Multi-source cooperative communications using low-density parity-check product codes

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***Abstract*—In this paper, we investigate the performance of low-density parity-check (LDPC) product codes in a multi-source relay network where multiple sources transmit data to a same destination with the help of a noisy relay. We consider an LDPC product code resulting from the concatenation of Multiple Serially Concatenated Multiple Parity-Check (M-SC-MPC) codes. Every source encodes its data using the same M-SC-MPC code and broadcasts the codeword to relay and destination. The relay decodes and stores all codewords from sources in the rows of a matrix and encodes the columns using another M-SC-MPC code. Only the redundancy part generated by the relay is forwarded to the destination. At the destination, the codewords from the sources and the redundancy part from the relay form an observation of a product codeword whose parity check matrix is sparse. This LDPC product code can be iteratively decoded at destination using the low complexity sum-product algorithm. Since there are errors at the input of the relay-destination channel, an appropriate log-likelihood ratio is used in the LDPC decoding at destination in order to reduce the error propagation effect. The system error performance is given on the additive white Gaussian noise (AWGN) channel and the Rayleigh flat fading channel.**

***Index Terms*—LDPC codes, product codes, relay, cooperative communications, error propagation, sensor network.**

I. Introduction

Based on the relay model studied by Cover and El. Gamal [1], cooperative communication in wireless networks has attracted much attention during the last years. Many cooperative methods such as Amplify-and-Forward, Decode-and-Forward and Coded Cooperation have been proposed [2].

In [3], the authors have studied a cooperative scheme for a wireless network with multiple sources, a relay and a destination. The relay uses decode-and-forward method to help the sources to transmit their data to the same destination. All sources use a same Bose-Chaudhuri-Hocquenghem (BCH) code to protect their data and broadcast the codewords to relay and destination. Instead of forwarding the whole decoded (detected) source codewords to destination, the relay stores all the codewords from sources in the rows of a matrix and encodes the columns using another BCH code and forwards only the relay-generated redundancy to the destination. At the destination, the codewords from sources and the redundancy from the relay are decoded iteratively using the turbo product code decoding algorithm [4]. Since the source-relay channel is noisy, there are residual errors after relay decoding (detection). After relay encoding, these errors are propagated to the redundancy part. In [3], the turbo decoding algorithm takes these errors into consideration and computes an appropriate log-likelihood ratio (LLR) for the observations coming from the relay-destination channel to alleviate the error-propagation problem. High coding gain has been obtained through the proposed cooperative scheme. The main advantage is that turbo code performance is achieved on the network without increasing transmission delay usually associated to the turbo-coding at each source. It will benefit the sensor networks where sensors transmit small data blocks at a low data rate.

In this paper, we investigate the error performance of this cooperative scheme with extension to the low-density parity-check (LDPC) codes [5]. Our study is different from [6] and [7] where cooperation is based on the bi-layer LDPC code, which is well designed to guarantee the perfect reception over the noisy source-relay channel. We concentrate on the imperfect source-relay transmission that will propagate errors at relay reception. In this paper, we use a special type of LDPC code defined as the LDPC product code [8] whose encoding can be implemented as a traditional product code. The component code of the LDPC product code is the multiple serially concatenated multiple parity-check (M-SC-MPC) code proposed in [9].

In our cooperative scheme, the motivation of using LDPC codes comes from the property of low-density generator matrix (LDGM) code. If the block code used at relay (the column code of the product code) has a sparse generator matrix, few parity bits are influenced by relay decoding errors. The error-propagation problem will be less severe than in the case of using a dense generator matrix.

