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# Vector gauge boson dark matter for the SU(N) gauge group model

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**Abstract:** The existence of dark matter is explained by a new stable neutral vector boson, C-boson, of mass (900GeV), predicted by the Wu mechanisms for mass generation of gauge field. According to the Standard Model (SM)  $W$ ,  $Z$ -bosons normally get their masses through coupling with the SM Higgs particle of mass 125GeV. We compute the self-annihilation cross section of the vector gauge boson C-dark matter and calculate its relic abundance. We also study the constraints suggested by dark-matter direct-search experiments.

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**Keywords:** Field theory, Dark matter, Gauge boson

## 1. Introduction

The nature of dark matter, proposed in 1933 to explain why galaxies in some clusters move faster than their predicted speed if they contained only baryonic matter [1], remains one of the open questions of modern physics. Several models of dark matter have been suggested, such as Light Supersymmetric Particles ([2], [3], [4], [5], [6], [7],[8],[9]); heavy fourth generation neutrinos ([10], [11]); Q-Balls ([12], [13]); mirror particles ([14], [15], [16], [17], [18]); and axion particles, introduced in an attempt to solve the Charge-Parity (CP) violation problem in particle physics ([19], [20]) .

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Recently, the Brane world ideal has been used to furnish new solutions to old problems in particle physics and cosmology ([21-35]). Scenarios in which all fields are allowed to propagate in the bulk are called Universal Extra Dimensions (UED) models ([36], [37]). UED models provide a viable dark matter candidate, namely the Lightest Kaluza Klein particle (LKP) ([38], [39]).

Nevertheless, it has been realized [40-42] that a conventional model [43], [41], [42] based on superstring-inspired  $E_6$  has exactly the ingredients which allow it to become a model of vector-boson dark matter. In such a model, the vector boson itself (X) comes from an  $SU(2)_N$  gauge extension of the Standard Model. [41]. Constraints to such a model posed by dark-matter direct-search experiments have been studied ([41]) and possible signals at the Large Hadron Collider (LHC) have been considered [44], [45].

The goal of the present paper is to investigate the possibility that be a stable neutral vector C-boson of mass 900 GeV (as predicted by the Wu mechanisms for mass generation of gauge fields: [46], [47], [48], [49], [50], [51]), proposed recently by the author [52], explains the existence of dark matter. According to the SM  $W$ ,  $Z$ -bosons normally acquire their masses through their coupling with the SM Higgs boson, mass 125 GeV [53], [54], [55], [56], [57]. Here we compute the self-annihilation cross section of the vector gauge C-boson -dark matter, calculate C-boson's relic abundance and study the modeling constraints generated by dark-matter direct-search experiments.

## 2 The Lagrangian of the model

Let us suppose that the gauge symmetry of the theory is  $SU(N) \times U(2)$  group, which is written specifically as follows [52]:

$$G = SU(N) \times U(2), \quad (1)$$

where  $SU(N)$  is the special unitary group of  $N$ -dimensions,  $\psi(x)$  is a  $N$ -component vector in the fundamental representative space of  $SU(N)$  group, and  $T_i$  ( $i = 1, 2, \dots, N^2 - 1$ ) denotes the representative matrices of the generators of  $SU(N)$  group. The latter are Hermit and traceless. They satisfy the condition:

$$[T_i, T_j] = if_{ijk}T_k, \quad Tr(T_i T_j) = \delta_{ij}K \quad (2)$$

where  $f_{ijk}$  are structure constants of the  $SU(N)$  group, and  $K$  is a constant independent of the indices  $i$  and  $j$  but dependent on the representation of the group. The representative matrix of a general element of the  $SU(N)$  group is expressed as:

$$U = e^{i\alpha^i T_i}, \quad (3)$$

with  $\alpha^i$  being the real group parameters. In global gauge transformations, all  $\alpha^i$  are independent of space-time coordinates, while in local gauge transformations  $\alpha^i$  are functions of space-time coordinates.  $U$  is a unitary  $N \times N$  matrix.

In order to introduce the mass term of gauge fields without violating local gauge symmetry at energy scale close to 12 TeV, two kinds of gauge fields are required:  $a_\mu$  and  $b_\mu$  [46],[52].

In this version of the Wu gauge model, the gauge fields  $a_\mu^i$  and  $b_\mu^i$  are introduced. Gauge field  $a_\mu^i$  is introduced to ensure the local gauge invariance of the theory. The generation of gauge field  $b_\mu^i$  is a purely quantum phenomenon:  $b_\mu^i$  is generated through non-smoothness of the scalar phase of the fundamental spinor fields [52], [58].

