes replacement (R) for deterioration states greater than x = 2. This replacement threshold is non-increasing in the used technology (j) because the profit function

Table 2: The optimal policy for Example 1.

$(k, j) = (1, 1)$ s	Action	$(k, j) = (2, 1)$ s	Action	$(k, j) = (2, 2)$ s	Action	$(k, j) = (3, 1)$ s	Action	$(k, j) = (3, 2)$ s	Action	$(k, j) = (3, 3)$ s	Action
0	$x = 0, 1$: *DN* $x \geq 2$: *R*	0	$x = 0$: *DN* $x \geq 1$: *R*	0	$x = 0$: *DN* $x = 1$: *R* ($i \leq 5$) *DN* ($i \geq 6$)	[0, 2]	$\forall x$: *R*	0	$x = 0$: *DN* $x \geq 1$: *R*	0	$x = 0$: *DN* $x \geq 1$: *R*
1	$x = 0$: *DN* $1 \leq x \leq 4$: *IM* $x = 5$: *R*	1	$x = 0$: *DN* $x = 1$: *R* ($i = 2, 3$) *IM* ($i \geq 4$) $x \geq 2$: *R*	1	$x = 0$: *DN* $1 \leq x \leq 4$: *IM* $x = 5$: *R*	[3, 4]	$3 \leq x \leq s$: *PM* $\forall x \in$ X\ [3, s]: *R*	1	$x = 0$: *DN* $1 \leq x \leq 3$: *IM* $x = 4, 5$: *R*	1	$x = 0$: *DN* $1 \leq x \leq 4$: *IM* $x = 5$: *R*
[2, 4]	$x = 0, 1$: *DN* $2 \leq x \leq s$: *PM* $s < x < 5$: *IM* $x = 5$: *R*	[2, 4]	$x = 0,1$: *DN* $2 \leq x \leq s$: *PM* $x > s$: *R*	[2, 4]	$x = 0,1$ *DN* $2 \leq x \leq s$: *PM* $s < x < 5$: *IM* $x = 5$: *R*	s ≥ 5	$x \leq 2$: *R* $x \geq 3$: *PM*	[2, 4]	$x = 0,1$ *DN* $2 < x \leq s$: *PM* $s < x < 4$: *IM* $x = 4, 5$: *R*	[2, 4]	$x = 0,1$ *DN* $2 \leq x \leq s$: *PM* $s < x < 5$: *IM* $x = 5$: *R*
s ≥ 5	$x = 0, 1$: *DN* $x \geq 2$ *PM*	[5, 10]	$x = 0,1$: *DN* $x \geq 2$ *PM*	s ≥ 5	$x = 0, 1$: *DN* $x \geq 2$ *PM*			[5, 13]	$x = 0,1$ *DN* $x > 2$ *PM*	[5, 8]	$x = 0, 1$: *DN* $x \geq 2$ *PM*
		s = 11	$x = 0$: *R* ($i \leq 4$) *DN*($\forall i \geq 5$) $x = 1$: *R* ($i \leq 3$) *DN*($\forall i \geq 4$) $x \geq 2$: *PM*					s = 14	$x = 0$: *R* $x = 1$: *R* ($i \leq 3$) *DN*($\forall i \geq 4$) $x \geq 2$: *PM*	s ≥ 8	$x = 0$: *DN* $x = 1$ *IM* $x \geq 2$ *PM*
		s = 12	$x = 0$: *R* ($i \leq 8$) *DN*($\forall i \geq 9$) $x = 1$: *R* ($i \leq 6$) *DN*($\forall i \geq 7$) $x \geq 2$: *PM*					s ≥ 15	$x = 0, 1$: *R* $x \geq 2$ *PM*		
		s ≥ 13	$x = 0, 1$: *R* $x \geq 2$: *PM*								

and the failure rate of components are improved over technological generations, the replacement option has more value than the do nothing option.

As illustrated by Table 2, the replacement threshold is $x = 2$ for $j = 1$ and $x = 1$ for $j = 3$. When the stock level $s > 0$, imperfect/perfect maintenance (*IM / PM*) is performed by using spare parts to restore the asset to the previous/initial deterioration state. Consider the case $s = 2$, the policy dictates the *DN* action when $x = \{0, 1\}$; *PM* when $x = \{2\}$, IM with $x = \{3, 4\}$ and finally replace with new asset at $x = 5$. Clearly, with a high stock level, *PM* demonstrates its dominance and replacement is not necessary. For example, with $(k, j) = (2, 2)$ when the stock level is greater than 5, the optimal policy prescribes DN until $x = 2$, then PM.

Now we consider the influence of stock level on the optimal policy in the obsolescence cases: $(k, j) = (2, 1)$; $(3, 1)$; $(3, 2)$. These are the cases where the available technology is greater than the one currently in use. With the chosen parameters in Table 1, the optimal decision is to replace the used asset by the latest available technology when replacement is prescribed.

