# Similarity analysis of a glottal-like jet 

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#### Abstract

This work discloses similarity properties of a high aspect ratio pulsating jet with lateral confinement exhibiting axis switching in a region close to the jet exit. The analysis is conducted separately within the major and minor planes of the jet source. The similarity relations derived from the in-plane flow equations are combined with experimental data collected in a recent threedimensional velocimetry study of a glottal-like jet. Empirically non-collapsing profiles are shown to rescale as collapsing profiles in the similarity variables within ranges of the streamwise coordinate that are determined by the analysis. The similarity properties are shown to be useful to make predictions on the jet axis switching characteristics.


Keywords glottal flow • self-similarity • high aspect ratio jets • axis switching

## 1 Introduction

Glottal flow is a pulsating jet with lateral confinement. Pulsation is produced by the flow-induced motion of the vocal folds which open and close repetitively during voice production, defining a time-varying jet exit section with a lenslike shape. Confinement is introduced by the presence of the vocal tract, which

[^0]extends from the glottis to the mouth, inhibiting the phenomenon of flow entrainment from the atmosphere as observed in free jets.

The glottal jet presents a number of characteristics that may be of interest for biomechanical modeling and other applications, such as the development of bio-inspired devices for flow control purposes. Recent studies [22,13] have stressed its three-dimensional axis switching properties: the jet has a crosssection evolution in which the minor and major axes switch identity as the downstream distance from the jet exit increases. This study deals with selfsimilarity properties that are found to hold locally, including the zone where axis switching occurs.

Why and how self-similarity properties can be exploited is thoroughly discussed in [5]. When the equations governing a phenomenon present a certain invariance, the invariance is also present in the solution, which can be expressed in terms of a reduced set of combination of the basic variables. The variables in this set are called similarity variables, and the analysis leading to them is called a similarity analysis. Once the similarity variables are known, alternative techniques such as numerical analysis can be applied more effectively and with a better basic understanding of the problem. Because the invariance requirements inevitably force the suppresion of certain parameters of the real problem, any claims for the generality of a similarity property must be tempered by comparison with experiment.

In the field of flow control, self-similarity has been reported both for continuous and synthetic jets [2]. Similarity analyses have been provided for continuous turbulent round jets, for instance in [4], and for elongated synthetic jets [14]. The elongated jet in [14] is modeled as a continuous planar jet without special consideration of the non planar properties associated to the axis switching phenomenon. This work presents a similarity analysis for a glottal-like jet taking into account its three-dimensional nature, and using experimental observations as guide.

Time-averaged profiles documented in [13] are such that a simple rescaling of the streamwise velocity yields a collapse onto a single curve for a wide range of streamwise distances. Since self-similarity is mostly deduced by the good collapse of mean velocities and Reynolds stresses, results in [13] suggest that a similarity analysis may lead to interesting results for the glottal jet.

Self-similarity can be defined in more than one way, including different types and degrees [7]. A strong kind of self-similarity, called self-preservation, is said to occur when a single length and velocity scale is sought to bring into congruence all the velocity profiles (or other quantities). A weaker type of self-similarity is said to occur if every dependent variable is allowed to have its own scale, with each scale evolving downstream in a manner that is determined by the equations of motion. In such cases, self-similarity results from the constraint that all terms in the governing equations should evolve in the same manner [8]. As in [4], this work will adopt the less restrictive definition.

For the high-aspect-ratio geometry of our case study, it is convenient to restrict the similarity analysis to certain privileged (symmetry) planes that
coincide, physiologically, with the mid-sagittal and mid-coronal planes. We introduce the term in-plane similarity to refer to cases in which self-similarity occurs at least within certain flow planes. This analysis discloses normalization scalings leading to the collapse of all the mean velocity components and of the mean Reynolds stresses profiles of the glottal jet data. In addition, it provides relationships indicating the different self-similar ranges and predicting characteristics of the axis switching phenomenon.

The article is organized as follows. Section 2 presents the source experiment and the particular features of the generated glottal jet, which pose the constraints of the similarity analysis that is undertaken. Section 3 derives the governing equations for a high-aspect-ratio jet in the two planes of symmetry of the flow in the assumption of self-similarity. Section 4 combines the experimental data with the scaling relations derived from the flow equations. Section 5 discusses how the similarity properties condition the axis switching phenomenon. Conclusions of the analysis are drawn in Section 6.

## 2 Glottal Flow

During voice production, the glottal source has the shape of an elongated orifice with sharp corners at the commissures, that presents a time-varying crosssection as the vocal folds oscillate. Vocal-fold vibration can include complete or incomplete glottal closure. If there is a phase of complete glottal closure, the airflow during this phase is interrupted. In the open phase, a high-aspect-ratio jet is formed that is laterally confined due to the presence of the vocal tract. For experimental characterization with velocimetry techniques, the glottal jet can be generated using a physical model of the vocal folds.

The experimental data used as guide for this work were obtained in a Stereoscopic Particle Image Velocimetry (SPIV) experiment presented in [13]. The jet in this study was generated with a life-size vocal fold model based on a design developed in $[19,18]$. A sketch of the experimental setup is given in Fig 1. During the experiment, the vocal-fold valve oscillated with a fundamental frequency of $f_{0} \sim 55 \mathrm{~Hz}$, a mean subglottal overpressure of $15 \mathrm{~cm} \mathrm{H}_{2} \mathrm{O}$, a mean flow rate of $5.5 \times 10^{-4} \mathrm{~m}^{3} / \mathrm{s}$, an open quotient of about 0.75 and a reference jet velocity at the flow exit of $U_{0} \sim \sqrt{2 \Delta p / \rho} \sim 52 \mathrm{~m} / \mathrm{s}$.