From the viewpoint of the gauge field  $b_\mu^i$  generation described here, the gauge principle is an “automatic” consequence of the non-smoothness of the field trajectory in the Feynman path integral [52], [58].

$a_\mu(x)$  and  $b_\mu(x)$  are vectors in the canonical representative space of  $SU(N)$  group. They can be expressed as linear combinations of generators, as follows:

$$a_\mu(x) = a_\mu^i(x)T_i \quad (25)$$

$$b_\mu(x) = b_\mu^i(x)T_i, \quad (26)$$

where  $a_\mu^i(x)$  and  $b_\mu^i(x)$  are component fields of the gauge fields  $a_\mu(x)$  and  $b_\mu(x)$ , respectively.

Corresponding to these two kinds of gauge fields, there are two kinds of gauge covariant derivatives:

$$D_\mu = \partial_\mu - ig_2 a_\mu \quad (27)$$

$$D_{b\mu} = \partial_\mu + icg_2 b_\mu \quad (28)$$

The strengths of gauge fields  $a_\mu(x)$  and  $b_\mu(x)$  are defined as

$$a_{\mu\nu} = \frac{1}{-ig} [D_\mu, D_\nu] = \partial_\mu a_\nu - \partial_\nu a_\mu - ig_2 [a_\mu, a_\nu] \quad (29)$$

$$b_{\mu\nu} = \frac{1}{icg} [D_{b\mu}, D_{b\nu}] = \partial_\mu b_\nu - \partial_\nu b_\mu + icg_2 [b_\mu, b_\nu] \quad (30)$$

respectively.

Similarly,  $a_\mu(x)$  and  $b_\mu(x)$  can also be expressed as linear combinations of generators:

$$a_{\mu\nu} = a_{\mu\nu}^i T_i \quad (31)$$

$$b_{\mu\nu} = b_{\mu\nu}^i T_i \quad (32)$$

Using relations (2) and (29), (30), we obtain

$$a_{\mu\nu}^i = \partial_\mu a_\nu^i - \partial_\nu a_\mu^i + g_2 f^{ijk} a_\mu^j a_\nu^k \quad (33)$$

$$b_{\mu\nu}^i = \partial_\mu b_\nu^i - \partial_\nu b_\mu^i - c g_2 f^{ijk} b_\mu^j b_\nu^k \quad (34)$$

The Lagrangian density of the model is

$$\begin{aligned} \mathfrak{L}_{Wu} = & -\bar{\psi}(\gamma^\mu D_\mu + m)\psi - \frac{1}{4K} \text{Tr}(a^{\mu\nu} a_{\mu\nu}) - \frac{1}{4K} \text{Tr}(b^{\mu\nu} b_{\mu\nu}) \\ & - \frac{\mu^2}{2K(1+c^2)} \text{Tr}[(a^\mu + cb^\mu)(a_\mu + cb_\mu)] \end{aligned} \quad (35)$$

where  $c$  is a constant.

The space-time metric is selected as  $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$ ,  $(\mu, \nu = 0,1,2,3)$ . According to relation (2), Lagrangian density  $L$  can be rewritten as:

$$\begin{aligned} \mathfrak{L}_{Wu} = & -\bar{\psi}[\gamma^\mu (\partial_\mu - ig_2 a_\mu^i T_i) + m]\psi - \frac{1}{4} a^{i\mu\nu} a_{i\mu\nu}^i - \frac{1}{4} b^{i\mu\nu} b_{i\mu\nu}^i \\ & - \frac{\mu^2}{2(1+c^2)} (a^{i\mu} + cb^{i\mu})(a_\mu^i + cb_\mu^i) \end{aligned} \quad (36)$$

This Lagrangian has strict local gauge symmetry [46], [52].

In equation (1), the gauge group  $U(2) = SU_L(2) \times U(1)_Y$  is the known SM of electroweak (EW) interactions [53], [54], [55], [56]. The generators of  $SU_L(2)$  correspond to the three components of weak isospin  $T_a$  ( $a = 1, 2, 3$ ). The  $U(1)_Y$  generator corresponds to the weak hypercharge  $Y$ . These are related to the electric charge by  $Q = \frac{T_3}{2} + Y$ .

