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# Evolution of Kerr superspinars due to accretion counterrotating thin discs

Zdeněk Stuchlík    Stanislav Hledík    Kamila Truparová

*Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava  
Bezručovo nám. 13, CZ-74601 Opava, Czech Republic*

*E-mail: zdenek.stuchlik@fpf.slu.cz, stanislav.hledik@fpf.slu.cz*

## Abstract

The string theory predicts existence of extremely compact objects spinning faster than the Kerr black holes. The spacetime exterior to such superspinars is described by the Kerr naked singularity geometry breaking the black hole limit on the internal angular momentum. We demonstrate that conversion of Kerr superspinars into a near-extreme black hole due to an accretion counterrotating Keplerian disc is much more effective in comparison with the case of a corotating one since both the accreted rest mass necessary for conversion and the evolution time of conversion are by orders smaller for counterrotating discs. The conversion time of Kerr superspinars is given for several accretion regimes, and it is shown that the self-regulated accretion flow implies fastest evolution to the black-hole state. In final stages of the conversion, Kerr superspinars can serve as very efficient particle accelerators in the region where the black-hole horizon forms.

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## 1 Introduction

The string theory predicts a very interesting possibility to be tested in the framework of relativistic astrophysics. As shown by Gimon and Hořava [1], Kerr superspinars with mass  $M$  and angular momentum  $J$  violating the general relativistic bound on the spin of compact objects ( $a \equiv J/M^2 > 1$ ) could be primordial remnants of the high-energy phase of very early period of the evolution of the Universe when the effects of the string theory were relevant. The spacetime outside a Kerr

superspinar with boundary at radius  $R$ , where the stringy effects become to be irrelevant, is assumed to be described by the standard Kerr geometry. The exact solution governing the interior of the superspinar is not known in the 3+1 theory. It is expected that extension of the internal stringy solution is limited to radii  $r < R < M$ , thus covering the region of causality violations (naked time machine) and still allowing for the presence of the relevant astrophysical phenomena related to the Kerr naked singularity spacetimes. There is an expectation that the pathological naked time machine is replaced by a correctly behaving stringy solution [1], being motivated by the resolution of the problems of 4 + 1 SUSY black hole solution [2] where the pathological time machine region is replaced by a portion of the Gödel universe [3, 4].

The Kerr superspinars (or Kerr naked singularity spacetimes) should have extremely strong gravity in their vicinity and some properties substantially differing from those of Kerr black holes [5–7]. The differences could be related both to the accretion phenomena [8–13] and optical effects [5, 8, 14–19]. Here we focus our attention to the evolution of Kerr superspinars due to accretion from Keplerian discs.

Evolution of supermassive black holes plays a crucial role in modelling of galactic structures in the expanding universe [20, 21]. It is well known that evolution due to accretion from geometrically thin discs could lead asymptotically to the extreme black hole states, but a conversion into a Kerr naked singularity spacetime is not allowed [22]. In fact, there is a general belief that the extreme black holes are not accessible by astrophysically relevant processes due to laws of the black hole physics [23] – attempts to find some counterexamples [24] put doubts on their physical plausibility.

Accretion onto black holes implies growing (lowering) of their spin for corotating (counterrotating) thin discs. On the other hand, properties of the geodesic structure of the Kerr naked singularity spacetimes imply lowering of their spin for both corotating and counterrotating discs and their final conversion into a black hole. It was shown that in the case of corotating discs the conversion has to cause changes in their structure in a spectacular way because of the jump in the properties of corotating circular geodesics of naked singularity and black hole spacetimes [25], while in the case of counterrotating discs we expect quite regular behaviour of the discs during the conversion due to regular behaviour of the counterrotating geodesics [5].

The energy efficiency of accretion in Keplerian discs around Kerr superspinars is given by the energy of the marginally stable circular geodesic  $E_{\text{ms}}$ . For corotating discs the efficiency can strongly exceed the efficiency of the Keplerian discs around near-extreme Kerr black holes. In the field of Kerr naked singularities with spin  $a$

very close to the minimal value of  $a = 1$  the radius of marginally stable geodesic approaches  $r_{\text{ms}} = M$  from below and the efficiency overcomes substantially even the annihilation efficiency, being  $\approx 158\%$ , and is much larger than the efficiency  $\approx 42.3\%$  of accretion in the field of near-extreme Kerr black holes [5]. For counterrotating discs there is  $r_{\text{ms}} > 9M$  and the efficiency of Keplerian accretion is lower than for near-extreme black holes [5].

The process of converting Kerr superspinars into near-extreme black holes due to accretion from a corotating Keplerian disc is very dramatic [25] and could be interesting with connection to the most extreme cases of Gamma Ray Bursts (GBR) since a large part of the accretion disc becomes to be dynamically unstable after the transition of the superspinner into a near-extreme black hole state and falls freely to the black hole causing a sharp decrease of the disc luminosity [25]. If such an extremely compact object violating the standard Kerr spin bound  $a < 1$  will be observed in the Universe, it could quite well be naturally interpreted in the framework of the string theory. The astrophysical effects related to Kerr superspinars could thus belong to relevant experimental tests of the string theory [1]. On the other hand, one has to be very careful in making any definite conclusions based on the Kerr spin bound, since breaching of the Kerr bound is also allowed for braneworld Kerr black holes endowed with a quite regular event horizon when the tidal charge, the braneworld parameter reflecting the brany black hole interaction with the external bulk space, takes negative values [26–31].

We shall discuss evolution of a Kerr superspinner due to accretion from a Keplerian, equatorial, geometrically thin disc. We shall focus our attention on the counterrotating discs and compare the results to those of the evolution of Kerr superspinars due to accretion corotating Keplerian discs [25] and to the evolution of black holes [21, 22]. The Keplerian discs have quasi-geodetical structure and their edge is located at the marginally stable geodesic where the relativistic phenomena are strong. In such discs, the total radiation rate (luminosity  $L$ ) is given by

$$L = (1 - E_{\text{ms}}) \frac{dm}{dt} c^2,$$

where  $1 - E_{\text{ms}}$  is the efficiency of converting accreting rest mass into outgoing radiation and  $dm/dt$  is the rest mass accretion rate. We assume the superspinner boundary surface to be located at  $r(\theta) = R = 0.1M$  which guarantees that the surface is well under the ISCO that cannot exceed  $r_{\text{ms}} = 2M/3$  [5]. Further, we assume that the boundary of the superspinner has properties similar to those of the horizon, i.e., it is not radiating and eats the accreted matter, in accord with some other related actual papers investigating the Kerr superspinars [1, 7, 13, 17]. Of course, all results presented here are valid also for evolution of Kerr naked singularities.

In Section 2 we present evolution equations of Kerr superspinars due to accretion from counterrotating Keplerian discs. In Section 3 we construct the time evolution in selected typical accretion regimes with specific laws for the accretion flow. In Section 4 we compare the results to those for the accretion corotating Keplerian discs. In Section 5 we present discussion, and in final Section 6 some conclusions are given.