The reminder of the paper is organized as follows. The M-SC-MPC codes and the LDPC product codes are introduced in section II. The network setup is introduced in section III. In section IV, we introduce the cooperative scheme and the corresponding LDPC decoding at the destination. The simulation results on the additive white Gaussian noise (AWGN) channel are given in section V and in section VI,

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$$\mathbf{H}_{p2} = \begin{bmatrix} \mathbf{h}_{b,1} & 0 & 0 & 0 & \mathbf{h}_{b,2} & 0 & 0 & 0 & \dots & \mathbf{h}_{b,n_b} & 0 & 0 & 0 \\ 0 & \mathbf{h}_{b,1} & 0 & 0 & 0 & \mathbf{h}_{b,2} & 0 & 0 & \dots & 0 & \mathbf{h}_{b,n_b} & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 & \ddots & 0 & \dots & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{h}_{b,1} & 0 & 0 & 0 & \mathbf{h}_{b,2} & \dots & 0 & 0 & 0 & \mathbf{h}_{b,n_b} \end{bmatrix} \quad (5)$$

we conclude this paper.

II. LDPC Product Codes

A. M-SC-MPC Codes

An M-SC-MPC code [9] is constructed as the serial concatenation of M component polynomial codes. The generator polynomial of the i-th component code with dimension k_i, length n_i and redundancy $r_i = n_i - k_i$ is:

$$g_i(x) = 1 + x^{r_i}. \quad (1)$$

Fig. 1 shows the encoder structure of the M-SC-MPC code. Every component encoder has an encoding matrix of r_i rows and $\lceil \frac{k_i}{r_i} \rceil + 1$ columns. In this matrix, the k_i input bits of the i-th component encoder are organized in the column order, from top left to bottom right, in condition that the first $(r_i - s_i)$ locations of the encoding matrix are not used, with $s_i = (k_i \bmod r_i)$. Every row of the matrix is encoded by a single parity-check (SPC) code and the parity bit is appended to the last column. The interleaver reads out the encoded bits in the encoding matrix of the preceding component encoder in column order, from top left to bottom right, and writes them into the new encoding matrix of the next component encoder in the same way. Since every component encoder is systematic, the M-SC-MPC code is also systematic.

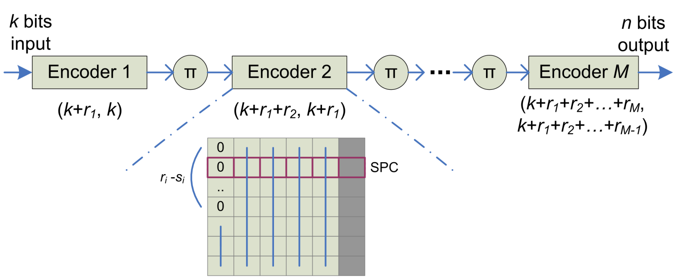

Fig. 1. M-SC-MPC encoder structure

The parity-check matrix of every component code defines a sub-matrix of the parity-check matrix of the M-SC-MPC code. With large enough values of r_i, the parity check matrix of the M-SC-MPC code is sparce, so the M-SC-MPC code is an LDPC code. In [9], it has been proved that the parity-check matrix of M-SC-MPC code is free of length-4 cycles if the code length n of M-SC-MPC code is smaller than $r_1 r_2 + \sum_{i=2}^{M} r_i$, where all r_i are distinctive, coprime and increasing with i. Using the above construction, we can find that the M-SC-MPC code is not only an LDPC code but also an LDGM code.

B. LDPC Product Codes

LDPC product codes can be constructed as a serial concatenation of M-SC-MPC codes. A construction method is given in [8]. Suppose that the parity-check matrices of two component M-SC-MPC codes are:

$$\begin{aligned} \mathbf{H}_a &= [\mathbf{h}_{a,1}, \mathbf{h}_{a,2}, \dots, \mathbf{h}_{a,n_a}]. \\ \mathbf{H}_b &= [\mathbf{h}_{b,1}, \mathbf{h}_{b,2}, \dots, \mathbf{h}_{b,n_b}]. \end{aligned} \quad (2)$$

where $\mathbf{h}_i$ represents the i-th column of the matrix. Then the parity-check matrix of the LDPC product code can be expressed as:

$$\mathbf{H}_p = \begin{bmatrix} (\mathbf{H}_{p1})_{r_a n_b \times n_a n_b} \\ (\mathbf{H}_{p2})_{r_b n_a \times n_a n_b} \end{bmatrix} \quad (3)$$

where $\mathbf{H}_{p1}$ is a block diagonal matrix:

$$\mathbf{H}_{p1} = \begin{bmatrix} \mathbf{H}_a & 0 & \dots & 0 \\ 0 & \mathbf{H}_a & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{H}_a \end{bmatrix} \quad (4)$$

and $\mathbf{H}_{p2}$ is given by (5) at the top of this page.

