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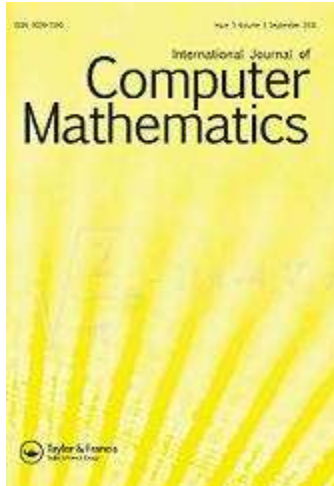
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Comments on “Inversion of A Generalized Vandermonde Matrix” by M. E. A. El-Mikkawy, 80 (2003) 759-765

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Abstract

In this paper, we give the comments on the article “Inversion of a Generalized Vandermonde Matrix” by M. E. A. El Mikkawy, Int. J. Computer Math., 80(2003) pp.759-765. The article gives an algorithm for the elementary symmetric functions calculation which contains a severe error. In these comments we proposed necessary corrections of that algorithm.

Keywords: Elementary Symmetric Functions, Numerical Recipes, Vandermonde Matrix, Confluent Vandermonde Matrix

1. Introduction

In this paper, we give the corrections of the numerical algorithm, presented in the article [1], for the calculation of the elementary symmetric functions. The elementary symmetric functions play an important role in algebra, particularly in the inversion of confluent Vandermonde matrices ([3]).

The paper is organized as follows: section 2 provides the necessary theoretical background, section 3 presents the algorithm from the work [1], section 4 shows, based on an example, why that algorithm leads to undefined results, section 5 proposes the corrected version of the algorithm, and section 6 shows its exemplary execution.

2. Theoretical background

At first let us present the notion of the symmetrical function. We can call a function symmetric, if and only if after any arbitrary permutation of its independent variables we receive the same function (Prasolov [2] pp.77-84). The notion of elementary symmetric functions is closely connected with the polynomial:

$$p_{nj}(x) = (x - c_1)(x - c_2) \dots (x - c_{j-1})(x - c_{j+1}) \dots (x - c_n) \quad (1)$$

After the expansion of the polynomial $p_{nj}(x)$ and sorting it with respect to decreasing powers of the x variable, we can receive:

$$p_{nj}(x) = x^{n-1} - \sigma_{2,j}^{(n)} x^{n-2} + \sigma_{3,j}^{(n)} x^{n-3} + \dots + (-1)^n \sigma_{n,j}^{(n)} \quad (2)$$

where $\sigma_{i,j}^{(n)}$ are just the elementary symmetric functions, defined as follows:

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2.1. Definition (El-Mikkawy [1] pp.759)

If the n parameters c_1, c_2, \dots, c_n are distinct, then the elementary symmetric functions $\sigma_{i,j}^{(n)}$ in $c_1, c_2, \dots, c_{j-1}, c_{j+1}, \dots, c_n$ are defined for $j = 1(1)n$ by:

$$\sigma_{1,j}^{(n)} = 1$$

$$\sigma_{i,j}^{(n)} = \sum_{\substack{r_1=1 \\ r_1 \neq j}}^n \sum_{\substack{r_2=r_1+1 \\ r_2 \neq j}}^n \dots \sum_{\substack{r_{i-1}=r_{i-2}+1 \\ r_{i-1} \neq j}}^n \prod_{m=1}^{i-1} c_{r_m}, \quad \text{for } i = 2(1)n \quad (3)$$

3. Source algorithm

The article [1] proposes the following algorithm for the elementary symmetric functions calculation:

Set $\sigma_{1,1}^{(1)} = 1$

For $i=2, 3, \dots, n$

$$\sigma_{i,1}^{(i)} = \sigma_{i-1,1}^{(i-1)} c_i$$

For $j=i-1, i-2, \dots, 2$

$$\sigma_{j,1}^{(i)} = \sigma_{j-1,1}^{(i-1)} c_i + \sigma_{j,1}^{(i-1)}$$

Next j

Next i

4. Example of the execution of the source algorithm

The algorithm 3 contains a severe error due to which it is impossible to use the algorithm. Let us try to calculate the series of the symmetric functions $\sigma_{i,1}^{(4)}$, $i = 1, 2, 3, 4$ by means of the algorithm 3. The iterations go as follows:

