Sensor fault and unknown input estimation based on proportional integral observer applied to LPV descriptor systems

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Sensor Fault and Unknown Input Estimation Based on Proportional Integral Observer Applied to LPV Descriptor Systems.

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Abstract: In this paper a detection, isolation and fault estimation scheme using a Proportional-Integral (PI) Observer that provides a robust estimation against measurements noise and unknown inputs is presented. The main contribution consists in the synthesis of a PI-Observer for descriptor LPV systems, can be useful not only for state estimation, but also to estimate sensor faults and unknown inputs. The proposed PI-Observer can provide an alternative fault diagnosis scheme. Once a fault is detected, a bank of observers is activated for the purpose of fault isolation. This scheme reconstructs the sensor faults based on augmented state equations where an auxiliary state is assigned to represent the dynamic behavior of the fault. An illustrative example is presented in simulation.

Keywords: Proportional-Integral Observer, Descriptor Systems, Linear Matrix Inequalities (LMIs), Fault Diagnosis and Isolation (FDI).

1. INTRODUCTION

Recent researches in Fault Detection and Diagnosis (FDD) design schemes, especially model-based FDD, are focused on systems modeled by coupled differential and algebraic equations (DAE's). Descriptor systems constitute an important class of systems of both theoretical and practical interest, which include, chemical and biological processes, interconnected large-scale systems, electrical circuits systems and robotic manipulators, among others. In this way, much research has aimed at generalizing existing theories, especially in the time domain, from normal systems to descriptor systems, which include controllability and observability (Yip and Sincovec (1981)), feedback control (Jiang et al. (2009)), observer design (Koenig (2006)) and robust control (Yang et al. (2006)).

Linear Parameter Varying (LPV) approach, can be used to convert a nonlinear system into a multiple model form. The transformation is realized without loss of information. LPV systems can be seen as nonlinear systems that are linearized along a trajectory determined by the parameter vector that it is assumed measurable (Toth et al. (2009)). Many physical systems exhibit parameter variations due to non-stationary or nonlinear behavior, or dependence on external variables. For such processes, the theory of LPV systems offers an attractive modeling framework. In this way, some authors have proposed to represent the systems using the LPV approach (Cerone et al. (2010)).

In LPV systems, the model parameters are assumed as functions of a time-varying signal, the called *scheduling-variable* ($\varepsilon(\theta)$). Based on such representation some authors as Ichalal et al. (2009), have developed methods for state estimation and fault diagnosis in nonlinear systems described by Takagi-Sugeno multiple models. Or as in Hamdi et al. (2009), where a polytopic Unknown Input Observer for LPV descriptor systems is designed to estimate states of the system in presence of unknown inputs. In the work of Astorga et al. (2009), a strategy for detection and isolation sensor faults using a bank of residuals within a Generalized Observer Scheme (GOS) is presented. This strategy for fault estimation doesn't take into account the effects of disturbances and other uncertain factors.

Starting from LPV descriptor representation of the nonlinear systems, the present paper proposes sufficient conditions for the design of a Proportional-Integral Observer (PIO) that estimates the states of the system and the fault, even in the presence of unknown inputs. In contrast to Hamdi et al. (2011), who only consider constant actuator faults and disturbances with slow variation, we consider here dynamic sensor faults in spite of unknown inputs which extends the range of possible scenarios in a real plant.

The model for sensor faults presented in this paper uses an increase in the system equations. This increase is an auxiliary state (or more in case of multiple faults) that represents the dynamic fault and in effect, converts the sensor fault as an actuator fault, which enables to estimate the fault as an new state. This paper presents a fault diagnosis scheme based on GOS for incipient or abrupt faults in descriptor systems, which is robust against unknown inputs because the disturbance distribution matrices are considered known, this fact allows its uncoupling. This observer is characterized by a integral term on the output error to estimate the unknown input. The stability and the convergence properties are ensured by using Linear Matrix Inequality (LMI's). The method used to model sensor faults, is based on the work of Park et al. (1994), where the sensor faults are presented as pseudo-actuator faults.

