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On Curvature and the Navier Stoke Equations. (A Letter)

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Abstract.

We introduce herein ideas surrounding curvature and its relation to the Navier Stoke equations. We will form the basis for the relation between curvature in the classical sense and conditions necessary for the existence of blow up solutions.

Introduction.

The major concern surrounding the Navier Stoke Equations (NS), which we remind the reader follows as:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + f \quad (1)$$

where (f, ρ, \mathbf{u}) represent arbitrary initial forces, pressure and the initial velocity vector field respectively, Is the possibility of there being blow up solutions associated with (ρ, \mathbf{u}) .

A blow up solution occurs when the solution to a system of partial differential equations produces functions that are asymptotic over some region of its domain. As these equations are of significance in fluid mechanics, smoothness requires that such blow up solutions are impossible[1]. Understanding what might cause such solutions is one part of understanding what it is that might be required of f , in order to produce such solutions. Knowing this one can check whether such forces are even possible.

There are many works that are related to understanding generalized solutions to such PDE's[2][3]. Attempts at understanding 3D-case of (NS) include Solutions of the Navier-Stokes and Euler equations with initial conditions (Cauchy problem) for 2D and 3D cases which were obtained in the converging series form by analytical iterative method using Fourier and Laplace transforms in a paper [4].

Our aim is to formulate the problem in a sense of curvature. More clearly, we will lay foundations for the investigation into the nature of such solutions via its association with curvature.

Outline

Many entangled factors come into play when considering the possibility of a

blow-up solution. The nature of f (arbitrary body forces), the initial velocity vector field and also of relevance is the nature and setup of the PDE's itself. To investigate this possibility in a broader sense, we assume a blow up solution that we associate with either of (ρ, \mathbf{u}) . For this to be the case, the following criterion is necessary.

Given a finite continuous region $R := \{\mathbf{r}_i \in \mathbb{R}^n\}$ over which either (ρ, \mathbf{u}) blows up, it is required that:

$$\frac{\mathbf{u}(\mathbf{r}_1) - \mathbf{u}(\mathbf{r}_2)}{\mathbf{r}_1 - \mathbf{r}_2} > \epsilon(\mathbf{r}) \quad (2)$$

$\forall \mathbf{r}_i, \mathbf{r}_j \in \Phi^{-1} | \Phi = \mathbf{u} \cap P$ where P is a vertical plane through $\mathbf{r}_k | \lim_{\mathbf{r} \rightarrow \mathbf{r}_k} \Phi(\mathbf{r}) \rightarrow \infty$.

Here, we associate $\epsilon(\mathbf{r})$ with some minimal curvature κ required of either (ρ, \mathbf{u}) over R .

The nature of such a vector field will naturally depend on its initial form and what we call its t -component. More specifically, the differential above pertains to t , and as such the requirements will be of the form: $(\mathbf{u}_x, \mathbf{u}_t)$ where it is the t -component that our requirement is relayed into.

A Logical Discourse

The behaviour of such PDE's can be described as follows:

Any magnitude \mathcal{M} at t_0 adjoined to a differential $\frac{\partial \mathbf{u}}{\partial t}$ is expressed as a functional of the variable with respect to which the functional is being differentiated, in this instance t . The nature of the functional $F(t)$ depends solely on the magnitude of the change $\sum_{\forall i} \partial^i \mathcal{M}$. This magnitude we know is entangled with $F(i)$ of neighbouring points $x, x + \delta x$. More specifically, the expression involving \mathbf{u}^o, t that describes the variation over $x, x + \delta x$ is incorporated into the magnitudes over $x + \delta x, x + \delta x + \delta x$. It is this entanglement that makes ascertaining the necessary curvatures required of $(\mathbf{u}^o, \mathbf{f})$ in order for the existence of a blow up solution difficult.