The SPIV measurements were conducted with the vocal fold valve attached to a uniform square-sectioned duct ( $20 \mathrm{~mm} \times 20 \mathrm{~mm} \times 17 \mathrm{~cm}$ ). The inspected flow region was 20 mm long and was chosen to lie as close as possible ( $x=$ 6 mm ) to the valve exit ( $x=0 \mathrm{~mm}$ ). Time averaged flow fields are the mean of the 1000 instantaneous velocity fields captured for each of the 9 coronal ( $x y$ ) slices throughout the SPIV experiment - data acquisition lasted 200 seconds per slice.

Previous studies regarding high-aspect-ratio jets, whether pulsating or nonpulsating [ $10,11,6,20,12,16,15]$, generally include a check for the collapse of the time-averaged streamwise velocity profile in the minor plane. These studies, however, use a single length and velocity scale in the normalization of


Fig. 1: Experimental setup used to generate the glottal jet (a), and close-up of artificial vocal folds at plane $x=0$ defining the open/closed glottal outlet.
the profiles, as mentioned in our reference to self preservation. Two remarks are necessary regarding these studies. On one hand, mean velocities are poor indicators of self-similarity, while other quantities, such as Reynolds stresses, provide a far more sensitive indication of the self-similar nature of a wide range of flows cf. [23,3,6]. On the other hand, in the absence of an analysis leading to plane-dependent length scales, no property of self-similarity comes out at all. This work will show that further insight on the similarity properties of this type of jet can be gained combining the in-plane flow equations with the experimental profiles of the three components of the mean velocity and Reynolds stresses.

An analytical derivation of the self-similarity from the equations of motion and conservation of momentum is seldom possible for complex flows [6]. Restricting the study to certain privileged planes will allow us to overcome this limitation for the investigated jet. Complexity in the glottal jet is associated to three aspects. The first one is geometrical in nature, the second one re-
gards the temporal evolution of the jet and the third one is associated to the boundary conditions. Let us make a brief review of how self-similarity may be affected by these flow features.
(a) Local Reynolds number:

Self-similarity is expected to occur differently if the local Reynolds number of the flow varies or not with downstream distance. The planar and the axisymmetric jet are examples of these two categories respectively. The glottal jet is neither axisymmetric nor planar: it is an example of local-Reynoldsvarying flow, but with different trends in the $y$ and $z$ directions. Being an axis-switching flow, the time-averaged local length scales are different in the major/minor axis. Local Reynolds increases with $x$ in the $y$ direction and decreases with $x$ in the $z$ direction.
(b) Time-varying nozzle:

The glottal outlet vibrates with an open and a close phase. The duration of the open phase with respect to the cycle duration is termed open quotient. During the open phase, the outlet presents a time-varying cross section, with an aspect ratio that is also time-varying. Jet aspect ratio may play an important role in determining, for instance, the switching location of an axisswitching jet [9]. In line with this trend, the three-dimensional reconstruction in [13] shows that the switching location of the glottal jet exhibits an evolution which varies in time in a manner that seems to follow the time-dependence of the jet aspect ratio. This suggests that the glottal jet keeps memory of the time-varying jet exit conditions, at least in the axis-switching region. A similarity analysis of the glottal jet in this region, even if conducted on the time-averaged fields, should comply with a non loss of memory of the averaged jet exit conditions.
(c) Boundary conditions:

In the jet developing region examined here, and further downstream, the glottal jet is laterally confined by the walls of the vocal tract. Confinement is known to affect the centerline position, the expansion rate and the stability of a jet $[1,17]$. In unconfined jets, the net rate of mass flow increases with downstream distance due to flow entrainment, thus favoring self-similarity. In laterally confined jets, the absence of this effect is expected to reinforce the jet memory of the source conditions.

The next section shows that an analytical treatment is possible conducting the analysis in the two planes where the similarity equations can be simplified invoking symmetry arguments. Taking up the term used in the introduction, the similarity analysis restricted to these symmetry planes is hereafter termed in-plane similarity.


Fig. 2: Symmetry planes considered in the derivation of the similarity equations of the jet: plane $(z=0)$, also called mid-coronal plane, and plane $(y=0)$ or mid-sagittal plane.

## 3 In-plane Similarity Equations

In this section we derive similarity equations of a high-aspect-ratio jet in the minor and major planes of the jet exit. To do this, we follow firstly the approach in [21] and secondly the approach in [7]. The starting point is to write out the standard Reynolds Decomposition for the time-averaged velocity in the two symmetry planes sketched in Fig 2. The standard Reynolds Decomposition is typically considered for turbulent flows, but has been previously used for cyclic jet flows $[2,14]$.

If $(U, V, W)$ denote the time-averaged flow velocities in $(x, y, z)$, we may assume that $W=0$ for plane $(z=0)$ and that $V=0$ for plane $(y=0)$. It is also assumed that the viscous terms can be dropped. If $P$ is the pressure and $\rho$ the density, the time-averaged streamwise momentum equation in these planes can be written as follows:

$$
\begin{align*}
(z=0) \quad U \frac{\partial U}{\partial x} & +V \frac{\partial U}{\partial y}+\frac{1}{\rho} \frac{\partial P}{\partial x}+  \tag{1}\\
& +\frac{\partial<u^{2}>}{\partial x}+\frac{\partial<u v>}{\partial y}+\frac{\partial<u w>}{\partial z}=0 \\
(y=0) \quad U \frac{\partial U}{\partial x} & +W \frac{\partial U}{\partial z}+\frac{1}{\rho} \frac{\partial P}{\partial x}+  \tag{2}\\
& +\frac{\partial<u^{2}>}{\partial x}+\frac{\partial<u v>}{\partial y}+\frac{\partial<u w>}{\partial z}=0
\end{align*}
$$

with the continuity equation:

$$
\begin{equation*}
\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0 \tag{3}
\end{equation*}
$$