## 2 Evolution equations

The Kerr superspinar spacetime is determined by the standard Kerr geometry [23, 32] characterized by the mass parameter  $M$  and the dimensionless spin  $a = J/M^2$  where  $J$  is the internal angular momentum of the superspinar; magnitude of its spin  $|a| > 1$ .

### 2.1 Kerr geometry

In the standard Boyer–Lindquist coordinates and geometric units ( $c = G = 1$ ), the Kerr geometry is given in the form

$$ds^2 = - \left( 1 - \frac{2Mr}{\Sigma} \right) dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{A}{\Sigma} \sin^2 \theta d\varphi^2 - \frac{4M^2 ar \sin^2 \theta}{\Sigma} dt d\varphi,$$

where

$$\Delta = r^2 - 2Mr + (aM)^2, \quad \Sigma = r^2 + (aM)^2 \cos^2 \theta,$$

$$A = [r^2 + (aM)^2]^2 - \Delta (aM)^2 \sin^2 \theta.$$

The physical ring singularity of the spacetime is located at  $r = 0$ ,  $\theta = \pi/2$ . There is no event horizon for the superspinar (naked singularity) spacetimes since  $\Delta > 0$  for  $|a| > 1$ . The pathological causality violation region is determined by the condition [23]

$$g_{\phi\phi} = \frac{A}{\Sigma} \sin^2 \theta = \left[ r^2 + (aM)^2 + \frac{2M^3 a^2 r \sin^2 \theta}{\Sigma} \right] \sin^2 \theta < 0,$$

and we can convince ourselves easily that causality violations can occur only at  $r < 0$  [33]. Physically realistic models of Kerr superspinars have to remove the causality violating region and the ring singularity. Therefore, the minimal condition for the boundary surface of Kerr superspinars reads  $r(\theta) = R = 0$ . In recent papers concerning the Kerr superspinars, usually the boundary at  $r(\theta) = R = 0.1M$  is assumed and we keep this assumption.

## 2.2 Keplerian discs

The structure of Keplerian, thin accretion discs is governed by the equatorial circular geodesic motion in the Kerr spacetime. In Kerr naked singularity spacetimes relevant for the external field of Kerr superspinars, there are two families of circular geodesics [5] – the 1st (2nd) family orbits are governing corotating (counterrotating) Keplerian discs.

The Carter equations imply the specific energy and specific angular momentum of the circular geodesics to be given by the relations [5, 34]

$$E_K = \frac{x^{3/2} - 2x^{1/2} \pm a}{x^{3/4} \sqrt{x^{3/2} - 3x^{1/2} \pm 2a}}, \quad \Phi_K = \pm \frac{x^2 + a^2 \mp 2ax^{1/2}}{x^{3/4} \sqrt{x^{3/2} - 3x^{1/2} \pm 2a}},$$

where  $x = r/M$  and the upper (lower) sign corresponds to the 1st (2nd) family orbits. The angular velocity with respect to static observers at infinity  $\Omega = d\phi/dt$  is given by the relation

$$\Omega_K = \pm \frac{1}{x^{3/2} \pm a}.$$

In theory of Keplerian discs a crucial role is devoted to the innermost (marginally) stable circular orbit (ISCO) in the field of a given spacetime that is usually considered to correspond to the inner edge of Keplerian discs [35]. In Kerr spacetimes the ISCO is determined by the relation [34]

$$x_{\text{ms}} = 3 + Z_2 \mp \sqrt{(3 - Z_2)(3 + Z_1 + 2Z_2)},$$

where

$$Z_1 = 1 + (1 - a^2)^{1/3} [(1 + a)^{1/3}(1 - a)^{1/3}], \quad Z_2 = \sqrt{3a^2 + Z_1^2}.$$

The marginally bound orbits with  $E_K = 1$  have radii given by

$$x_{\text{mb}} = 2 + a \mp 2(1 + a)^{1/2}.$$

The 1st family orbits are corotating relative to distant observers ( $\Omega_K > 0$ ) – such orbits are locally corotating ( $\Phi_K > 0$ ) in regions distant from superspinars, but could be locally counterrotating ( $\Phi_K < 0$ ) in vicinity of superspinars with the spin parameter  $a < a_0 = 3^{3/2}/4 \approx 1.299$ . For superspinars with spin  $a < a_c = 2^{5/2}/3^{3/2} \approx 1.089$ , the 1st family orbits with  $\Phi_K < 0$  could have negative energy ( $E < 0$ ), while being located close enough to the superspinar boundary – the marginally stable circular orbit is located under  $x = 1$  for  $a < 5/3$  and  $x_{\text{ms}} \rightarrow 1$  from below for  $a \rightarrow 1$  from above [5]. The 1st family orbits reveal a strong

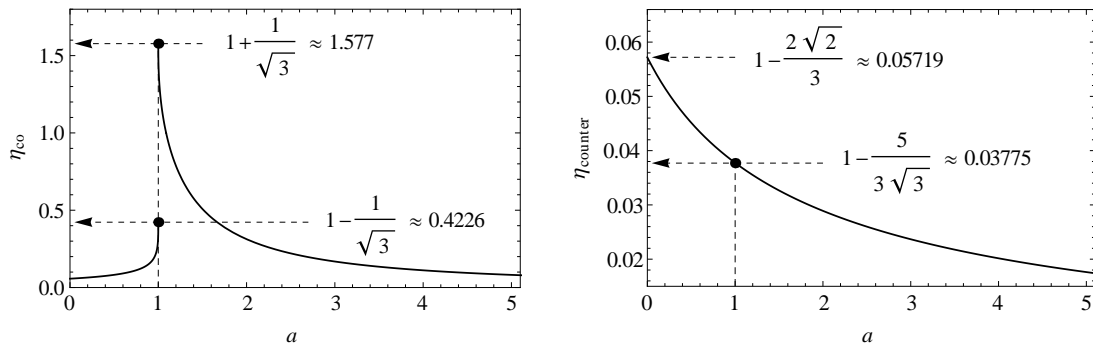


Figure 1: The efficiency of Keplerian accretion discs orbiting Kerr superspinars and black holes. *Left panel:* Corotating discs. *Right panel:* Counterrotating discs.

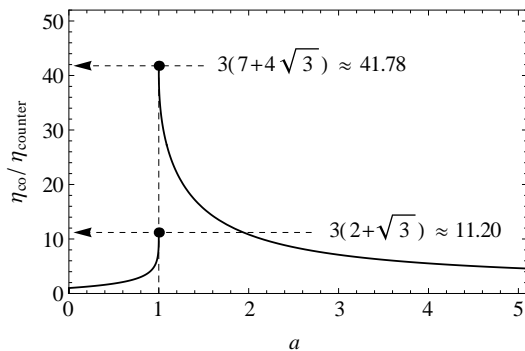


Figure 2: The ratio of efficiencies of corotating and counterrotating discs.

jump in their properties when transition from a naked singularity spacetime to a black hole spacetime occurs. On the other hand, the 2nd family orbits are counterrotating relative to distant observers ( $\Omega_K < 0$ ) and locally counterrotating ( $\Phi_K < 0$ ) everywhere for all Kerr superspinars. The Keplerian energy  $E_K$  and angular momentum  $\Phi_K$  radial profiles change smoothly when the transition across  $a = 1$  occurs.