The  $SU_L(2) \times U(1)_Y$  invariant Lagrangian is given as follows:

$$\mathfrak{L}_{EW} = \bar{\psi} \gamma^\mu [\partial_\mu - ig_1 \frac{1}{2} \tau \cdot A_\mu + i \frac{g_1'}{2} B_\mu] \psi + \bar{e}_R \gamma^\mu [\partial_\mu + ig_1' B_\mu] e_R - \frac{1}{4} F_{\mu\nu}^\alpha F_{\alpha}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \quad (37)$$

with field strength tensors:

$$F_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha - g_1 \varepsilon^{\alpha\beta\gamma} A_\mu^\beta A_\nu^\gamma \quad (38)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (39)$$

for the three non-Abelian fields of  $SU(2)_L$  and the single Abelian gauge field associated with  $U(1)_Y$ , respectively.

The covariant derivative is:

$$D_\mu = \partial_\mu - \frac{1}{2} ig_1 A_\mu^\alpha T_\alpha + \frac{1}{2} ig_1' B_\mu \quad (40)$$

with  $g_1, g_1'$  being the  $SU(2)_L$ , and  $U(1)_Y$  the coupling strength, respectively.

This Lagrangian is invariant under the infinitesimal local gauge transformations for  $SU(2)_L$  and  $U(1)_Y$  independently. Being in the adjointed representation, the  $SU(2)_L$  massless gauge fields forms a weak isospin triplet, with the charged fields being defined by

$$W_\mu^\pm = (A_\mu^1 \mp iA_\mu^2) / \sqrt{2} \quad (41)$$

The neutral component of  $A_\mu^3$  mixes with the Abelian gauge field  $B_\mu$  to form the physical states:

$$Z_\mu = A_\mu^3 \cos \theta_w - B_\mu \sin \theta_w \quad (42)$$

$$A_\mu = B_\mu \cos \theta_w + A_\mu^3 \sin \theta_w, \quad (43)$$

where  $\tan \theta_w = \frac{g_1'}{g_1}$  is the weak mixing angle.

Based on the gauge group  $SU(N) \times U(2)$ , the final Lagrangian of the model is given as follows [52]:

$$\begin{aligned} \mathfrak{L}_{Model} = & \bar{\psi} \gamma^\mu [\partial_\mu - ig_1 \frac{1}{2} \tau \cdot A_\mu + i \frac{g_1'}{2} B_\mu - ig_2 a_\mu^i T_i] \psi + \bar{e}_R \gamma^\mu [\partial_\mu + ig_1' B_\mu] e_R \\ & - \frac{1}{4} F_{\mu\nu}^\alpha F_{\alpha}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} a^{i\mu\nu} a_{\mu\nu}^i - \frac{1}{4} b^{i\mu\nu} b_{\mu\nu}^i \\ & - \frac{\mu^2}{2(1+c^2)} (a^{i\mu} + cb^{i\mu})(a_\mu^i + cb_\mu^i) \end{aligned} \quad (44)$$

where  $c$  is a constant.

### 3 The masses of gauge fields

Two obvious characteristics of the Wu Lagrangian equation (36) is that the mass term of the gauge fields is introduced into the Lagrangian, and that this term does not affect the symmetry of the Lagrangian. It has been proved that this Lagrangian has strict local gauge symmetry [46]. Since both vector fields  $a_\mu$  and  $b_\mu$  are standard gauge fields, this model is a gauge field model which describes gauge interactions between gauge fields and matter fields [46].

The mass term of gauge fields can be written as follows [46]:

$$\frac{1}{2} (a^\mu, b^\mu) M \begin{pmatrix} a^\mu \\ b^\mu \end{pmatrix}, \quad (45)$$

where  $M$  is the mass matrix:

$$M = \frac{1}{1+c^2} \begin{pmatrix} \mu^2 & c\mu^2 \\ c\mu^2 & c^2\mu^2 \end{pmatrix}. \quad (46)$$

Physical particles generated from gauge interactions are eigenvectors of mass matrix, and the corresponding masses of these particles are eigenvalues of mass matrix. The mass matrix  $M$  has two eigenvalues:

$$m_1^2 = \mu^2 \quad , \quad m_2^2 = 0 \quad . \quad (47)$$

The corresponding eigenvectors are:

$$\begin{pmatrix} \cos \theta_{wu} \\ \sin \theta_{wu} \end{pmatrix} \quad \begin{pmatrix} -\sin \theta_{wu} \\ \cos \theta_{wu} \end{pmatrix} \quad (48)$$

where

$$\cos \theta_{wu} = \frac{1}{\sqrt{1+c^2}} \quad , \quad \sin \theta_{wu} = \frac{c}{\sqrt{1+c^2}} \quad . \quad (49)$$

We define

$$\begin{aligned} C_\mu &= \cos \theta_{wu} a_\mu + \sin \theta_{wu} b_\mu , \\ F_\mu &= -\sin \theta_{wu} a_\mu + \cos \theta_{wu} b_\mu \end{aligned} \quad (50)$$

$C_\mu$  and  $F_\mu$  are eigenstates of mass matrix: they describe the particles generated from gauge interactions.

