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► **To cite this version:**

Jonatan Álvarez-Muñoz, Fermi Guerrero Castellanos, Sylvain Durand, Nicolas Marchand, G. Mino-Aguilar. Control and Model-Aided Inertial Navigation of a Nonholonomic Vehicle. CLCA 2012 - 15th IFAC Latinamerican Control Conference, Oct 2012, Lima, Peru. pp.n/c. hal-00727706

**HAL Id: hal-00727706**

**<https://hal.science/hal-00727706>**

Submitted on 28 Aug 2019

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# Control and Model-Aided Inertial Navigation of a Nonholonomic Vehicle

J. U. Álvarez-Muñoz \* J. F. Guerrero-Castellanos \*  
S. Durand \*\* N. Marchand \*\*\* G. Mino-Aguilar \*

\* *Facultad de Ciencias de la Electrónica, BUAP, Puebla, Mexico*  
(e-mail: fguerrero@ece.buap.mx, junmunoz@ece.buap.mx)

\*\* *UMI LAFMIA, CNRS - CINEVESTAV, Mexico D.F., Mexico*  
(e-mail: sylvain@durandchamontin.fr)

\*\*\* *Automatic Control Department, GIPSA-lab, Grenoble, France*  
(e-mail: Nicolas.Marchand@inpg.fr)

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**Abstract:** The present work deals with the control and localization problem of wheeled-mobile robots with nonholonomic constraints. In the proposed method a simple nonlinear control law, composed of a position and heading direction controller, is designed to asymptotically stabilize the position error. The control law takes into account the constraints on the control signals in order to avoid saturation of the actuators. Furthermore, this paper considers a method of using the dynamic vehicle model and vehicle's nonholonomic constraints in order to aid position and attitude estimates provided by an Inertial Navigation System (INS). It is shown that dynamic model and vehicle's nonholonomic constraints can reduce the error growth in robot position estimates. Simulations are included to confirm the effectiveness of the proposed scheme.

*Keywords:* Posture stabilization, nonholonomic constraints, model-aided inertial navigation.

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## 1. INTRODUCTION

Wheeled Mobile Robots (WMRs) are considered as the most widely used class of mobile robots. This is due to their fast manoeuvring, simple controllers and energy saving characteristics. These robots are increasingly present in industrial and service robotics, particularly when autonomous motion capabilities are required over reasonably smooth grounds and surfaces. Several mobility configurations (wheel number and type, their location and actuation, single or multi-body vehicle structure) can be found in the applications, Bloch and Reyhanoglu (1991). The most common for single-body robot is differential drive which is kinematically equivalent to a unicycle, tricycle or car-like drive, and omnidirectional steering. Many studies have targeted different aspects of WMRs such as kinematics, dynamics and controller design, Kolmanovsky and McClamroch (1995). Also, WMRs have nonholonomic constraints since they have restricted mobility in that the wheels roll without slipping. In particular in Brockett et al. (1983), a linearized mobile robot model is shown to have deficiency in controllability and the linear control method cannot be employed. Thus, nonlinear control for this class of systems has been studied extensively from several points of view, including setpoint stabilization, Campion et al. (1991); Pomet and Campion (1992); Canudas de Wit and Sordalen (1992); Park et al. (2000), trajectory-tracking, path-following and others, as depicted in Kanayama et al. (1990); Aicardi et al. (1995); Lee et al. (2001); Jiang et al. (2001); Sira-Ramirez et al. (2011).

On the other hand, localization is a key problem in autonomous mobile robotics. Different techniques have been

developed to tackle this problem. They can be sorted into two main categories: **1) Relative (local) localization** consists of evaluating the position and the orientation through integration of information provided by encoders or inertial sensors with knowledge of initial conditions; **2) Absolute (global) localization** is the technique that permits the vehicle to determine its position directly using navigation beacons, active or passive landmarks, map matching or a Global Position System (GPS) ( see Borenstein et al. (1996) and references therein).

The Inertial Navigation System (INS) has commonly been used as a mean of localisation for various autonomous vehicles including land, underwater and aerial vehicles, see Meyer-Hilberg and Jacob (1994). It consists of gyroscopes (rate gyros) and accelerometers that provide angular rate and velocity rate information. By integrating this information, the position and orientation of the vehicle is calculated. The disadvantage in the use of an INS, particularly when using low-cost sensors, is due to the unbounded error in pose estimates due to the dead-reckoning nature of the sensor. This was highlighted in Sukkarieh et al. (1999).

In order to limit the error growth, absolute localization techniques using global sensors measurements have been used. This can increase the accuracy of the estimate and keep the uncertainty within certain bounds. However, these external sensors have several practical disadvantages mainly relating to a reliance on external information, such as reception of satellite transmissions or reliably observable terrain features. One source of information that can be used to aid in the localisation of the vehicle, without the need for external sensing, is the one from knowledge of the vehicles motion. Such information can be represented in two ways: **1) Vehicle Model Constraints,**

where specific constraints on the pose of the vehicle – i.e. a wheeled vehicles sideways velocity – will be zero, like in Dissanayake et al. (2001); **2) Vehicle Dynamic Modelling**, where a motion model of the vehicle give a history of the vehicle pose, control inputs and external forces acting on the vehicle, like in Koifman and Bartzhack (1999); Xin et al. (2003).

