# A new approach in distributed multisensor tracking systems based on Kalman filter methods

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**Abstract –** *In multisensor tracking systems, the state fusion also known as "track to track" fusion is a crucial issue where the derivation of the "best" track combination is obtained according to a stochastic criteria in a minimum variance sense. Recently, sub-optimal weighted combination fusion algorithms involving matrices and scalars were developed. However, hence they only depend on the initial parameters of the system motion model and noise characteristics, these techniques are not robust against erroneous measures and unstable environment. To overcome this drawbacks, this work introduces a new approach to the optimal decentralized state fusion that copes with erroneous observations and system shortcomings. The simulations results show the effectiveness of the proposed approach. Moreover, the reduced complexity of the designed algorithm is well suited for real-time implementation.*

**Keywords:** State fusion,Kalman filter,decentralized fusion.

## 1 Introduction

In recent years, there has been a growing interest to improve the performance of fusion process in multisensor systems. Its applications are spread over many fields including, guidance, robotic, target tracking and signal processing [8, 3, 11, 12]. The key architectures involved in such applications are centralized and decentralized schemes [9]. In the former, the measurements are treated by a central processor and the resulting fusion process is referred to as data fusion. While in the latter, these observations are proceeded locally, then a global state estimate is obtained by a linear combination of the local state estimates. The information fusion instead of data fusion exhibits several advantages like reduced computational load and low transmission bandwidth but it provides a sub-optimal global state estimation in compare to the centralized fusion scheme.

Since it has been proven, for many years, that local estimates have correlated error [1], many work addressed the problem of optimal estimation that minimizes a stochastic quadratic criteria as the Linear Minimum Mean Square Error (LMMSE), the Least Mean Square (LMS) or the Linear Unbiased Minimum Variance (LUMV), which basically is the minimum norm of the variance-covariance matrix of the global estimation error.

More recently, In [4] and [5], optimal distributed estimation algorithms which are for finding the optimal linear combination of the local estimates weighted by matrices and scalars were developed. In the first one, the computational complexity is very high, in particular when the number of sensors is large because it involves the inversion of an $MN-by-MN$ matrix. While, in the latter the solution involves the inversion of a matrix of dimension $N$ but provides lower performance.

The above state fusion methods have the disadvantage that they require a large computational burden or give a globally sub-optimal state estimate. Moreover, they are not robust against erroneous observations because local errors measures effects are not passed to the error estimation matrix since the weighting coefficients depend only on the system model and the noise characteristics [6]. To overcome this drawbacks, we propose to reformulate the problem of finding the optimal coefficients that minimizes a stochastic criteria in a minimum variance sense into an instantaneous deterministic criteria based the minimum error of the realizations "measures" with probability one revealed in [7].

Furthermore, the state fusion algorithms based on Kalman filter methods are only effective when the model parameters and the noise statistics are exactly known, which is hardly satisfied in practical. Thus, an on line estimator of the variance matrix for each local sensor is implemented using the correlation method in order to pre-weighten the local measurement then provide an optimal fused measurement which has the same dimension of the local measurements. This approach has two main advantages: the first one is that it provides a general sub-optimal solution to the problem of decentralized state fusion. The second advantage is that it adapts to the sensor performances and copes with unstable environment that characterizes the target tracking applications.

In the following paragraphs, we review the optimal fusion algorithms using matrices and scalars to show the complexity of the state fusion problem formulation. Then, an efficient algorithm that take into consideration the reliability of the measurements using a decentralized fusion scheme is presented. Finally, the results of simulations involving a target tracking scenario in multisensor environment are presented to assess the effectiveness of the proposed algorithm.

# 2 Background

## 2.1 Optimal fusion using matrices

From [5], given the unbiased local estimates $\hat{x}_1, ..., \hat{x}_M$ provided by $M$ decentralized Kalman filter, we want to find an optimal fused estimate in the sense of minimum variance of the estimation error among all the linear unbiased estimates. The corresponding criteria is given by:

$$J = trace(E[(\hat{x} - x)(\hat{x} - x)^T]) \quad (1)$$

where:

$$\hat{x} = B + W^T X \quad (2)$$

and $B$ and $W^T$ are constant vector and matrix respectively

$$X = \begin{pmatrix} \hat{x}_1 \\ . \\ . \\ . \\ \hat{x}_M \end{pmatrix} W = \begin{pmatrix} w_1^T \\ . \\ . \\ . \\ w_M^T \end{pmatrix}, w_i \in R^{N \times N} \quad (3)$$

with:

$$E(\hat{x} - x) = 0 \quad (4)$$

Tacking expectation of (1) and because of the unbiasedness of the local estimates i.e. $E(\hat{x}_i) = E(x)$ yields to:

$$E(x) = B + W^T E(X) = B + \sum_{i=1}^{M} w_i^T E(x) \quad (5)$$

and

$$B + (\sum_{i=1}^{M} w_i^T - I)E(x) = 0 \quad (6)$$

E(x) is unknown. Thus, this equation is true only if $B = 0$ and $\sum_{i=1}^{M} w_i^T - I = 0$, therefore (2) became:

$$\hat{x} = W^T X \quad (7)$$

The minimization of the variance of the estimation error $E[(\hat{x} - x)(\hat{x} - x)^T]$ became a quadratic optimization problem subject to linear constraint given by:

$$J = arg \ \ min_{AW=I_n} = E[(W^T X - x)(W^T X - x)] \quad (8)$$

Where $A = [I...I]$, I is an $N - by - N$ identity matrix and the general solution of (8) is given by:

$$W = (I - (PCP)^{\dagger} C)A^{\dagger} + PZ \quad (9)$$

Where

$$P = I - A^{\dagger} A$$

$Z$ is an arbitrary $M$-by-$N$ matrix satisfying $D^T P Z = 0$, $D$ is the square root factor of covariance matrix $C$ i.e. $C = DD'$.
When $C$ is not singular, the weighting matrix $W$ has an explicit expression as:

$$W = C^{-1} A^T (AC^{-1} A^T)^{-1} \quad (10)$$

Therefore, each element $w_i$ is given by

$$w_i = \sum_{k=1}^{M} C_{i,k}^{-1} ( \sum_{j,k=1}^{M} C_{j,k}^{(-1)})^{-1} \quad (11)$$

Where $C_{i,j}^{-1}$ is the $(j,k)^{th} N - by - N$ sub matrix of $C^{-1}$. The weighting matrix given by equation (11) requires the inversion of an $MN$-by-$MN$ matrix.
Although, the study presented in [5] develops an efficient iterative algorithm which reduces the computational complexity induced by the inversion of a large dimension matrix. But, it does not provide a general formula like the well known two sensor state fusion Bar-Shalom and Compo formula [2].

## 2.2 Optimal fusion using scalars

From [5], let's consider the multisensor discrete time-varying linear stochastic system:

$$x(t+1) = A(t)x(t) + G(t)w(t) \quad (12)$$

$$y_i(t) = H_i(t)x(t) + v_i(t) \quad (13)$$

where $x(t) \in R^n$ is the state vector, $A(t), G(t), H_i(t)$ are known time-varying matrices with compatibles dimensions and $w(t), v_i(t)$ are respectively the process noise and the local sensor noises for $i = 1, ..., M$.
**Assumption 1.** $w(t), v_i(t)$ are zeros mean, correlated, white Gaussian noises, and

$$E\{[\begin{pmatrix} w(t) \\ v_i(t) \end{pmatrix}][w^T(k) v_i^T(k)]\} = [\begin{pmatrix} Q(t) & S_i(t) \\ S_i^T(t) & R_i(t) \end{pmatrix} \delta_{tk}$$

$$E[v_i(t) v_j^T(k)] = S_{ij} \delta_{tk}, \quad fori \neq j \quad (14)$$

where $\delta_{tk}$denote the Kronecker delta function
**Assumption 2.** the initial state $x(0)$ is independent of $w(t)$ and $v_i(t)$ and

$$Ex(0) = \mu_0, \ \ E[(x(0) - \mu_0)((x(0) - \mu_0)^T] = P_0^x \quad (15)$$

Given the problem formulation above, we want to find an optimal fused estimate in a minimum variance sense of estimation error among all the linear unbiased estimates. But instead of using matrices, we will consider scalars.Thus, the resulting global state estimate will be in the form of :

$$\hat{x} = \alpha_1 \hat{x}_1 + \alpha_2 \hat{x}_2 + ... + \alpha_M \hat{x}_M \quad (16)$$

where $\alpha_i$ are scalars.
Let the fused estimate variance of the estimation error $\tilde{x} =$

$x - \hat{x}$ be $P^x$. From the unbiased assumption $E(\hat{x}_i) = E(x)$, and taking expectation of both side of (16), we obtain the normalization condition equation given by:

$$\alpha_1 + \alpha_2 + ... + \alpha_M = 1 \tag{17}$$

Thus, from (16), (17) we have that
$\tilde{x} = x - \hat{x} = \sum_{i=1}^{M} \alpha_i (x - \hat{x}_i) = \sum_{i=1}^{M} \alpha_i \tilde{x}_i$
and the variance of the error estimation will be in form of:

$$P^x = E(\tilde{x}\tilde{x}^T) = \sum_{i,j=1}^{l} \alpha_i \alpha_j P_{ij}^x \tag{18}$$