The jump in the structure of corotating discs is most profoundly represented by the efficiency factor of the Keplerian accretion, given by the relation

$$\eta = 1 - E_{\text{ms}}$$

that is illustrated in Fig. 1. We immediately see the large jump in the efficiency of corotating discs from the factor  $\eta_{\text{ebh}} \approx 0.423$  to  $\eta_{\text{ens}} \approx 1.58$ . For counterrotating discs the efficiency factor matches smoothly at  $a = 1$  and is by almost two order smaller than for corotating discs. For completeness we also compare the efficiency

factor of corotating and counterrotating discs introducing the ratio

$$\mathcal{R} = \frac{\eta_{\text{co}}(a)}{\eta_{\text{counter}}(a)}$$

plotted in Fig. 2 and clearly demonstrating the ratio of energy outputs in the two different accretion regimes.

### 2.3 Evolution of spin and mass of accreting Kerr superspinars

Evolution of a Kerr superspinar (naked singularity) is described in terms of the evolving spin  $a$  and is finished when  $a = 1$  and the superspinar is converted into a near-extreme Kerr black hole. We shall assume positive values of the superspinar spin during its evolution.

The inner boundary of the counterrotating thin (Keplerian) disc is located at the marginally stable counterrotating circular geodesic  $x_{\text{ms}}$  satisfying the condition  $x_{\text{ms}} > 9$ ; for  $x_{\text{ms}}$  rising the superspinar spin grows to higher values of  $a > 1$  while the energy  $E_{\text{ms}}$  of the ISCO counterrotating orbit falls with spin growing [5]. A mass element located at the disc inner boundary has a specific energy  $E_{\text{ms}}$  and a specific angular momentum  $\Phi_{\text{ms}}$  that are functions of the superspinar spin  $a$ . Matter falls freely from  $x_{\text{ms}}$  transporting energy  $E_{\text{ms}}$  and angular momentum  $\Phi_{\text{ms}}$  into the superspinar, thus changing its parameters. If a rest mass  $\Delta m$  is captured, the superspinar mass  $M$  and angular momentum  $J$  changes are given by the relations

$$\Delta M = E_{\text{ms}} \Delta m, \quad \Delta J = \Phi_{\text{ms}} \Delta m.$$

The superspinar evolution is then determined by two differential equations giving evolution of the spin  $a$  and mass  $M$  due to the amount of the rest mass of accreting matter  $m$  (see [36])

$$\frac{da}{d \ln M} = \frac{1}{M} \frac{\Phi_{\text{ms}}}{E_{\text{ms}}} - 2a, \quad \frac{dM}{dm} = E_{\text{ms}}. \quad (1)$$

It is convenient to solve the evolutionary differential equations using radius of the marginally stable circular orbit as a variable. Therefore, we express the variables  $a$ ,  $E_{\text{ms}}$ ,  $\Phi_{\text{ms}}$  in terms of the dimensionless radius  $x_{\text{ms}} = r_{\text{ms}}/M$  to obtain, in the case of counterrotating discs, the relations

$$\begin{aligned} a &= \frac{1}{3} \sqrt{x_{\text{ms}}} (-4 + \sqrt{3x_{\text{ms}} - 2}), \\ E_{\text{ms}} &= \sqrt{1 - \frac{2}{3x_{\text{ms}}}}, \\ \Phi_{\text{ms}} &= -\frac{2M}{3\sqrt{3}} (1 + 2\sqrt{3x_{\text{ms}} - 2}). \end{aligned}$$



Clearly, the angular momentum of the matter accreted from counterrotating discs is always negative, while its energy is positive and the r.h.s. of evolution equation is always negative indicating evolution of a Kerr superspinar to black-hole states and its conversion into a near-extreme black hole. For corotating discs the evolution is more complex, but again leads to the conversion into a near-extreme black hole state as shown in [25].

Expressing the evolutionary equation in the form

$$M \frac{da}{dx_{\text{ms}}} \frac{dx_{\text{ms}}}{dM} = \frac{1}{M} \frac{\Phi_{\text{ms}}}{E_{\text{ms}}} - 2a \quad (2)$$

and using the relations  $a(x_{\text{ms}})$ ,  $E_{\text{ms}}(x_{\text{ms}})$ ,  $\Phi_{\text{ms}}(x_{\text{ms}})$ , we arrive to

$$\frac{dx_{\text{ms}}}{x_{\text{ms}}} = -2 \frac{dM}{M}.$$

Variations of  $x_{\text{ms}}$  and mass  $M$  are thus given by

$$\frac{x_{\text{ms}}}{x_{\text{msi}}} = \left( \frac{M_i}{M} \right)^2,$$

where the index “i” means initial values of the variables. For counterrotating discs this simple formula enables to present the integrated evolution laws of the superspinar spin and mass in the following simple form

$$a = \frac{\sqrt{x_{\text{msi}}}}{3} \frac{M_i}{M} \left( -4 + \sqrt{3x_{\text{msi}} \frac{M_i^2}{M^2} - 2} \right), \quad (3)$$

and

$$\frac{M}{M_i} = \sqrt{\frac{3x_{\text{msi}}}{2} - 1} \sin \left( \sqrt{\frac{2}{3x_{\text{msi}}} \frac{\Delta m}{M_i}} \right) + \cos \left( \sqrt{\frac{2}{3x_{\text{msi}}} \frac{\Delta m}{M_i}} \right).$$

Now we can directly find that a Kerr superspinar of initial mass  $M_i$  and spin  $a_i$  related to the marginally stable orbit radius  $x_{\text{msi}}$  through Eq. (3) will be converted into a Kerr black hole with  $a = 1$  by accretion of the rest mass  $\Delta m_c$  determined by the relation

$$\frac{\Delta m_c}{M_i} = \sqrt{\frac{3x_{\text{msi}}}{2}} \arcsin \left[ \frac{1}{9} \sqrt{\frac{2}{x_{\text{msi}}}} (\sqrt{3x_{\text{msi}} - 2} - 5) \right]. \quad (4)$$

The rest mass necessary for conversion  $\Delta m_c$  and the mass  $M_c$  of the resulting black hole expressed in terms of the superspinar initial mass  $M_i$  are represented as

