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# Rank-Constrained Fundamental Matrix Estimation by Polynomial Global Optimization Versus the Eight-Point Algorithm

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## Abstract

The fundamental matrix can be estimated from point matches. The current gold standard is to bootstrap the eight-point algorithm and two-view projective bundle adjustment. The eight-point algorithm first computes a simple linear least squares solution by minimizing an algebraic cost and then computes the closest rank-deficient matrix. This article proposes a single-step method that solves both steps of the eight-point algorithm. Using recent result from polynomial global optimization, our method finds the rank-deficient matrix that exactly minimizes the algebraic cost. The current gold standard is known to be extremely effective but is nonetheless outperformed by our rank-constrained method bootstrapping bundle adjustment. This is here demonstrated on simulated and standard real datasets. With our initialization, bundle adjustment consistently finds a better local minimum (achieves a lower reprojection error) and takes less iterations to converge.

Keywords : Global optimization, convex optimization, linear matrix inequality, fundamental matrix.

## 1 Introduction

The fundamental matrix has received a great interest in the computer vision community (see for instance [24, 2, 33, 13, 31, 7, 3]). This  $(3 \times 3)$  rank-two matrix encapsulates the epipolar geometry, the projective motion between two uncalibrated perspective cameras, and serves as a basis for 3D reconstruction, motion segmentation and camera self-calibration, to name a few. Given  $n$  point matches  $(\mathbf{q}_i, \mathbf{q}'_i)$ ,  $i = 1, \dots, n$  between two images, the fundamental matrix is estimated in two phases. The initialization phase finds some suboptimal estimate while the refinement phase iteratively

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minimizes an optimal but nonlinear and nonconvex criterion. The gold standard uses the *eight-point algorithm* and *projective bundle adjustment* for these two phases, respectively. A ‘good enough’ initialization is necessary to avoid local minima at the refinement phase as much as possible. The main goal of this article is to improve the current state of the art regarding the initialization phase.

The eight-point algorithm follows two steps [24]. In its first step, it relaxes the rank-deficiency constraint and solves the following convex problem:

$$\tilde{\mathbf{F}} = \arg \min_{\mathbf{F} \in \mathbb{R}^{3 \times 3}} C(\mathbf{F}) \text{ s.t. } \|\mathbf{F}\|^2 = 1, \quad (1)$$

where  $C$  is a convex, linear least squares cost, hereinafter called the *algebraic cost*:

$$C(\mathbf{F}) = \sum_{i=1}^n \left( \mathbf{q}_i^T \mathbf{F} \mathbf{q}_i \right)^2. \quad (2)$$

This minimization is subject to the normalization constraint  $\|\mathbf{F}\|^2 = 1$ . This is to avoid the trivial solution  $\mathbf{F} = \mathbf{0}$ . Normalization will be further discussed in section 3. The estimated matrix  $\tilde{\mathbf{F}}$  is thus *not* a fundamental matrix yet. In its second step, the eight-point algorithm computes the closest rank-deficient matrix to  $\tilde{\mathbf{F}}$  as:

$$\mathbf{F}_{8\text{pt}} = \arg \min_{\mathbf{F} \in \mathbb{R}^{3 \times 3}} \|\mathbf{F} - \tilde{\mathbf{F}}\|^2 \text{ s.t. } \det(\mathbf{F}) = 0. \quad (3)$$

Both steps can be easily solved. The first step is a simple linear least squares problem and the second step is solved by nullifying the least singular value of  $\tilde{\mathbf{F}}$ . It has been shown [13] that this simple algorithm performs extremely well in practice, provided that the image point coordinates are *standardized* by simply rescaling them so that they lie in  $[-\sqrt{2}; \sqrt{2}]^2$ .

Our main contribution in this paper is an approach that solves for the fundamental matrix minimizing the algebraic cost. In other words, we find the global minimum of:

$$\tilde{\mathbf{F}} = \arg \min_{\mathbf{F} \in \mathbb{R}^{3 \times 3}} C(\mathbf{F}) \text{ s.t. } \det(\mathbf{F}) = 0 \text{ and } \|\mathbf{F}\|^2 = 1. \quad (4)$$

Our algorithm uses polynomial global optimization [23, 15]. Previous attempts [36, 6, 19] in the literature differ in terms of optimization strategy and parameterization of the fundamental matrix. None solves problem (4) optimally for a general parameterization: they either do not guarantee global optimality [6, 19] or prescribe some camera configurations [36, 6, 19] (requiring typically that the epipole in the first camera does not lie at infinity).

Our experimental evaluation on simulated and real datasets compares the difference between the eight-point algorithm and ours used as initialization to bundle adjustment. We observe that (i) bundle adjustment consistently converges within less iterations with our initialization and (ii) bundle adjustment always achieves an equal or lower reprojection error with our initialization. We provide numerous examples of real image pairs from standard datasets. They all illustrate practical cases for which our initialization method allows bundle adjustment to reach a better local minimum than the eight-point algorithm.

## 2 State of the Art

Accurately and automatically estimating the fundamental matrix from a pair of images is a major research topic. We first review a four-class categorization of existing meth-

ods, and specifically investigate the details of existing global methods. We finally state the improvements brought by our proposed global method.

## 2.1 Categorizing Methods

A classification of the different methods in three categories –linear, iterative and robust– was proposed [2]. Linear methods directly optimize a linear least squares cost. They include the eight-point algorithm [24], SVD resolution [2] and variants [33, 13, 31, 7]. Iterative methods iteratively optimize a nonlinear and nonconvex cost. They require, and are sensitive to the quality of, an initial estimate. The first group of iterative methods minimizes the distances between points and epipolar lines [17, 5]. The second group minimizes some approximation of the reprojection error [25, 37, 34, 8]. The third group of methods minimizes the reprojection error, and are equivalent to two-view projective bundle adjustment. Iterative methods typically use a nonlinear parameterization of the fundamental matrix which guarantees that the rank-deficiency constraint is met. For instance, a minimal 7-parameter update can be used over a consistent orthogonal representation [3]. Finally, robust methods estimate the fundamental matrix while classifying each point match as inlier or outlier. Robust methods use M-Estimators [12], LMedS (median least squares) [37] or RANSAC (random sampling consensus) [32]. Both LMedS and RANSAC are stochastic.

To these three categories, we propose to add a fourth one: *global methods*. Global methods attempt at finding the global minimum of a nonconvex problem. Convex relaxations have been used to combine a convex cost with the rank-deficiency constraint [6]. However, the relaxations do not converge to a global minimum and the solution’s optimality is not certified.

## 2.2 Global Methods

In theory, for a constrained optimization problem, global optimization methods do not require an initial guess and are guaranteed to reach the global minimum. There are two ways to obtain this certificate of optimality. The first way consists in describing the search space the most exhaustively as possible in order to test as many candidate solutions as possible. In this category, there are methods such Monte-Carlo sampling, which tests random elements of space constraints, and reactive tabu search [9, 11], which continues searching even after a local minimum has been found. The major drawback of these methods is mainly in the prohibitive computing time, required to guarantee a sufficiently high probability of success. The second class of methods provides a certificate of optimality using the mathematical theory from which they are built. Branch and Bound algorithms [20] or global optimization by interval analysis [10, 27] are examples of methods lying in this second family. However, although these methods can be faster than those of the first category, their major drawback is their lack of generality. Indeed, these approaches are usually dedicated to one particular type of cost function because they use highly specific computing mechanisms to be as efficient as possible. A review of global methods may be found in [35].

A good deal of research has been conducted over the last few decades on applying global optimization methods in order to solve polynomial minimization problems under constraints. The major drawback of these applications has been the difficulty to take constraints into account. But, through simplification of the problem, these approaches have mainly been used to find a starting point for local iterative methods.

However, recent results in the areas of convex and polynomial optimization have facilitated the emergence of new approaches. These have attracted great interest in the computer vision community. In particular, global polynomial optimization [23, 15] has been used in combination with a finite-epipole nonlinear parameterization of the fundamental matrix [36]. This method does not cover camera setups where the epipole lies at infinity. A global convex relaxation scheme [23, 15] was used to minimize the Sampson distance [19]. Because this implies minimizing a sum of many rational functions, the generic optimization method had to be specifically adapted. The solution’s global optimality can thus not be certified.