The resulting criteria for minimizing the variance of the global estimate $\hat{x}$ which is the trace of $P^x$ becomes

$$J = \alpha^T A \alpha \tag{19}$$

where $\alpha = [\alpha_1, \alpha_2, ..., \alpha_M]$.
The fusion problem reduces to find the scalar weights $\alpha_i$ under the restriction (17) that minimizes the performance index $J$
Applying the Lagrange multiplier method, we introduce the auxiliary function

$$f(\lambda, \alpha) = J + 2\lambda(\alpha^T e - 1) \tag{20}$$

where $\lambda$ is a real number.
Setting $\partial f / \partial \alpha|_{\alpha = \bar{\alpha}} = 0$ we have that:

$$A\bar{\alpha} + \lambda e = 0 \tag{21}$$

Merging (21) and (17) yields to the matrix equation:

$$\begin{pmatrix} A & e \\ e^T & 0 \end{pmatrix} \begin{pmatrix} \bar{\alpha} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{22}$$

where $A = trace(P_{ij}^x)$, $\alpha = [\alpha_1, \alpha_2, ..., \alpha_M]$ and $e = [1, 1, ..., 1]^T$. Then, hence $e^T A^{-1} e \neq 0$, and using the inverse formula we obtain that:

$$\begin{pmatrix} \bar{\alpha} \\ \lambda \end{pmatrix} = \frac{1}{-e^T A^{-1} e} \begin{pmatrix} -A^{-1}e \\ 1 \end{pmatrix} \begin{pmatrix} A & e \\ e^T & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{23}$$

Since $\frac{\partial f^2}{\partial \alpha^2} = A > 0$, the optimal fusion scalar weights $\alpha_i$ that minimize the performance index $J = tr(P^z)$ are given by:

$$\alpha_i = \frac{A^{-1} e}{e^T A^{-1} e} \tag{24}$$

In case of uncorrelated estimation error $\tilde{z}_i, \tilde{z}_j$ $i \neq j$; the coefficients $\alpha_i$ are given by the following equation

$$\alpha_i = \frac{\rho}{tra(P_i^z)} \qquad for i = 1, 2, ..., M \tag{25}$$

where:

$$\rho = (\sum_{j=1}^{M} \frac{1}{tr(P_j^z)})^{-1} \tag{26}$$

The equation (24) completely define the optimal scalars coefficients that minimizes the fusion criterion in a minimum variance sense.
The above techniques provides a suboptimal global estimate in compare to the optimal centralized Kalman fusion filter. However, they are more robust against transmission errors. But, hence the optimal coefficients either scalars or matrices can be pre-computed because they only require the system model and the noises characteristics to be known, they don't cope with unreliable measurements and unstable environments that characterizes practical scenarios; especially in tracking applications. To overcome these drawbacks, a new approach which integrate information about the local measurements reliability into the fusion formula is presented.

# 3 New approach

The key idea of the proposed approach is to reformulate the stochastic performance criteria into an instantaneous deterministic one according the concept of the convergence in realization revealed in [7]. But, unlike the former, our interest is mainly focused on the convergence of a weighted combination of the realizations, related to the local estimates, to an asymptotically optimal fused measurement. In addition, it addresses the problem of unstable environments by an on line estimation of the local variance matrices. Figure 1 shows how this approach involves:

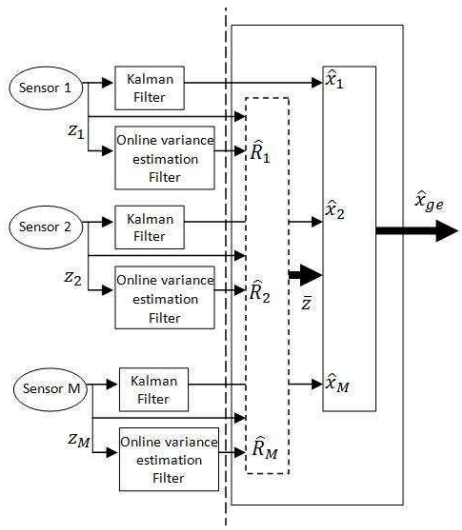


Figure 1: Schematic diagram of the proposed algorithm

## 3.1 Online sensor variance estimation

The decentralized fusion based on Kalman filter requires an exact prior knowledge of the model parameters and noise statistics. Although, in several studies, these characteristics are assumed to be invariant. But, in practical applications, especially in target tracking, the target model and noise statistics are partially known and may vary over the observation period. In [7], an on line estimation of the noise

variance matrices based on the correlation function is applied to cope with both stable and unstable systems based on a new stationary measurement process as described in the following paragraph.
From [7], lets consider the multisensor linear time-invariant discrete system

$$\begin{aligned} x(t+1) &= Ax(t) + Gv(t) \\ y_i(t) &= Hx(t) + w_i(t) \end{aligned} \tag{27}$$