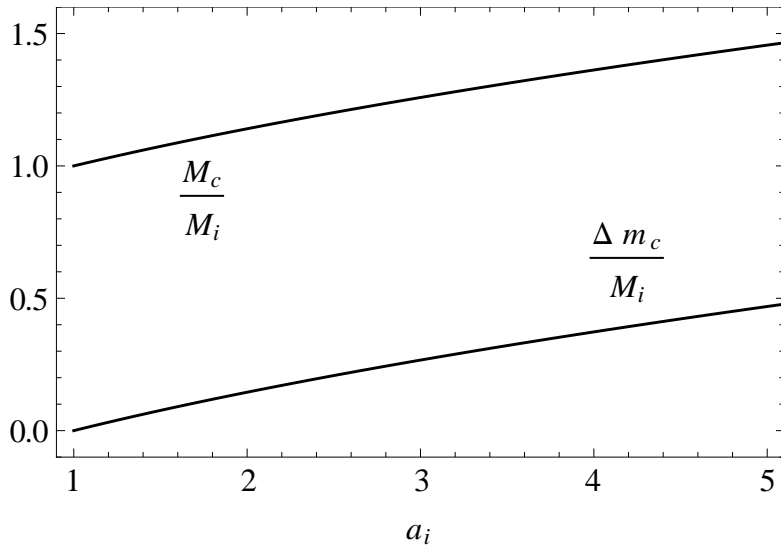


Figure 3: Conversion due to accretion in counterrotating discs. Amount of the rest mass  $\Delta m_c$  necessary for conversion of a Kerr superspinner with initial values of parameters  $M_i$ ,  $a_i$ , and the mass  $M_c$  of the resulting Kerr black hole, expressed in terms of initial mass  $M_i$ .

functions of the initial spin  $a_i$  in Fig. 3. Both  $\Delta m_c/M_i$  and  $M_c/M_i$  grow smoothly with the initial spin  $a_i$ . The conversion of superspinners due to accretion from counterrotating discs is very efficient process since the magnitude of the angular momentum of matter falling from the counterrotating marginally stable circular geodesic is much higher than in the case of corotating disc. Consequently, the superspinner mass increase and the rest mass necessary for the conversion are relatively small – for example, conversion of a superspinner having initial spin  $a_i = 3$  needs accretion of rest mass  $\Delta m_c \simeq 0.25M_i$  and leads to resulting black hole mass  $M_c \simeq 1.24M_i$ .

### 3 Time evolution for typical accretion regimes

Time evolution of a Kerr superspinner with initial values of mass and spin  $M_i$ ,  $a_i$  can be found by integrating the evolution equation (1) expressed in the form

$$\frac{dM}{dt} = E_{\text{ms}} \frac{dm}{dt}, \quad (5)$$

assuming that the accretion rate relative to observers at infinity  $dm/dt$  is given. In general situations corresponding to non-Keplerian accretion discs, the time evolution of the superspinner parameters can only be determined numerically [20, 21].

Here we apply four properly chosen assumptions on the nature of the accretion rate in Keplerian discs that enable integration of the evolution equations in terms of elementary functions and can thus serve as a test bed for more complicated situations. We choose three standard assumptions [8, 9] completed by the fourth case related to the time evolution due to accretion rate in the spherical accretion [22] that could correspond to accretion of matter accumulated in large distances to the superspinar.

### 3.1 Constant flow

The simplest possibility is represented by the constant flow that can be expressed in the form

$$\frac{dm}{dt} = k_1 M_i = \text{const}.$$

Using Eq. (5), the time evolution of the superspinar mass is determined by the formula

$$\frac{M}{M_i} = \sqrt{\frac{3x_{\text{msi}}}{2} - 1} \sin\left(\sqrt{\frac{2}{3x_{\text{msi}}}} k_1 t\right) + \cos\left(\sqrt{\frac{2}{3x_{\text{msi}}}} k_1 t\right).$$

Time evolution of its spin is given by the formula (3) where the last expression for  $M/M_i$  is used. The time necessary for the conversion into the black hole state is simply determined by

$$t_c = \frac{1}{k_1} \frac{\Delta m_c}{M_i}$$

with  $\Delta m_c/M_i$  given by Eq. (4).

### 3.2 Mass-proportional flow

For accretion flow proportional to the superspinar mass

$$\frac{dm}{dt} = k_2 M$$

we find, using Eq. (5), the time evolution of the superspinar mass in the form

$$\frac{M}{M_i} = \frac{\left(1 + \sqrt{1 - \frac{2}{3x_{\text{msi}}}}\right) e^{-k_2 t}}{\left(1 - \frac{1}{3x_{\text{msi}}} + \sqrt{1 - \frac{2}{3x_{\text{msi}}}}\right) e^{-2k_2 t} + \frac{1}{3x_{\text{msi}}}}.$$

Consequently, the superspinar spin evolution is again determined by the Eq. (3). The conversion time is then given by the formula

$$t_c = \frac{1}{k_2} \ln \left| \left(1 + \sqrt{1 - \frac{2}{3x_{\text{msi}}}}\right) \frac{\sqrt{3x_{\text{msi}}}}{\sqrt{27 + 5}} \right|.$$

### 3.3 Self-regulated flow

The critical accretion rate of self-regulated accretion discs that can be important in close binary systems or in dense active galactic nuclei with large amount of accreting matter is determined by the total disc luminosity that is self-regulated at the Eddington critical rate  $L_{\text{Ed}} = 1.3 \times 10^{31} \text{ W } (M/M_{\odot})$  [35]. It is expressed in the form

$$\frac{dm}{dt} = k_3 \frac{M}{1 - E_{\text{ms}}}$$

The time evolution of the superspinar mass (and consequently of its spin) can be obtained by integrating the evolution equation (5) with given accretion flow and leads to the formula

$$\frac{M}{M_i} = \sqrt{\frac{3x_{\text{msi}}}{2}} \left( 1 + \sqrt{1 - \frac{2}{3x_{\text{msi}}}} \right) e^{-k_3 t} \sqrt{2 e^{k_3 t} \left( 1 + \sqrt{1 - \frac{2}{3x_{\text{msi}}}} \right)^{-1} - 1}$$

The conversion time then reads

$$t_c = \frac{1}{k_3} \ln \left| \frac{1 + \sqrt{1 - \frac{2}{3x_{\text{msi}}}}}{1 + \frac{5}{\sqrt{27}}} \right|$$

### 3.4 Bondi flow

Finally we consider the situation when the accretion rate is controlled by large-distance influence of the superspinar that could be relevant when the surrounding matter disposable for accretion is limited. We assume the matter flow of the Bondi type that is inspired by the case of adiabatic spherical accretion [37, 38]. The accretion rate can then be given in the form

$$\frac{dm}{dt} = \tilde{k} M^2,$$

where the constant  $\tilde{k}$  is of different dimensionality in comparison with constants used in the previous three accretion rate laws. The time evolution of the superspinar mass – and its spin due to Eq. (3) – is then given by the formula

$$\frac{M}{M_i} = \left[ 1 + 2\sqrt{1 - \frac{2}{3x_{\text{msi}}}} \tilde{k} M_i t + (\tilde{k} M_i t)^2 \right]^{-1/2}$$

and the conversion time can be expressed in the form

$$t_c = \frac{1}{\tilde{k} M_i} \sqrt{\frac{2}{3x_{\text{msi}}}} \left( \sqrt{\frac{3x_{\text{msi}}}{2}} - 1 - \frac{5}{\sqrt{2}} \right)$$