### 2.3 The proposed method.

The proposed method lies in the fourth category: it is a global method. Similarly to the eight-point algorithm, it minimizes the algebraic cost, under the nonlinear rank-deficiency constraint. Contrarily to previous global methods [36, 6, 19], ours handles all possible camera configurations (it does not make an assumption on the epipoles being finite or infinite) and certifies global optimality.

## 3 Polynomial Global Optimization

### 3.1 Background on the Optimization Method

Our optimization method is based on an idea first described in [21]. It consists in reformulating a nonconvex global optimization problem  $P$  with polynomial data (i.e., minimization of a polynomial objective function subject to polynomial inequalities and/or equations) as an equivalent convex linear programming (LP) problem over probability measures. Instead of optimizing over a vector in a finite-dimensional Euclidean space, we thus optimize over probability measures, i.e., in an infinite-dimensional space. The unknown measures are supported on the feasibility set of the optimization problem which in our case is a basic semi-algebraic set, i.e., a set defined by finitely many polynomial equalities and inequalities. In addition, we assume that the set is compact. More concretely, a probability measure is understood as a linear functional acting on the space of continuous functions, and we manipulate a measure only through its moments which are images of monomials (dense in the space of continuous functions with compact support). Using results from Functional Analysis (in duality with counterpart results of Real Algebraic Geometry), a sequence of numbers are moments of a probability measure on a compact basic semi-algebraic set  $K$  if and only if this sequence satisfies countably many semidefinite constraints on so-called moment and localizing matrices of increasing size. Then, accordingly, we construct a hierarchy of semidefinite programs  $Q_d$ ,  $d \in \mathbb{N}$ , where each  $Q_d$ ,  $d \in \mathbb{N}$ , is concerned with moment and localizing matrices of fixed size  $d$ , and is a convex relaxation of the original problem  $P$ . The sequence of corresponding optimal values is monotone non decreasing and converges to the global optimal value of  $P$ . In fact, solving a linear program on the space of finite measures on a compact basic semi-algebraic set boils down to solving a hierarchy of semidefinite programming (SDP) problems, also called linear matrix inequalities (LMIs).

Practice reveals that convergence is fast and very often finite. In fact, finite convergence is guaranteed in a number of cases, e.g., discrete optimization and some convex problems as well. Moreover there is a sufficient condition (on the rank of moment ma-

trices) which permits to detect whether the optimal value is attained at some step in the hierarchy and to extract corresponding global optimizers. We then obtain a numerical certificate of global optimality of the solution(s). Moreover, very recent results by Nie [28] show that finite convergence is even *generic*! This approach has been successfully applied to globally solve various polynomial optimization problems (see [22] for an overview of results and applications). In computer vision this approach was first introduced in [18] and used in [19].

Concretely, we formulate each of our polynomial optimization problems as an LP on measures which in turn reduces to solving a hierarchy of finite-dimensional SDP problems. Each SDP can be formulated with the help of the Matlab interface GloptiPoly 3 [16] and then is solved by using public-domain implementations of primal-dual interior point algorithms for semidefinite programming. These algorithms rely on a suitable logarithmic barrier function for the SDP cone, and they proceed by iteratively reducing the duality gap between the primal problem and its dual (which is also an SDP problem). Each iteration consists of solving a linear system of equations, involving the gradient and the Hessian of a Lagrangian built from the barrier function. Most of the computational burden comes from the construction and the storage of the Hessian matrix, and problem sparsity can be largely exploited at this stage. For more information on SDP and related optimization methods, see [4] and [1].

### 3.2 Application to Fundamental Matrix Estimation

An important feature of our approach is that the rank-2 constraint can be directly included in the problem description and moreover we do *not* need an initial estimate as in other methods. Nevertheless, the problem being homogeneous, an additional normalization constraint is needed to avoid the trivial solution  $\mathbf{F} = 0$ . This is generally done by setting one of the  $f_{ij}$  coefficients of the  $\mathbf{F}$  matrix to 1. However with such a normalization there is no guarantee that the feasible set is compact which is a necessary condition for the convergence of our polynomial optimization method. Thus, to guarantee compactness of the feasible set and to avoid the trivial solution, we include the additional normalization constraint  $\|\mathbf{F}\|^2 = 1$ .

Alternatively, the rank-2 constraint can be inferred by parameterizing the  $\mathbf{F}$  matrix using one or two epipoles. For instance, in [10] the authors have tried to use polynomial optimization to estimate the  $\mathbf{F}$  matrix, parameterized by using a single epipole. However such an approach has several drawbacks. Firstly, the coefficients of  $\mathbf{F}$  are not bounded and the convergence of the method is not guaranteed. Secondly, using the epipole explicitly increases the degree of the polynomial criterion and consequently, the size of the corresponding relaxations in the hierarchy. This results in a significant increase of the computational time. Finally, the choice of this parametrization is arbitrary and does not cover all camera configurations.

Notice also that even though the parametrization based on two epipoles is theoretically optimal, this method is not practical as it would be necessary to cover all the 36 possible parameter sets. Therefore it is preferable to introduce the rank constraint directly in the problem description rather than via some parametrization of  $\mathbf{F}$ . A similar approach has been developed in [11] by using a specific hierarchy of convex relaxations, a similar technique. But the optimization variables are not bounded since like in [10], the constraint “ $\|\mathbf{F}\|^2 = 1$ ” is removed and a normalization constraint is introduced by fixing one of the  $f_{ij}$  coefficients to 1. Moreover, the constraint “ $\det(\mathbf{F}) = 0$ ” is taken into account by introducing additional optimization variables and there is no proof that this specific hierarchy of convex relaxations converges to the global minimum.

Finally, in [12] the authors minimize the Sampson distance via solving a hierarchy of convex relaxations built upon an epigraph formulation. Nevertheless, even if the rank correction is achieved using the constraint “ $\det(\mathbf{F}) = 0$ ”, the normalization is done by setting the coefficient  $f_{33}$  to 1, and so the other coefficients of  $\mathbf{F}$  are not bounded. Moreover there is no guarantee of convergence to global minimum.

Our method is summarized in Algorithm 1 below. Its main features are:

- In contrast with [18, 19] the optimization problem is formulated with an *explicit* Frobenius norm constraint on the decision variables. This enforces compactness of the feasibility set which is included in the Euclidean ball of radius 1. We have observed that enforcing this Frobenius norm constraint has a dramatic influence on the overall numerical behavior of the SDP solver, especially with respect to convergence and extraction of global minimizers.
- We have chosen the SDPT3 solver [29, 30] since our experiments revealed that for our problem it was the most efficient and reliable solver;
- We force the interior-point algorithm to decrease the duality gap as much as possible, overruling the default parameter tuning in SDPT3;
- Generically, polynomial optimization problems on a compact set have a unique global minimizer; this is confirmed in our numerical experiments as the moment matrix has almost always rank-one (which certifies global optimality) at the second SDP relaxation of the hierarchy. In some (very few) cases the global optimum is not fully accurate, but yet largely satisfactory; if a more accurate optimum is required, a local refinement based on Newton’s method can be tried (which we did not do in our implementation).

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**Algorithm 1** Polynomial optimization for fundamental matrix estimation

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**Require:** Matched points  $(\mathbf{q}_i, \mathbf{q}'_i)$ ,  $i = 1, \dots, n$

- 1: Create the cost function:
 

```
mpol('F',3,3);
for k = 1:size(q1)
    n(k) = (q2'*F*q1)^2;
end;
Crit = sum(n);
```
  - 2: Create the constraints:
 

```
K_det = \det(F) == 0; K_fro trace(F*F') == 1;
```
  - 3: Tune duality gap:
 

```
pars.eps = 0; mset(pars);
```
  - 4: Change the solver to SDPT3:
 

```
mset('yalmip',true); mset(sdpsettings('solver','sdpt3'));
```
  - 5: Form the problem and call the solver:
 

```
P = msdp(min(crit),K_det,K_fro);msol(P);
```
-

## 4 Experimental Results

This section presents results obtained by the test procedure presented below with the 8-point method and our global method. First, criteria to evaluate the performance of a fundamental matrix estimate are described. Next, the evaluation methodology is detailed. Experiments were then carried out on synthetic data to test the robustness to noise and to the number of extracted points. Finally, experiments on real data were performed to confirm previous results and to study the influence of the motion between the two images.