Taking equation (50) into account, the Wu Lagrangian density  $L$  given by (36) changes into:

$$\mathfrak{L}_{Wu} = -\bar{\psi}(\gamma^\mu \partial_\mu + m)\psi - \frac{1}{4} C_0^{i\mu\nu} C_{0\mu\nu}^i - \frac{1}{4} K_0^{i\mu\nu} K_{0\mu\nu}^i - \frac{\mu^2}{2} C^{i\mu} C_\mu^i , \quad (51)$$

In relation (51) we have used the following simplified notations:

$$C_{0\mu\nu}^i = \partial_\mu C_\nu^i - \partial_\nu C_\mu^i , \quad (52)$$

$$K_{0\mu\nu}^i = \partial_\mu F_\nu^i - \partial_\nu F_\mu^i \quad . \quad (53)$$

From equation (51) it is deduced that the mass of field  $C_\mu$  is  $\mu$  and the mass of gauge field  $F_\mu$  is zero. That is:

$$M_C = \mu \quad , \quad M_F = 0 \quad , \quad (54)$$

Transformations (50) are pure algebraic operations which do not affect the gauge symmetry of the Lagrangian [46]. They can, therefore, be regarded as redefinitions of gauge fields. The local gauge symmetry of the Lagrangian is still strictly preserved after field transformations. In other words, the symmetry of the Lagrangian before transformations is absolutely the same with the symmetry of the Lagrangian after transformations. We do not introduce any kind of symmetry breaking at energy scales close to 12 TeV [52].

Fields  $C_\mu$  and  $F_\mu$  are linear combinations of gauge fields  $a_\mu$  and  $b_\mu$ . The forms of local gauge transformations of fields  $C_\mu$  and  $F_\mu$  are, therefore, determined by the forms of local gauge transformations of gauge fields  $a_\mu$  and  $b_\mu$ . Since  $C_\mu$  and  $F_\mu$  consist of gauge fields  $a_\mu$  and  $b_\mu$  and transmit gauge interactions between matter fields, for the sake of simplicity we also call them gauge fields, just as  $W$  and  $Z$  are called gauge fields in the electroweak model ([53], [54], [55], [56]). This gauge field theory, therefore, predicts the existence of two different kinds of force transmitting vector fields: a massive and a massless one.

Taking the Higgs mechanism [53-56] into account, in the vacuum energy scale of 179 GeV the  $W^\pm$  and  $Z^0$  become massive, while the photon  $A$  remains massless. The symmetry  $SU(N)$  does not break down, since the gauge bosons  $C^0$  and  $F$  derive their masses by the Wu mechanisms [46], [47], [48], [49], [52] in the vacuum energy scale of 12TeV.

The most general Lagrangian consistent with  $SU(2) \times U(1)$  gauge invariance, Lorentz invariance, and renormalizability is ([59]):

$$\mathfrak{L}_\phi = -\frac{1}{2} |(\partial_\mu - iA_\mu \cdot t^{(\phi)} - iB_\mu y^{(\phi)})\phi|^2 - \frac{\mu^2}{2} \phi^* \phi - \frac{\lambda}{4} (\phi^* \phi)^2, \quad (55)$$

where

$$t^{(\phi)} = \frac{g_1}{2} \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}, \quad y^{(\phi)} = -\frac{g'_1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (56)$$

are the generators of the  $(\phi^+, \phi^0)$ ,  $\lambda > 0$ , and

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \quad (57)$$

For  $\mu^2 < 0$ , there is a tree-approximation vacuum expectation value at the stationary point of the Lagrangian

$$\langle \phi \rangle^* \langle \phi \rangle = v^2 = |\mu^2| / \lambda. \quad (58)$$