Since $\mathbf{H}_{p1}$ and $\mathbf{H}_{p2}$ are redundant, we should eliminate the last $r_a r_b$ rows of $\mathbf{H}_{p1}$ or $\mathbf{H}_{p2}$, to obtain the full rank parity-check matrix $\mathbf{H}_p$. With the sparse parity-check matrix $\mathbf{H}_p$, the LDPC product code can be decoded iteratively using LDPC decoding algorithms such as the sum-product algorithm [10].

III. Network Setup

The network setup is illustrated in Fig. 2. There are k' sources transmitting independent data to the same destination $\mathbf{D}$ with the help of the relay $\mathbf{R_1}$. We assume that all the source-destination channels have the same average signal to noise ratio (SNR_{sd}) and so have the source-relay and relay-destination channels (SNR_{sr} and SNR_{rd} respectively). The transmission of the sources and the relay is scheduled by the time-division multiple-access (TDMA) mode and the multiple-access interference is not considered.

The sources are sensors transmitting at low data rate but thanks to the large number of sensors, the network sum-rate is high. The relay with better computing and energy capability is located close to the sensors to help the transmission. We suppose that all sources and the relay locate at the same distance to the destination, so $d_{rd} = d_{sd}$ and $SNR_{rd} = SNR_{sd}$. Generally, $SNR_{sr} = SNR_{sd} + \Delta SNR$. The value of ΔSNR depends on the channel model and the distance relation between source-relay and relay-destination. It can be expressed as:

$$\Delta SNR = 10 \log_{10} \left(\frac{d_{sd}}{d_{sr}} \right)^{l}. \tag{6}$$

In the case of multi-path propagation, we set $l = 3.5$. If $d_{sd}/d_{sr} = 2$, ΔSNR equals about 10.5dB.

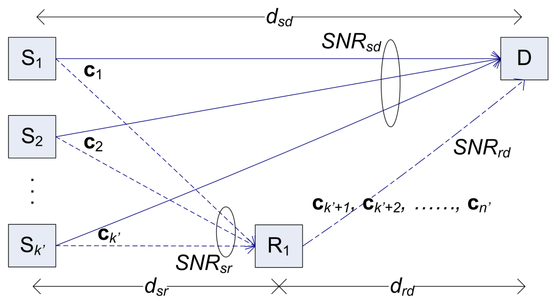

Fig. 2. Multi-source cooperative network

IV. Cooperative Scheme & Iterative Decoding at Destination

The LDPC product code described in section II is applied to the relay cooperative network in Fig. 2. The k' sources encode their independent data using the same M-SC-MPC (n, k) code and broadcast the codewords to destination and relay. There are several strategies that can be used at relay such as hard detection, hard decoding and soft decoding. After decoding (detecting) all k' codewords from sources, the relay stores them in the first k' rows of a matrix as illustrated in Fig. 3 and encodes the columns of the matrix with a second M-SC-MPC code. By column encoding, the relay obtains a codeword of a product code (with some erroneous bits) whose parity-check matrix $\mathbf{H}_p$ is sparse. Only the relay generated column redundancy part is forwarded to the destination. The destination receives the noisy codewords coming from sources as well as the noisy column redundancy from relay.