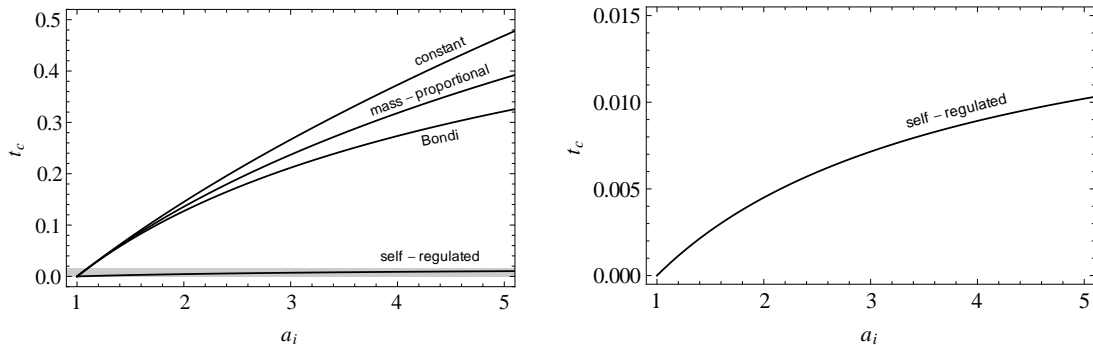


Figure 4: *Left panel:* Time evolution due to accretion in counterrotating discs. Time  $t_c$  necessary for conversion of a Kerr superspinner into a Kerr black hole is given as a function of the initial value of spin  $a_i$  for four typical regimes of accretion: constant accretion flow, flow proportional to the mass of the Kerr superspinner, self-regulated critical flow, and Bondi flow. *Right panel:* Zoom of the shaded region in the left panel.

The time evolution of a superspinner with initial spin  $a_i$  up to the extreme black hole state ( $a = 1$ )<sup>1</sup> is illustrated in Fig. 4 for all of the four accretion rate laws. For purposes of the comparison of different accretion flows influence on the conversion time  $t_c$ , we put  $k = k_1 = k_2 = k_3 = \tilde{k}M_i$ . Of course, in astrophysically realistic situations magnitude of the accretion constants have to be related to concrete astrophysical conditions that generally imply different regimes of accretion with substantially different values of the constant  $k$ .

Clearly, the self-regulated accretion leads to extremely fast conversion of a Kerr superspinner into a near-extreme black hole state, but it assumes a large amount of matter in close vicinity of accreting superspinner. These results are quite different from those obtained in the case of accretion from corotating thin discs [25].

## 4 Comparison with accretion from corotating Keplerian discs

Accretion from a corotating Keplerian disc into a Kerr superspinner (naked singularity) is more complex in comparison to the case of counterrotating discs, because of the unusual behaviour of corotating circular geodesics in the Kerr naked singularity spacetimes [5]. It is reflected by the fact that for Kerr spacetimes with spin  $a < a_0 = 3^{3/2}/4 \approx 1.299$  counterrotating orbits with  $\Phi_{\text{ms}} < 0$

1. Note that in real situations a near-extreme black hole is created due to the conversion process as discussed in [25].

appear in the innermost parts of the corotating Keplerian discs and the accretion reduces its angular momentum directly. Moreover, in the spacetimes with spin  $a < a_c = 2^{5/2}/3^{3/2} \approx 1.089$  the counterrotating orbits near the disc edge have negative energy and  $E_{\text{ms}} < 0$ . Therefore, the rotation energy of such superspinars has to be extracted by the accretion process, lowering thus mass of the superspinar.

We shall briefly summarize evolution laws of the Kerr superspinars due to corotating discs [25]. The evolution from an initial state with  $M_i$ ,  $a_i$  is expressed by the formulae for spin

$$a = \frac{\sqrt{x_{\text{msi}}} M_i}{3} \frac{M_i}{M} \left( 4 + \epsilon \sqrt{3x_{\text{msi}} \frac{M_i^2}{M^2} - 2} \right),$$

and mass

$$\frac{M}{M_i} = \epsilon \sqrt{\frac{3x_{\text{msi}}}{2} - 1} \sin \left( \sqrt{\frac{2}{3x_{\text{msi}}} \frac{\Delta m}{M_i}} \right) + \cos \left( \sqrt{\frac{2}{3x_{\text{msi}}} \frac{\Delta m}{M_i}} \right)$$

that is expressed in terms of the amount of accreted rest mass  $\Delta m$ . The parameter  $\epsilon$  takes value of  $\epsilon = -1$  for superspinar states with  $a < a_c$  and  $\epsilon = 1$  for states with  $a > a_c$ .

The accreted rest mass  $\Delta m_c$  necessary for conversion of a Kerr superspinar with initial mass  $M_i$  and spin  $a_i$  into a near-extreme black hole state is determined by the relation

$$\frac{\Delta m_c}{M_i} = \sqrt{\frac{3x_{\text{msi}}}{2}} \arcsin \left[ \frac{1}{3} \sqrt{\frac{2}{x_{\text{msi}}}} (1 + \epsilon \sqrt{3x_{\text{msi}} - 2}) \right] \quad (6)$$

for  $a_i < 2\sqrt{2}$ , where the parameter  $\epsilon$  has the same meaning as before, and by the relation

$$\frac{\Delta m_c}{M_i} = \sqrt{\frac{3x_{\text{msi}}}{2}} \left\{ \pi - \arcsin \left[ \frac{1}{3} \sqrt{\frac{2}{x_{\text{msi}}}} (1 + \sqrt{3x_{\text{msi}} - 2}) \right] \right\} \quad (7)$$

for  $a_i > 2\sqrt{2}$ .

The resulting black hole mass  $M_c/M_i$ , and the rest mass  $\Delta m_c/M_i$  required for the conversion, are shown in Fig. 5. Deceleration of decrease of the superspinar spin in the region  $1 < a < a_c$  occurs due to the energy extraction and superspinar's mass decrease due to accretion. For any superspinar with  $a_i < 5/3$  the resulting black hole mass is lower than the initial mass. The largest part of the initial mass is extracted from a superspinar with  $a_i = a_c$  being