2. LPV DESCRIPTOR SYSTEMS

Consider the following continuous-time nonlinear descriptor system:

$$\begin{aligned} E\dot{x} &= F(x(t), u(t), d(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ with $(m < n)$ is the control input vector. $y(t) \in \mathbb{R}^p$ and $d(t) \in \mathbb{R}^l$ are the measurement output and the unknown bounded process disturbance vectors. $E \in \mathbb{R}^{q\times n}$ is a singular matrix with $rankE \leq n$ and $F(\cdot)$ is a smooth function. The linearization of the function $F(\cdot)$ by Taylor series around ε_i operation points (x_i, u_i) gives a set local linear descriptor models with the following descriptor linear system representation (Hamdi et al. (2010)):

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) + Rd(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

where $E, A \in \mathbb{R}^{q\times n}$, $B \in \mathbb{R}^{k\times n}$, $R \in \mathbb{R}^{l\times n}$ and $C \in \mathbb{R}^{m\times n}$ are known constant matrices. With the following assumptions as in Darouach et al. (1996):

Assumption 1: It is assumed that the triplet (E, A, C) is observable if:

$$\text{rank} \begin{bmatrix} sE - A \\ C \end{bmatrix} = n \qquad \forall s \in \mathcal{C}$$

where $\mathcal{C}$ is the set of complex numbers.

Assumption 2: It is assumed that the impulsive terms of the system are observable (the triplet (E, A, C) is Impulse-Observable) if:

$$\text{rank} \begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = n + rank(E)$$

As is proposed in Hamdi et al. (2010), (1) can be represented by the polytopic LPV descriptor structure as follows:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{Q} \varepsilon_i(\theta(t)) \left[A_i x(t) + B_i u(t) + R_i d(t) + \Delta x_i\right] \\ y(t) &= Cx(t) \end{aligned} \tag{3}$$

where $E, A_i \in \mathbb{R}^{q\times n}$, $B_i \in \mathbb{R}^{k\times n}$, $R_i \in \mathbb{R}^{l\times n}$ and $C \in \mathbb{R}^{m\times n}$ are known constant matrices. Δx_i represents the contribution of small signals due to the linearization of the system, i.e., is a vector that depends of the $i^{th}th$ operating point. This representation is defined as a multi-linear system where the matrices are set by known operation points.

LPV descriptor system is represented into a polytopic domain. The vertices of this polytope are called the submodels of this representation, i.e., the parameter $\theta(t)$ varies in a convex polytope of vertices θ_i such that $\theta(t) \in Co\{\theta : \theta_1, \theta_2, ..., \theta_Q\}$ with Q as the total number of local models. These submodels are combined by weighing functions to yield a global model. With respect to LPV systems, an additional assumption is cited:

Assumption 3: Let us consider that:
i). $\theta(t)$ is bounded, and
ii). $\theta(t)$ is on-line accessible and fault-free.

$\varepsilon_i(\theta(t))$ is the weighting function that defines the relative contribution of each local model to build the global model. This weighting function is constructed in such a manner that complies with follow property of the convex sum:

$$\sum_{i=1}^{Q} \varepsilon_i(\theta(t)) = 1, \;\; 0 \leq \varepsilon_i(\theta(t)) \leq 1 \tag{4}$$

This polytopic structure makes it possible to represent any nonlinear behavior. The next section is dedicated to develop a method to detect and estimate the sensor fault in order to provide an efficient monitoring tool in the operator's decision.