Lets assume that $v(t)$ and $w_i(t)$ are uncorrelated Gaussian noises with zero mean, a process variance matrix $Q > 0$ and unknown measure variances $R_i > 0$. One can write:

$$y_i(t) = H(I_n - q^{-1}A)^{-1}Gq^{-1}v(t) + w_i(t) \tag{28}$$

where $q^{-1}$ is the backward shift operator,$I_n$ is the identity matrix of dimension $n$.
Using the left co prime factorization we can write that

$$H(I_n - q^{-1}A)^{-1}Gq^{-1} = P^{-1}(q^{-1})B(q^{-1}) \tag{29}$$

Where $P(q^{-1})$ and $B(q^{-1})$ are polynomial matrices having the form of

$$X(q^{-1}) = X_0 + X_1q^{-1} + ... + X_nq^{-n} \tag{30}$$

With $P_0 = I_L$, L is the measurement vector dimension, $B_0 = 0$ and $X_n \neq 0$,$X_j = 0(j > n)$.
Replacing (29) into (28) yields

$$z_i(t) = B(q^{-1})v(t) + P(q^{-1})w(t) \tag{31}$$

$$z_i(t) = P(q^{-1})y_i(t) \tag{32}$$

Applying (31) yields that $z_i(t)$ is a stationary process because it is a linear combination of two stationary process.
The correlation function of $z_i(t)$ is denoted as

$$R_{zi}(k) = E[z_i(t)z_i(t-k)] \qquad k = 0, 1, ..., n_{zi} \tag{33}$$

$$R_{zi}(k) = 0, \qquad k > n_{zi} \tag{34}$$

Where $n_{zi}$ is the highest order between the two polynomials $P(q^{-1})$ and $B(q^{-1})$.
Then, the sampled estimate of $R_{zi}$ is given by

$$\hat{R}^t_{zi}(k) = \frac{1}{t}\sum_{j=1}^{t} z_i(j)z_i^T(j-k) \tag{35}$$

Where $k = 0, 1, ..., n_{zi}$, $t = 1, 2, ...t_f$. And the correspondent recursive formula will be

$$\hat{R}^t_{zi}(k) = \hat{R}^{t-1}_{zi} + \frac{1}{t}[z_i(t)z_i^T(t-k) - \hat{R}^{t-1}_{zi}(k)] \tag{36}$$

With the initial value $\hat{R}^1_{zi}(k) = z_i(1)z_i^T(1-k)$.
From [7] and according to the ergodicity of the stationary stochastic process $z(t)$ we have that

$$\hat{R}^t_{zi}(k) \rightarrow R_{zi}(k) \;\; t \rightarrow \infty \;\; w.p.1. \tag{37}$$

The correlation function (33) yields

$$R_{zi}(k) = \sum_{j=k}^{n_b} B_jQB^T_{j-k} + \sum_{j=k}^{n_p} P_jR_iP^T_{j-k} \tag{38}$$

for $k = 1, ..., n_{zi} \quad i = 1, ..., M$ and $B_j$and$P_j$ are known.
Expanding equation (38) for each element of the matrices will provide a set of linear equations in the form of

$$\Omega_i\theta_i = \omega_i \tag{39}$$

where $\Omega_i$ and $\omega_i$ are known and $\theta_i$ represent the matrices components of $Q$ and $R_i$ . If $\Omega_i$ has a full rank for a fixed number of unknown variance matrices components, we can select $n_i$ linear equations as:

$$\Omega_{i0}\theta_i = \omega_{i0} \tag{40}$$

where $\Omega_{i0}$ is a non singular matrix.
Hence $\theta_i$ can be solved as

$$\theta_i = \Omega_{i0}^{-1}\omega_{i0} \tag{41}$$

using the estimates $\hat{R}^t_{ri}(k)$ we can write

$$\hat{\theta}_i = \Omega_{i0}^{-1}\hat{\omega}_{i0} \tag{42}$$

Which defines the variance matrix components of the $ith$ sensor.

### 3.2 New performance criteria

So far, the stochastic convergence analysis provided a complex general solution to the state fusion problem as in [5]. However, in practical applications, the cross-covariance matrices may not be calculated easily. This statement leads us to investigate a new performance criteria and develop an efficient state fusion algorithm based the concept of convergence in realization revealed in [7].
From the viewpoint of a stochastic process, a sample function of the time t, obtained from a given random experiment, is called a realization of a stochastic process. Reversely, a family of realizations can be seen as a stochastic process. Moreover, an event with probability one will be inferred as sure in a given random experiment so that the convergence with probability one yields to the convergence in realization. Furthermore, in [10], it was established that the centralized fusion measurement can be viewed as a fused measurement model of all local measurements. Precisely, in [14] the author shows that the optimal fused measurement equation is equivalent to the measurement equation of the centralized fusion scheme, which is globally optimal [15].
So if we consider that the "realization" of the optimal centralized fusion scheme is available at each time increment $k$ we can reformulate the state fusion problem as follows:
Given the locally unbiased states estimates $\hat{x}_1, ..., \hat{x}_M$ obtained from local Kalman filters, what are the weighting state coefficients that minimize the error between the optimal fused measurement and the combination of the local

measurements related to the state estimations at each time increment $k$?.

The resulting quadratic criteria will be in the form of

$$J = \|\bar{z}(k) - \sum_{i=1}^{M} \hat{z}_i(k)\|^2 \quad (43)$$

Where

$$\hat{z}_i(k) = H_i(k)\alpha_i\hat{x}_i(k) \quad (44)$$

And $\hat{x}_i(k)$ is the local unbiased state estimate provided by the $ith$ local Kalman filter, $\alpha_i$ is the corresponding coefficient and $H_i$ is the local observation matrix of sensor $i$.