The main contributions in this paper are:

- The development of a simple nonlinear control law (composed of a position controller and heading direction controller) in order to stabilize a WMR with differential drive. The proposed control law exploits the fact that the kinematic model can be broken down into two subsystems, one defining the translation movement and the other one the rotation movement. These subsystems are coupled in cascade since the translational subsystem depends on the rotational one, but the rotational subsystem is independent of the translational one. The proposed controller is extremely simple and it is suitable for embedded implementation.
- The development of a method to aid in estimating the errors in the inertial navigation solution of a WMR with differential drive, using vehicle dynamic model and vehicle model constraints. The main idea is to contribute further to the understanding of the value of dynamic vehicle model aiding.

The paper is organized as follows. In section 2, the dynamic models of Inertial Navigation System and Wheeled Mobile Robot are recalled. The control law for stabilization purposes is established in section 3. In section 4, we present the model-aided inertial navigation method. Some simulation results using the control law and the aided INS are presented in section 5. Finally some discussions conclude the paper.

## 2. SYSTEM DESCRIPTION

A classical method for aiding an INS using some external measurements coming from GPS and magnetometers consists in using a Kalman filter. In this case, the INS reads the acceleration and rotation rate of the vehicle, provided by an on-board Inertial Measurement Unit (IMU). Then, the Kalman filter is used to compute the errors in the INS from an observation of the differences between the INS and GPS with magnetometer. This framework is illustrated in Fig. 1(a). As it was mentioned in the introduction, an alternative method can be using also the dynamic model of the vehicle to control, such as suggested in Fig. 1(b). The dynamic model of the vehicle is used to compute velocity, position and rotation rates of the vehicle. A Kalman filter is used to compute the total error states of the system, including errors in velocity, position, orientation and accelerometer and gyro biases. The system is thus aided with GPS and magnetometer readings, but when these signals are noisy or unavailable the system is aided by the dynamic vehicle model. Moreover, in this paper we focus in particular on nonholonomic vehicles, and we propose to take into account such constraints in order to improve the estimation.

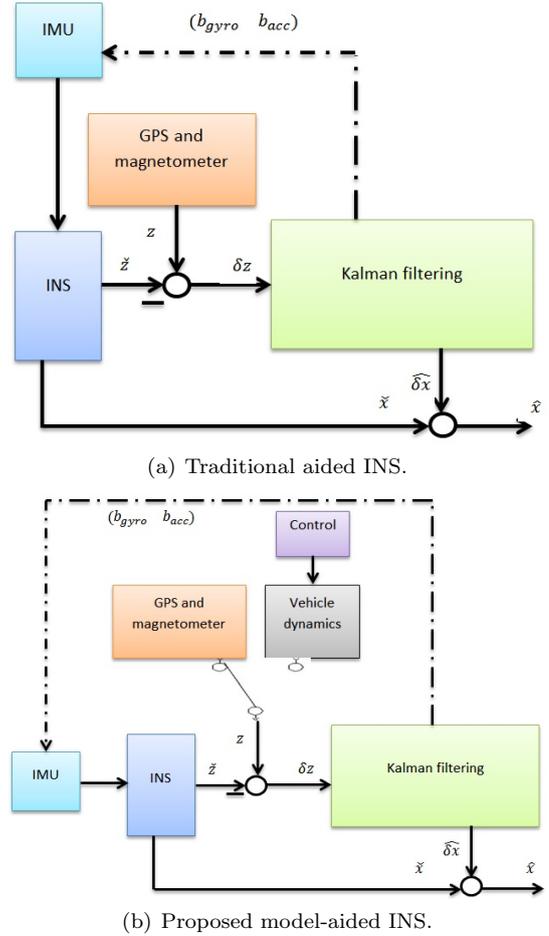


Fig. 1. High-level configuration of the system.

### 2.1 Mobile Robot Dynamic Model

In this subsection, dynamics and kinematics of wheeled-mobile robots are shown under the nonholonomic constraints as in . The dynamic equations under nonholonomic constraints can be described by Euler-Lagrange formulation as:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = B(q)\tau + J^T(q)\lambda \quad (1)$$

where  $q \in \mathbb{R}^n$  is generalized coordinates,  $\tau \in \mathbb{R}^n$  is a control input vector,  $\lambda$  is the Lagrange multipliers of constrained forces,  $M \in \mathbb{R}^{n \times n}$  is a symmetric and positive definite inertia matrix,  $C \in \mathbb{R}^{n \times n}$  is a centripetal and Coriolis matrix,  $G \in \mathbb{R}^n$  is a gravitation vector,  $B \in \mathbb{R}^{n \times r}$  is an input transformation matrix ( $n < r$ ), and  $J \in \mathbb{R}^{n \times r}$  is a matrix related with nonholonomic constraints. Pose vector of robot in the surface is defined as  $q = (x_r, y_r, \theta_r)$  where  $x_r$  and  $y_r$  are the coordinates of point  $CG$  center of axis of wheels, and  $\theta_r$  is the orientation angle of robot in the inertial frame  $\mathbf{E}^f(e_1^f, e_2^f)$  (see Fig. 2). One can write the dynamic equations of mobile robot according to (1), using the fact that  $G(q)$  and  $C(q, \dot{q})$  are zero.

$$\begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{pmatrix} \begin{pmatrix} \ddot{x}_r \\ \ddot{y}_r \\ \ddot{\theta}_r \end{pmatrix} = \frac{1}{R} \begin{pmatrix} \cos \theta_r & \cos \theta_r \\ \sin \theta_r & \sin \theta_r \\ L & -L \end{pmatrix} \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} + \begin{pmatrix} \sin \theta_r \\ -\cos \theta_r \\ 0 \end{pmatrix} \lambda \quad (2)$$

and

$$\lambda = -m(\dot{x}_r \cos \theta_r + \dot{y}_r \sin \theta_r) \dot{\theta}_r \quad (3)$$

Where  $\tau_1$  and  $\tau_2$  are the torques of right and left motors,  $m$  and  $I$  present the mass and inertia of robot respectively.  $R$  is the radius of wheels and  $2L$  is the distance of rear wheels.