3. REPRESENTATION OF THE SENSOR FAULTS

Let us consider the sensor fault $f_s(t)$ and the disturbances $d(t)$ given by:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{Q} \varepsilon_i(\theta(t)) \left[A_i x(t) + B_i u(t) + R_i d(t) + \Delta x_i\right] \\ y(t) &= Cx(t) + Jf_s(t) \end{aligned} \tag{5}$$

where $f_s(t) \in \mathbb{R}^{n_f}$ is a fault vector which represents the evolution of the fault and J represents the sensor fault distribution matrix. Let us consider $f_s(t) = \beta(t_f)f(t)$ characterizes the time profile of a fault that occurs at some unknown time t_f. More specifically, $\beta(t_f)$ is a smooth function modeled by:

$$\beta(t - t_f) = \begin{cases} 0, & t < t_f \\ 1 - e^{-\varsigma(t - t_f)}, & t \geq t_f \end{cases} \tag{6}$$

where the scalar $\varsigma > 0$ denotes the unknown fault evolution rate. Small values of ς characterize slowly developing faults, also known as *incipient faults*. For large values of ς, the time profile approaches a step function, which models *abrupt faults*. Based on Park et al. (1994), the LPV descriptor system (5) can be represented such as:

$$\begin{aligned}\bar{E}\dot{\bar{x}}(t) &= \sum_{i=1}^{Q} \varepsilon_i(\theta(t)) \left[\bar{A}_i\bar{x}(t) + \bar{B}_i\bar{u}(t) + \bar{R}_i d(t) + \Delta\bar{x}_i\right] \\ y(t) &= \bar{C}\bar{x}(t)\end{aligned} \tag{7}$$

where $\bar{x}(t) \in \mathbb{R}^{n+n_f}$ is the augmented state vector defined as $\bar{x}(t) = \begin{bmatrix} x(t) \\ f_s(t) \end{bmatrix}$, the control vector is defined as $\bar{u} = [\, u \ \xi \,]$ and the matrices are given by:

$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & I_{n_f} \end{bmatrix} \quad \bar{C} = [\, C \ J \,] \quad \bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & \alpha_{n_f} \end{bmatrix}$$

with a sensor input defined as: $\xi = \dot{f}_s - \alpha f_s$, and

$$\bar{B}_i = \begin{bmatrix} B_i & 0 \\ 0 & I_{n_f} \end{bmatrix} \quad \bar{R}_i = \begin{bmatrix} R_i \\ 0 \end{bmatrix} \quad \Delta\bar{x}_i = \begin{bmatrix} \Delta x_i \\ 0 \end{bmatrix}$$

where 0 is a zero matrix of appropriate dimension. The term α_{n_f} represents a diagonal matrix of appropriate dimension where $\alpha > 0$ can be considered as an additional degree of freedom in the observer design. The fault can be modeled by a linear system of arbitrary order, but this fact depends of the characteristics of the fault.

It is a well-known result that the necessary and sufficient condition for detectability, is that the fault event vector is detectable, i.e., if and only if $(\bar{A}_i, \bar{C})$ is observable (Park et al. (1994)). In addition, this requirement may be assumed in terms of the original system, i.e., if (A_i, C) of the system (5) is observable.

4. PROPORTIONAL INTEGRAL OBSERVER (PIO)

The fault reconstruction is achieved considering a descriptor system represented by the polytopic LPV descriptor model (7), the equations of the PIO are (Hamdi et al. (2011)):

$$\begin{aligned}\dot{z}(t) &= \sum_{i=1}^{Q} \varepsilon_i(\theta(t)) \left[N_i z(t) + G_i\bar{u}(t) + L_i y(t) + H_i\hat{d}(t) + \Delta z_i\right] \\ \hat{\bar{x}}(t) &= z(t) + \tilde{M}y(t) \\ \dot{\hat{d}}(t) &= \sum_{i=1}^{Q} \varepsilon_i(\theta(t))\Phi_i(y(t) - \hat{y}(t))\end{aligned} \tag{8}$$

where $\hat{\bar{x}}(t), z(t) \in \mathbb{R}^{n+n_f}$ and $\hat{d}(t) \in \mathbb{R}^p$ are the estimate state vector, state observer vector and estimate unknown input respectively. $N_i, G_i, L_i, H_i, \Delta z_i, \tilde{M}$ and Φ_i are matrices for the PI observer. From (7) the estimation error is equal to:

$$\begin{aligned}\bar{e}(t) &= \bar{x}(t) - \hat{\bar{x}}(t) \\ \bar{e}(t) &= (I_{n+n_f} - \tilde{M}\bar{C})\bar{x}(t) - z(t)\end{aligned} \tag{9}$$

where I_{n+n_f} represents the identity matrix of order $n+n_f$, then on can define a real matrix $U \in \mathbb{R}^{(n+n_f)\times(n+n_f)}$ calculated as:

$$U\bar{E} = I_{n+n_f} - \tilde{M}\bar{C} \tag{10}$$

and, for $[\bar{E}\ \bar{C}]^T$ as full rank column, is possible to calculate U and $\tilde{M}$, such that:

$$\left[\, U \ \tilde{M} \,\right] = \begin{bmatrix} \bar{E} \\ \bar{C} \end{bmatrix}^+ \tag{11}$$

where the superscript $^+$ represents the inverse generalized matrix, and the estimation error can be rewritten as:

$$\bar{e}(t) = U\bar{E}\bar{x}(t) - z(t) \tag{12}$$

A restriction of the PIO is to suppose that the unknown inputs are bounded and their dynamic is slow, i.e., $\dot{d}(t) \simeq 0$. Then, for $\delta(t) = d(t) - \hat{d}(t)$, the unknown input derive is defined as:

$$\dot{\delta}(t) = -\dot{\hat{d}}(t) \tag{13}$$

The estimation error dynamic is written as:

$$\begin{aligned}\dot{\bar{e}}(t) = \sum_{i=1}^{Q} \varepsilon_i(\theta(t)) \big[&(U\bar{A}_i - L_i\bar{C} - N_iU\bar{E})\bar{x}(t) + \\ &(U\bar{B}_i - G_i)\bar{u}(t) + (U\bar{R}_i - H_i)d(t) + \\ &(U\Delta\bar{x}_i - \Delta z_i) + H_i\delta(t) + N_i\bar{e}(t)\big]\end{aligned} \tag{14}$$

where the following conditions can be defined:

$$U\bar{A}_i = N_iU\bar{E} - L_i\bar{C} \tag{15}$$

$$G_i = U\bar{B}_i \tag{16}$$

$$H_i = U\bar{R}_i \tag{17}$$

$$\Delta z_i = U\Delta\bar{x}_i \tag{18}$$

From (3), (13) and (14), the estimation error and the unknown input dynamic is:

$$\dot{\bar{e}}(t) = \sum_{i=1}^{Q} \varepsilon_i(\theta(t))(N_i\bar{e}(t) + H_i\delta(t)) \tag{19}$$

$$\dot{\delta}(t) = \sum_{i=1}^{Q} \varepsilon_i(\theta(t))(-\Phi_i\bar{C})\bar{e}(t) \tag{20}$$

and the following function can be established:

$$\begin{bmatrix} \dot{\bar{e}}(t) \\ \dot{\delta}(t) \end{bmatrix} = \sum_{i=1}^{Q} \varepsilon_i(\theta(t)) \begin{bmatrix} N_i & H_i \\ -\Phi_i\bar{C} & 0 \end{bmatrix} \begin{bmatrix} \bar{e}(t) \\ \delta(t) \end{bmatrix} \tag{21}$$

Then, the state estimation error (21) converges asymptotically to zero if the real part of the eigenvalues of $\begin{bmatrix} N_i & H_i \\ -\Phi_i\bar{C} & 0 \end{bmatrix} < 0$, i.e., are stables. From (10) and (15), the matrices N_i, can be determined as follows:

$$N_i = U\bar{A}_i - (L_i - N_i\tilde{M})\bar{C} \tag{22}$$

and defining $K_i = L_i - N_i\tilde{M}$, the matrices L_i can be calculated. Then, (21) can be rewritten as:

$$\begin{bmatrix} \dot{\bar{e}}(t) \\ \dot{\delta}(t) \end{bmatrix} = \sum_{i=1}^{Q} \varepsilon_i(\theta(t))(\breve{A}_i - \breve{K}_i\breve{C}) \begin{bmatrix} \bar{e}(t) \\ \delta(t) \end{bmatrix} \tag{23}$$

where $\breve{A}_i = \begin{bmatrix} U\bar{A}_i & H_i \\ 0 & 0 \end{bmatrix}$, $\breve{K}_i = \begin{bmatrix} K_i \\ \Phi_i \end{bmatrix}$ and $\breve{C} = [\bar{C}\ \ 0]$.