We can always perform an  $SU(2) \times U(1)$  gauge transformation to a unitary gauge, in which  $\phi^+ = 0$  and  $\phi^0$  is Hermitian, with positive vacuum expectation value. In unitarily gauge the vacuum expectation values of the components of  $\phi$  are:

$$\langle \phi^+ \rangle = 0, \quad \langle \phi^0 \rangle = v > 0. \quad (59)$$

The scalar Lagrangian (55) then yields a vector boson mass term:

$$\begin{aligned} -\frac{1}{2} |(\partial_\mu - iA_\mu \cdot \tau^{(\phi)} - iB_\mu y^{(\phi)})\phi|^2 &= -\frac{1}{2} \left| \left( \frac{g_1}{2} A_\mu T^a - \frac{g'_1}{2} B_\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\ &= -\frac{v^2 g_1^2}{4} W_\mu^* W^\mu - \frac{v^2}{8} (g_1^2 + g_1'^2) Z_\mu Z^\mu, \end{aligned} \quad (60)$$



The masses are given as follows

$$M_A = 0, M_W = \frac{1}{2} g_1 v, M_Z = \frac{1}{2} \sqrt{g_1^2 + g_1'^2}. \quad (61)$$

The gauge fields and masses predicted by this model are summarized in Table.1.

**Table.1.** Gauge fields and masses predicted by the proposed model ([52]).

Gauge fields	Masses (M)-GeV	Symmetry patter
C (New)	900	Without symmetry breaking
F (New)	0	Without- symmetry breaking
W	81	With -symmetry breaking
Z	91	With - symmetry breaking
A	0	With- symmetry breaking

#### 4 The Renormalization of the Model

In this paper, we use two mechanisms that can make gauge field to gain nonzero mass. One is the Wu mechanism ([46], [47], [48], [49], [50], [51], [52]), by which the mass term of the gauge field is introduced by using another set of gauge field. In this mechanism, the mass term of the gauge field does not affect the symmetry of the Lagrangian. We can imagine the new interaction picture: when matter fields take part in gauge interactions, they emit or absorb one kind of gauge field which is not eigenstate of mass matrix. Perhaps this is the gauge field consisted with the existence of dark matter in our Universe. This gauge field would appear in two states, a massless and a massive one, which correspond to two kinds of vector fields.

The second mechanism of mass generation is the Higgs mechanism ([53], [54], [56], [57]), which can make gauge field -W,Z to gain nonzero mass and to guarantee renormalizability by means of the interactions of the Higgs boson with gauge bosons [52]:

$$\begin{aligned} \frac{\eta^2 + 2v\eta}{v^2} \left( Z_\mu Z^\mu M_Z^2 + C_\mu C^\mu M_C^2 + W_\mu^+ W^{\mu-} M_W^2 \right) &= \left( \frac{M_Z^2}{v^2} \right) \eta^2 Z_\mu Z^\mu + \left( \frac{M_C^2}{v^2} \right) \eta^2 C_\mu C^\mu \\ + \left( \frac{M_W^2}{v^2} \right) \eta^2 W_\mu^+ W^{\mu-} &+ 2 \left( \frac{M_Z^2}{v} \right) \eta Z_\mu Z^\mu + 2 \left( \frac{M_C^2}{v} \right) \eta C_\mu C^\mu + 2 \left( \frac{M_W^2}{v} \right) \eta W_\mu^+ W^{\mu-} \end{aligned} \quad (62)$$

where

$$M_Z^2 / v^2, M_C^2 / v^2, M_W^2 / v^2, 2M_Z^2 / v, 2M_C^2 / v, 2M_W^2 / v \quad (63)$$

are the dimensionless and dimension of mass coupling constants.

For instance, the C-boson readily derives its mass by the Wu mechanism and yet renormalizability is ensured via the Higgs mechanism [52].

However, as we have stated above, the Wu gauge field theory has maximum local SU (N) gauge symmetry [46, 52]. When we quantize the Wu gauge field theory in the path integral

formulation, we have to select gauge conditions first [46, 52]. To fix the degree of freedom of the gauge transformation, we must select two gauge conditions simultaneously: one for the massive gauge field  $C_\mu$  and another for the massless gauge field  $F_\mu$ . For example, if we select temporal gauge condition for massless gauge field  $F_\mu$ ,

$$F_4 = 0 , \quad (64)$$

there still exists a remainder gauge transformation degree of freedom, because the temporal gauge condition is unchanged under the following local gauge transformation:

$$F_4 \rightarrow UF_\mu U^\dagger + (1 / ig_2 \sin \theta)U \partial_\mu U^\dagger , \quad (65)$$

where

$$\partial_t U = 0 , U = U(\vec{x}) . \quad (66)$$

In order to render this remainder gauge transformation degree of freedom completely fixed, we have to select another gauge condition for gauge field  $C_\mu$ , for instance