Estimating the orders of magnitude of the various terms, it is possible to obtain equations relating the velocity scales $\left(U_{s}, V_{s}, W_{s}\right)$ with the length scales ( $L_{x}, L_{y}, L_{z}$ ) and the fluctuating components of the velocities $\left(u_{s}, v_{s}, w_{s}\right)$.

$$
\begin{align*}
& (z=0) \quad \frac{U_{s}^{2}}{L_{x}}+\frac{U_{s} V_{s}}{L_{y}}+\frac{1}{\rho} \frac{P_{s}}{L_{x}}+\frac{u_{s}^{2}}{L_{x}}+\frac{u_{s} v_{s}}{L_{y}}+\frac{u_{s} w_{s}}{L_{z}} \sim 0  \tag{4}\\
& (y=0) \quad \frac{U_{s}^{2}}{L_{x}}+\frac{U_{s} W_{s}}{L_{z}}+\frac{1}{\rho} \frac{P_{s}}{L_{x}}+\frac{u_{s}^{2}}{L_{x}}+\frac{u_{s} v_{s}}{L_{y}}+\frac{u_{s} w_{s}}{L_{z}} \sim 0 \tag{5}
\end{align*}
$$

Notice that the fluctuating velocities are condensing both, phase-averaged and turbulent fluctuations. From (3):

$$
\begin{equation*}
\frac{U_{s}}{L_{x}}+\frac{V_{s}}{L_{y}}+\frac{W_{s}}{L_{z}} \sim 0 \tag{6}
\end{equation*}
$$

Similar scaling identities can be written using the cross-stream momentum equations. Taking into account that $W$ is constant and zero in plane $(z=0)$, and that $V$ is constant and zero in plane $(y=0)$, we obtain:

$$
\begin{array}{ll}
(z=0) & \frac{1}{\rho} \frac{\partial P}{\partial z}+\frac{\partial<u w>}{\partial x}+\frac{\partial<v w>}{\partial y}+\frac{\partial<w^{2}>}{\partial z}=0 \\
(y=0) \quad & \frac{1}{\rho} \frac{\partial P}{\partial y}+\frac{\partial<u v>}{\partial x}+\frac{\partial<v w>}{\partial z}+\frac{\partial<v^{2}>}{\partial y}=0 \tag{8}
\end{array}
$$

so that,

$$
\begin{array}{ll}
(z=0) & \frac{1}{\rho} \frac{P_{s}}{L_{z}}+\frac{u_{s} w_{s}}{L_{x}}+\frac{v_{s} w_{s}}{L_{y}}+\frac{w_{s}^{2}}{L_{z}} \sim 0 \\
(y=0) & \frac{1}{\rho} \frac{P_{s}}{L_{y}}+\frac{u_{s} v_{s}}{L_{x}}+\frac{v_{s}^{2}}{L_{y}}+\frac{v_{s} w_{s}}{L_{z}} \sim 0 \tag{10}
\end{array}
$$

Replacing (7) and (8) into (4) and (5) respectively:

$$
\begin{align*}
& (z=0) \quad \frac{P_{s}}{\rho} \sim-w_{s}^{2}-\frac{L_{z}}{L_{x}} u_{s} w_{s}-\frac{L_{z}}{L_{y}} v_{s} w_{s}  \tag{11}\\
& (y=0) \quad \frac{P_{s}}{\rho} \sim-v_{s}^{2}-\frac{L_{y}}{L_{x}} u_{s} v_{s}-\frac{L_{y}}{L_{z}} v_{s} w_{s} \tag{12}
\end{align*}
$$

Now, (11-12) can be used to eliminate the pressure scale from (4-5), so that:

$$
\begin{align*}
& (z=0) \quad \frac{U_{s}^{2}}{L_{x}}+\frac{V_{s} U_{s}}{L_{y}}+\frac{u_{s}^{2}}{L_{x}}+\frac{u_{s} v_{s}}{L_{y}}+  \tag{13}\\
& +\left(1-\frac{L_{z}^{2}}{L_{x}^{2}}\right) \frac{u_{s} w_{s}}{L_{z}}-\left(\frac{L_{z}}{L_{x}}\right) \frac{v_{s} w_{s}}{L_{y}}-\frac{w_{s}^{2}}{L_{x}} \sim 0 \\
& (y=0) \quad \frac{U_{s}^{2}}{L_{x}}+\frac{W_{s} U_{s}}{L_{y}}+\frac{u_{s}^{2}}{L_{x}}+\frac{u_{s} w_{s}}{L_{z}}+  \tag{14}\\
& +\left(1-\frac{L_{y}^{2}}{L_{x}^{2}}\right) \frac{u_{s} v_{s}}{L_{y}}+\left(\frac{L_{y}}{L_{x}}\right) \frac{v_{s} w_{s}}{L_{z}}-\frac{v_{s}^{2}}{L_{x}} \sim 0
\end{align*}
$$

The analysis is undertaken at some distance from the exit orifice for regions in which $L_{y} \sim L_{z} \ll L_{x}$. This leads to a simplified equation for the streamwise time-averaged momentum equations in the mid-coronal $(z=0)$ and the midsagittal $(y=0)$ planes:

$$
\begin{align*}
(z=0) \quad U \frac{\partial U}{\partial x} & +V \frac{\partial U}{\partial y}+  \tag{15}\\
& +\frac{\partial<u^{2}>}{\partial x}+\frac{\partial<u v>}{\partial y}+\frac{\partial<u w>}{\partial z}-\frac{\partial<w^{2}>}{\partial x}=0 \\
(y=0) \quad U \frac{\partial U}{\partial x} & +W \frac{\partial U}{\partial z}+  \tag{16}\\
& +\frac{\partial<u^{2}>}{\partial x}+\frac{\partial<u v>}{\partial y}+\frac{\partial<u w>}{\partial z}-\frac{\partial<v^{2}>}{\partial x}=0
\end{align*}
$$

Notice that if we had considered the planar jet assumption for planes $(z=0)$ and $(y=0)$, the simplified equations would differ from the above equations in the last terms.