$\bar{z}(k)$ is the instantaneous fused measurement that minimizes the effect of the unreliable measurements according to their estimated variances matrices $\hat{R}_i(k)$.

In the general case, the optimal fused measurement is given by the following equation

$$\bar{z}(k) = [\sum_{i=1}^{M} H_i^T \hat{R}_i(k)^{-1} H_i]^{-1} \sum_{i=1}^{M} H_i^T \hat{R}_i(k)^{-1} z_i(k) \quad (45)$$

Where $z_i$ is the local measurement, $H_i \neq H_j$ for $i \neq j$ $i = 1, ..., M$. $M$ is the sensors number.

Thus, the global state estimate is given by the following formula

$$x_{ge} = \sum_{i=1}^{M} \alpha_i \hat{x}_i \quad (46)$$

To ensure the unbiasedness of the resulting global estimate, the constraint $\sum_{i=1}^{M} \alpha_i = I_N$ had to be satisfied.

**proof.**

Assuming that $E(x_i - x_{ge}) = 0$

$$E(\sum_{i=1}^{M} \alpha_i \hat{x}_i) = \sum_{i=1}^{M} \alpha_i E(\hat{x}_i) = \sum_{i=1}^{M} \alpha_i E(x_{ge}) \quad (47)$$

Thus $\sum_{i=1}^{M} \alpha_i = I_N$ is a prerequisite to the unbiasedness of the fused estimate.

And the statistical criteria given in [4], [5] reduces to minimize an instantaneous quadratic error obtained by replacing (44) in (43) as

$$arg \quad min_{\sum_{i=1}^{M} \alpha_i = I_N} J(k) = \|\bar{z}(k) - \sum_{i=1}^{M} H_i \alpha_i \hat{x}_i(k)\|^2 \quad (48)$$

the $\|.\|$ refers to the Euclidean norm of a vector given by the formula

$$\|x\| = \sqrt{x^T x} \quad (49)$$

Setting

$$\bar{z} = \frac{1}{M} \sum_{i=1}^{M} \bar{z} \quad (50)$$

And replacing in (48) yields

$$arg \quad min_{\sum_{i=1}^{M} \alpha_i = I_N} J(k) = [\sum_{i=1}^{M} (\frac{1}{M} \bar{z}(k) - H_i \alpha_i \hat{x}_i(k)]^2 \quad (51)$$

Hence

$$\sum_{i=1}^{M} min[(\frac{1}{M}\bar{z}(k) - H_i\alpha_i\hat{x}_i(k)]^2$$
$$\to min[\sum_{i=1}^{M}(\frac{1}{M}\bar{z}(k) - H_i\alpha_i\hat{x}_i(k)]^2 \quad (52)$$

The problem becomes: finding the coefficients $\alpha_i$ that minimize the error between a linear combination of the local measurements $h_i\alpha_i\hat{x}_i$ and the expected local optimal measurement $\bar{z}_i = \frac{1}{M}\bar{z}$ under the constraint $\sum_{i=1}^{M} \alpha_i = I_N$

Moreover (48) will equal zero if :

$$\frac{1}{M}\bar{z} = h_i\alpha_i\hat{x}_i \quad (53)$$

Finally, the whitening coefficients will be in the form of

$$\alpha_i = \frac{1}{M}(H_i^T H_i)^{-1} H_i^T \bar{z}\hat{x}_i^T(\hat{x}_i\hat{x}_i^T) \quad (54)$$

Equation (54) completely defines the generalized coefficients formula for a decentralized state fusion problem.

Thus we can resume the algorithm steps in table 1:

| |
|---|
| A) Estimate the online variance matrices for each sensor<br>using the correlation method described in subsection 3.1<br>Compute the optimal fused measurement $\bar{z}(k) = \sum_{i=1}^{M} w_i(k) z_i(k)$<br>with the relevant weights<br>$w_i(k) = [\sum_{i=1}^{M} H_i^T \hat{R}_i(k)^{-1} H_i]^{-1} \sum_{i=1}^{M} H_i^T \hat{R}_i(k)^{-1}$<br>M is the number of sensors |
| B) Proceed the local Kalman filters independently<br>Calculate the unbiased optimal local state estimates $\hat{x}_i(k)$<br>using Kalman filter [13] |
| C) Calculate the corresponding weights<br>$\alpha_i = \frac{1}{M}(H_i^T H_i)^{-1} H_i^T \bar{z}\hat{x}_i^T(\hat{x}_i\hat{x}_i^T)$ |
| E) Compute the global state estimate as<br>$\hat{x}_{ge}(k) = \sum_{i=1}^{M} \alpha_i(k)\hat{x}_i(k)$ |