Then, the PI observer (8) for a LPV descriptor system with inputs unknown (3) exists and their estimation error converges asymptotically to zero, if and only if, the pairs $(\breve{A}_i, \breve{C})$ are detectable $\forall i = 1, 2, ..., Q$. This observer is asymptotically stable if exists a positive definite symmetric matrix P and matrices $W_i = P\breve{K}_i$ such that the following LMI holds:

$$(\breve{A}_i^T P + P\breve{A}_i - \breve{C}^T W_i^T - W_i\breve{C}) < 0, \ \ \forall i \in 1, 2, ..., Q. \tag{24}$$

Observer gains can be calculated from $\breve{K}_i = P^{-1}W_i$. For ensuring the stability and convergence of the observation error, it is possible to define in the left part of the complex plane a bounded area S with a line of abscissa $(-\sigma)$ where $\sigma \in \mathbb{R}^+$, and the LMI's defined in (24) must be replaced by the following inequalities:

$$\begin{aligned}&(\breve{A}_i^T P + P\breve{A}_i - \breve{C}^T W_i^T - W_i\breve{C}) + 2\sigma P < 0,\\ &\forall i \in 1, 2, ..., Q.\end{aligned} \tag{25}$$

then, consequently $\hat{\bar{x}}(t)$ will asymptotically converge to $\bar{x}(t)$ and $\hat{d}(t)$ to $d(t)$.

5. ILLUSTRATIVE EXAMPLE

Consider a continuous-time LPV descriptor nonlinear system (1), described by:

$$\begin{aligned}&\dot{x}_1(t) = -1.5x_1^2(t) + 0.2x_3(t)x_4(t) + d(t)\\ &\dot{x}_2(t) = -u_1(t)x_1^2(t) - x_4(t)x_3^2(t) - 0.5x_2(t)\\ &0 = 0.5x_2(t) - x_3(t) + 0.2x_4(t)\\ &0 = -x_2^2(t) + x_3^2(t) - 2x_4(t) + u_2(t)\\ &y(t) = \begin{bmatrix} 1 & 1 & 0 & 1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix} x(t) + v_k,\end{aligned} \tag{26}$$

where $u_1(t)$ and $u_2(t)$ are constant signal of magnitude 10 and 7 respectively. $v_k \in \mathbb{R}^p$ is the measurement noise that is modeled as Gaussian white noise with variance center 0.01. For simulation purposes, an unknown input is modeled as a rectangular signal applied for $10 \leq t \leq 25$. LPV multi-model representation of the nonlinear dynamic system is given by the follow set of matrices:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 0 & 1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}, R_i = R = \begin{bmatrix} 1\\ 0\\ 0\\ 0\end{bmatrix}$$

$$A_1 = \begin{bmatrix} -0.8775 & 0 & 0.526 & -0.0274\\ -5.8500 & -0.5 & 0.1481 & 0.0026\\ 0 & 0.5 & -1 & 0.2\\ 0 & 2.6522 & -0.274 & -2\end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0\\ -0.0856 & 0.01\\ 0 & 0\\ 0 & 1\end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.6375 & 0 & 0.6166 & 0.0307\\ -4.2500 & -0.5 & 0.2176 & -0.0036\\ 0 & 0.5 & -1 & 0.2\\ 0 & 1.8526 & 0.3068 & -2\end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0\\ -0.0452 & 0.01\\ 0 & 0\\ 0 & 1\end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.6357 & 0 & 0.6047 & 0.0226\\ -4.2380 & -0.5 & 0.1162 & -0.0015\\ 0 & 0.5 & -1 & 0.2\\ 0 & 1.9660 & 0.2264 & -2\end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 & 0\\ -0.0449 & 0.01\\ 0 & 0\\ 0 & 1\end{bmatrix}$$

$$\Delta x_1 = \begin{bmatrix} 0.2004\\ 1.6753\\ 0\\ 1.7397\end{bmatrix}, \quad \Delta x_2 = \begin{bmatrix} -0.0268\\ 0.8083\\ 0\\ 0.8346\end{bmatrix}, \quad \Delta x_3 = \begin{bmatrix} -0.0011\\ 0.8505\\ 0\\ 0.9535\end{bmatrix}$$