The mechanism associated with any differential is simple, it describes a difference as measured of functionals over neighbouring points in some $R \in \mathbb{R}^n$ via the use of a functional of the form $F(t)$. For instance, if the magnitudes were representable via t as t^4 then the change would roughly be of the form: ct^3 . What we are interested in is a blow up initiator point, specifically one that results in a change that is describable as $\frac{1}{t}$. This is because, any non-reciprocal polynomial like $F(t)$ adjoined with \mathbf{u}^o will again result (logically speaking) in another polynomial like expression expressing the difference over neighbouring points. This can be seen more clearly if one notices that there are only so many ways in which one can adjoin a polynomial like

$F(t)$ to \mathbf{u}^o , and once accomplished, a difference, which is essentially what a differential is, is incapable of reducing polynomial expressions to reciprocal ones. Thus it is easy to see that blow up solutions can only occur in such constructions if $\sum_{\forall i} \partial^i \mathcal{M}$ at some point is specifically of the general magnitude of the form $S := \{\frac{1}{t}\} | t \in R \in \mathbb{R}^n$. knowing the magnitudes of $\mathcal{M}(x) - \mathcal{M}(x + \delta x)$ i.e. specifically of \mathbf{u}^o, \mathbf{f} , one can ascertain the general nature of the functional $F(t)$. Logically, this will tell us a lot about whether a blow up ripple boundary of points having differences in the proportion of $\frac{i}{k}$ exists, and if so we need only notice that in such setups, since differences of the form : $u_x P(t) + .. - u_x \frac{1}{k}$ can never result in polynomial forms which indicates that at such ripples blow up solutions are possible.

We will now attempt outlining the above in a formal mathematical nature.

Let us denote $P_{DE}(\overline{X}, t_o)$ by \mathfrak{S}_o Logically the change: $\Delta t(P_{DE}(\overline{X}, t_i)) = \{\mathfrak{S}_o(\overline{X} + \delta) \otimes_1 \mathcal{T} + \epsilon_{\phi 1} - \mathfrak{S}_o(\overline{X}) \otimes_2 \mathcal{T}_{\phi 2}\}$

Where the t -forms $(\mathcal{T}_{\phi 1}, \mathcal{T}_{\phi 2})$ denote some functionals $(F_1(t), F_2(t + \epsilon))$ that via some operational set (\otimes_1, \otimes_2) on $(\mathfrak{S}_o(\overline{X} + \delta), \mathfrak{S}_o(\overline{X}))$, describe the change $P_{DE}(\overline{X}, t_o)$. Additionally, we have $\mathfrak{S}_o(\overline{X} + \delta) \otimes_1 \mathcal{T}(t + \epsilon)_{\phi 1} = \{\mathfrak{S}_o(\overline{X}) \otimes_2 \mathcal{T}(t)_{\phi 1} - \mathfrak{S}_o(\overline{X} - \delta) \otimes_3 \mathcal{T}(t - \epsilon)_{\phi 2}\}$

The following can be observed from here is that the previous describes the mechanical principles by which the functional solution to a PDE is formed.

We have on the first such difference from the boundary $\mathfrak{S}_o(\overline{X}, t_0)$ is expressed as : $\mathfrak{S}_o(\overline{X} + \delta, t_0 + \epsilon) - \mathfrak{S}_o(\overline{X}, t_0) = \mathfrak{S}_o(\overline{X} + \delta + \delta) \otimes \mathcal{T}(t + \epsilon + \epsilon)_{\phi}$ This gives indication that every convolution of the form $\mathfrak{S}_o \otimes \mathcal{T}_{\phi}$ leaves \mathfrak{S}_o unaltered and convolutes \mathcal{T}_{ϕ} into $\mathfrak{S}_o \otimes \mathcal{T}_{\phi}$ arithmetically. By this we mean that the difference is expressed as arithmetic operations used between $F(t), \mathfrak{S}_o$ forming an expression.

We can use the above to form the conclusion that the 'cause' of an explosive solution over \mathbb{R}^m must lie in the nature of \mathcal{T}_{ϕ} and since no set of expressions involving non asymptotic expressions is capable of producing one such asymptotic function upon its difference, it is required that somewhere along \mathfrak{S}_o , a region $R \in \mathbb{R}^m$ exists over which the curvature associated with \mathfrak{S}_o lies in or exceeds $\epsilon'(r)$.

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