Table 1: The Proposed Algorithm steps

# 4 Simulations

To show the efficiency of the proposed algorithm we compare the Root Square Error (RSE) of the estimated position in Cartesian coordinates with the one obtained using the decentralized state fusion formula using matrices as proposed in [5] and the optimal centralized fusion filter based on Kalman filter methods. The simulation scenario involves three sensors assumed to have processing capabilities. The target is defined as a 2D non-maneuvering model with the following system state transition and observation equations:

$$x(k+1) = \mathbf{A}x(k) + \mathbf{G}v(k)$$
$$z_i(k) = \mathbf{H}_i x(k) + w_i(k) \tag{55}$$

where :

$$A = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} G = \begin{pmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{pmatrix} H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}^T \tag{56}$$

$v(k)$ and $w_i(k)$ are respectively the additive process noise and sensor noises with zeros mean and known process variance $Q$, but unknown sensors variances $R_i$. The measurements which are the target positions according to the sensor geographical positions are provided in Cartesian coordinates (X,Y) with probability one (no miss) at a constant rate of $5s$ (scan period). The simulation parameters are identical for all the three algorithms and include the sensor standard deviations according to $X, Y$ coordinates $(250m, 250m), (300m, 300m), (200m, 200m)$. The initial target position and speed are respectively $400m, 100m$, $100m/s, 100m/s$. The initial covariance estimation error matrix is $diag(P_0) = \{100, 10, 100, 10\}$ and the simulations results are collected using 500 Monte-Carlo runs.

To show the robustness of our algorithm against inconsistent measures, two cases are considered. The first one occurs when the output of one or several sensors don't provide any information about the target position. To simulate this phenomenon, the measurements of the $ith$ sensor are replaced by a Gaussian noise with zero mean and a variance matrix $R_i = R_f$ where $R_f >> R_i$ since $t = 50T$.
While in the second case, we consider the rejection of a glint noise that affects one or several sensors. These "glint measures" are generated according the Scaled-Contaminated Model (SCM) which is a mixture of two Gaussian random processes

$$N_v(0, \sigma_0^2, \sigma_1^2) = \epsilon N(0, \sigma_0^2) + (1-\epsilon) N(0, \sigma_1^2) \tag{57}$$

Where $\sigma_0 \ll \sigma_1$ and $\epsilon < 1$.
We notice here that if $\epsilon = 0$ the SCM reduces to the Gaussian distribution.
***case 1.*** The first step of the algorithm is to compute the optimal fused measurement according to the estimated variances $\hat{R}_1, \hat{R}_2, \hat{R}_3$ as defined in subsection 3.1.
In figure 2 the estimated variance matrix component according to the decoupled coordinates $X, Y$ is plotted.
Figure 3 shows that the fused measure is not affected by the erroneous measurement of sensors 2, while the corresponding state estimate diverges as noticed in figure 4. Also, in figure 5, we can see that the fused trajectory obtained using

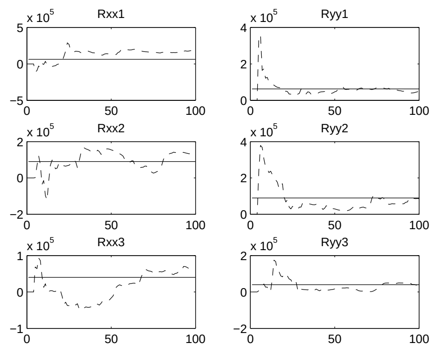


Figure 2: on line variances estimation in X,Y coordinates

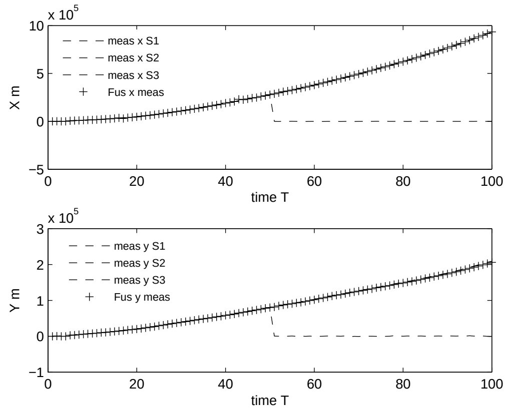


Figure 3: Fused measurement and local measurements

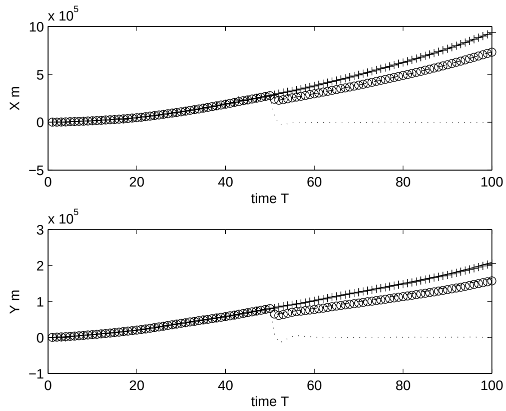


Figure 4: Fused state estimates and local estimates

the optimal centralized and decentralized state fusion algorithm presented in [5] are affected by the inconsistent measures of sensor 2, while the fused trajectory generated by the proposed algorithm is closer to the reel trajectory. This is also verified when we compare the RSE according the coordinates $X, Y$ in figure 6.