$$\Delta M = M_i \left( 1 - \sqrt{\frac{2}{3}} \right) = 0.1835 M_i,$$

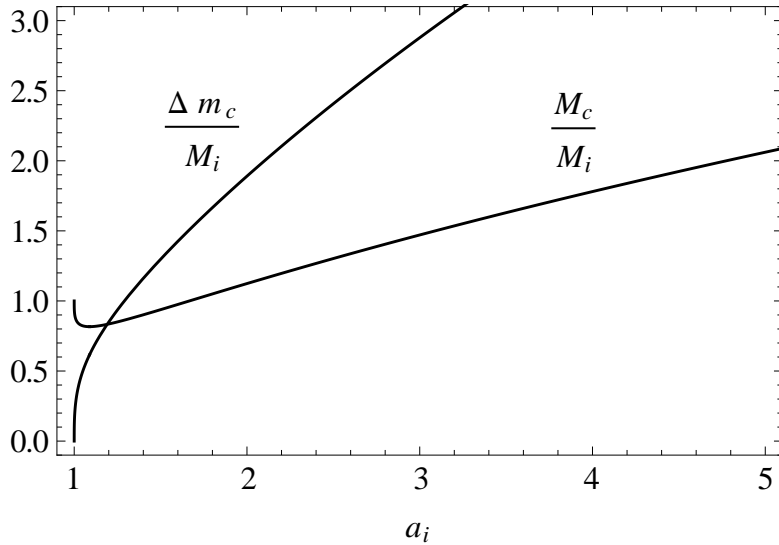


Figure 5: Conversion due to accretion in corotating discs. Amount of the rest mass  $\Delta m_c$  necessary for conversion of a Kerr superspinner with initial values of parameters  $M_i$ ,  $a_i$ , and the mass  $M_c$  of the resulting Kerr black hole. The minimum of  $M/M_i|_{\min} = \sqrt{2/3} \approx 0.8165$  is located at  $a_i = a_c = 2^{5/2}/3^{3/2} \approx 1.089$ . There is  $M/M_i = 1$  at  $a_i = 5/3$ .

with corresponding accreted rest mass

$$\Delta m_c = M_i \arcsin \frac{1}{\sqrt{3}} \approx 0.6155 M_i.$$

Note that in such a situation both  $\Delta M$  and  $\Delta m_c$  are converted into outgoing radiation.

#### 4.1 Conversion time

We present the dependence of the conversion time  $t_c$  on the superspinner initial spin  $a_i$  for all four types of the accretion flow considered in the case of counterrotating discs.

For the constant flow we arrive again to the simple formula

$$t_c = \frac{1}{k_1} \frac{\Delta m_c}{M_i}$$

with  $\Delta m_c/M_i$  given now by Eqs (6) and (7).

For the mass-proportional flow we obtain

$$t_c = \begin{cases} \frac{1}{k_2} \ln \left[ \left(1 + \frac{1}{\sqrt{3}}\right) \frac{1}{\sqrt{x_{\text{msi}}}} \left(1 + \sqrt{1 - \frac{2}{3x_{\text{msi}}}}\right)^{-1} \right] & \text{for } a_i < a_c, \\ \frac{1}{k_2} \ln \left[ \frac{3}{2} \left(1 + \frac{1}{\sqrt{3}}\right) \sqrt{x_{\text{msi}}} \left(1 + \sqrt{1 - \frac{2}{3x_{\text{msi}}}}\right) \right] & \text{for } a_i > a_c. \end{cases}$$

For the self-regulated accretion the conversion time reads

$$t_c = \begin{cases} \frac{1}{k_3} \ln \left[ \left(1 + \frac{1}{\sqrt{3}}\right) \frac{1}{x_{\text{msi}}} \left(1 + \sqrt{1 - \frac{2}{3x_{\text{msi}}}}\right)^{-1} \right] & \text{for } a_i < a_c, \\ \frac{1}{k_3} \ln \left[ \frac{3}{2} \left(1 + \frac{1}{\sqrt{3}}\right) \left(1 + \sqrt{1 - \frac{2}{3x_{\text{msi}}}}\right) \right] & \text{for } a_i > a_c. \end{cases}$$

The Bondi flow implies the conversion time in the form

$$t_c = \begin{cases} \frac{1}{\tilde{k}M_i} \left( \frac{1}{\sqrt{3x_{\text{msi}}}} - \sqrt{1 - \frac{2}{3x_{\text{msi}}}} \right) & \text{for } a_i < a_c, \\ \frac{1}{\tilde{k}M_i} \left( \frac{1}{\sqrt{2}} + \sqrt{1 - \frac{2}{3x_{\text{msi}}}} \right) & \text{for } a_i > a_c. \end{cases}$$

All the expressions for the conversion time are illustrated in Fig. 6 assuming (as in Fig. 4) that  $k = k_1 = k_2 = k_3 = \tilde{k}M_i$ . We see that the time evolution of a superspinner again strongly depends on the assumed law for accretion rate. For small values of  $a_i < 1.6$  the differences of conversion time are relatively small, but strongly grow for larger values of  $a_i$ .

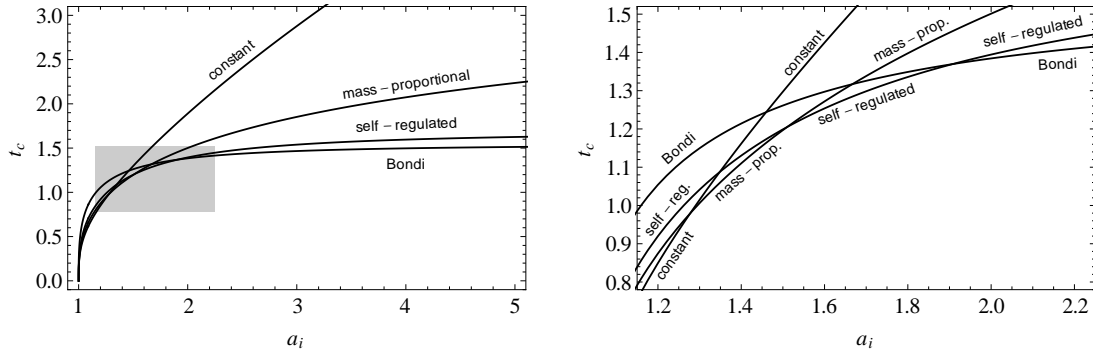


Figure 6: *Left panel:* Time evolution due to accretion in corotating discs. Time  $t_c$  necessary for conversion of a Kerr superspinner into a Kerr black hole is given as a function of the initial value of spin  $a_i$  for four typical regimes of accretion: constant accretion flow, flow proportional to the mass of the Kerr superspinner, self-regulated critical flow, and Bondi flow. Differences of about one order between these laws occur for  $a_i \sim 10$ . *Right panel:* Zoom of the shaded region in the left panel.



## 5 Discussion

Using the evolution equations of Kerr superspinars due to the corotating and counterrotating Keplerian discs, we are able to put some relevant estimates on limits of survival period of these hypothetical objects, the energy output during their evolution, and processes that could be relevant at the conversion of a superspinar into a near-extreme black hole. Of course, of special interest are the astrophysical phenomena enabling clear observational distinguishing of Kerr superspinars and black holes.

### 5.1 Survival period of Kerr superspinars

The survival time of superspinars is determined by amount of matter available for accretion, the Kerr accretion regime (co- or counterrotating) and the initial values of their mass  $M_i$  and spin  $a_i$ . Clearly, the strongest limit is implied by the most efficient, self-regulated accretion that we consider for our estimates.