The nonholonomic constraint, the no slip condition, is written in the form of

$$\dot{x}_r \sin \theta_r + \dot{y}_r \cos \theta_r = 0 \quad (4)$$

This equation is not integrable, so the feasible trajectory of robot is limited.

On the other hand, the kinematic relationship is

$$\dot{q} = \begin{pmatrix} \cos \theta_r & 0 \\ \sin \theta_r & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_r \\ \omega_r \end{pmatrix} \quad (5)$$

After some manipulations is easy to shown that:

$$\dot{v}_r = \frac{F}{m} = \frac{\tau_1 + \tau_2}{mR} \quad \dot{\omega}_r = \frac{\Gamma}{I} = \frac{(\tau_1 - \tau_2)L}{IR} \quad (6)$$

where  $v_r$  and  $\omega_r$  are the linear and angular velocity of the mobile robot.

Actually, (5) can be used in order to design a nonlinear kinematic controller to stabilize the configuration variables. This will be discussed in the next section.

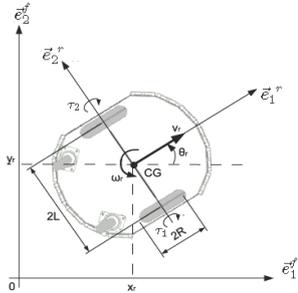


Fig. 2. Coordinate plane of the mobile robot.

## 2.2 Inertial Navigation System

The system uses two kinds of inertial sensors; an accelerometer and a rate gyro whose measurements are mathematically represented by

$$a_c = a_r + b_a + \mu_a \quad (7)$$

where  $a_c$  is the measurement from the accelerometer,  $a_r$  is the truth acceleration of the vehicle,  $b_a$  is the accelerometer bias and  $\mu_a$  is assumed to be zero-mean Gaussian white-noise process with covariance  $N_a$ . The output signal of a rate gyro is given by

$$\omega_g = \omega_r + b_g + \mu_g \quad (8)$$

where  $\omega_g$  is the measurement from the rate gyro,  $\omega_r$  is the truth angular rate of the vehicle and  $b_g$  is the rate gyro bias. As in the case of the accelerometer  $\mu_g$

is assumed to be zero-mean Gaussian white-noise process with covariance  $N_g$ . In this work the signals  $b_a$  and  $b_g$  are considered unknown but constants.

Then, position  $\bar{p}_f = (\bar{x}_r \ \bar{y}_r)^T$ , velocity  $\bar{v}_f = (\dot{x}_r \ \dot{y}_r)^T$  and orientation  $\bar{\theta}_r$  of the mobile robot can be computed integrating the above equations. They are given by

$$\begin{aligned} \dot{\bar{p}}_f &= \bar{v}_f \\ \dot{\bar{v}}_f &= (a_c \cos \theta_g \ a_c \sin \theta_g)^T \\ \dot{\bar{\theta}}_r &= \omega_g \end{aligned} \quad (9)$$

Due to inherent errors in the gyros and accelerometers, the INS navigation solution will have an unbounded drift, where the divergence rate depends on the quality of the IMU. Since the INS is a divergence system, it requires an aiding system to avoid the growth of its errors.

## 3. NONLINEAR CONTROLLER

Lets introduce some definition and lemmas before detailing the nonlinear control strategy.

### 3.1 Mathematical preliminaries

*Definition 3.1.* For a given positive constant parameter  $M_i$ , the function  $\sigma_{M_i} : \mathbb{R} \rightarrow \mathbb{R}$  is a linear function of saturation of  $M_i$  if it is continuous, no decreasing, such that

$$\sigma_{M_i}(s) := \min(M_i, \max(-M_i, s)) \quad (10)$$

and satisfies

$$s\sigma_{M_i}(s) > 0, \quad \forall s \neq 0 \quad (11)$$

The functions of saturation are quite interesting in systems which are subject to some restrictions on the input amplitude. In such cases, the limits  $M_i$  allow to bound the amplitude of the control signal and, consequently, the signal applied to the actuators.

*Lemma 3.2.* Let consider the double integrator system defined by

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= u \end{aligned} \quad (12)$$

and the control law given by

$$u = -\sigma_{M_2}(a_1 y_2 + \sigma_{M_1}(a_2 y_2 + a_1 a_2 y_1)) \quad (13)$$

where  $\sigma_{M_i}$  are defined in Definition 3.1, with  $M_2 > 2M_1$  and  $a_1, a_2 > 0$  are some tunable parameters, then the double integrator closed-loop system is globally and asymptotically stable (GAS) and locally exponentially stable (LES).