### 4.1 Evaluation Criteria

Various evaluation criteria were proposed in the literature [37] to evaluate the quality of a fundamental matrix estimate. Driven by practice, a fundamental matrix estimate  $\mathbf{F}$  is evaluated with respect to the behavior of its subsequent refinement by projective bundle adjustment. It is worth of note that a pair of uncalibrated perspective cameras are, up to some projective basis, equivalent to the fundamental matrix. In other words, a fundamental matrix is an implicit projective 3D reconstruction.

Bundle Adjustment from two uncalibrated views is described as a minimization problem. The cost function is the mean value of reprojection errors of measurement points  $(\mathbf{q}_i, \mathbf{q}'_i)_{i=1, \dots, n}$ . The unknowns are the 3D points  $(\mathbf{Q}_i)_i$  and the projection matrices  $\mathbf{P}$  and  $\mathbf{P}'$ .

- The first criterion is then the initial reprojection error written  $e_{\text{Init}}(\mathbf{F})$ .
- The second criterion is the value of the cost function achieved at the optimum (i.e. the final reprojection error  $e_{\text{BA}}(\mathbf{F})$ ).
- The third criterion is the number of iterations taken to converge,  $\text{Iter}(\mathbf{F})$ .

These three criteria assess whether the estimate provided by the two methods, denoted by  $\mathbf{F}_{8pt}$  and  $\mathbf{F}_{Gp}$ , is in the ‘good’ basin of attraction. Indeed, the number of iterations gives an indication of the distance between the estimate and the optimum while  $e_{\text{BA}}(\mathbf{F})$  gives an indication on the quality of the optimum.

### 4.2 Evaluation Method

The following algorithm summarizes our evaluation method:

#### Evaluate( $\mathbf{F}$ )

1. **Inputs:** fundamental matrix estimate  $\mathbf{F}$ ,  $n$  point matches  $(\mathbf{q}_i, \mathbf{q}'_i)$ ,  $i = 1, \dots, n$
2. Form initial projective cameras [26]:
  - Find the second epipole from  $\mathbf{F}^\top \mathbf{e}' \sim 0_{(3 \times 1)}$
  - Find the canonical plane homography  $\mathbf{H}^* \sim [\mathbf{e}]_\times \mathbf{F}$
  - Set  $\mathbf{P} \sim [\mathbf{I}_{(3 \times 3)} \ 0_{(3 \times 3)}]$  and  $\mathbf{P}' \sim [\mathbf{H}^* \ \mathbf{e}']$
3. Form initial 3D points [14]:
  - Triangulate each point independently by minimizing the reprojection error



4. Compute  $e_{\text{Init}}(\mathbf{F})$
5. Run two-view uncalibrated bundle adjustment [12]
6. Compute  $e_{\text{BA}}(\mathbf{F})$  and  $\text{Iter}(\mathbf{F})$
7. **Outputs:**  $e_{\text{Init}}(\mathbf{F})$ ,  $e_{\text{BA}}(\mathbf{F})$  and  $\text{Iter}(\mathbf{F})$

### 4.3 Experiments on Simulated Data

#### 4.3.1 Simulation Procedure

For each simulation series and for each parameter of interest (noise, number of points and number of motions), the same methodology is applied with the following four steps:

1. For two given motions between two successive images ( $[\mathbf{R}_k t_k]$ ) and for a given matrix of internal parameters, a set of 3D-points  $(\mathbf{Q}_i)_i$  ( $i = 1 \dots N$ ) is generated and two projection matrices  $\mathbf{P}^1$  and  $\mathbf{P}^2$  are defined. In practice, the rotations matrices,  $\mathbf{R}_1$  and  $\mathbf{R}_2$ , of two motions are defined by:

$$\mathbf{R}_k \triangleq \begin{bmatrix} \cos(\theta_k) & 0 & \sin(\theta_k) \\ 0 & 1 & 0 \\ -\sin(\theta_k) & 0 & \cos(\theta_k) \end{bmatrix} \text{ with } \begin{cases} \theta_1 = \frac{\pi}{3} \\ \text{and} \\ \theta_2 = \frac{\pi}{6} \end{cases} \quad (5)$$

and their translation vectors by  $t_1 = (20, 0, 5)^\top$  and  $t_2 = (6, 0, 0)^\top$ . These matrices are chosen such that  $[\mathbf{R}_1, t_1]$  is a large movement and  $[\mathbf{R}_2, t_2]$  is a small movement (see Figure 1). We simulated points lying in a cube with 10 meter side length. The first camera looks at the center of the cube and it is located 15 meters from the center of the cube. The focal length of the camera is 700 pixels and the resolution is  $640 \times 480$  pixels.

2. Thanks to projection matrices  $\mathbf{P}^1$  and  $\mathbf{P}^2$ , the set of 3D-points  $(\mathbf{Q}_i)_i$  is projected onto the two images as  $(\mathbf{q}_i, \mathbf{q}'_i)_i$ . At each of their pixel coordinates, a centered Gaussian noise with a variance  $\sigma^2$  is added. In order to have statistical evidence, the results are averaged over  $M$  trials.
3. The resulting noisy points  $(\tilde{\mathbf{q}}_i, \tilde{\mathbf{q}}'_i)_i$  are used to estimate  $\mathbf{F}$  by our method  $\mathbf{F}_{Gp}$  and the reference 8-point method  $\mathbf{F}_{8pt}$ .
4. Finally, via our evaluation procedure we evaluate the estimation error with respect to the variance of the noise ( $\sigma^2$ ) and the number of points ( $N$ ).

#### 4.3.2 Robustness to Noise

We tested in two simulation series the influence of the variation of the noise standard deviation  $\sigma$  ranging from 0 to 2 pixels. The number of simulated points is 50. The number of trials  $M$  is 100. The first (resp. second) simulation serie is based on the first motion  $[\mathbf{R}_1 t_1]$  (resp. the second motion  $[\mathbf{R}_2 t_2]$ ). Figure 2 gathers the influence of noise on evaluation criteria. The first line shows the reproduction errors before,  $e_{\text{Init}}(\mathbf{F})$ , and after  $e_{\text{BA}}(\mathbf{F})$  refinement through Bundle Adjustment with respect to the noise standard deviation. The second line shows the number of iterations  $\text{Iter}(\mathbf{F})$  of the

Bundle Adjustment versus the noise standard deviation. The first (resp. second) row concerns the first (resp. second) motion.

For the two motions, re-projection errors,  $e_{\text{Init}}(\mathbf{F})$  or  $e_{\text{BA}}(\mathbf{F})$ , increase with the same slope when the noise level increases. Notice that for both movements, the Bundle Adjustment step does not improve the results. Indeed, the noise gaussian noise is added to the projections  $(\mathbf{q}_i, \mathbf{q}'_i)_i$ . So this is noise which in practice would be produced by the extraction points process. Thus the solution produced by the resolution of the linear system is very close to the optimum and does not need to be refined. The initial solution provided by the triangulation step is then very close to a local minimum of the Bundle Adjustment problem. Moreover, the variation of the errors of initial re-projection before (*8pt – Init* et *Gp – Init*) and after (*8pt – BA* et *Gp – BA*) Bundle Adjustment versus the noise standard deviation is linear. However, the number of iterations needed for convergence is different in the two methods. The initial estimate of the triangulation computed from  $\mathbf{F}_{Gp}$  is closer to the local minimum than that obtained from  $\mathbf{F}_{8pt}$ . For the first motion (large displacement between camera 1 and 2), the number of iterations of the global method (in green) remains smaller than for the 8-point method (in blue) even though their difference seems to decrease when the noise level is high ( $\sigma > 1$ ). For a significant displacement the quality of the estimate  $\mathbf{F}$  by the global method remains better even though the difference in quality diminishes with the noise level. Conversely, for the second motion (small displacement between the camera 1 and 2) both methods are equivalent since the difference in quality is only significant for a high level of noise ( $\sigma > 1$ ). This is logical as the movement is less important. As a conclusion, the 8-point method provides a solution equivalent to that obtained with the global method when the displacement is not too important. For more significant movements the provided solution is not so close even though still in the same basin of attraction of a local minimum.