To seek for a similarity solution, one assumes that:

$$
\begin{align*}
U & =U_{s}(x) f(\xi, \eta)  \tag{17}\\
V & =V_{s}(x) h(\xi, \eta)  \tag{18}\\
W & =W_{s}(x) b(\xi, \eta)  \tag{19}\\
<u^{2}> & =R_{u u}(x) g_{u u}(\xi, \eta)  \tag{20}\\
<u v> & =R_{u v}(x) g_{u v}(\xi, \eta)  \tag{21}\\
<u w> & =R_{u w}(x) g_{u w}(\xi, \eta) \tag{22}
\end{align*}
$$

where $f, h, b, g_{u u}, g_{u v}, g_{u w}$ are some functions (to be determined empirically), $\xi, \eta$ are the normalized cross-stream coordinates $\left(\xi=y / L_{y}, \eta=z / L_{z}\right)$ and $U_{s}, V_{s}, W_{s}, R_{u u}, R_{u v}, R_{u w}$ are functions of $x$ only.

For the fluctuations, different length scales $l_{y}$ and $l_{z}$ could be used so that $g_{u u}\left(\xi^{\prime}, \eta^{\prime}\right), g_{u v}\left(\xi^{\prime}, \eta^{\prime}\right), g_{u w}\left(\xi^{\prime}, \eta^{\prime}\right)$, with $\xi^{\prime}=y / l_{y}, \eta^{\prime}=z / l_{z}$. Introducing this distinction does not affect the subsequent derivation of the similarity equations as far as $\left[l_{y, x} / l_{y}\right] \sim\left[L_{y, x} / L_{y}\right]$ and $\left[l_{z, x} / l_{z}\right] \sim\left[L_{z, x} / L_{z}\right]$.

On substituting expressions (17-22) into (15-16), one obtains:

$$
\begin{align*}
(z=0) \quad & {\left[U_{s} U_{s, x}\right] f^{2}+\left[\frac{U_{s}^{2}}{L_{y}} L_{y, x}\right]\left(-\xi f f_{, \xi}\right)+}  \tag{23}\\
& {\left[\frac{U_{s} V_{s}}{L_{y}}\right]\left(h f_{, \xi}\right)+\left[\frac{R_{u v}}{L_{y}}\right] g_{u v, \xi}+\left[\frac{R_{u w}}{L_{z}}\right] g_{u w, \xi}+} \\
& {\left[\frac{R_{u u}}{L_{y}} L_{y, x}\right]\left(-\xi g_{u u, \xi}\right)+\left[R_{u u, x}\right] g_{u u}-\left[R_{w w, x}\right] g_{w w}+} \\
& {\left[\frac{R_{w w} L_{y, x}}{L_{y}}\right]\left(\xi g_{w w, \xi}\right)=0 } \\
(y=0) \quad & {\left[U_{s} U_{s, x}\right] f^{2}+\left[\frac{U_{s}^{2}}{L_{z}} L_{z, x}\right]\left(-\eta f f_{, \eta}\right)+} \\
& {\left[\frac{U_{s} W_{s}}{L_{z}}\right]\left(b f_{, \eta}\right)+\left[\frac{R_{u v}}{L_{y}}\right] g_{u v, \eta}+\left[\frac{R_{u w}}{L_{z}}\right] g_{u w, \eta}+}  \tag{24}\\
& {\left[\frac{R_{u u}}{L_{z}} L_{z, x}\right]\left(-\eta g_{u u, \eta}\right)+\left[R_{u u, x}\right] g_{u u}-\left[R_{v v, x}\right] g_{v v}+} \\
& {\left[\frac{R_{v v} L_{z, x}}{L_{z}}\right]\left(\eta g_{v v, \eta}\right)=0 }
\end{align*}
$$

where commas before $x, \xi$ or $\eta$ denote differentiation with respect to these variables. The terms in brackets group the terms that are functions of $x$ only.

For self-similarity to hold, we do not require as in standard similarity analysis applied to round or planar jets (see for instance [2]) that the terms in brackets should be constant. The approach in [21] is here abandoned by the proposal in [7] where a less restrictive condition is required, namely, that all the terms in brackets should have the same $x$-dependence. Equating the terms in brackets and dividing all the terms by $U_{s}^{2}$, we obtain:

$$
\begin{align*}
&(z=0) \quad {\left[\frac{U_{s, x}}{U_{s}}\right] \propto\left[\frac{L_{y, x}}{L_{y}}\right] \propto\left[\frac{R_{u v}}{L_{y} U_{s}^{2}}\right] \propto }  \tag{25}\\
& \propto\left[\frac{V_{s}}{L_{y} U_{s}}\right] \propto\left[\frac{R_{u w}}{L_{z} U_{s}^{2}}\right] \propto\left[\frac{R_{u u} L_{y, x}}{U_{s}^{2} L_{y}}\right] \propto \\
& \propto\left[\frac{R_{u u, x}}{U_{s}^{2}}\right] \propto\left[\frac{R_{w w, x}}{U_{s}^{2}}\right] \propto\left[\frac{R_{w w} L_{y, x}}{L_{y} U_{s}^{2}}\right] \\
&(y=0) \quad\left[\frac{U_{s, x}}{U_{s}}\right] \propto\left[\frac{L_{z, x}}{L_{z}}\right] \propto\left[\frac{R_{u w}}{L_{z} U_{s}^{2}}\right] \propto  \tag{26}\\
& \propto\left[\frac{W_{s}}{L_{z} U_{s}}\right] \propto\left[\frac{R_{u v}}{L_{y} U_{s}^{2}}\right] \propto\left[\frac{R_{u u} L_{z, x}}{U_{s}^{2} L_{z}}\right] \propto \\
& \propto\left[\frac{R_{u u, x}}{U_{s}^{2}}\right] \propto\left[\frac{R_{v v, x}}{U_{s}^{2}}\right] \propto\left[\frac{R_{v v} L_{z, x}}{L_{z} U_{s}^{2}}\right]
\end{align*}
$$