**Proof.** Let  $\xi = Ty$  be a linear transformation, with

$$T = \begin{pmatrix} a_1 a_2 & a_2 \\ 0 & a_1 \end{pmatrix} \quad (14)$$

and  $y = (y_1, y_2)^T$ . Applying this to the double integrator system (12) yields

$$\begin{aligned} \dot{\xi}_1 &= a_2 \xi_2 + a_2 u \\ \dot{\xi}_2 &= a_1 u \end{aligned} \quad (15)$$

with  $\xi = (\xi_1, \xi_2)^T$  and

$$u = -\sigma_{M_2}(\xi_2 + \sigma_{M_1}(\xi_1)) \quad (16)$$

Then, the closed-loop system becomes

$$\begin{aligned}\dot{\xi}_1 &= a_2 [\xi_2 - \sigma_{M_2}(\xi_2 + \sigma_{M_1}(\xi_1))] \\ \dot{\xi}_2 &= -a_1 \sigma_{M_2}(\xi_2 + \sigma_{M_1}(\xi_1))\end{aligned}\quad (17)$$

Now we can analyze the convergence of this system.

1) Firstly, let consider the dynamics of state  $\xi_2$  and the candidate Lyapunov function

$$V_{\xi_2} = \xi_2^2 \quad (18)$$

The time derivative of  $V_{\xi_2}$  along the trajectory  $\xi_2$  is given by

$$\begin{aligned}\dot{V}_{\xi_2} &= 2\xi_2\dot{\xi}_2 \\ &= -2a_1\xi_2\sigma_{M_2}(\xi_2 + \sigma_{M_1}(\xi_1))\end{aligned}\quad (19)$$

Then, assuming  $|\xi_2| > 2M_1$  implies  $|\xi_2 + \sigma_{M_1}(\xi_1)| \geq M_1 + \varepsilon$  with  $\varepsilon > 0$  sufficiently small. Also, one can easily show  $\xi_2 + \sigma_{M_1}(\xi_1)$  is of the same sign that  $\xi_2$  and, consequently,  $\dot{V}_{\xi_2} < 0$ . This implies  $\xi_2$  decreases until it enters to the set

$$\Phi_2 = \{\xi_2 : |\xi_2| \leq 2M_1\} \quad (20)$$

in a finite time  $t_1$  and stay in this set for all  $t > t_1$ . Furthermore, Lemma 4 in Marchand and Hably (2005) ensures  $\xi_1$  is bounded. Once this convergence is achieved, then  $\xi_2 + \sigma_{M_1}(\xi_1) \in [-2M_1, 2M_1]$ , and so is not saturated  $\sigma_{M_2}(\cdot)$ . The system (17) hence becomes

$$\begin{aligned}\dot{\xi}_1 &= -a_2\sigma_{M_1}(\xi_1) \\ \dot{\xi}_2 &= -a_1[\xi_2 + \sigma_{M_1}(\xi_1)]\end{aligned}\quad (21)$$

2) Secondly, let consider the dynamics of  $\xi_1$  and the candidate Lyapunov function

$$V_{\xi_1} = \xi_1^2 \quad (22)$$

whose time derivative along trajectory  $\xi_1$  is

$$\begin{aligned}\dot{V}_{\xi_1} &= 2\xi_1\dot{\xi}_1 \\ &= -2a_2\xi_1\sigma_{M_1}(\xi_1)\end{aligned}\quad (23)$$

From the definition of the function of saturation, it follows  $\dot{V}_{\xi_1} < 0$  and, consequently,  $\xi_1$  decreases till it enters to the set

$$\Phi_1 = \{\xi_1 : |\xi_1| \leq M_1\} \quad (24)$$

in a finite time  $t_2$  and stay in it for all  $t > t_2$ , and so is not saturated  $\sigma_{M_1}(\cdot)$ . The system (21) hence becomes

$$\begin{aligned}\dot{\xi}_1 &= -a_2\xi_1 \\ \dot{\xi}_2 &= -a_1[\xi_2 + \xi_1]\end{aligned}\quad (25)$$

which, in a matrix form, yields

$$\begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{pmatrix} = \begin{bmatrix} -a_2 & 0 \\ -a_1 & -a_1 \end{bmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \quad (26)$$

This result is of the form  $\dot{\xi} = A\xi$ , whose solution  $\xi = e^{At}\xi(0)$  vanishes when  $t \rightarrow \infty$ , if  $A$  is Hurwitz. As a consequence, the double integrator system (12) is GAL and LES when applying the control law (13). This ends the proof.

Now let consider the nonlinear system defined by

$$\begin{aligned}\dot{y}_1 &= y_2 \\ \dot{y}_2 &= \sigma_{M_3}(u) \\ z &= y_1\end{aligned}\quad (27)$$

where the objective is to make  $z$  tracking a desired trajectory  $z_d$ , given for  $z_d$ ,  $\dot{z}_d$  and  $\ddot{z}_d$ .

*Lemma 3.3.* If  $|\ddot{z}_d| \leq M_3 - \delta$  for all  $t > t_0$  and for some  $\delta > 0$ , with  $M_2 \leq \delta$ ,  $M_2 > 2M_1$  and the control law given by

$$u = \ddot{z}_d - \sigma_{M_2}(a_1\ddot{z}_2 + \sigma_{M_1}(a_2\ddot{z}_2 + a_1a_2\ddot{z}_1)) \quad (28)$$

where  $\sigma_{M_i}$  are defined in Definition 3.1,  $\ddot{z}_1 = y_1 - z_d$  and  $\ddot{z}_2 = y_2 - \dot{z}_d$ , then the reference tracking is asymptotic.