### 4.3.3 Influence of the Number of Points

In this experiment, we kept the noise level constant with a standard deviation  $\sigma^2 = 0.5$  pixels. We tested the influence of the number of matches  $(\mathbf{q}_i, \mathbf{q}'_i)_i$  on the quality of the resulting estimate of  $\mathbf{F}$ . The number of points  $N$  varied from 10 to 100. Two simulation series are also carried out with the two motions.

Figure 3 brings with the same organization the evaluation criteria. It displays the influence of the number of matches for estimating  $\mathbf{F}$  on the re-projection errors and on the number of iterations. For both motions and for a sufficiently high number of matches ( $N > 50$ ), re-projection errors, before and after refinement with Bundle Adjustment, or the number of iterations versus the number of matches converge to the same asymptote. From a high number of matches, the initial estimate from triangulation computed with  $\mathbf{F}_{8pt}$  and with  $\mathbf{F}_{Gp}$  are both in the same basin of attraction for the Bundle Adjustment problem. However, for a number of matches smaller than 50, the number of iterations to converge is smaller for given re-projection errors. The quality of the estimation by the global method seems better. The initial estimate from triangulation computed with  $\mathbf{F}_{8pt}$  goes away from the basin of convergence whereas the one computed with  $\mathbf{F}_{Gp}$  remains in the basin.

### 4.3.4 Influence of the Number of Points with Wide Baseline

In order to sustain the previous behavior, for a noise standard deviation of  $\sigma^2 = 1$  pixel and for the significant displacement  $[\mathbf{R}_1 \ t_1]$ , the influence of the number of matching

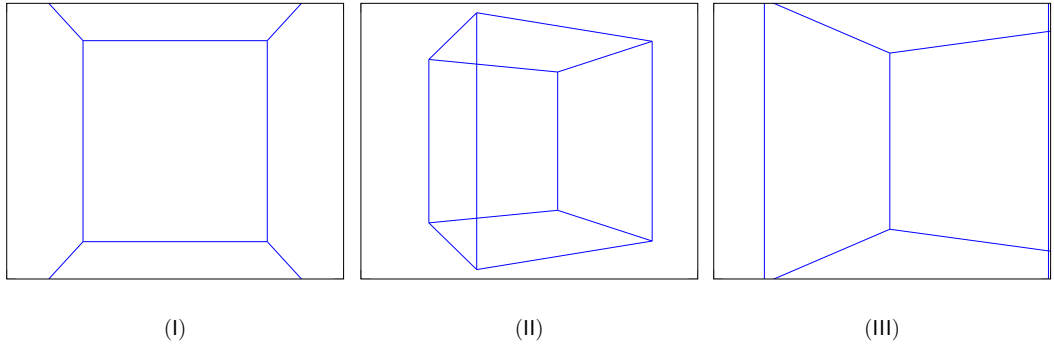


Fig. 1: Projection of the cube in the camera on initial position (I) and in the camera after applying the rigid transformation  $[\mathbf{R}_1 t_1]$  (II) and the rigid transformation  $[\mathbf{R}_2 t_2]$  (III).

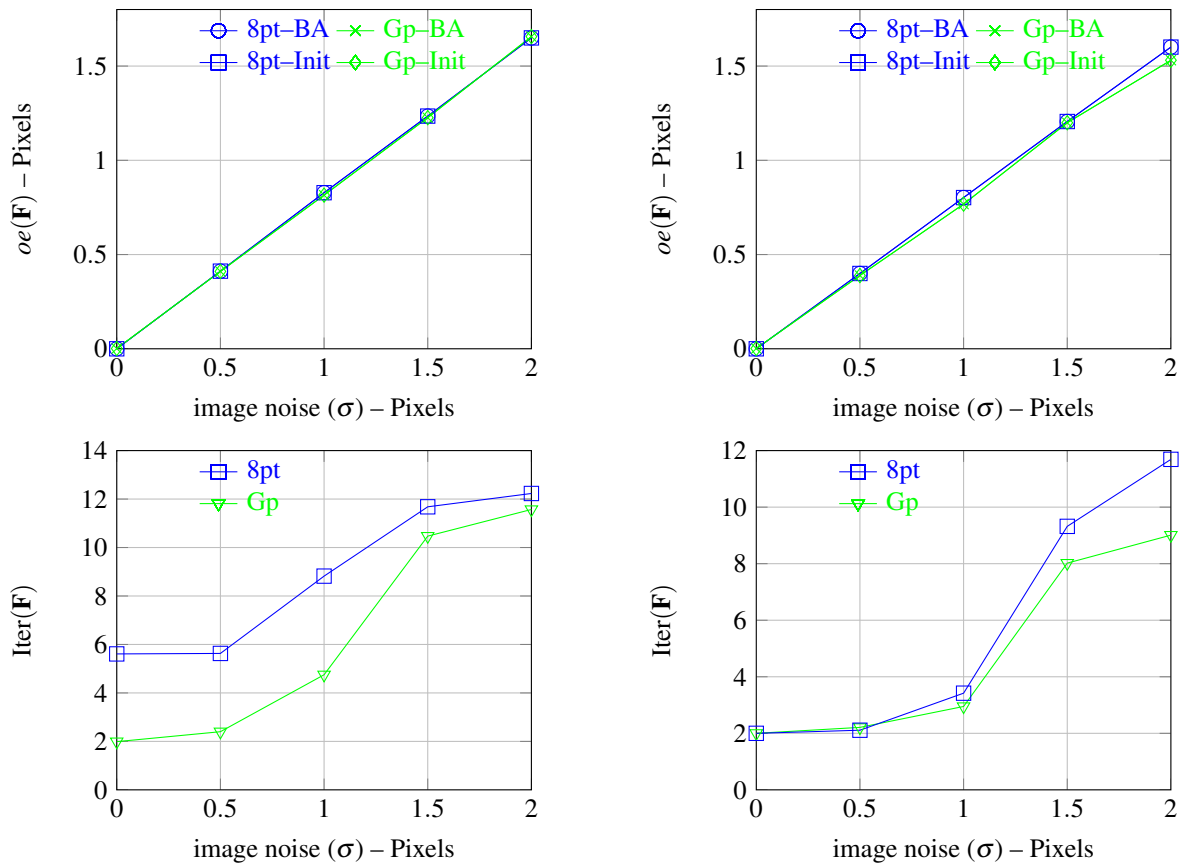


Fig. 2: For two movements,  $[\mathbf{R}_1 t_1]$  (left column) and  $[\mathbf{R}_2 t_2]$  (right column), reprojection errors and number of iterations measured against image noise.