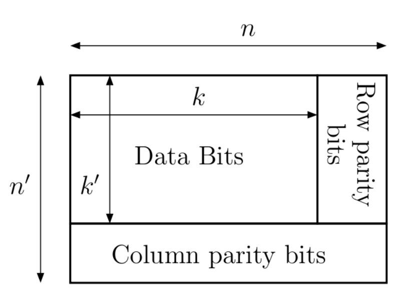

Fig. 3. Product code matrix

The parity-check matrix $\mathbf{H}_p$ of the product code is given by equation (3). The destination can decode the data using the sum-product algorithm. We consider the AWGN and Rayleigh fading channel models with the BPSK modulation. The received product codeword observation is modeled as:

$$r_i = \alpha_i \varepsilon_i e_i + b_i. \tag{7}$$

where α_i is the real Rayleigh fading coefficient and $\alpha_i = 1$ in the case of AWGN channel. $e_i = 2c_i - 1$ is a BPSK symbol, where c_i is the binary coded bit. b_i is the AWGN noise with zero mean and variance σ^2. $\varepsilon_i \in \{+1, -1\}$ is a random variable representing the error events at the output of the relay encoder with a corresponding bit error probability $\Pr\{\varepsilon_i = -1\} = p$. Perfect channel state information is assumed at the reception.

For every transmitted bit, the decoder at destination calculates the channel output LLR given by:

$$L(x_i) = \log \frac{\Pr\left(e_i = +1 | r_i,\, \alpha_i\right)}{\Pr\left(e_i = -1 | r_i,\, \alpha_i\right)}. \tag{8}$$

For the source-destination channel, there is no error event at the source, so $p = 0$ and the corresponding $L(x_i) = \frac{2\alpha_i r_i}{\sigma^2}$. For the observations coming from the relay-destination channel, it has been proved that [3]:

$$\begin{aligned} L(x_i) &= \frac{2\alpha_i r_i}{\sigma^2} + \ln \frac{1 + \frac{p}{1-p} \exp\left(\frac{-2\alpha_i r_i}{\sigma^2}\right)}{1 + \frac{p}{1-p} \exp\left(\frac{+2\alpha_i r_i}{\sigma^2}\right)} \\ &\approx \operatorname{sgn}(r_i) \cdot \min\left(\frac{2\alpha_i |r_i|}{\sigma^2},\, -\ln \frac{p}{1-p}\right). \end{aligned} \tag{9}$$

We denote $x = -\ln \frac{p}{1-p}$ as a limiter threshold on the LLR values at the output of the relay-destination channel.

During the LDPC iterative decoding, we apply a similar limiter denoted y to the LLRs of the message passed from check nodes to bit nodes such that the check node is associated to the column redundancy part of the product codeword.

V. Simulation Results

In the simulations, the $k' = 49$ sources use the M-SC-MPC (64, 49) code and the relay uses the same M-SC-MPC (64, 49) code. The M-SC-MPC (64, 49) code has two component encoders ($M = 2$) and its total redundancy bit number is 15 with $r_1 = 7$ and $r_2 = 8$ for each component encoder. Through cooperation, a LDPC product (4096, 2401) code is constructed. The destination uses the sum-product algorithm with limiters $(x,\, y)$ on LLRs as described above to decode the observed LDPC product codeword. The iteration number is fixed to 50. The network error performance measured in terms of BER and FER are given versus different values of E_b/N_0 where E_b is the average information bit energy received at destination including the signal from the relay [3]. We consider the hard detection at the relay.

A. AWGN Channel

For AWGN channels, we fix $(E_c/N_0)_{sr} = 6\text{dB}$ on the source-relay channel, where E_c is the average encoded bit energy received at relay. The average SNR on source-relay channel doesn't change with the increase of the average SNR of source-destination channel, which is a pessimist configuration in practice but considered for illustration purpose. When the relay uses hard detection strategy to receive the source codewords, the bit error probability p at relay output is about 2.3×10^{-2} and the corresponding theoretical limiter value x is about 3.7.