**Proof.** Expressing system (27) in terms of  $\ddot{z}_1$  and  $\ddot{z}_2$  yields

$$\begin{aligned}\dot{\ddot{z}}_1 &= \ddot{z}_2 \\ \dot{\ddot{z}}_2 &= -\ddot{z}_d + \sigma_{M_3}(u)\end{aligned}\quad (29)$$

Then, applying the control law (28) and  $M_2 \leq \delta$ , one can show that the function of saturation  $\sigma_{M_3}(\cdot)$  ever operates in its linear region and, consequently, the closed-loop system becomes

$$\begin{aligned}\dot{\ddot{z}}_1 &= \ddot{z}_2 \\ \dot{\ddot{z}}_2 &= -\sigma_{M_2}(a_1\ddot{z}_2 + \sigma_{M_1}(a_2\ddot{z}_2 + a_1a_2\ddot{z}_1))\end{aligned}\quad (30)$$

which is equivalent to system (13) in Lemma 3.2 where the stability conditions were established. As a result,  $\ddot{z}_{1,2} \rightarrow 0$  and, consequently,  $z(t) = y_1(t) \rightarrow z_d(t)$  when  $t \rightarrow \infty$ . This concludes the proof for asymptotic tracking.

### 3.2 Control strategy

Actually, the mobile robot system in (5) can be divided into two subsystems, afterwards denoted  $\Sigma_1$  and  $\Sigma_2$ , which represent the equations of translation and rotation movement respectively. Let define  $\theta := \theta_r$ ,  $v := v_r$  and

$$x := \begin{pmatrix} x_{1_1} \\ x_{1_2} \\ x_{2_1} \\ x_{2_2} \end{pmatrix} = \begin{pmatrix} \int x_r \\ x_r \\ \int y_r \\ y_r \end{pmatrix} \quad (31)$$

Then, the system (5) can be broken down into two subsystems  $\Sigma_1 := f(x, \theta)$  and  $\Sigma_2 := g(\theta)$  such that

$$\Sigma_1 : \begin{cases} \dot{x}_{1_1} = x_{1_1} \\ \dot{x}_{1_2} = v \cos \theta \\ \dot{x}_{2_1} = x_{2_1} \\ \dot{x}_{2_2} = v \sin \theta \end{cases} \quad (32)$$

$$\Sigma_2 : \dot{\theta} = \omega \quad (33)$$

One could note that  $\Sigma_2$  is independent of  $x_1$  and  $x_2$  since it is a simple integrator with respect to  $\omega$ . On the other hand,  $\Sigma_1$  is composed of two independent double integrators with respect to  $v$  and  $\theta$ .

In the sequel,  $\theta$  is considered as a virtual control, as suggested in Olfati-Saber (2000); Hably et al. (2006), and joint with  $v$  both will control the subsystem  $\Sigma_1$ . Firstly, we assume that a control  $\omega$  exists in such a way it drives  $\theta$  to a desired angle  $\theta_d$  given by

$$\theta_d := \arctan\left(\frac{r_2}{r_1}\right) \quad (34)$$

where  $r_1$  and  $r_2$  are some parameters next defined in (38). Also, this desired angle is such that

$$\cos(\theta_d) = \frac{r_1}{\sqrt{r_1^2 + r_2^2}} \quad \text{and} \quad \sin(\theta_d) = \frac{r_2}{\sqrt{r_1^2 + r_2^2}} \quad (35)$$

The application of the control signal  $v$

$$v = \sqrt{r_1^2 + r_2^2} \quad (36)$$

when  $\theta = \theta_d$ , will transform subsystem  $\Sigma_1 := f(x, \theta)$  (32) into the form of two independent chain of integrators (32)

$$\begin{cases} \dot{x}_{1_1} = x_{1_1} \\ \dot{x}_{1_2} = r_1 \\ \dot{x}_{2_1} = x_{2_1} \\ \dot{x}_{2_2} = r_2 \end{cases} \quad (37)$$

*Theorem 3.4.* Let consider the system (5), the velocity control signal  $v$  defined in (36), with  $r_1$  and  $r_2$  given by

$$\begin{aligned} r_1 &= -\sigma_{M_2} \left( a_1 x_{1_2} + \sigma_{M_1} (a_2 x_{1_2} + a_1 a_2 x_{1_1}) \right) \\ r_2 &= -\sigma_{M_2} \left( b_1 x_{2_2} + \sigma_{M_1} (b_2 x_{2_2} + b_1 b_2 x_{2_1}) \right) \end{aligned} \quad (38)$$

and the angular rate control signal  $\omega$  defined by

$$\omega = \dot{\theta}_d - \sigma_{M_\theta} (\theta - \theta_d) \quad (39)$$

with  $M_2 > 2M_1$ , where  $\sigma_{M_1}$ ,  $\sigma_{M_2}$  and  $\sigma_{M_\theta}$  are some saturated linear functions of  $M_1$ ,  $M_2$  and  $M_\theta$  defined in Definition 3.1. Then, for a given initial state  $x(t=0) \in \mathbb{R}^4$ , with  $x$  defined in (31), we have

$$\lim_{t \rightarrow \infty} x(t) = 0, \quad \forall t \geq 0 \quad (40)$$

with  $0 \leq \sqrt{2}M_2 \leq v$ .