It is important to notice that the requirement that the terms in brackets should be constant would lead to a linear growth rate in both cross-stream directions ( $y$ and $z$ ). This does not hold for the glottal jet data: as shown in the following section, such a condition is fulfilled in the $y$ direction but not in $z$ direction. The looser requirement we adopt allows, instead, for many more classes of functions satisfying the self-similarity constraint.

Combining (25) and (26) one obtains the following requirement:

$$
\begin{equation*}
\left[L_{y, x} / L_{y}\right] \propto\left[L_{z, x} / L_{z}\right] \propto\left[U_{s, x} / U_{s}\right] \tag{27}
\end{equation*}
$$

Note however that relations in (27) need not hold simultaneously in the whole range of $x$ values.

Relations (25-26) yield the scalings that will be used to rescale the profiles in the minor and major planes. For example, $V_{s}$ is such that $V_{s} \propto U_{s} L_{y, x}$ in $z=$ 0 , and hence, $V\left(x_{i}, y, 0\right)$ at different streamwise distances $x_{i}$ with $i=1,2, \ldots$ can be normalized dividing by $U_{s} L_{y, x}$. This is not the only normalization scale compatible with (25). Another possible choice for $V$ in $z=0$ would be $V_{s} \propto U_{s, x} L_{y}$. If there is in-plane self-similarity, these scalings should make the profiles for different $x_{i}$ with $i=1,2, \ldots$ collapse onto a single curve within a certain range of the $x$ axis. Because the dependent variable $V$ of this example scales with $U_{s} L_{y, x}$, requirement (27) reduces to $\left[L_{y, x} / L_{y}\right] \propto\left[U_{s, x} / U_{s}\right]$.

Similar scalings can be derived for other quantities (velocities or Reynolds stresses) as shown in Table 1. Notice that the similarity scalings are profileand plane-dependent, differing from the conventional use of a single length and velocity scale.

| Profiles | In-Plane Similarity Scaling |  |
| :---: | :---: | :---: |
|  | $z=0$ | $y=0$ |
| $U$ | $U_{s}$ | $U_{s}$ |
| $V$ | $U_{s} L_{y, x}$ | - |
| $W$ | - | $U_{s} L_{z, x}$ |
| $\langle u u\rangle$ | $U_{s}^{2}$ | $U_{s}^{2}$ |
| $\langle v v\rangle$ | $U_{s}^{2}$ | $U_{s}^{2}$ |
| $\langle w w\rangle$ | $U_{s}^{2}$ | $U_{s}^{2}$ |
| $\langle u v\rangle$ | $U_{s}^{2} L_{y, x}$ | $U_{s, x} U_{s} L_{y}$ |
| $\langle u w\rangle$ | $U_{s, x} U_{s} L_{z}$ | $U_{s}^{2} L_{z, x}$ |

Table 1: Normalization similarity scalings for velocities and Reynolds stresses.

## 4 In-plane Similarity Analysis of the Glottal Jet Data

In this section we apply the in-plane similarity relations derived in the previous section to the glottal jet data.

For high aspect ratio nozzles, data are often rendered dimensionless using the jet exit velocity $U_{0}$ and the equivalent diameter $D_{e}$. The latter is defined as the diameter of a circular jet with momentum flux equal to that of the elongated jet [11]. For the nozzle in motion of the present study, $D_{e}$ can be defined following [15]. With this definition, the glottal jet experiment has an equivalent diameter $D_{e} \sim 4 \mathrm{~mm}$ [13].

In the context of the similarity analysis we wish to undertake, a natural choice for the velocity scale is the centerline velocity $U_{c}$, since $U_{c}(x)$ is the maximum velocity of the profile at any $x$ position. This choice already assures that all $U / U_{s}$ profiles will collapse at least on one point, located at the profile's maximum. The in-plane similarity analysis calls for one length scale per symmetry plane. These two length scales, labeled $L_{y}$ and $L_{z}$ in the previous section, need not coincide. For the glottal jet data, $L_{y}$ and $L_{z}$ can be conveniently chosen following two alternatives. The first one is to fix a second point of collapse for the $U / U_{s}$ profiles, given for instance by the jet half width. The second alternative is to infer $L_{y}$ and $L_{z}$ from the proportionality relations in the previous section, namely:

$$
\begin{align*}
L_{y} & \propto \frac{V_{s}}{U_{s, x}}  \tag{28}\\
L_{z} & \propto \frac{W_{s}}{U_{s, x}} \tag{29}
\end{align*}
$$

with $V_{s}(x)$ and $W_{s}(x)$ defined as the maxima of the $V(x)$ and $W(x)$ profiles. The first choice leads to more accurate results in combination with our data (the error in the determination of the maxima of $W(x)$ is large in our experiment due to the separation between the different measurement planes). In this section $L_{y}$ and $L_{z}$ are defined following the first choice, but further considerations regarding the second choice will be provided in Section 5.