The critical, Eddington luminosity governing the self-regulated accretion flow reads [35]

$$L_{s-r} = 1 \times 10^{38} \left( \frac{M}{M_\odot} \right) \text{ erg/s} = \eta c^2 \left( \frac{dm}{dt} \right)_{s-r} .$$

Using the relation for the self-regulated flow we arrive to

$$k_{s-r} \frac{M}{\eta} = \frac{L_{s-r}}{\eta c^2}$$

and we find the constant giving the time scale of the flow to be

$$k_{s-r} = \frac{L_{s-r}}{M c^2} \simeq 10^{-9} \text{ yr}^{-1} .$$

The self-regulated flow is governed by both mass of the central object and accretion efficiency  $\eta$  that is determined by the spin of the central object.

Then the typical black hole self-regulated accretion rate ( $\eta \simeq 0.1$ ) reads

$$\left. \frac{dm}{dt} \right|_{\text{crit}} = 10^{-8} M_\odot \text{ yr}^{-1} \left( \frac{M}{M_\odot} \right) ,$$

and puts limit on the existence of Keplerian accretion discs [35]. Such an accretion rate can be relevant for corotating discs orbiting Kerr superspinars with spin  $a > 3$ , while for superspinars with spin  $a < 2$  ( $\eta \sim 1$ ), the self-regulated accretion rate is rather given by

$$\left. \frac{dm}{dt} \right|_{\text{crit}} = 10^{-9} M_\odot \text{ yr}^{-1} \left( \frac{M}{M_\odot} \right) .$$

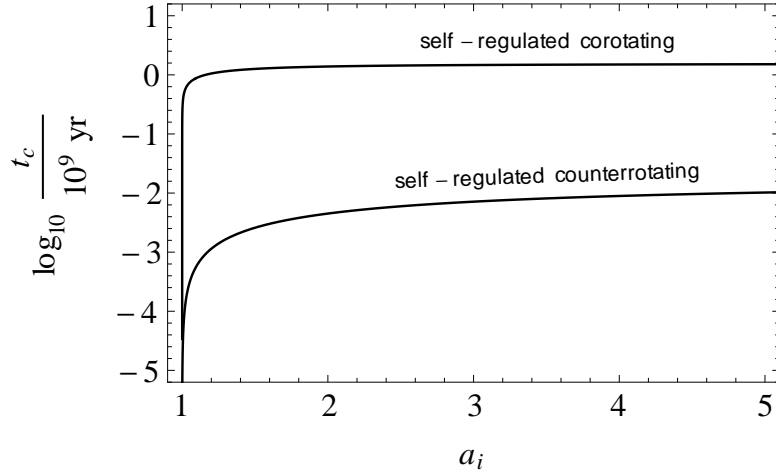


Figure 7: Conversion time of Kerr superspinars due to accretion Keplerian discs in self-regulated accretion regime given as a function of their initial spin. Notice that for large values of  $a_i$  the conversion time of corotating discs  $t_{c(c)}$  converges to  $1.5 \times 10^9$  yr and the conversion time of counterrotating discs  $t_{c(r)}$  converges to  $1.9 \times 10^7$  yr.

For Keplerian counterrotating discs around Kerr superspinars of all  $a < 10$ , the efficiency  $\eta \sim 0.01$ , and the self-regulated flow reads

$$\left. \frac{dm}{dt} \right|_{\text{crit}} = 10^{-7} M_{\odot} \text{yr}^{-1} \left( \frac{M}{M_{\odot}} \right),$$

i.e., it is by two orders larger in comparison with the self-regulated flows in corotating discs around near-extreme superspinars.

The survival period of Kerr superspinars is given by the characteristic time of evolution of corotating discs  $t_{s(c)} \sim k_{s-r}^{-1} \sim 10^9$  yr, since for counterrotating (retrograde) Keplerian discs the characteristic time is significantly shortened to  $t_{s(r)} \sim 10^{-2} k_{s-r}^{-1} \sim 10^7$  yr. The survival time depends explicitly on the initial spin  $a_i$  of the superspinar, but it is independent of its initial mass  $M_i$ . The mass parameter  $M_i$  is relevant only in determining the possibility of the superspinar conversion due to the relation to accreted rest mass  $\Delta m_c$  necessary for the conversion. Detailed information on the survival (conversion) time is represented in Fig. 7, where the conversion time  $t_c$  expressed in units of  $k_{s-r}^{-1} \sim 10^9$  yr is given as a function of  $a_i$ . The survival time of a “primordial” superspinar, created in the very early stages of the Universe evolution, should correspond to the age of the Universe, i.e.,  $t_s \sim 10^{10}$  yr. We can see that self-regulated accretion rate excludes

possibility of superspinars reaching the recent era  $t_c \sim 10^{10}$  yr, since there is

$$t_{c(c)}(a \rightarrow \infty) = \frac{1}{k_{s-r}} \ln \left[ 3 \left( 1 + \frac{1}{\sqrt{3}} \right) \right] \simeq 1.5 \times 10^9 \text{ yr} .$$

On the other hand, it is quite interesting that the conversion time  $t_c \sim 1.5 \times 10^9$  yr, corresponding to the era of high-redshift quasars and active galactic nuclei (AGN) is reached by superspinars with quite non-extreme initial spin  $a_i \sim 2$ . Superspinars with  $a_i \gtrsim 2$  would have their spin at the era of high-redshift quasars very close to the extreme value of  $a \sim 1$  and could thus demonstrate phenomena related to the conversion process. On the contrary, we could state that if no superspinars are observed in the high-redshift quasars, and some primordial superspinars were created, then their initial spin  $a_i < 2$ .

Concerning the counterrotating accretion, we see immediately that it works on scales by two orders smaller in comparison with the corotating accretion, and there is no realistic possibility for primordial Kerr superspinars to survive during such an accretion regime, if there is rest mass enough for the conversion process; note that such a rest mass is by one order smaller in comparison with initial mass of the superspinars. In fact, there is

$$t_{c(r)}(a \rightarrow \infty) = \frac{1}{k_{s-r}} \ln \left( \frac{2\sqrt{27}}{5 + \sqrt{27}} \right) \simeq 1.9 \times 10^7 \text{ yr} .$$

It is clear that if we consider sub-subsequent periods of co- and counterrotating self-regulated accretion, then the full time of evolution is essentially given by the corotating periods, if the accreted masses of each period are comparable.

Generally, the frequency of accretion episodes and the amount of available accreting matter are crucial for the survival time of Kerr superspinars. These depend on the conditions in the early universe [21] and on the ratio of the central object mass (a superspinner or a black hole) to the galaxy mass which is in the high-redshift quasars by almost two orders larger than in low-redshift AGN (0.01–0.1 to 0.001) [39, 40]. Further, we can expect that superspinars efficiently regulate the accretion rate on large scales, comparable or even larger than in the case of supermassive black holes where the scales reach the dark matter halo extension (100 kpc) [41].