We find that if the technological improvement is not significant, the firm tends to take advantage of the performance gained by available equipment and their spare parts in stock before investing in technological innovation. There exists an optimal inventory level to balance the loss due to later investment in new technology: replacement should be made when the stock level is lower than the defined threshold (s_1) or greater than (s_2). For example, with (k, j) = (2, 1), the profit function and the failure rate are improved, i.e., $g_2(x) / g_1(x) = 1.01$ and $\alpha_2 / \alpha_1 = 0.9$. If stock level $s \in [5, 10]$, the asset operates normally until deterioration state $x = 2$, then is maintained perfectly to restore in the initial sate. At low stock level, $x < 5$, compared with the cases ($k = j$), the firm tends to replace earlier. For example, with $s = 2$, the replacement threshold in obsolescence case (k, j) = (2, 1) is $s = 3$ while it is $s = 5$ in case (k, j) = (1, 1). The greater the stock level is, the higher the holding cost is, so a high inventory level is not needed in the obsolescence case. As illustrated in Table 2, given (k, j) = (2, 1) with $s \geq 13$, spare parts are utilized to perform PM when $x > 1$. This restores the asset to an initial state and a maximal profit $g(0)$ is incurred during the next period. If the deterioration state is $x = 0$ or $x = 1$, it is optimal to sell the asset in order to obtain the maximal residual values $b_{i,j,k}(0)$ or $b_{i,j,k}(1)$ and invest in new technology; else ($x > 1$) we continue to perform PM to reduce the deterioration state and stock level.

In the case where the latest available technology is much greater improvement over the one currently in place, e.g., (k, j) = (3, 1), the firm tends to invest early in new technology. We replace the current asset by latest available technology when the stock level is less than 2. If stock level $s \geq 3$, with $x \geq 3$, we utilize spare parts in stock for PM to restore the asset to as-good-as-new before selling it at the next epoch (when its deterioration state $x \leq 2$ or stock level $s \leq 2$).

Table 3: The optimal policy for the first ten decision epochs in the obsolescence case (3, 1)

$\delta = 0.99$		0.8		0.6	
s	Action	s	Action	s	Action
[0, 2]	$\forall x$: R	0	$x < 2$: DN $x \geq 2$: R	0	$x < 3$: DN $x \geq 3$: R
		1	$x = 0$: DN $x = 1, 2$: IM $x \geq 3$: R	1	$x < 2$: DN $x = 2, 3$: IM $x \geq 4$: R
		2	$x < 2$: DN $x = 2$: PM $x = 3$: IM $x \geq 4$: R	2	$x < 2$: DN $x = 2$: PM $x = 3,4$: IM $x = 5$: R
[3, 4]	$3 \leq x \leq s$: PM $\forall x \in X \setminus [3, s]$: R	[3, 4]	$x = 0,1$ DN $2 \leq x \leq s$: PM $x > s$: R	[3, 4]	$x = 0,1$ DN $2 \leq x \leq s$: PM $s < x < 5$: IM $x = 5$: R
$s \geq 5$	$x \leq 2$: R $x \geq 3$: PM	$s \geq 5$	$x = 0,1$ DN $x \geq 2$: PM	$s \geq 5$	$x = 0,1$ DN $x \geq 2$: PM

Next we consider the rapid speed of technological evolution and its affects on the optimal strategy. Note that this speed is represented by the appearance probabilities of new technology in next the period p_{i+1}^{k+1}. As illustrated by the numerical examples in the case (k, j) = (3, 1), when ε does not change, $\delta = 0.99$, 0.8 and 0.6, consider the first ten decision epochs (Table 3). We find that the smaller δ is, the greater the value of DN or IM/PM are because the firm tends to keep the used asset for waiting the appearance of new technology when its appearance probability during the next period is high. Consider, for example, when $\delta = 0.99$, the replacement option has significantly more value and we replace immediately the asset at the low stock levels $s \leq 2$. Contrarily, the do nothing and maintenance options demonstrate their dominance when $\delta = 0.6$, 0.8.

4 CONCLUSION

In this paper, we proposed a model that allows us to take into account the spare part inventory in the maintenance/replacement problem of a stochastically deteriorating system under technological change. It determines the maintenance/replacement strategy based on the parametric performance of the system and technological environment. We have combined several aspects never seen before in the same model: equipment replacement, maintenance, technological evolution, and spare parts inventory.

Through our numerical examples we have shown the influence of the spare parts inventory level and technological change on the maintenance/replacement strategy. In the non-obsolescence case, it is obvious that replacement is done only at low stock levels; on the contrary, at high stock levels, the maintenance options demonstrate their dominance. In the obsolescence case, the replacement option with new technology is motivated, but the trade-off between the benefits of utilizing spare parts in store and the revenues of investment in new technology is also considered. Therefore, replacement is not done when the stock level is in the interval that is determined by the parameters of the model. The better the technological improvement is the greater value the replacement option is of. However, in the case of rapid technological change, the do nothing and repair options have significantly more value as they allow for the appearance of even better technologies in the future.