$$\partial^\mu C_\mu = 0 . \quad (67)$$

If we select two gauge conditions simultaneously, when we quantize the theory in path integral formulation, there will be two gauge fixing terms in the effective Lagrangian. The effective Lagrangian can then be written as:

$$\mathfrak{S}_{eff} = \mathfrak{S} - \frac{1}{2\alpha_1} f_1^a f_1^a - \frac{1}{2\alpha_2} f_2^a f_2^a + \bar{\eta}_1 M_{f_1} \eta_1 + \bar{\eta}_2 M_{f_2} \eta_2 , \quad (68)$$

where

$$f_1^a = f_1^a(F_\mu), f_2^a = f_2^a(C_\mu) . \quad (69)$$

If we select

$$f_2^a = \partial^\mu C_\mu^a , \quad (70)$$

then the propagator for massive gauge field  $C_\mu$  is:

$$\Delta_{F\mu\nu}^{ab}(k) = -i\delta^{ab} / (k^2 + \mu^2 - i\varepsilon) \times [g_{\mu\nu} - (1 - \alpha_2)k_\mu k_\nu / (k^2 + \alpha_2\mu^2)] \quad (71)$$

If we let  $k$  approach infinity, then

$$\Delta_{F\mu\nu}^{ab}(k) = \frac{1}{k^2} \quad (72)$$

In this case, according to the power-counting law, the Wu gauge field theory suggested in this paper is a kind of renormalizable theory [46, 52].

## 5 The R-parity of the vector gauge boson C- dark matter model

Internal global symmetries are related to conserved quantum numbers such as the baryonic number and electric charges. The symmetry generators  $B_l$  are Lorentz scalars and for Lie algebra, with structure constants  $C_{lm}^k$ , it is:

$$[B_l, B_m] = iC_{lm}^k B_k \quad (73)$$

The suggested generic field theory has internal symmetries, with generators  $B_l$ . Let us consider an Abelian group. In accordance with this theory, this Abelian group has the following commutators:

$$\begin{aligned} [Q_a, B_l] &= Q_a \\ [\bar{Q}_a, B_l] &= -\bar{Q}_a \end{aligned} \quad (74)$$

Clearly, only one independent combination of the Abelian generators actually has a nonzero commutator with  $Q_a$  and  $\bar{Q}_a$ ; let us denote this U(1) generator by R:

$$\begin{aligned} [Q_a, R] &= Q_a \\ [\bar{Q}_a, R] &= -\bar{Q}_a \end{aligned} \quad (75)$$

It follows that the suggested field theory in general possesses an internal global U(1) symmetry known as R-symmetry. Here, the generators have R-charge +1 and -1 respectively.

These particles cannot have a zero mass, since C particles acquire their mass via the Wu mechanism, in which case the continuous R-symmetry of equation.75 must be broken. However, it turns out that, despite this breaking, discrete R-symmetry remains, described by the  $Z_N$  group [42], [43], [70].

This leads us to the multiplicatively conserved quantum number called R-parity. Formally, one can define the R-parity of any particle of spin  $j$ , baryon number  $B$  and lepton number  $L$  to be:

$$R = (-1)^{2j+3(B-L)} \quad (76)$$

Particles within the same quantum numbers do not have the same R-parity: all ordinary particles and the F gauge bosons are assigned with an R-parity +1, whereas the C particle has an R-parity of -1. If R-parity is exactly conserved, then there can be no mixing between the C and ordinary particles and every interaction vertex will contain an even number of  $R = -1$  particles. The R-parity assignment is very useful for phenomenology, since it leads to two three important consequences:

1. C particles can be produced only in pairs;

2. C particles cannot decay; they must be stable. Since C is electrically neutral [52], it interacts weakly with ordinary matter. The C-particle, therefore, is a plausible candidate for Cold Dark Matter (CDM);

We will consider that the proposed model conserves R-parity. The gauge fields and R-parity predicted by this model are summarized in Table.2.