As in [13], the half widths are computed from the 2D gaussian profiles that best fit the experimental values of $U$ for each $x$ position. The empirical $x$-dependence of $L_{y}$ and $L_{z}$ follows respectively a linear and a quadratic law, as shown in Fig 3. The same holds if one computes $l_{y}$ and $l_{z}$ as the half widths of the 2D gaussian profiles that best fit the experimental values of $\left.<u^{2}\right\rangle$ for each $x$ position.


Fig. 3: Length and velocity scales for the glottal jet data used in the similarity analysis. (a) Length scales $L_{y}, L_{z}, l_{y}$ and $l_{z}$ as a function of the streamwise distance $x$. (b) Streamwise component of the centerline velocity $U_{c}$ of the glottal jet as a function of the streamwise distance $x$.

Using the scales shown in Fig 3, it is possible to plot the empirical values of the ratios appearing in (27) for the inspected $x$ values. The $x$-dependence of these similarity ratios, shown in Fig 4 , can be used to locate the ranges in $x$ where in-plane self-similarity is expected to hold following relations (25) and (26). For the glottal jet data, the $x$-dependence of the similarity ratios is not the same throughout the whole inspected range, but the scalings involved in the similarity ratios are not required to hold simultaneously. Table 1 shows

Similarity ratios $\left[\mathrm{mm}^{-1}\right]$


Fig. 4: Similarity ratios appearing in (25) and (26) ( $U_{s, x} / U_{s}, L_{y, x} / L_{y}, L_{z, x} / L_{z}$, $l_{y, x} / l_{y}$ and $\left.l_{z, x} / l_{z}\right)$ as a function of $x$ for the glottal-jet data. Ratios $L_{z, x} / L_{z}$ and $l_{z, x} / l_{z}$ are shown multiplied by $(-1)$ (and shifted with a constant in the second case) to achieve a better visibility in the plot.
the scales to be used in the normalization of the different glottal jet profiles for each symmetry plane.

The next two subsections (4.1) and (4.2) show how the similarity scalings lead to a collapse of the mean velocities and Reynolds stresses in the minor and major planes, within the $x$-ranges where self-similarity is expected to hold.

A remark is necessary before these tasks are undertaken. The in-plane analysis has the zero velocity condition for the velocity component that is normal to the correspondent symmetry plane, but the time-averaged jet centerline has a slight tilt (of 5 degrees at most) with respect to the $x$ axis. Because of this, the orientation and origin of the local coordinate system satisfying the in-plane condition at each $x$ position will not exactly coincide with the fixed laboratory coordinate system defining the mid-sagittal $(y=0)$ and mid-coronal $(z=0)$ planes. Coordinates $\bar{y}$ and $\bar{z}$ are defined to include the shifts $y_{c}(x)$ and $z_{c}(x)$ correcting the centerline position of the jet. These coordinates compensate for the error in the origin of the local coordinate system, but not for the error in the orientation of the symmetry planes. The latter correction would have required a larger densification in the measurement planes of the inspected volume, but in view of the reduced tilt, this should not significantly alter the presented profiles.

### 4.1 Minor plane results

For the analysis of the profiles, let us first take the minor plane, that coincides with the physiological mid-coronal plane $(z=0)$. As mentioned above, the
shift introduced by the centerline position of the jet, which is slightly tilted as the jet spreads, is corrected in the rescaled profiles using $\bar{y}=y-y_{c}(x)$.

Using the similarity scalings, collapsing profiles are obtained for both nonzero velocity profiles $U$ and $V$, as shown in Fig 5 and Fig 6 respectively. The collapse sweeps the range $x>10 \mathrm{~mm}$ where $U_{s, x} / U_{s}$ and $L_{y, x} / L_{y}$ show a similar $x$-dependence.

The asymmetry in the profiles with respect to the vertical axis can be attributed to more than one cause. There is, of course, the above-mentioned usage of a fixed coordinate system and the time-averaged jet tilt, but there is also the existence of moderate asymmetries in the aperture defined by the model folds during regular oscillations. This can be appreciated in Fig 1b: notice that a vertical line joining the two commissures does not separate the slit into two exactly equal black sections. The same holds for a centered horizontal line dividing the image into an upper and a lower part.

Collapsing profiles are also obtained for the mean Reynolds stresses rescaled in the $y$ and $z$ axes using length scales $l_{y}$ and $l_{z}$. These length scales are applied because they are available from glottal jet data, but the collapse is not significantly modified if $L_{y}$ and $L_{z}$ are used instead. For $\left\langle u^{2}\right\rangle$ two subregions can be distinguished, as shown in Fig 7: subregion (I) $10 \mathrm{~mm}<x<14 \mathrm{~mm}$ and subregion (II) $16 \mathrm{~mm}<x<21 \mathrm{~mm}$. The profile for $<u^{2}>$ changes shape approximately where $L_{z}(x)$ reaches its minimum value. Profiles $<u v>$ are normalized using $R_{u v}$, that scales as $U_{s}^{2} L_{y, x}$. The range of validity is once again $x>10 \mathrm{~mm}$, as shown in Fig 8. The profile for $\langle u w\rangle$ is not shown because the measured values are too small (within the error bars of the experiment) to allow for a similarity analysis.

In spite of the moderate lack of symmetry of the minor plane profiles for the glottal jet, their shape compares well with the shape of the cross-stream distributions reported, for instance, for the high-aspect-ratio rectangular synthetic jet studied in [20]. In the $\left\langle u^{2}\right\rangle$ profile, where two profile shapes are distinguished, it is the shape in subregion (I) that compares well with Fig. 12c in [20].