**Proof.** First, we consider the stabilization of the rotation movement. Let  $\tilde{\theta} = \theta - \theta_d$  be the error between the measured/estimated and desired angle. The dynamics of this error is  $\dot{\tilde{\theta}} = \omega - \dot{\theta}_d$ , from (33). Then, using the control law defined in (39) results in  $\dot{\tilde{\theta}} = -\sigma_{M_\theta}(\tilde{\theta})$ . Now, considering the candidate Lyapunov function

$$V_{\tilde{\theta}} = \tilde{\theta}^2 \quad (41)$$

and evaluating its time derivative along the trajectory  $\tilde{\theta}$  yields

$$\dot{V}_{\tilde{\theta}} = -2\tilde{\theta}\sigma_{M_\theta}(\tilde{\theta}) < 0, \quad \forall \tilde{\theta} \neq 0 \quad (42)$$

This means  $\tilde{\theta} \rightarrow 0$  when  $t \rightarrow \infty$  and, consequently,  $\theta \rightarrow \theta_d$  when  $t \rightarrow \infty$ . Then, as soon as  $\theta = \theta_d$  the system (32) with  $v$  defined in (36) and  $r_1, r_2$  given by (38) becomes

$$\begin{cases} \dot{x}_{1_1} = \dot{x}_{1_1} \\ \dot{x}_{1_2} = -\sigma_{M_2} \left( a_1 \dot{x}_{1_2} + \sigma_{M_1} (a_2 x_{1_2} + a_1 a_2 x_{1_1}) \right) \\ \dot{x}_{2_1} = \dot{x}_{2_1} \\ \dot{x}_{2_2} = -\sigma_{M_2} \left( b_1 x_{2_2} + \sigma_{M_1} (b_2 x_{2_2} + b_1 b_2 x_{2_1}) \right) \end{cases} \quad (43)$$

Therefore, from Lemma 3.3 the system (43) is GAS and LES. As a result,  $\lim x(t) = 0$  when  $t \rightarrow \infty$  under the restriction  $0 \leq \sqrt{2}M_2 \leq v$ . This ends the proof.

*Remark 3.5.* For simplicity, the convergence was proved for the equilibrium  $x(t) = 0$ . However, the proof can be extended to any equilibrium point of the whole coordinate plane in making the appropriate variable translation, e.g.  $\vartheta = x - x_d$  where  $x_d$  is the equilibrium.

*Remark 3.6.* The stabilization of the whole system during the transition  $\theta(t) \rightarrow \theta_d(t)$  can be guaranteed by the result of Sontag (1989).

*Remark 3.7.*  $\theta_d$  is calculated through

$$\theta_d = \text{atan2}(r_2, r_1) \quad (44)$$

where  $\theta_d \in [0, 360]$  represents the relative North, with positive clockwise rotation, e.g.  $\theta_d = 90$  degrees corresponds to the East direction.

*Remark 3.8.* From (34), the time derivative of the desired angle is given by

$$\dot{\theta}_d = \frac{\dot{r}_2 r_1 - r_1 \dot{r}_2}{v^2} \quad (45)$$

For this reason, the saturation function  $\sigma_{M_i}$  con  $i = 1, 2$  has to be differentiable. An easily implementable solution is given by (see Hably et al. (2006))

$$\sigma(s) = \begin{cases} -1 & \text{if } s = -1 - \alpha \\ p_1(s) & \text{if } s \in [-1 - \alpha, -1 + \alpha] \\ s & \text{if } s \in [-1 + \alpha, 1 + \alpha] \\ p_2(s) & \text{if } s \in [1 - \alpha, 1 + \alpha] \\ +1 & \text{if } s = 1 + \alpha \end{cases} \quad (46)$$

with  $p_1(s) = e_1 s^2 + e_2 s + e_3$  and  $p_2(s) = -e_1 s^2 + e_2 s - e_3$ , where  $e_1 = \frac{1}{4\alpha}$ ,  $e_2 = \frac{1}{2} + \frac{1}{2\alpha}$  and  $e_3 = \frac{\alpha^2 - 2\alpha + 1}{4\alpha}$ . Such a function is twice differentiable. Finally, this function of saturation bounding to  $M_i$  and  $-M_i$  with  $j = 1, 2$  is defined by  $\sigma_{M_i}(\cdot) := M_i \sigma(\cdot)$ .

#### 4. MODEL-AIDED INERTIAL NAVIGATION

This section reports the concept and development of an integrated model-aided INS, applied to a nonholonomic vehicle navigation. The basic idea and concept of using the dynamic vehicle model for aiding the INS was illustrated in Fig. 1(b), where the output from the dynamic vehicle model is treated like the ones of external aiding sensors (GPS and magnetometer). The interest in studying the integration of a vehicle model is due to GPS outages, sensor dropouts or emergency navigation. Note that the integration of a vehicle model in the navigation system does not require any additional instrumentation.

The Kalman filter is carried out in discrete-time framework. Hence, measurements are taken at each instant time  $t_j$  with  $j \in \mathbb{Z}$  denoting  $\Delta t = t_{j+1} - t_j$  the sample time. A more detailed outline of the model-aided INS evaluated herein is shown in Fig. 3. In this paper, external velocity and position measurements are not included while utilizing the output from the vehicle model, and the other way around when using external velocity measurements. This is illustrated with a switch in Fig. 1 and 3. In order to aid

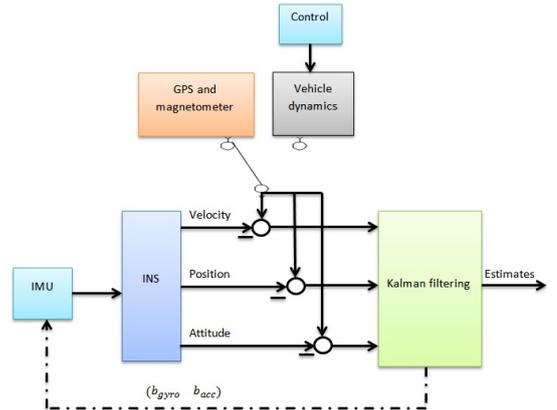


Fig. 3. Block diagram of the model-aided INS.