**Table.2.** Gauge fields and R-parity predicted by the proposed model.

Gauge fields	R-parity	Decay patter
C (New)	odd	Stable
F (New)	Even	Unstable
W	Even	Unstable
Z	Even	Unstable
A	Even	Unstable

## 6 Annihilation cross section of vector gauge boson C- dark mater

The C-boson is the lightest particle having odd R. It is thus stable and a possible candidate for dark matter. In the early Universe,  $CC^-$  pair will annihilate to particles of even R, i.e.  $d\bar{d}$ , through Higgs h exchange and direct interaction. However, the corresponding process for dark matter direct search, i.e.  $Cd \rightarrow C\bar{d}$  through h exchange, may also be important, as discussed in the next section. We show the various annihilation diagrams, result in the no relativistic cross section  $\times$  relative velocity given by

$$\sigma u_{rel} = \frac{g_c^4 m_c^2}{72\pi} \left[ \sum_h \frac{3}{(m_h^2 + m_c^2)^2} + \frac{3}{8m_c^4} \right] \quad (77)$$

where the sum over h is for 3 families [42],[43]. The factor of 3 for h is the number of colors. Assuming as we do that  $m_c$  is the smallest mass in equation (77), we must have

$$\sigma u_{rel} \approx \frac{g_c^4}{m_c^2} \quad (78)$$

In particular, for vector gauge boson C has a mass of 900 GeV, as predicted by the Wu mechanisms for mass generation of gauge field (see also [52]). The self- annihilation cross section should be

$$\sigma u_{rel} \approx 0.86 pb \quad (79)$$

where  $g_c^2 = 0.4$ .

## 7 Relict densicty of vector gauge boson C-dark matter

Let us now review the standard calculation of the relic abundance of a particle species (see [60, 61] for more details) denoted C-particle which was at thermal equilibrium in the early

universe and decoupled when it was non relativistic. The evolution of its number density  $n$  in an expanding universe is governed by the Boltzmann equation:

$$\frac{dn}{dt} + 3Hn = -\langle\sigma u\rangle(n^2 - n_{eq}^2), \quad (80)$$

where  $H = (8\pi\rho / 3M_{pl})^{1/2}$  is the expansion rate of the universe,  $n_{eq}$  the number density at thermal equilibrium and  $\langle\sigma u\rangle$  is the thermally averaged annihilation cross section times the relative velocity. We are eventually concerned with a massive CDM candidate, for which the equilibrium density is given by the non relativistic limit:

$$n_{eq} = g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}, \quad m \gg T \quad (81)$$

$$\langle\sigma u\rangle = a + b\langle u^2\rangle = a + 6\frac{b}{x}, \quad x = \frac{m}{T}, \quad (82)$$

where  $a$  and  $b$  are S and P wave contributions.

The freeze-out condition is:

$$n\langle\sigma u\rangle \leq H(m), \quad (83)$$

$$x_F = \frac{m}{T} \ln \left[ c(c+2) \sqrt{\frac{45}{8}} \frac{g}{2\pi^3} \frac{mM_{pl}(a+6b/x_F)}{\sqrt{g_* x_F}} \right] \quad (84)$$

where  $H(m)$  the expansion of the Universe and  $x_F$  dimensionless variable,  $c$  is an order 1 constant fixed by matching later and early time evolution. The relic density of a particle species- $x$  is given by

$$\Omega_x h^2 \approx \frac{1.07 \times 10^9 \text{ GeV}^{-1}}{M_{pl}} \frac{x_F}{\sqrt{g_*}} \frac{1}{a+3b/x_F}. \quad (85)$$

Typically, for  $x_F \approx 10$  and  $g_* \approx 100$ , one can estimate

$$\Omega_x h^2 \approx \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle\sigma u\rangle} \approx \frac{0.1 \text{ pb}}{\langle\sigma u\rangle}. \quad (86)$$

For vector gauge boson  $C$  mass of 900 GeV, predicted by the Wu mechanisms for mass generation of gauge field, the relic density should be

$$\Omega_c h^2 = \frac{0.1 \text{ pb}}{\langle\sigma u\rangle} = 0.11 \quad (87)$$

where  $\langle \sigma u \rangle$  is calculated by the equation (79).

The analysis of the three-year Wilkinson Microwave Anisotropy Probe (WMAP) data tells us that the density of dark matter is  $\Omega_{dm} h^2 = 0.102 \pm 0.009$  where  $\Omega_{dm}$  is  $\rho_{dm} / \rho_{crit}$ ,  $\rho_{crit}$  is the density corresponding to a flat universe [62] and  $h$  is the Hubble constant in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$  [63].

A cold dark matter candidate produced at the Large Hadron Collider (LHC) should therefore have this annihilation cross section. This quantity leads us to the second method of measuring the coupling of dark matter from Standard Model particles, namely through the search for the annihilation or decay products of dark matter coming from high-density regions such as the centre of galaxies [64]. Since the WMAP results provide rather good information about  $\langle \sigma u \rangle$ , the uncertainties in this approach lie in the lack of knowledge of the exact density of dark matter in dense regions such as the centre of galaxies and in the separation of the signal from dark-matter annihilation from possible background signals.