### 4.2 Major plane results

The major plane $(y=0)$ is not a measurement plane in the source experiment [13]. Nevertheless, this near-symmetry plane can be reconstructed extracting the lines of data collected for the nine inspected coronal planes that approximately correspond to the mid-sagittal position. The fact that the major plane has to be reconstructed poses two difficulties that were absent for the minor plane: the lines of data will not necessarilly coincide exactly with the major axis position, and the major plane profiles will be made up of only nine points per $x$ position. This section shows that the collapsing tendency is nonetheless verified when velocities and Reynolds stresses are rescaled in the similarity variables. As before, the shift introduced by the mean centerline position of


Fig. 5: Dimensionless time-averaged profiles for $U\left(x_{i}, y, z=0\right)$ using different scalings. (a) Scaling based on the jet exit velocity $U_{0}$ and the equivalent diameter $D_{e}[15,13]$. (b) In-plane similarity scaling: the jet centerline velocity $U_{s}=U_{c}$ and $L_{y}$ for lengths.
the jet, which is slightly tilted as the jet progresses downstream, is corrected in the rescaled profiles using $\bar{z}=z-z_{c}(x)$.

The major plane results are shown in Fig 9, 10 and 11. The similarity scalings lead to a reorganization of the otherwise scattered points of the plots, condensing them as in Fig 10, or re-ordering them as in Fig 11. Notice that in Fig 11, the data reorganization introduced by the similarity scaling compenses for the small number of available $z$-positions, unveiling the profile shape, which appears to be similar to that of Fig 7b. The inspected subrange in $x$


Fig. 6: Dimensionless time-averaged profiles for $V\left(x_{i}, y, z=0\right)$ using different scalings. (a) Scaling based on the jet exit velocity $U_{0}$ and the equivalent diameter $D_{e}[15,13]$. (b) In-plane similarity scaling: $U_{s} L_{y, x}$ for velocities and and $L_{y}$ for lengths.
where such shape is found ( $10 \mathrm{~mm}<x<14 \mathrm{~mm}$ ) is the same in both, major and minor planes.

The ranges in $x$ where similarity applies in the major plane need not coindice with those of the minor plane. Fig 10 presents the range $9 \mathrm{~mm}<$ $x<12 \mathrm{~mm}$ where the collapse is optimal. This is consistent with the fact that scales $L_{z}, l_{z}$ and $U_{s}$ exhibit an $x$-dependence within this range approximating the requirement in (27). The same holds for Fig 11. Profiles $\langle u v\rangle$ and $<u w\rangle$ are not shown in this plane because the values for the glottal jet data


Fig. 7: Dimensionless time-averaged profiles for $\left\langle u^{2}\right\rangle\left(x_{i}, y, z=0\right)$ using different scalings. (a) Scaling based on the squared jet exit velocity $U_{0}^{2}$ and the equivalent diameter $D_{e}[15,13]$. (b-c) In-plane similarity scaling: $U_{s} L_{y, x}$ for velocities and $l_{y}$ for lengths. Profiles are shape-preserving if two subregions in $x$ are distinguished as shown in (b) and (c).


Fig. 8: Dimensionless time-averaged profiles for $\left\langle u v>\left(x_{i}, y, z=0\right)\right.$ using different scalings. (a) Scaling based on the squared jet exit velocity $U_{0}^{2}$ and the equivalent diameter $D_{e}[15,13]$. (b) In-plane similarity scaling: $U_{s}^{2} L_{y, x}$ for Reynolds stresses and $l_{y}$ for lengths.
are too small (within the error bars of the experiment) to allow for a similarity analysis.

## 5 Axis-switching prediction

Axis switching is the most salient feature of the three-dimensional study presented in [13]. The effect is typical of non circular jets [9]. For jets in which the exit cross section has two symmetry axes, axis switching means that the major and minor axes invert their roles as the downstream coordinate $x$ increases.

| $\mathrm{X} / \mathrm{D}_{\mathrm{e}}$ |
| :---: |
| -2.51 |
| - 2.58 |
| - 2.64 |
| $\pm 2.70$ |
| -2.77 |
| $\bigcirc 2.83$ |
| $\square 2.89$ |
| $\checkmark 2.96$ |
| $\triangle 3.02$ |
| $\nabla 3.09$ |
| -3.15 |
| - -3.21 |
| -3.28 |
| - 3.34 |
| - 3.40 |
| $\bigcirc 3.47$ |
| $\square 3.53$ |
| * 3.59 |


(a)

(b)

Fig. 9: Dimensionless time-averaged profiles for $U\left(x_{i}, y \sim 0, z\right)$ using different scalings. (a) Scaling based on the jet exit velocity $U_{0}$ and the equivalent diameter $D_{e}[15,13]$. (b) In-plane similarity analysis: the jet centerline velocity $U_{s}=U_{c}$ and $L_{z}$ for lengths.

This defines a point $x_{S L}$, termed switching location or crossover position, at which the half width of the jet is coincident in both $y$ and $z$ directions. The mean field of the investigated jet switches orientation at $\left\langle x_{S L}\right\rangle$, the timeaveraged switching location, indicated by the point where the jet half-widths in $y$ and $z$ intersect, as shown in Fig 3, where $<x_{S L}>=8.6 \mathrm{~mm}$. The fact that $x_{S L}$ is time-varying is associated to the pulsating nature and time-varying exit cross section of the jet - further details on the temporal variation of $x_{S L}$ can be found in [13]. This section reconsiders the axis switching phenomenon in the context of the present similarity analysis.


Fig. 10: Dimensionless time-averaged profiles for $W\left(x_{i}, y \sim 0, z\right)$ using different scalings. (a) Scaling based on the jet exit velocity $U_{0}$ and the equivalent diameter $D_{e}[15,13]$. (b) In-plane similarity analysis: $U_{s} L_{z, x}$ for velocities and $L_{z}$ for lengths.