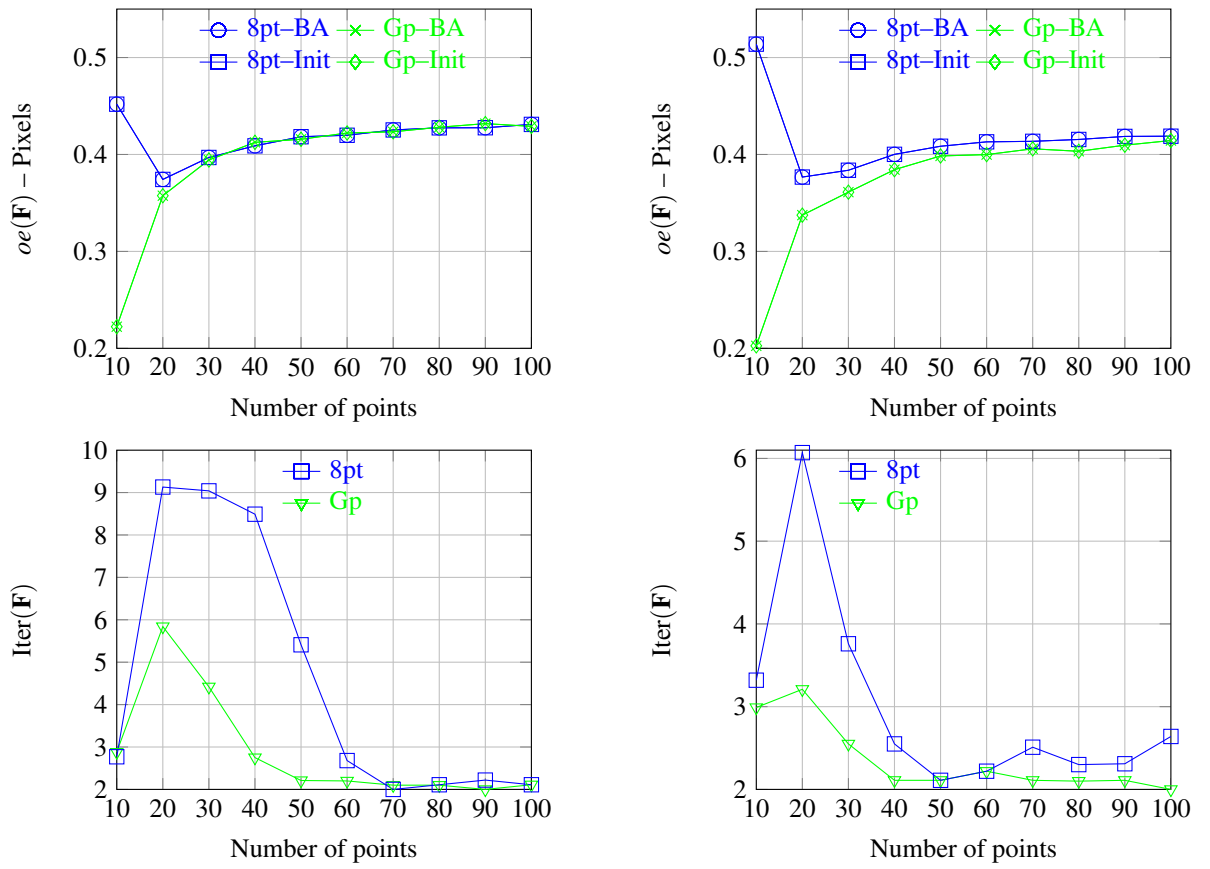


Fig. 3: For two movements,  $[\mathbf{R}_1 t_1]$  (left column) and  $[\mathbf{R}_2 t_2]$  (right column), reprojection errors and number of iterations measured against number of points for a gaussian noise with a variance fixed to 0.5.

points on the re-projection errors and on the number of iterations was tested. In this difficult context, Figure 4 demonstrates that the initial estimate computed from  $\mathbf{F}_{Gp}$  is always closer to the local minimum than that computed from  $\mathbf{F}_{8pt}$ . No matter what is the number of matching points, the number of iterations needed to converge is always smaller.

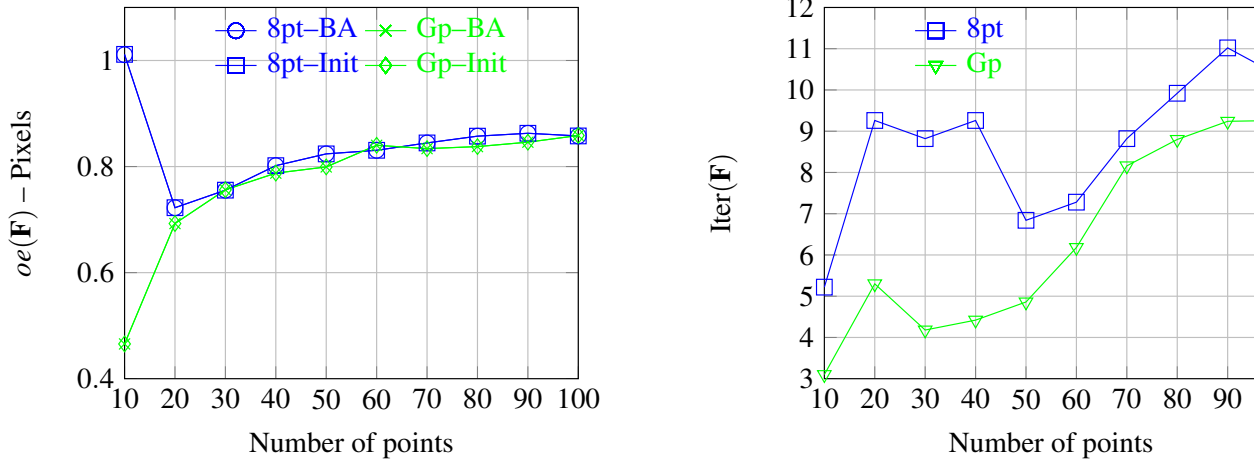


Fig. 4: For the movement  $[\mathbf{R}_1 t_1]$ , reprojection errors and number of iterations measured against number of points for a gaussian noise with a variance fixed to 1 (left and right).

As a conclusion, the quality of solutions obtained by both methods is almost identical when the movement is not too important, the number of matching points is sufficiently large, and the noise level is not too high. However, when one of these three parameters varies then the 8-point method lacks precision whereas the global method still allows Bundle Adjustment to convergence to the global minimum. The 8-point method computes the projection of an unconstrained local minimizer on the feasible set whereas the global method provides a global minimizer of the constrained optimization problem. It is already surprising that even for good values of the three parameters the resulting solutions are not too far apart. But for bad values of the parameters it would be even more surprising.

#### 4.4 Experiments on Real Data

The evaluation criteria remain the same,  $e_{\text{Init}}(\mathbf{F})$ ,  $e_{\text{BA}}(\mathbf{F})$  and  $Iter(\mathbf{F})$  and the computation time is added. Two experiments were carried out with two sets of images that illustrate different motions between two successive images.

##### 4.4.1 Experiment 1

The first set of four images (see Table 5) shows all possible epipolar configurations (right or left epipole at infinity ...). With four images, six motions between a pair of images are possible: A – B, A – C, A – D, B – C, B – D and C – D. For every pair of images, 60 matches are available to compute an estimate of  $\mathbf{F}$ . The values of the evaluation criteria are summarized in table 5. No matter what pair of images is used,

the re-projection errors and the number of iterations are almost always better when  $\mathbf{F}_{Gp}$  is used as initial guess. In addition, for three motions (A – C, A – D and C – D), in contrast with the initial guess  $\mathbf{F}_{Gp}$ , the initial guess from the 8-point method is not in a better basin of attraction. This may explain why the initial re-projection errors  $e_{\text{Init}}(\mathbf{F})$  are sometimes larger for  $\mathbf{F}_{Gp}$  as the initial guess may be in a good basin of attraction but with a larger re-projection error. For the four motions A – B, B – C, B – D and B – D, both initializations are in the same basin of attraction but the number of iterations demonstrates that the initial guess from the global method is always closer to the local minimizer. Finally, even though the computation time of the latter is significantly larger than for the 8-point method, it still remains compatible with a practical use.

#### 4.4.2 Experiment 2

The second experiment compares the two methods on large motions. It is based on many series of images. First we test our algorithm with the classic series *Library*, *Merton*, *dinosaur* and *house* that are available at [www.robots.ox.ac.uk/~vgg/data/data-mview.html](http://www.robots.ox.ac.uk/~vgg/data/data-mview.html). For the set of three images of *Library* and *Merton* serie, Table 6, 7, 8 and Table 9 demonstrate that the quality of the solution achieved by the global method is always better than with the 8-point method (in some cases both solutions are very close).

We also conducted the same tests on other pairs of images. For the first pair, we used images from a standard cylinder graciously provided by the company NOOMEO (<http://www.noomeo.eu/>). This cylinder is use to evaluate the accuracy of 3D reconstructions. Matched points are calculated with digital image correlation method. They are located in a window inside the cylinder. Thus, we have 6609 pairs  $(\mathbf{q}_i, \mathbf{q}_i)_i$  matched to sub-pixel precision. Results are presented in the Table 10. We observe that the computation time of the 8-point method exceeds one second. This is due to the large number of matched points which leads to the resolution of a large linear system. However, as the points are precisely matched, this system is well conditioned. But the quality of the fundamental matrix estimated with the 8-point method is not sufficient to properly initialize the Bundle-Adjustment because the final re-projection error is 1.47 pixels. At the same time, even if the number of iterations is larger, our global method supplies a good estimation because the final re-projection error is 0.25 pixels. Furthermore, the calculation time remains constant in approximately 2 seconds. For the second pair, we use images taken by an endoscope. Table 11 shows the results obtained on this difficult case. As for the previous example, we observe that the fundamental matrix estimated by our global method is good quality because the final error is 0.93 pixels. At the same time, Bundle Adjustment puts more iterations to converge on a less precise solution when we use  $\mathbf{F}_{8pt}$  to initialize it.