The weighting functions are calculated according to (4), $\forall i = 1, \ldots, Q$ with Q as the total number of local models considered, in this case $Q = 3$. The weighting functions $\varepsilon_i(\theta(t))$ characterize the dynamic behavior of the descriptor nonlinear system and its evolution depends of parameters that are functions of the state $x_3(t)$. These functions are defined as:

$$\varepsilon_i(t)(x_3(t)) = \frac{\mu_i(x_3(t))}{\sum_{i=1}^{3}\mu_i(x_3(t))} \tag{27}$$

Assumption 4: The state $x_3(t)$ is defined as free of faults and always measurable.

The parameters trajectory is determined by the behavior of the system variables as:

$$\begin{aligned}\mu_1(x_3(t)) &= \exp\left(-\frac{1}{2}\left(\frac{x_3(t)+5}{2}\right)^2\right)\\ \mu_2(x_3(t)) &= \exp\left(-\frac{1}{2}\left(\frac{x_3(t)}{2}\right)^2\right)\\ \mu_3(x_3(t)) &= \exp\left(-\frac{1}{2}\left(\frac{x_3(t)-5}{2}\right)^2\right)\end{aligned} \tag{28}$$

5.1 Fault-free case.

For initial conditions, $x^0 = [0.4 \quad -0.004 \quad 0.755 \quad 3.785]^T$ and $\hat{x}^0 = [0 \quad 0 \quad 0 \quad 0]^T$, the nonlinear states and its estimated are depicted in the Fig. 1 in fault free case. In this figure the quick convergence of the observer to LPV descriptor nonlinear system can be seen.

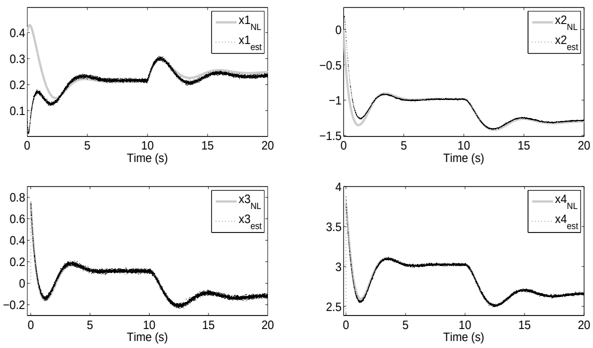

Fig. 1. States of nonlinear system and their estimates.

5.2 Sensor-fault case.

A vector $f_s(t) \in R^{n_f}$ with $n_f = 1$ and $\beta(t - t_f) = 1 - e^{\varsigma(t-t_f)}$ with $\varsigma = 0.01$, is applied for y_2 in time $21 \leq t \leq t_f$ and $f_s(t) = (2sin(5t - 10))$. To design the observer for the LPV descriptor system (5) the augmented system should be constructed as (7). Then, the matrices U and $\tilde{M}$ can be calculated by (11) such that $U\bar{E} + \tilde{M}\bar{C} = I_{n+n_f}$. Using the Yalmip Toolbox (Lofberg (2004)) or the LMI Control Toolbox (Gahinet et al. (1995)), it is possible to compute a solution satisfying the inequality (25), and to determine the observer matrices K_i and Φ_i respectively:

$$K_1 = \begin{bmatrix} 1.5644 & 4.6018 & -1.110\\ 31.943 & 6.6880 & 43.336\\ -2.219 & -4.373 & 0.7062\\ -31.15 & -14.15 & -36.61\\ 44.68 & 27.736 & 46.799\end{bmatrix} \quad \Phi_1 = \begin{bmatrix} -0.8181 & 1.4884 & -2.1623\end{bmatrix}$$