Information can be extracted from the derived similarity relations in connection with the axis switching phenomenon in more than one way. Let us consider two of them.

- In section 4, it was mentioned that functions $L_{y}(x)$ and $L_{z}(x)$ could be estimated defining $V_{s}(x)$ and $W_{s}(x)$ as the maxima of the $V(x)$ and $W(x)$ profiles and using the proportionality relations (28) and (29). Fig 12 shows a least-squares fit of the two functions $V_{s} / U_{s, x}$ and $W_{s} / U_{s, x}$ to which $L_{y}$ and $L_{z}$ are supposed to be respectively proportional if there is self-similarity. With a knowledge of the ordinates of the fitting functions, which fix $L_{y}(0)$

(a)

(b)

Fig. 11: Dimensionless time-averaged profiles for $<u^{2}>\left(x_{i}, y \sim 0, z\right)$ using different scalings. (a) Scaling based on the squared jet exit velocity $U_{0}^{2}$ and the equivalent diameter $D_{e}[15,13]$. (b) In-plane similarity scaling: $U_{s}^{2}$ for Reynolds stresses and $l_{z}$ for lengths.
and $L_{z}(0)$, the mean switching location $<x_{S L}>$ can be predicted without resorting to the jet half-widths.

- From the continuity equation along the jet centerline, we have:

$$
\begin{equation*}
\frac{\partial U_{c}}{\partial x}+\frac{\partial V_{c}}{\partial y}+\frac{\partial W_{c}}{\partial z}=0 \tag{30}
\end{equation*}
$$

When $V(y)$ and $W(z)$ are rescaled in the similarity variables, the slopes of the profiles at the centerline (the slopes in Fig 6 at $y=0$ and in Fig 10 at $z=0$ ) are well defined constants: $c_{1} \sim 0.63$ and $c_{2} \sim 1.8$. These constants


Fig. 12: Axis switching prediction from the similarity analysis. (a) Prediction of length scales $L_{y}(x)$ and $L_{z}(x)$ from the $x$-dependence of the velocity scales $\left(U_{s}, V_{s}, W_{s}\right)$ calculated with the three centerline velocity components. Points are used for experimental data and lines for least-squares fits. (b) Prediction of length scale $L_{z}(x)$ by numerical integration of equation (31). Points are used for experimental data $\left(L_{y}\right)$ and numerically integrated data $\left(L_{z}\right)$. Lines are used for least-square fits.
can be used to rewrite the continuity equation along the centerline (divided by $U_{s}$ ) as follows:

$$
\begin{equation*}
\frac{U_{s, x}}{U_{s}}+c_{1} \frac{L_{y, x}}{L_{y}}+c_{2} \frac{L_{z, x}}{L_{z}}=0 \tag{31}
\end{equation*}
$$

The first term of (31) is a function of $x$ that can be fitted from the jet centerline velocity data. If the jet spread in the mid-coronal plane $L_{y}$ is
available, one can use equation (31) to obtain $L_{z}$ by integration. The result of the numerical integration of $L_{z}$ is shown in Fig 12. Note that the similarity analysis has the property of yielding the jet contraction/spread rate in one of the symmetry planes (the mid-sagittal plane) from data collected in the other symmetry plane (the mid-coronal plane). This procedure does not only allow for a prediction of the approximate axis switching location, but also of the position where the jet contracts most in the $z$ direction, i.e. of the minimum of $L_{z}(x)$.

## 6 Conclusions

Glottal-like flow is subjected to a similarity analysis that has two specificities. In the first place, the analysis is restricted to the two privileged planes of the flow: the minor plane and the major plane of the jet exit. In physiological terms these planes correspond to the mid-sagittal and mid-coronal planes. Such in-plane similarity analysis allows for a simplification of the derivation of the similarity variables. In the second place, the self-similarity condition that is used to derive the similarity variables is looser than the standard requirement: the dependent variables involved in the problem are allowed to have their own scale, with each scale evolving downstream in a manner that is determined by the in-plane equations of motion. These two specific features associate conveniently in the analysis of a high aspect ratio jet, rendering selfsimilarity compatible with the quadratic contracting-spreading rates observed in the minor plane of jets presenting axis switching.

Imposing a similarity solution to the simplified flow equations, one obtains a series of proportionality relations holding within the minor and major planes. These relationships provide the similarity variables that can be used, in combination with glottal-like jet data, to infer the ranges where self-similarity may be expected to occur. Using the similarity scalings to normalize the timeaveraged profiles, and restricting the plots to the ranges in which self-similarity is expected, empirically non-collapsing mean profiles are shown to rescale as collapsing profiles. This is verified for the time-averaged streamwise velocity, the cross-stream velocity components and the Reynolds stresses, in both the minor and the major planes.

In combination with data, the similarity constraints are shown to predict characteristics of the axis switching phenomenon observed in the investigated jet. These characteristics include the mean crossover position at which the switch occurs and the location at which the jet contracts most in the midsagittal plane. Once self-similarity is assured as a property of a certain jet region, the degrees of freedom of the problem are reduced and therefore, information obtained at different measurement points becomes redundant. Experimental efforts can thus be economized and numerical validation can be simplified.

The in-plane similarity variables obtained in this work are not exclusive to glottal-like jets. They can be applied, more generally, to the time averaged
velocities and Reynolds stresses of high aspect ratio jets. The rationale applied to this case study could be of use for flows presenting, for instance, several symmetry planes. On the other hand, the characterization of an axis-switching jet in terms of similarity properties can be exploited in complement with other techniques towards a better understanding of this kind of flows.

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