In Fig. 4, the network BER is plotted versus E_b/N_0. The curve 'M-SC-MPC' corresponds to the error performance of the M-SC-MPC (64, 51) code with direct transmission from sources to the destination without relay cooperation. The curve labeled "LDPC Product" corresponds to BER performance of the LDPC product (4096, 2401) code with error-free source-relay channel. It serves as the reference lower bound and a coding gain of about 3dB can be observed at a BER of 10^{-5}. This gain shows the benefits of the cooperative scheme. The curves labeled 'LDPC 1R, no Limit' corresponds to the cooperative case with the hard detection at relay and the destination uses the original sum-product decoding algorithm without limiters on the LLRs. For this curve, we observe the error floor effect which is caused by the residual errors at the relay. The curves labeled 'LDPC 1R (x, y)' also correspond to the cooperative case with the limiters (x, y) as described in section IV in the LDPC decoding at destination. We can observe that the error floor is largely alleviated by applying the limiter $x = 3.7$ on the input LLRs of the LDPC decoder. By adding the second limiter $y = 2$ on the LLR of the message passed from check node to bit node, the error floor is further reduced and the LDPC decoder becomes more robust in the presence of relay errors. The coding gain versus the direct transmission of M-SC-MPC code is of about 2dB at a BER of 10^{-5}. The value of y is obtained through simulations and needs further investigation.

Similar behavior can be observed for the FER performance in Fig. 5. In order to provide a fair comparison on FER, the FER corresponds to the direct transmission of M-SC-MPC (64, 51) code is calculated for a frame consisting of 49 M-SC-MPC (64, 51) codewords, which equals to the number of sources in our application.

B. Fast Rayleigh Fading Channel

For the fast Rayleigh fading channel, we consider that $SNR_{sr} = SNR_{sd} + 10\text{dB}$ with a hard-detection relay. When E_b/N_0 increases from 0dB to 16dB, the corresponding bit error probability p at the output of relay encoding decreases from 2.2×10^{-1} to 7.8×10^{-3}. We choose an average limiter $x = 3$ to be applied on the input LLRs of LDPC decoder. In Fig. 6 and Fig. 7, BER and FER are plotted versus E_b/N_0 at destination. The curve labeled 'LDPC Product' corresponds to the case of perfect decoding at the relay. It gives the lower bound for the network error rate. Compared

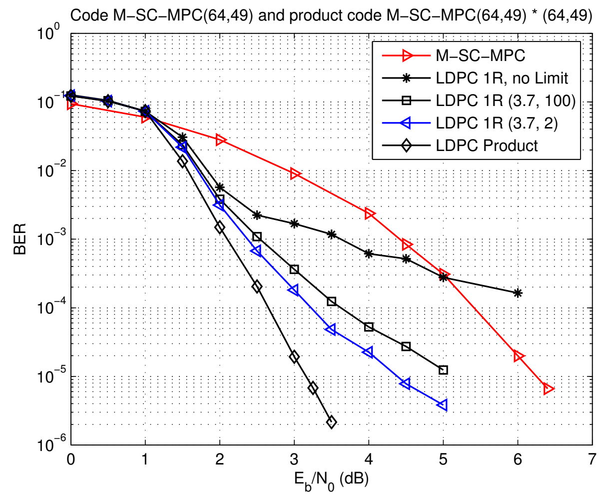

Fig. 4. BER performance on AWGN channel with hard-detection relay, $(E_c/N_0)_{sr} = 6\text{dB}$. 50 iterations

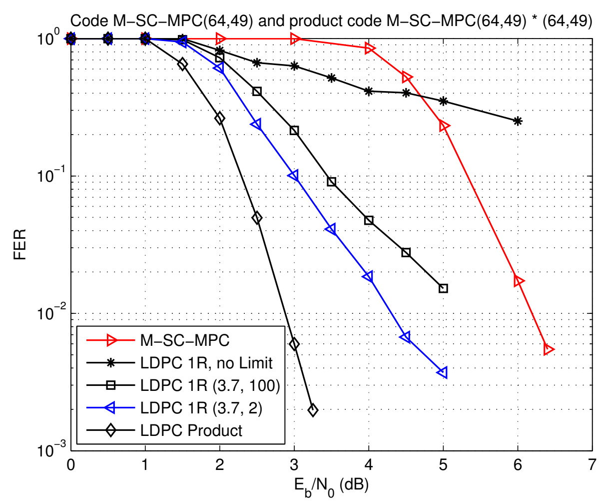

Fig. 5. FER performance on AWGN channel with hard-detection relay, $(E_c/N_0)_{sr} = 6\text{dB}$. 50 iterations