the INS system, the estimated state vector  $\hat{x}$  is composed of seven states, that are two velocities, two positions, one attitude, one accelerometer bias and one gyro bias, which yields

$$\hat{x} = (\hat{p}_r \ \hat{v}_r \ \hat{\theta}_r \ b_a \ b_g)^T \quad (47)$$

where  $\hat{p}_r$  is the position of the vehicle in  $\mathbf{E}^T(e_1^r, e_2^r)$ . In accordance to Fig. 1 and conventional Kalman Filter (KF)

notation, the general discrete input to the KF is defined as

$$\delta_{z_k} = z_k - \check{z}_k \quad (48)$$

where the accent ( $\check{\cdot}$ ) denotes a calculated variable, in this case from the INS. Also, the process model for the errors is expressed as

$$\dot{\delta \hat{x}} = F \delta \hat{x} + G \mu \quad (49)$$

where  $F$  describes the INS error dynamics,  $G$  is the noise input matrix and  $\mu$  is an uncorrelated zero-mean process noise vector of dimension 8 and covariance  $Q$ , representing the inertial sensor process noise. The error dynamics matrix  $F$  is calculated as follows

$$F = \begin{bmatrix} 1 & 0 & 0 & -\Delta t & 0 \\ \Delta t & 1 & 0 & -\frac{\Delta t^2}{2} & 0 \\ 0 & 0 & 1 & 0 & -\Delta t \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (50)$$

Also, the input matrix  $G$  is

$$G = \begin{bmatrix} \frac{\Delta t^2}{2} & \Delta t & \Delta t & 0 & 0 \end{bmatrix}^T \quad (51)$$

A virtual observation  $z_{obs}$  is made as the difference between the velocity and position predicted by the INS and the dynamic vehicle model. The relationship between the estimated errors and observation is given by the observation model  $H$ , as follows

$$z_{obs} = \begin{bmatrix} v_{INS} - v_{dvm} \\ p_{INS} - p_{dvm} \end{bmatrix} \quad (52)$$

$$= H \delta x + \nu \quad (53)$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (54)$$

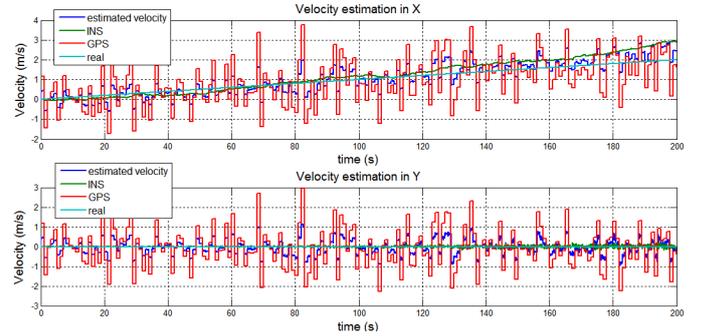
where  $\nu$  is an uncorrelated, zero-mean observation noise vector.

## 5. SIMULATION RESULTS

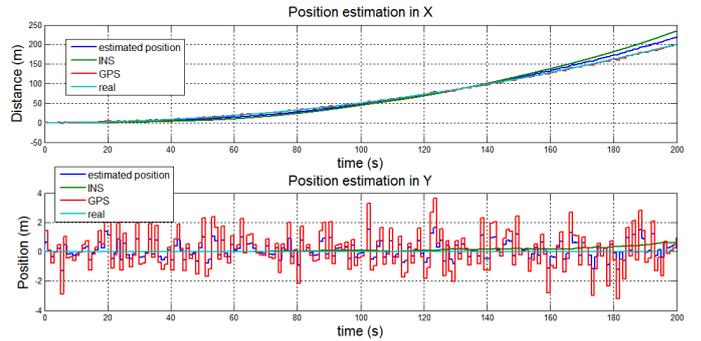
In this latter section, we propose to test our proposed strategy in simulation using *Matlab/Simulink*. In a first time, we consider two scenarios when the mobile robot is running on a linear way.

- In the first one, the vehicle is using GPS aided INS. The simulation results are represented in Fig. 4. One can see how is shattered the estimated (velocities, position and attitude) values with respect to the GPS information. Also, one can see the drift due to the INS system.
- In the second case, the system losses satellite transmission and can no longer provide positioning fixes to the INS. As a result, the on-board vehicle navigation system starts to aid the INS with the dynamic model information at the time of satellite signal loss. The results are depicted in Fig. 5. The estimation is highly improved since no shattering occurs anyway (in particular for the attitude estimation in Fig. 5(c)). Moreover, one can really appreciate how the estimated values are closer to the real ones, which means the INS drift is reduced.

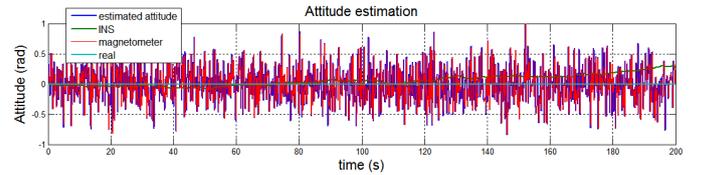
The same simulation is then performed in the case the mobile runs in a circle way. This is shown in Fig. 6 where the proposal is still performing.