For the set of 36 images of the *Dinosaur* series and 9 images of the *house* series, we tested the influence of motion amplitude between a pair of image on the quality of the resulting estimates obtained by both methods. For this purpose, we had both estimates with all possible motions  $((0, 1), (1, 2), (2, 3), \dots)$  with 1-image distance, then all possible motions  $((0, 2), (1, 3), (2, 4), \dots)$  with 2-image distance, and so on. With this process, we can measure the influence of the average angle on the quality of the fundamental matrix estimated by both methods. Figures from 12 to 17 shows the average of re-projection errors and the average of number of iterations with respect to average angle for the two series. The re-projection error after Bundle Adjustment is always smaller with the global method and with always a smaller number of iterations. Next, the larger the movement the more the solution by both methods deteriorates. But

the deterioration is larger for the 8-point method than for the global method. One may also observe that in some cases the re-projection error before Bundle Adjustment is in favor of the 8-point method. In analogy with the real cases studied before, this may be due to the fact that for these cases the initial guess  $\mathbf{F}_{Gp}$  is in a basin of attraction with a better local minimum than in the basin of attraction associated with  $\mathbf{F}_{8Pt}$ , but the ‘distance’ between the initial guess and the corresponding local minimizer is larger for  $\mathbf{F}_{Gp}$  than for  $\mathbf{F}_{8Pt}$ . Indeed in such cases the number of iterations is larger for  $\mathbf{F}_{Gp}$  than for  $\mathbf{F}_{8Pt}$ .



— A —



— B —



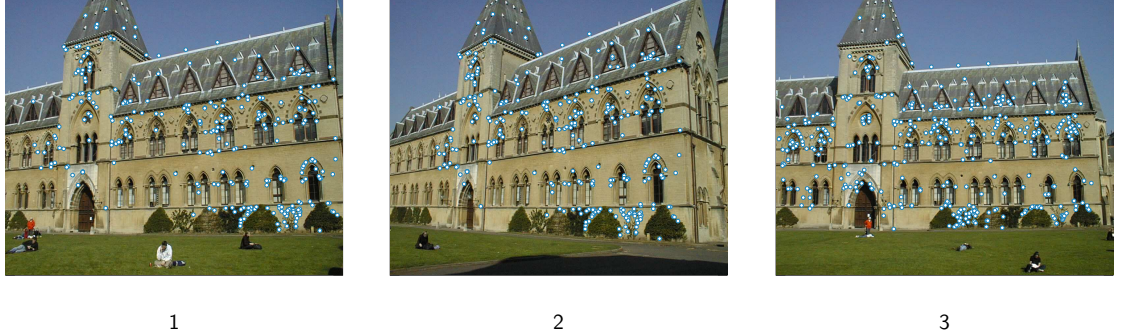
— C —



— D —

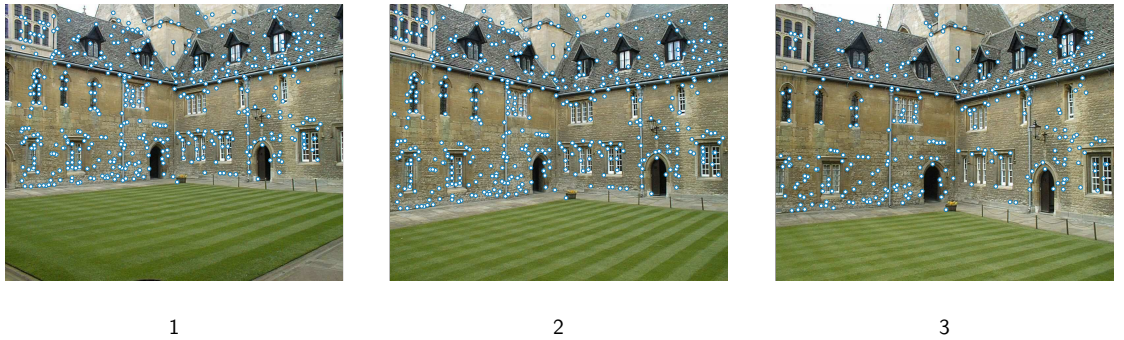
Views		Epipoles		$e_{\text{Init}}(\mathbf{F})$		$e_{\text{BA}}(\mathbf{F})$		Iter( $\mathbf{F}$ )		Time	
		$\mathbf{e}$	$\mathbf{e}'$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$
A	B	$\infty$	$\infty$	0.597617	0.65270	0.00252	0.00252	6	6	0.017	2.39
A	C	$\infty$	$\infty$	5.61506	5.61996	2.48258	0.00342	122	175	0.017	1.98
A	D	$\infty$	$\infty$	21.0855	21.5848	4.74837	0.00344	105	30	0.017	2.12
B	C	$\infty$	$\infty$	2.49098	1.91136	0.00260	0.00260	17	12	0.018	1.97
B	D	$\infty$	$\infty$	22.0071	23.6253	0.00268	0.00268	122	81	0.018	1.92
C	D	$\infty$	$\infty$	28.6586	28.6174	16.6507	0.25921	39	1001	0.018	2.1

Fig. 5: Reprojection Error before ( $e_{\text{Init}}(\mathbf{F})$ ) and after Bundle Adjustment ( $e_{\text{BA}}(\mathbf{F})$ ), Number of Iterations (Iter( $\mathbf{F}$ )), and CPU time to compute  $\mathbf{F}$  (Time), obtained when combining pairs of images to obtain epipoles close to the images or toward infinity



Views	nb Points	$e_{\text{Init}}(\mathbf{F})$		$e_{\text{BA}}(\mathbf{F})$		Iter( $\mathbf{F}$ )		Time		
		$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	
1	2	310	18.27449	18.32095	11.39809	11.287591	65	57	0.035	2.05
2	3	439	61.84817	63.79435	41.81854	40.2224	29	28	0.020	2.22
1	3	439	42.67438	42.23867	28.65971	27.90128	40	32	0.043	2.10

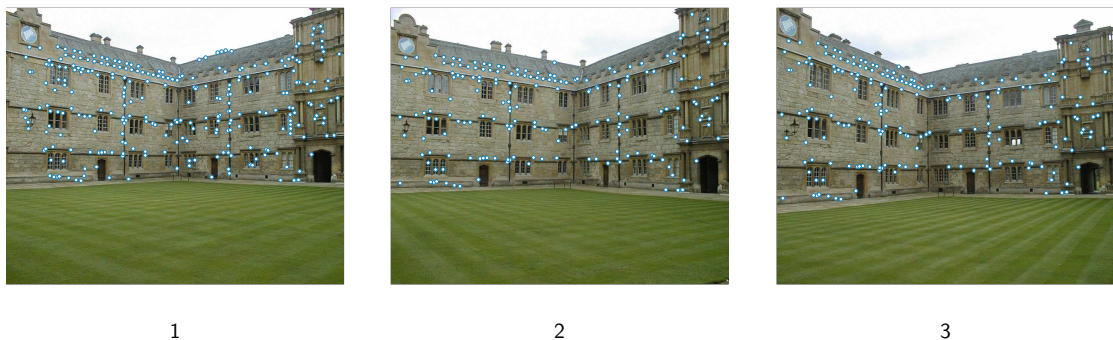
Fig. 6: Reprojection Error before ( $e_{\text{Init}}(\mathbf{F})$ ) and after Bundle Adjustment ( $e_{\text{BA}}(\mathbf{F})$ ), Number of Iterations (Iter( $\mathbf{F}$ )), and CPU time to compute  $\mathbf{F}$  (Time), obtained when combining pairs of images of the *Library* series