to the direction transmission using M-SC-MPC code without cooperation, the coding gain is of 9.3dB at BER=10^{-5}. The curve labeled 'LDPC 1R no Limit' corresponds to the case with a hard-detection relay without limiters on LLRs at the destination. We observe a severe error floor for high SNRs. If we apply the average limiters $(x, y) = (3, 2)$ on LLRs, the error floor phenomenon is alleviated and the coding gain of the cooperative scheme is of 6dB at a BER of 10^{-5}, as shown by the curve 'LDPC 1R (3, 2)'.

VI. Conclusion

In this paper, we have investigated the extension of the cooperative scheme proposed in [3] to LDPC product codes constructed by serial concatenation of M-SC-MPC codes. The simulation results show that this cooperative scheme can offer good coding gain on both AWGN and Rayleigh fading

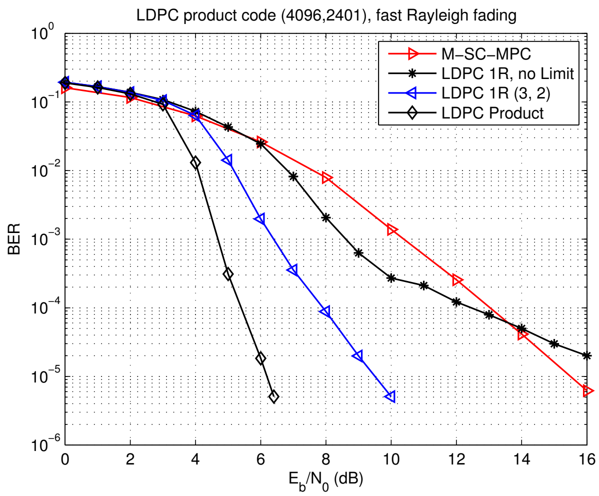

Fig. 6. BER performance on fast Rayleigh fading channel with hard-detection relay, $(E_b/N_0)_{sr} = E_b/N_0 + 10$dB. 50 iterations

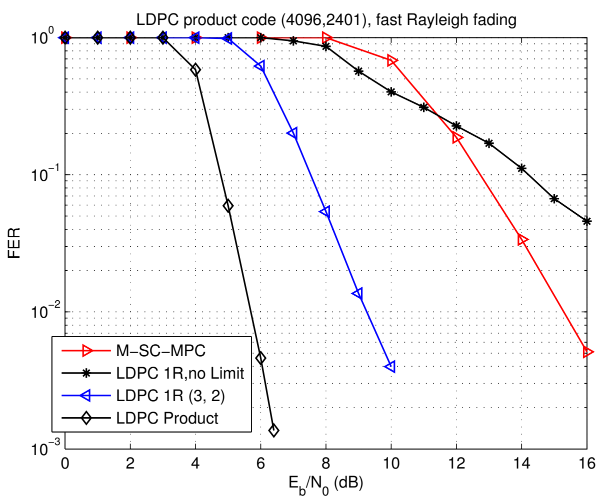

Fig. 7. FER performance on fast Rayleigh fading channel with hard-detection relay, $(E_b/N_0)_{sr} = E_b/N_0 + 10$dB. 50 iterations

channels and is flexible in practice as we can use either turbo product code or LDPC codes for the transmission. The use of LLR-limiters in the presence of residual errors at the relay can alleviate the error-propagation problem.

The M-SC-MPC code used in this paper is not only the LDPC code but also the low-density generator matrix code. Since it has a sparse generator matrix, small amount of parity bits will be influenced by the relay errors. To reduce the error floor effect, it is more interesting to use the M-SC-MPC code than the BCH code used in [3]. But since the small M-SC-MPC code is not efficient in terms of error correction, the proposed LDPC-based scheme does not outperform the BCH-based scheme of [3].

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