- $\sigma_{1,1}^{(1)} = 1$

- $i = 2$

$$\sigma_{2,1}^{(2)} = \sigma_{1,1}^{(1)} c_2 = 1 c_2 = c_2$$

- $i = 3$

$$\sigma_{3,1}^{(3)} = \sigma_{2,1}^{(2)} c_3 = c_2 c_3$$

$$j = 2: \sigma_{2,1}^{(3)} = \sigma_{1,1}^{(2)} c_3 + \sigma_{2,1}^{(2)} = \langle \text{undefined} \rangle \cdot c_3 + c_2 = \langle \text{undefined} \rangle$$

- $i = 4$

$$\sigma_{4,1}^{(4)} = \sigma_{3,1}^{(3)} c_4 = c_2 c_3 c_4$$

$$j = 3: \sigma_{3,1}^{(4)} = \sigma_{2,1}^{(3)} c_4 + \sigma_{3,1}^{(3)} = \langle \text{undefined} \rangle \cdot c_4 + c_2 c_3 = \langle \text{undefined} \rangle$$

$$j = 2: \sigma_{2,1}^{(4)} = \sigma_{1,1}^{(3)} c_4 + \sigma_{2,1}^{(3)} = \langle \text{undefined} \rangle \cdot c_4 + \langle \text{undefined} \rangle = \langle \text{undefined} \rangle$$

We can see that the execution of the algorithm proposed by the article [1] leads to the undefined results. In the next section we made the proper correction of the algorithm 3.

5. Corrected algorithm for the elementary symmetric functions calculation

A detailed analysis of the algorithm 3 leads to the conclusion that the improper results it gives are caused by the lack of the proper initiation of several variables. Namely, the algorithm does not initialize the series of the variables $\sigma_{i,1}^{(i)}$ for $i = 2, 3, \dots, n$. The correct version of the algorithm for determining the elementary symmetric functions has the following form:

```

1
2
3 Set  $\sigma_{1,1}^{(1)} = 1$ 
4
5 For  $i=2, 3, \dots, n$ 
6   Set  $\sigma_{1,1}^{(i)} = 1$ 
7
8    $\sigma_{i,1}^{(i)} = \sigma_{i-1,1}^{(i-1)} c_i$ 
9
10  For  $j=i-1, i-2, \dots, 2$ 
11     $\sigma_{j,1}^{(i)} = \sigma_{j-1,1}^{(i-1)} c_i + \sigma_{j,1}^{(i-1)}$ 
12
13  Next  $j$ 
14 Next  $i$ 

```

The code line we added is bold.

6. Example of the execution of the corrected algorithm

Let us calculate the series of the symmetric functions $\sigma_{i,1}^{(4)}$, $i=1,2,3,4$ by the corrected algorithm 5. The iterations go as follows:

- $\sigma_{1,1}^{(1)} = 1$
- $i = 2$
 $\sigma_{1,1}^{(2)} = 1$
 $\sigma_{2,1}^{(2)} = \sigma_{1,1}^{(1)} c_2 = 1c_2 = c_2$
- $i = 3$
 $\sigma_{1,1}^{(3)} = 1$
 $\sigma_{3,1}^{(3)} = \sigma_{2,1}^{(2)} c_3 = c_2 c_3$
 $j = 2: \sigma_{2,1}^{(3)} = \sigma_{1,1}^{(2)} c_3 + \sigma_{2,1}^{(2)} = 1c_3 + c_2 = c_2 + c_3$
- $i = 4$
 $\sigma_{1,1}^{(4)} = 1$
 $\sigma_{4,1}^{(4)} = \sigma_{3,1}^{(3)} c_4 = c_2 c_3 c_4$
 $j = 3: \sigma_{3,1}^{(4)} = \sigma_{2,1}^{(3)} c_4 + \sigma_{3,1}^{(3)} = (c_2 + c_3) \cdot c_4 + c_2 c_3 = c_2 c_3 + c_2 c_4 + c_3 c_4$
 $j = 2: \sigma_{2,1}^{(4)} = \sigma_{1,1}^{(3)} c_4 + \sigma_{2,1}^{(3)} = 1 \cdot c_4 + (c_2 + c_3) = c_2 + c_3 + c_4$

Summarizing the corrected algorithm 5 leads to the following, correct values of the symmetric functions $\sigma_{i,1}^{(4)}$, $i=1,2,3,4$:

$$\begin{cases} \sigma_{1,1}^{(4)} = 1 \\ \sigma_{2,1}^{(4)} = c_2 + c_3 + c_4 \\ \sigma_{3,1}^{(4)} = c_2 c_3 + c_2 c_4 + c_3 c_4 \\ \sigma_{4,1}^{(4)} = c_2 c_3 c_4 \end{cases}$$

7. Summary

In this paper, we give the correct version of the algorithm for the elementary symmetric functions calculating. The proposed algorithm is based on the algorithm from the work [1].

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