Views	nb Points	$e_{\text{Init}}(\mathbf{F})$		$e_{\text{BA}}(\mathbf{F})$		Iter( $\mathbf{F}$ )		Time		
		$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	
1	2	485	15.1771	15.3126	9.8859	9.7360	77	55	0.048	2.02
2	3	439	61.84817	63.79435	41.81854	40.2224	29	28	0.020	2.22
1	3	384	51.3128	2.3690	7.5245	0.56180	5	52	0.040	2.71

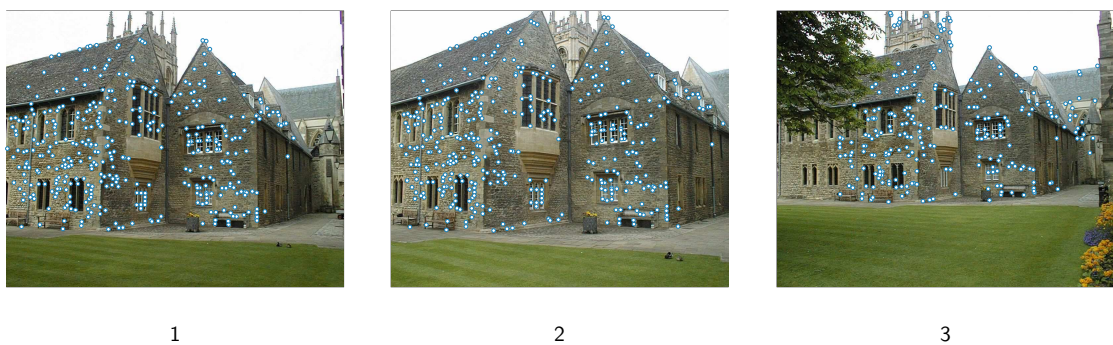
Fig. 7: Reprojection Error before ( $e_{\text{Init}}(\mathbf{F})$ ) and after Bundle Adjustment ( $e_{\text{BA}}(\mathbf{F})$ ), Number of Iterations (Iter( $\mathbf{F}$ )), and CPU time to compute  $\mathbf{F}$  (Time), obtained when combining pairs of images of the *Merton1* series





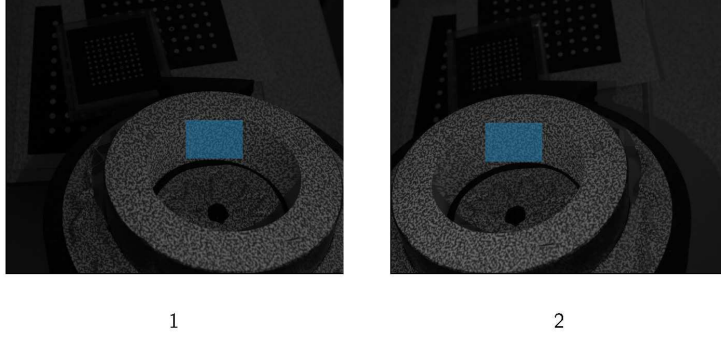
Views	nb Points	$e_{\text{Init}}(\mathbf{F})$		$e_{\text{BA}}(\mathbf{F})$		Iter( $\mathbf{F}$ )		Time		
		$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	
1	2	345	24.7158	29.6530	14.0481	2.7599	69	42	0.037	2.91
2	3	197	143.431	154.880	73.8776	72.8977	14	19	0.026	2.52
1	3	270	59.8109	77.9734	30.3824	15.6367	38	32	0.031	2.37

Fig. 8: Reprojection Error before ( $e_{\text{Init}}(\mathbf{F})$ ) and after Bundle Adjustment ( $e_{\text{BA}}(\mathbf{F})$ ), Number of Iterations (Iter( $\mathbf{F}$ )), and CPU time to compute  $\mathbf{F}$  (Time), obtained when combining pairs of images of the *Merton2* series



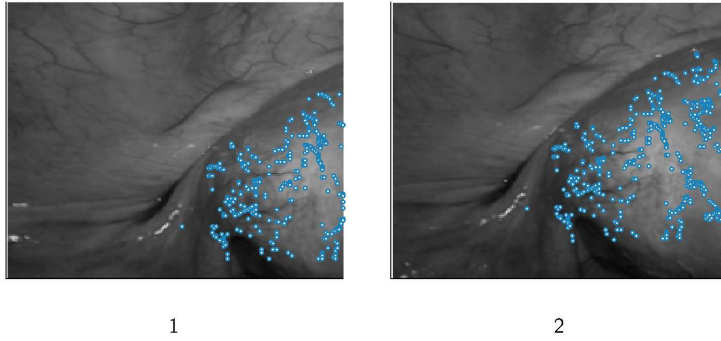
Views	nb Points	$e_{\text{Init}}(\mathbf{F})$		$e_{\text{BA}}(\mathbf{F})$		Iter( $\mathbf{F}$ )		Time		
		$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	
1	2	400	62.7290	68.6030	52.6557	23.1740	5	32	0.041	3.03
2	3	197	135.950	140.801	83.3365	76.8614	13	14	0.025	2.52
1	3	264	116.7659	118.1754	24.7184	13.5496	9	11	0.032	2.39

Fig. 9: Reprojection Error before ( $e_{\text{Init}}(\mathbf{F})$ ) and after Bundle Adjustment ( $e_{\text{BA}}(\mathbf{F})$ ), Number of Iterations (Iter( $\mathbf{F}$ )), and CPU time to compute  $\mathbf{F}$  (Time), obtained when combining pairs of images of the *Merton3* series



Views	nb Points	$e_{\text{Init}}(\mathbf{F})$		$e_{\text{BA}}(\mathbf{F})$		Iter( $\mathbf{F}$ )		Time		
		$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	
1	2	6609	6.3142	6.4021	1.4701	0.2531	401	237	1.23	2.33

Fig. 10: Reprojection Error before ( $e_{\text{Init}}(\mathbf{F})$ ) and after Bundle Adjustment ( $e_{\text{BA}}(\mathbf{F})$ ), Number of Iterations (Iter( $\mathbf{F}$ )), and CPU time to compute  $\mathbf{F}$  (Time), obtained when combining pairs of images of the *Cylinder* series. The matched points are located in blue bounding boxes.



Views	nb Points	$e_{\text{Init}}(\mathbf{F})$		$e_{\text{BA}}(\mathbf{F})$		Iter( $\mathbf{F}$ )		Time		
		$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	$\mathbf{F}_{8Pt}$	$\mathbf{F}_{Gp}$	
1	2	730	5.6624	5.6366	3.8566	0.9386	163	401	0.07	1.69

Fig. 11: Reprojection Error before ( $e_{\text{Init}}(\mathbf{F})$ ) and after Bundle Adjustment ( $e_{\text{BA}}(\mathbf{F})$ ), Number of Iterations (Iter( $\mathbf{F}$ )), and CPU time to compute  $\mathbf{F}$  (Time), obtained when combining pairs of images of the *Endoscope* series

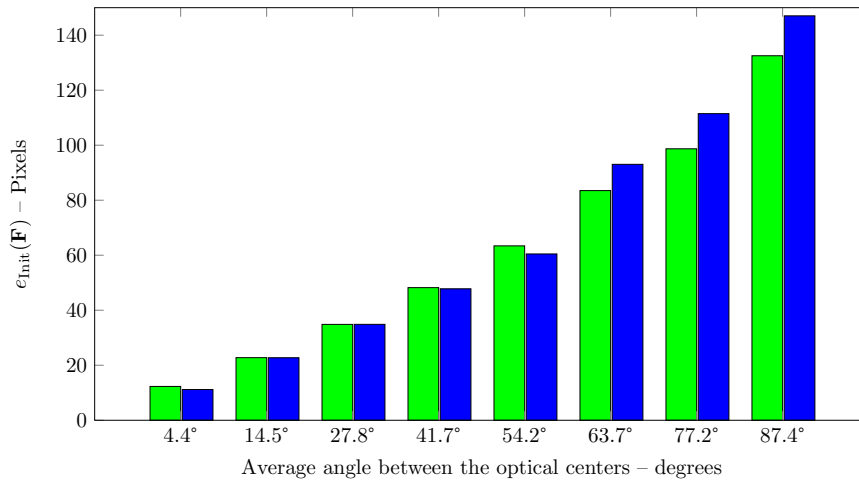


Fig. 12: Initial re-projections errors measured against movement amplitude for the *dinosaur* series