$$K_2 = \begin{bmatrix} 1.1564 & 3.9228 & -0.634 \\ 33.144 & 8.3304 & 41.232 \\ -1.866 & -3.833 & 0.3387 \\ -31.46 & -14.40 & -35.62 \\ 44.475 & 27.167 & 46.192 \end{bmatrix} \quad \Phi_2 = \begin{bmatrix} -0.9540 & 1.2413 & -1.9486 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 1.1362 & 3.8887 & -0.616 \\ 33.326 & 8.4962 & 41.215 \\ -1.846 & -3.802 & 0.3169 \\ -31.59 & -14.50 & -35.64 \\ 44.592 & 27.229 & 46.249 \end{bmatrix} \quad \Phi_3 = \begin{bmatrix} -0.9692 & 1.2212 & -1.9514 \end{bmatrix}$$

In sensor fault case, the states and its estimated are depicted in the Fig. 2. In this figure only the state x_3 is well estimated, this is because it is the only measurement that is considered not affected by the fault. For simulation purposes, an unknown input is modeled as a rectangular signal applied for $10 \leq t \leq 25$. The estimation of the unknown input is illustrated in the Fig. 3.

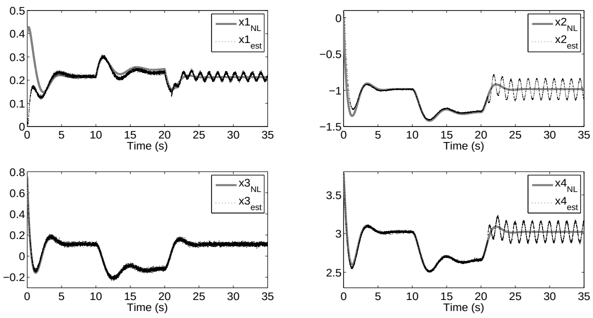

Fig. 2. States of nonlinear system and their estimates.

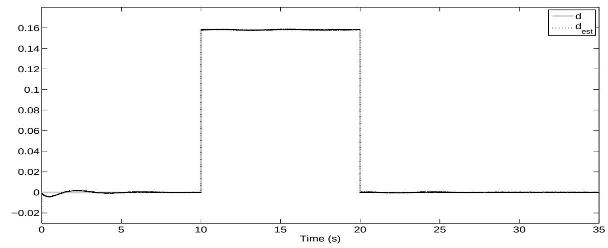

Fig. 3. Unknown input estimated (disturbance).

From the procedure of augmented system, given in the Section 3, is possible to use the PIO for estimate the dynamic behavior of the sensor fault as an additional state. Fig. 4 shows the signal of the sensor fault and their estimation.

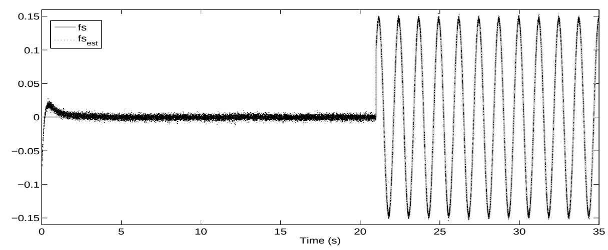

Fig. 4. Sensor fault and corresponding estimated.

For the purpose of fault isolation, a bank of observers is built as a generalized LPV descriptor observer scheme (based on GOS Frank (1994)). This scheme provides an estimator dedicated to a certain sensor is driven by all outputs except that of the respective sensor, i.e., where each one of the them is driven by all inputs and all outputs except the k^{th} measurement variable. The measure y_k is not used in the k observer due to the fact that y_k is assumed corrupted. This scheme allows one to detect and isolate only a single fault in any of the sensors, however, with increased robustness with respect to unknown inputs. For the bank of observers, the following LPV descriptor system in considered:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{M} \varepsilon_i(\theta(t)) \left(A_i x(t) + B_i u(t) + R_i d(t) + \Delta x_i\right) \\ \zeta_k(t) &= \tilde{C}_j x(t) + \tilde{J}_j f_s(t) \end{aligned} \tag{29}$$

with $\tilde{C}_j$ and $\tilde{J}_j$ are the matrix and sensor fault distribution vector respectively, without the k^{th} row. The bank of observer generates an incidence matrix (Table 1) where a signal that is obtained from the residuals defining the effects associated with the fault.