When this phenomenon affects 2 sensors simultaneously at

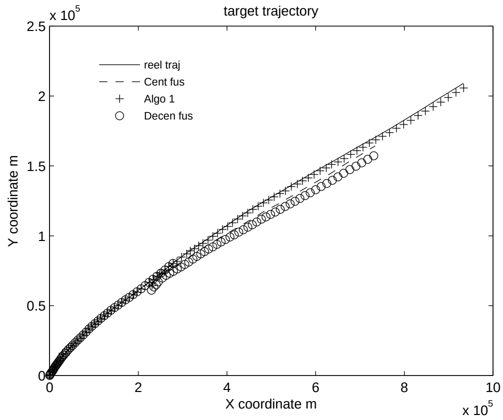


Figure 5: Target trajectory in X,Y coordinates

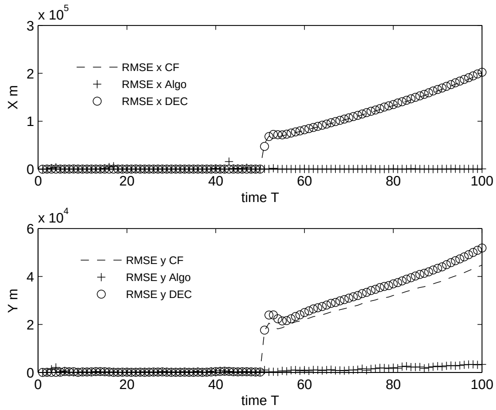


Figure 6: RSE in X,Y

$t = 80s$ which means that over $50\%$ of the sensors are providing erroneous measures, our algorithm is still efficient, hence the effect of the erroneous measures is mitigated while processing the fused measurement. Thus, the corresponding state weight will tend to zero leading to inhibit its contribution to the global state estimation.

***case 2.*** The "glint" noise replaces the reel measurements generated at time intervals equal to $10T$. We show in figures $8$ and $9$ the resulting RSE for the three algorithms according to the Cartesian coordinates $X, Y$ when this phenomenon affects one then two sensors.

*Remark. 1* The unreliable observations originate from an undetectable sensor component failure or a bad calibration procedure that may occur in practical applications. In this work three cases are considered, the first one occurs when the sensor outputs provide no information about the target position i.e the observation are modeled as a pure noise. Thus, the contribution of the corespondent measurement is negligible

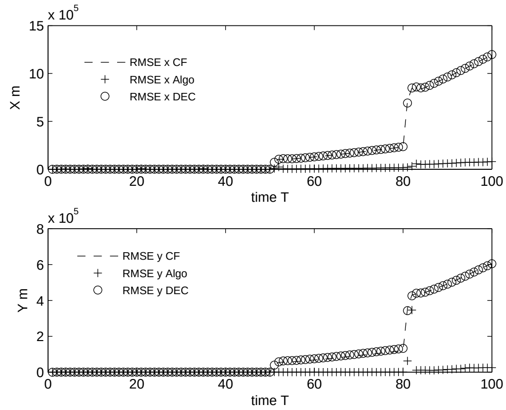


Figure 7: RSE in X,Y in case of 2/3 sensors are affected by erroneous measures

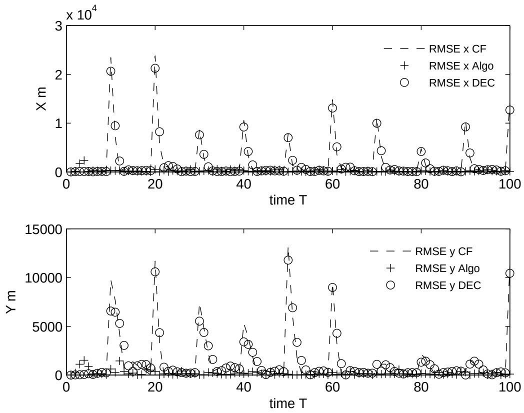


Figure 8: RSE in X,Y in case of glint noise affecting sensor 2

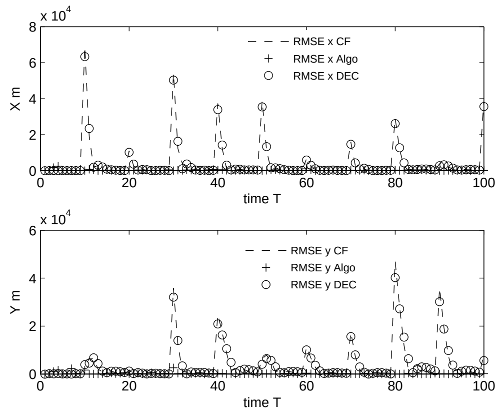


Figure 9: RSE in X,Y in case of glint noise affecting sensors 2 and 3

in compare to the remaining sensors so its effect is mitigated by itself. The second case is when the sensor provides a constant value in addition to a noise process (in the simulations, the constant was chosen equal to the last reliable measurement). The third case occurs when the output is a pure random noise with variance $R >> R_2$. The obtained results show that our algorithm outperforms the remaining ones in terms of rejecting these phenomenons.