Some proposed assumptions can be seen as limitations of our model such as the expectation of purchase price and improvement of new technology. In fact, these can be stochastic and difficult to capture. An extension of our model could reflect the stochastic characteristics of these parameters. Furthermore, the stochastic efficiency of the imperfect maintenance action could also be included in our model or the non-stationary properties of the deterioration.

The principal objective of this paper is to consider simultaneously the influences of technological evolution and spare parts levels on the optimal maintenance/replacement strategy; so we have simplified the inventory problem by assuming that spare parts stock cannot be replenished without purchasing a new asset and that the quantity of spare parts supplied is determined by the manufacturer. In further research, we could take into account the replenishment capacity of stock and also examine the optimal inventory policy.

5 REFERENCES

1) Bethuyne, G. 2002. The timing of technology Adoption by a cost-minimizing firm. *Journal of Economics* 76: 123-154.

2) Borgonovo, E; Marseguerra, M & Zio, E. 2000. A Monte Carlo methodological approach to plant availability modeling with maintenance, aging and obsolescence. *Reliability Engineering and System Safety* 67: 61-73.

3) Chelbi, A & Aït-Kadi, D. 2001. Spare provisioning strategy for preventively replaced systems subjected to random failure. *Int. J. Production Economics* 74: 183-189.

4) Chien, Y.H. 2009. A number-dependent replacement policy for a system with continous preventive maintenance and random lead times. *Applied Mathematical Modelling* 33: 1708-1718.

5) Clavareau, J & Labeau, P.E. 2009a. A Petri net-based modeling of replacement strategies under technological obsolescence. *Reliability Engineering and System Safety* 94: 357- 369.

6) Clavareau, J & Labeau, P.E. 2009b. Maintenance and replacement policies under technological obsolescence. *Reliability Engineering and System Safety* 94: 370-381.

7) Cobbaert, K & Oudheusden, D. V. 1996. Inventory models for fast moving spare parts subject to “sudden death” obsolescence. *Int. J. Production Economics* 44: 239-248.

8) De Smidt-Destombes, K. S; Van der Heijden, C.M & Haten, A. V. 2006. On the interaction between maintenance, spare part inventories and repair capacity for k-out-of-N system with wear-out. *European Journal of Operational Research* 174: 182-200.

9) Dogramaci, A & Fraiman, N.M. 2004. Replacement decisions with maintenance under uncertainty: an imbedded optimal control Model. *Operations Research* 52: 785-794.

10) Hopp, W.J & Nair, S.K. 1994. Maintenance and replacement policies under technological obsolescence. *Reliability Engineering and System Safety* 94: 370-381.

11) Kabir, A.B.M.Z & Al-Olayan, A.S. 1996. A stocking policy for spare part provisioning under age based preventive replacement. *European Journal of Operational Research* 90: 171-181.

12) Karsak, E.E & Tolga, E. 1998. An overhaul-replacement model for equipment subject to technological change in an inflation-prone economy. *Int. J. Production Economics* 56-57: 291-301.

13) Kennedy, W.J; Wayne Patterson, J & Fredendall, L.D. 2002. An overview of recent literature on spare parts inventories. *Int. J. Production Economics* 76: 201-215.

14) Kim, J.S; Shin, K.C; Yu, H.K. 1996. Optimal algorithm to determine the spare inventory level for a repairable-item inventory system. *Computers Ops Res* 23: 289-297.

15) Mercier, S. 2008. Optimal replacement policy for obsolete components with general failure rates submitted to obsolescence. *Applied Stochastic Models in Business and Industry* 24: 221-235.

16) Nair, S.K. 1995. Modeling strategic investment decisions under sequential technological change. *Management Science* 41: 282-297.

17) Nair, S.K. 1997. Indentifying technology Horizons for the strategic investment decisions. *IEEE Transactions on Engineering management* 44: 227-236.

18) Natali, H & Yatsenko, Y. 2007. Optimal equipment replacement without paradoxes: A continous analysis. *Operations Research Letters* 35: 245-250.

19) Natali, H & Yatsenko, Y. 2008b. The dynamics of asset lifetime under technological change. *Operations Research Letter* 36: 565-568.

20) Natali, H & Yatsenko, Y. 2008a. Properties of optimal service life under technological change. *Int. J. Production Economics* 114: 230-238.

21) Torpong, C & Smith, R.L. 2003. A paradox in equipment replacement under technological improvement. *Operations Research Letters* 31: 77-82.

22) Saker, R & Haque, A. 2000. Optimization of maintenance and spare provisioning policy using simulation. *Applied Mathematical Modelling 24*: 751-760.

23) Sheu, H.S & Chien, Y.H. 2004. Optimal age-replacement policy of a system subject to shocks with random lead-time. *European Journal of Operational Research* 159: 132-144.

24) Vaughan, T.S. 2005. Failure replacement and preventive maintenance spare parts ordering policy. *European Journal of Operational Research* 161: 183-190.