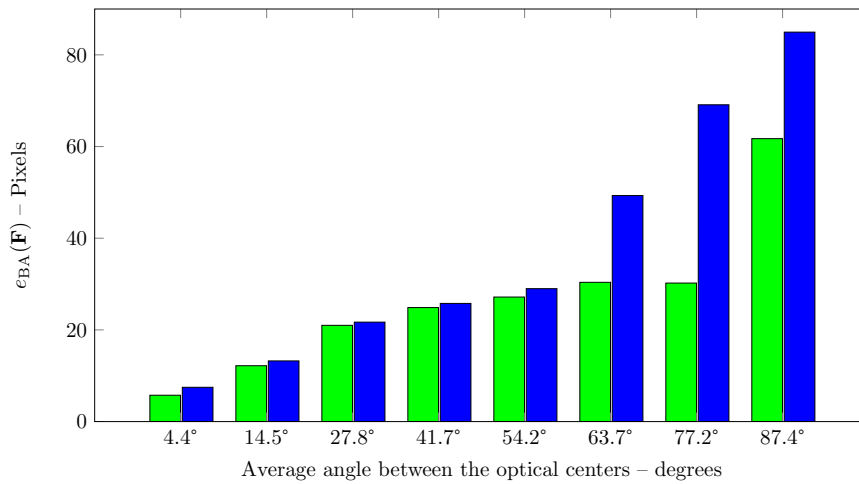


Fig. 13: Final re-projections errors measured against movement amplitude for the *dinosaur* series

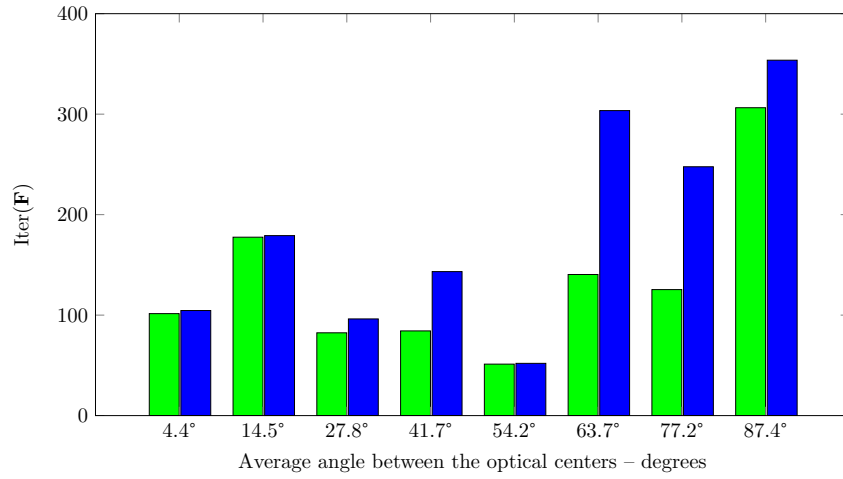


Fig. 14: Number of iterations performed by Bundle-Adjustment to converge measured against movement amplitude for the *dinosaur* series

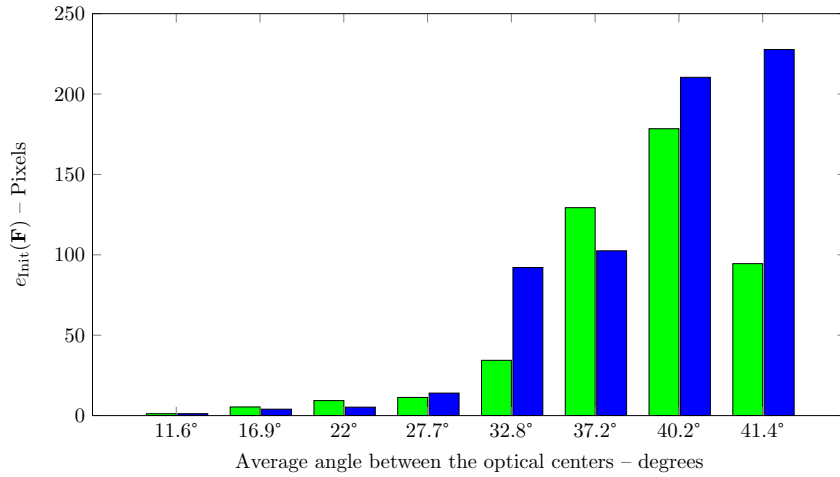


Fig. 15: Initial re-projections errors measured against movement amplitude for the *House* series

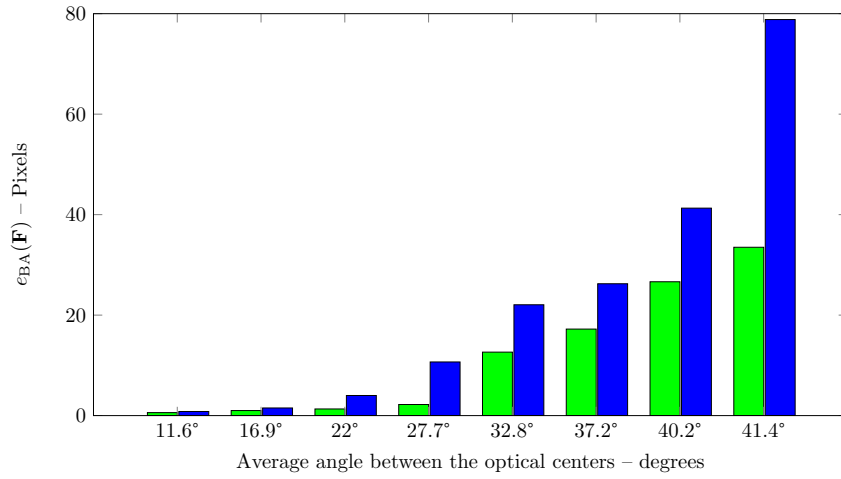


Fig. 16: Final re-projections errors measured against movement amplitude for the *House* series

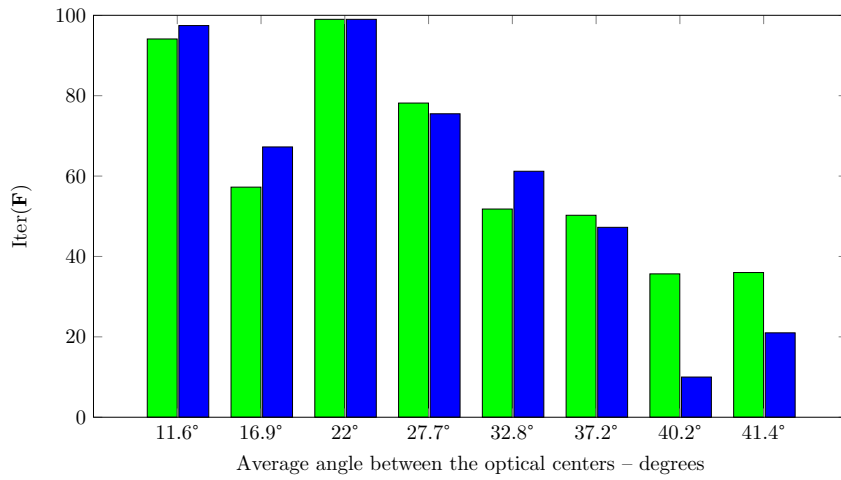


Fig. 17: Number of iterations performed by Bundle-Adjustment to converge measured against movement amplitude for the *House* series

## 5 Conclusion

We have studied the problem of estimating globally the fundamental matrix over nine parameters and under rank and normalisation constraints. We have proposed a polynomial-based approach which enables one to estimate the fundamental matrix with good precision. More generally, we have shown how to modify the constraints on the numerical certificate of optimality to obtain fast and robust convergence. The method converges in a reasonable amount of time compared to other global optimization methods.

From computational experiments conducted on both simulated and real data we conclude that the global method always provides an accurate initial estimation for the subsequent bundle adjustment step. Moreover, we have shown that if the eight-point method has a lower computational cost, its resulting estimate is frequently far from the global optimum obtained by the global method.

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