Table 1. Incidence Matrix

Fault	F_1	F_2	F_3
$\|\|y-\hat{y}\|\|$	1	1	1
$\|\|\zeta_1-\hat{\zeta}_1\|\|$	0	1	1
$\|\|\zeta_2-\hat{\zeta}_2\|\|$	1	0	1

The bank generates residuals different to zero, otherwise, only the observer which is insensitive to a sensor fault F_k effects generates a unique residual with a media zero. The performance of the fault diagnosis system is illustrated for faults on the sensors y_1 and y_2, the sensor y_3 is considered free of faults. Figs. 5-6, shows the results of the bank observers according to the incidence matrix, only the observer designed insensitive to a sensor fault provides a residual vector equal to zero means. Only a single sensor fault can be detected at a time.

This approach mainly considers the case of sensor fault. It is possible also to consider the actuator fault case, but it is important that into the LPV model of the system, the parameters considered to build the weighting functions cannot depend of the inputs of the system or any dependent variable of the inputs, i.e., the parameters should be free faults according to assumption A3. If this is the case, actuator faults can be estimated also using the same PI-Observer if they are modeled as unknown inputs of the system.

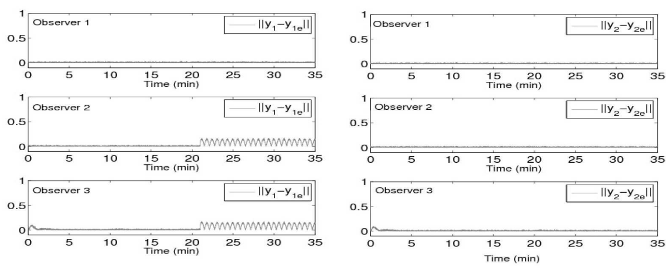

Fig. 5. Residual vector with a fault in the first sensor.

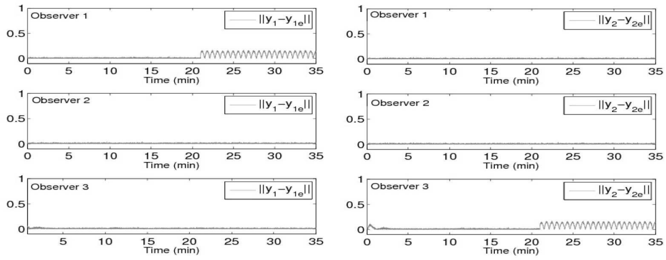

Fig. 6. Residual vector with a fault in the second sensor.

6. CONCLUSIONS

This paper has presented a sensor fault diagnosis method to detect and isolate sensor faults in LPV descriptor systems. This proposed method is easy to implement because only matrix manipulations were made to represent the system and its dynamics completely (states, unknown inputs and faults) into a LPV model. This allows the observer designed based on the LPV model, to provide reliable information on the status of the process. Through PIO approach, the error dynamics converges to zero, as well as the sensor faults $f_s(t)$ and the unknown inputs are estimated simultaneously. The diagnosis scheme is based on a PIO presented in Hamdi et al. (2011), who only considered constant actuator faults, while we considered here dynamic sensor faults in spite of unknown inputs. Conditions to ensure the existence and the stability of the proposed scheme by using LMI formulation were established. The PIO was designed to estimate simultaneously the sensor fault and the unknown input and also, it was used to build a bank of observers to isolate adequately sensor faults. The effectiveness of this algorithm was evaluated via simulations using a numerical example.

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