C-particle is stable and since it also interacts weakly with baryonic matter it can be a good Weakly Interacting Massive Particles (WIMP) candidate. The proposed C-dark matter model also provides the correct abundance of dark matter in the universe. The C-particle mass around 900 GeV predicted by the model is an encouraging theoretical suggestion, testable through LHC studies.

## 8 The direct C-dark matter search

Taking into account twist-2 operators and gluonic contributions calculated recently [65] and assuming that  $m(h_d) = m(h_s) = m(h_b) = m_h$ , we find

$$\begin{aligned} \frac{f_p}{m_p} = & 0.052 \left[ -\frac{g_N^2}{m_\eta^2} - \frac{g_N^2}{16} \frac{m_h^2}{(m_h^2 - m_h^2)^2} \right] + \frac{3}{4} (0.222) \left[ -\frac{g_N^2}{4} \frac{m_h^2}{(m_h^2 - m_h^2)^2} \right] \\ & - (0.925) \left( (1.19) \frac{g_N^2}{54 m_\eta^2} + \frac{g_N^2}{36} \left[ (1.19) \frac{m_h^2}{6(m_h^2 - m_c^2)^2} + \frac{1}{3(m_h^2 - m_c^2)^2} \right] \right) \end{aligned} \quad (88)$$

To obtain  $f_n / m_n$ , the numerical coefficients (0.052, 0.222, 0.925) [42] in the above equation are replaced with (0.061, 0.330, 0.922) [42]. The spin-independent elastic cross section for C-dark matter scattering off a nucleus of  $Z$  protons and  $A - Z$  neutrons normalized to one nucleon is then given by

$$\sigma_0 = \frac{1}{\pi} \left( \frac{m_N}{m_c} \right)^2 \left| \frac{Z f_p + (A - Z) f_n}{A} \right|^2, \quad (89)$$

where  $m_N$  and  $m_c$  are the masses of the nucleon and C-dark matter respectively.

Here we use  $^{73}\text{Ge}$  with  $Z = 32$  and  $A - Z = 41$  to compare with the recent Cold Dark Matter Search (CDMS) result [66]. For C-particle of mass  $m_c = 900 \text{ GeV}$ , the experimental upper bound is very well approximated by [67]

$$\sigma_0 \approx 2.2 \times 10^{-7} \text{ pb} (m_c / 1\text{TeV})^{0.86}. \quad ([42]) \quad (90)$$

Next, we study the scalar  $\eta$  as a possible dark-matter candidate, predicted by the Wu mechanisms for mass generation of gauge field [47-49].

For the scalar ( $\eta$ ) scattering off a nucleus by the Higgs exchange [68], [43] the terms  $f_p$  and  $f_n$  in cross section (89) become

$$\begin{aligned} \frac{f_p}{m_p} &= \left( -0.075 - \frac{0.925(3.51)(2)}{21} \right) \frac{\sqrt{2}}{m_\eta^2}, \\ \frac{f_n}{m_n} &= \left( -0.078 - \frac{0.922(3.51)(2)}{27} \right) \frac{\sqrt{2}}{m_\eta^2}. \end{aligned} \quad ([43]) \quad (91)$$

Assuming an effective  $m_\eta = 125$  GeV and using  $Z = 54$  and  $A - Z = 77$  for  $^{131}\text{Xe}$ , we find that  $\sigma_0 \approx 3.2 \times 10^{-10}$  pb, which is far below the current upper bound of the 2011 XENON100 experiment [69].

## 9 Conclusion

We show that a new neutral  $C$ - boson of mass (900GeV), predicted by the Wu mechanisms for mass generation of gauge field, can explain the dark matter in our Universe.

In this model, the Standard Model (SM)  $W, Z$ -bosons acquire their masses through the coupling with the SM Higgs boson of mass 125 GeV.

The  $C$ -dark matter readily derives its mass by the Wu mechanism and yet the renormalizability is ensured via the Higgs mechanism.

The thermally averaged cross section for  $C$ -dark matter self-annihilation into SM particles is computed and the relic abundance of  $C$ -dark matter is calculated. We have also studied the constraints from dark-matter direct-search experiments, and the scalar ( $\eta$ ) as a possible dark-matter candidate.

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