(a) Velocity estimation with GPS signal.



(b) Position estimation with GPS signal.



(c) Orientation estimation with magnetometer signal.

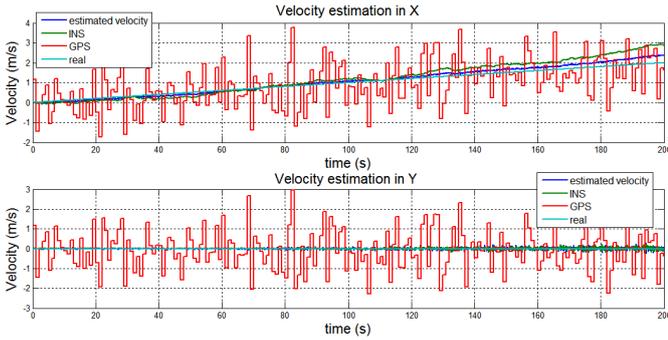
Fig. 4. Velocity, position and attitude estimations of the mobile when the system is aided with the GPS and magnetometer, in the case of a linear way.

## CONCLUSIONS

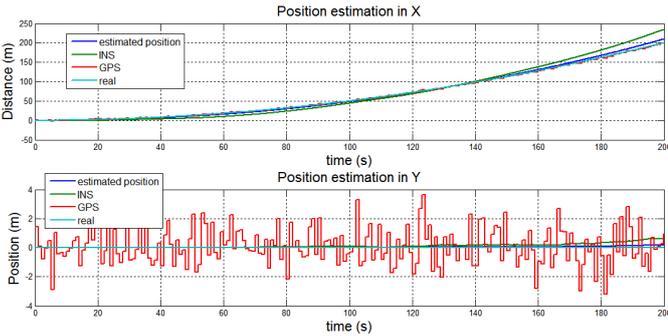
In this paper a simple nonlinear control law was designed to asymptotically stabilize the position of Wheeled Mobile Robots (WMRs) with nonholonomic constraints. Since input constraints exist in the actuators, the control law considers too the actuator saturation. Moreover, this work has presented a method for aiding the solution of an INS using vehicle model and nonholonomic constraints information. Simulation results show that the navigation system performance has been increased from un-aided system when vehicle model information is used. Real-time implementation will be pursued as a further work.

## ACKNOWLEDGMENT

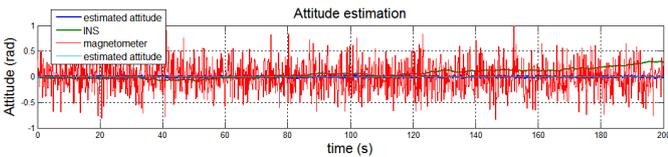
This work is part of a student exchange program between the Facultad de Ciencias de la Electrónica in BUAP (Puebla, Mexico) and GIPSA-lab, Control System Department (Grenoble, France). It is partially supported by *i*) CONACYT with scholarship number 252636351 *ii*) VIEP-BUAP under grant GUCJ-ING12-I.



(a) Velocity estimation with dynamic model information.



(b) Position estimation with dynamic model information.



(c) Orientation estimation with dynamic model information.

Fig. 5. Velocity, position and attitude estimations of the mobile when the system is aided with the information from the dynamic vehicle model, in the case of a linear way.

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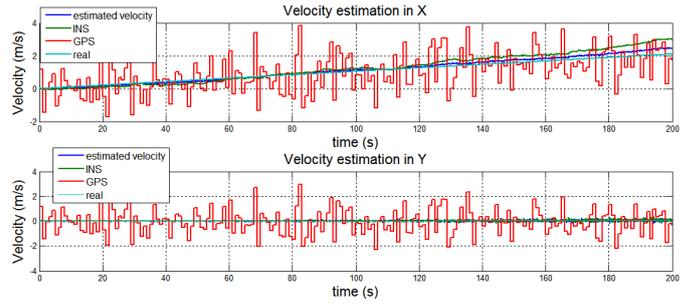
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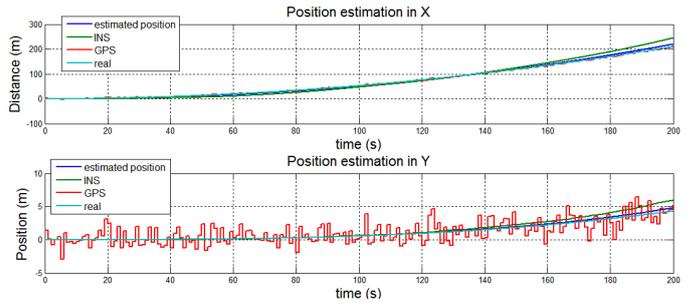
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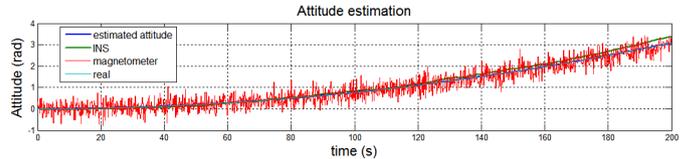
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(a) Velocity estimation with dynamic model information.



(b) Position estimation with dynamic model information.



(c) Orientation estimation with dynamic model information.

Fig. 6. Velocity, position and attitude estimations of the mobile when the system is aided with the information from the dynamic vehicle model, in the case of a circle way.

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