*Remark. 2* The proposed algorithm processes in two main steps. The first one can be assimilated to a data fusion process and the second to a state fusion process. Thus, the effect of an unreliable measure will be mitigated by the corresponding weight in the first step, while a bad estimation state will provide an expected measure with a large residual error in compare to the optimal fused measure leading to a negligible state coefficient.Thus it does not affect the global estimation state as shown in figures $5$ and $6$. Contrariwise, the centralized fusion scheme uses the local measures as they are acquired. Thus, local erroneous measure lead to the divergence of the global estimate. Also, in case of decentralized fusion [5], the local erroneous estimate resulting from sensor $(2)$ affects the global estimate because the corresponding weighting coefficient will not mitigate the effect of the local estimate.

*Remark. 3* In practical it is difficult to evaluate the computational cost of these algorithms because some sections of the resulting program code are executed in parallel mode (simultaneously in locals processors) while others are executed sequentially by a single processor at the fusion center. Thus, additional simulations that take into consideration the execution sequences of the resulting programs code are required.

*Remark. 4* The on line estimation of the sensors variances requires the processing of the variance matrices of the local sensors, this process occurs when all the variances matrices are available simultaneously. Also, the fused measurement has to be processed when all the measurements are available that's why we will assume here that the system is synchronized and the transmission delays are negligible.

# 5 Conclusion

In this work, a new approach to the decentralized state fusion problem is proposed, it provides a general fusion formula based on a new performance criteria in a realization sense. The key idea of the developed algorithm is to strength the robustness of a decentralized fusion scheme by integrating an updated information about the reliability of the sensing system. The obtained results show that our algorithm outperforms the related fusion approaches to reject the unreliable measures and copes with unstable environments encountered in target tracking applications.

# References


[1] Y. Bar-Shalom,"On the track-to-track correlation problem,"*IEEE Trans. Autom.Control*,vol. AC-26,no.2,pp571-572, Apr 1981.

[2] Y. Bar-Shalom and L. campo,"The effect of common process noise on the two-sensor fused-track covariance,"*IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-22,no.6, pp .803-805,Nov.1986.

[3] K. H. Kim,"Development of track to track fusion Algorithm," *Procced. of the American Control Conference*, Maryland, June 1994.

[4] S. S. Li, D. Z. Li ,"Multi-Sensor Information Fusion Kalman Filter Weighted by Scalars for Systems With Colored Measurement Noises,"*Journ. of dynam. syst., Measurem., and contr.*, vol. 127,no.12, pp. 663-667,2005.

[5] J. Zhou,Y. Zhu,Z. You and E. Song,"An Efficient Algorithm for optimal Linear Estimation Fusion in Distributed Multisensor Systems,"*IEEE Trans. syst.,Man, Cybern.*, vol.36,no.5,pp1000-1009,2006.

[6] S. H. Lee,D. Y. Kim,N. Nguyen and V. Shin,"Comparison of Multi-sensor fusions Filters Weighted by Scalars and Matrices,"*Internatio. Conf. on Contr., Automat. and syst.*, Seoul, Korea 2007.

[7] Y. Gao, W Jia,X. Sun,Z. Deng,"Self-Tuning Multisensor Weighted Measurement Fusion Kalman filter,"*IEEE Trans. Aero. and Electr. Syst.*, vol. 45,no.1 Jan. 2009.

[8] Y. Bar-shalom (Ed.),"Multitarget Multisensor Tracking: Advanced Apllications,"*Artech House, Norwood*, MA,1990.

[9] M. E. Liggins,D. Hall, J. Llinas (Ed.),"Handbook of Multisensor Data Fusion: Theory and Practice,"*Second Edition, CRC Press*,2009.

[10] C. Ran, Z. Deng,"Reduced Dimension weighted measurement fusion Kalman filtering algorithm,"*Control and Decision Conference, 2009. CCDC apos*,09. Chinese Volume , Issue , 17-19 June 2009 Page(s):2196 - 2200.

[11] J. Manyika and H. Durrant-Whyte,"Data Fusion and Sensor Management: a decentralized information-theoretic approach,"New Work, Ellis Horwood,1994.

[12] D.L. Hall and J. Llinas,"Handbook of Multisensor data fusion,"*New York : CRC Press*,2001.

[13] E. Brookner ,"Tracking and Kalman filtering made easy,"*John Wiley & Sons*,1998.

[14] Q. Gan and C.J. Harris,"Comparison of two measurement fusion methods for Kalman-filter-based multisensor data fusion,"*IEEE Transaction on Aerospace and Electronic Systems*, 273-279,2001.

[15] Z. L. Deng,Y. Gao,L. Mao,Y. Li,G. Hao,"New approach to information fusion steady-state kalman filtering,"*Automatica*, 1695-1707, 2005.