## 5.2 Conversion processes

The energy radiated by the thin accretion disc during conversion of a Kerr superspinner is given by

$$E_{\text{rad}}(a_i) = \Delta m_c(a_i) - \Delta M(a_i) ,$$

where  $\Delta M(a_i) \equiv M_c(a_i) - M_i$ . Clearly, in the case of corotating discs,  $\Delta M(a_i)$  is negative for  $a_i < 5/3$ , since in the final stages of the accretion process the rotational energy is directly extracted from superspinars with  $a < a_c$ .

### 5.2.1 Conversion parameters

Due to accretion from a Keplerian counterrotating disc the Kerr superspinner mass has to grow continuously, contrary to the case of a corotating disc when the mass falls in the final phases of the evolution to a near-extreme black-hole state because of extraction of the superspinner rotational energy. The conversion process is much more efficient for counterrotating discs – the accreted rest mass necessary for conversion is by more than one order smaller in comparison with that related to corotating discs (cf. Figs 3 and 5). This is demonstrated for representative values of  $a_i$  in Table 1 for  $\Delta m_c/M_i$ ,  $M_c/M_i$ , and  $E_{\text{rad}}/M_i$ . In Table 2, the conversion time  $t_c$  is given in all four accretion regimes assuming a common characteristic accretion flow constant; however, the flow constants depend on the amount of matter accessible for accretion and, therefore, the accretion regime. The highest flow can be expected in the case of self-regulated accretion implying the fastest evolution.

For accretion from counterrotating discs the conversion is much (by orders) faster in comparison with the corotating discs (see Figs 4 and 6) and there are strong differences in the conversion time in the four types of accretion in whole the range

Table 1: Comparison of accreted rest mass necessary for conversion, resulting conversion mass, and total radiated energy related to the conversion for corotating and counterrotating discs.

$a_i$	$\Delta m_c/M_i$		$M_c/M_i$		$E_{\text{rad}}/M_i$	
	corot	ctrot	corot	ctrot	corot	ctrot
1.5	1.2974	0.0764	0.9377	1.0737	1.35963	$2.688 \times 10^{-3}$
3.0	2.8782	0.2665	1.4708	1.2584	2.40737	$8.112 \times 10^{-3}$

Table 2: Comparison of conversion time for corotating and counterrotating discs in all four regimes.

$a_i$	$t_c$							
	constant flow		mass-prop. flow		self-reg. flow		Bondi flow	
	corot	ctrot	corot	ctrot	corot	ctrot	corot	ctrot
1.5	1.2974	0.0764	1.1969	0.0737	1.2612	0.0026	1.1989	0.0712
3.0	2.8782	0.2665	1.8523	0.2370	1.4665	0.0072	1.5389	0.2117

of the spin. Extremely short conversion can be realized in the case of the self-regulated accretion (with high accretion flows) which is by several orders faster in comparison with the evolution due to corotating discs. It is of crucial importance that accreted mass necessary for conversion of a superspinner with  $a_i > 2$  is more than one order smaller for the counterrotating accretion as compared to the corotating one. Therefore, the counterrotating accretion is much more effective in the conversion process even from this point of view.

### 5.2.2 Transition to the black hole state

Transition to the black hole state in the counterrotating accretion disc regime is quite smooth since the properties of counterrotating circular geodesics in the field of extreme Kerr black holes and Kerr superspinners are smoothly matched for  $a \rightarrow 1$ . The conversion time  $t_c \sim 10^7$  yr. After the transition, the created black hole can evolve to states with decreasing spin and can even change the spin orientation and reach the near-extreme state with inverted spin, if there is accreted mass enough for the spin inversion. Time necessary for the spin inversion depends on the accretion flow regime, and in the fastest case of the self-regulated flow it takes  $t_{\text{inv}} \sim 10^8\text{--}10^9$  yr. Therefore, such black holes created from a primordial Kerr superspinner can be near-extreme in the era of high-redshift quasars. However, the rest mass necessary for the black hole spin inversion is more than one order larger ( $> 25$ ) [22] than the one necessary for the superspinner conversion. This implies that the spin inversion is not much likely in practice.

Transition due to corotating accretion discs is, on the other hand, very dramatic, because of the strong discontinuity of corotating circular geodesics in black hole

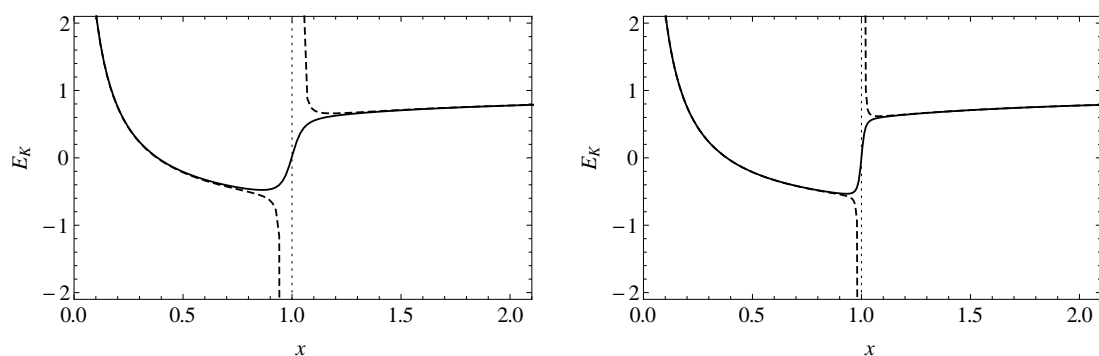


Figure 8: Corotating Keplerian energy  $E_K(x; a)$  radial profiles constructed for the field of near-extreme Kerr black holes and Kerr superspinners; we have chosen  $a = 1 \pm \delta$  with  $\delta = 10^{-3}$  (left panel) and  $\delta = 10^{-4}$  (right panel) in order to clearly demonstrate the energy gap of marginally stable circular orbits.

and naked singularity spacetimes with spin  $a \rightarrow 1$ . In the field of near-extreme Kerr superspinars, the circular orbits are located above and under the radius  $r = 1$  (along a long tube of the proper radial distance, as described in [42]), similarly to the circular geodesics in the field of near-extreme black holes where the orbits located above the horizon are astrophysically relevant. However, in the field of a Kerr superspinner with  $a = 1 + \delta$  ( $\delta \ll 1$ ), the Keplerian energy profile  $E_K(r)$  near the radius  $r = 1$  is completely different the profile constructed for the Kerr black hole field with  $a = 1 - \delta$  (see Fig. 8). The transition from a near-extreme superspinner state into the near-extreme black hole state is realized for an appropriately given  $\delta \ll 1$ . After the transition, a substantial part of the corotating disc, with energy lower than the energy of the marginally stable orbit of the resulting near-extreme black hole becomes to be unstable and plunges freely into the black hole, implying thus some dynamical observational consequences, e.g., a strong decrease of the disc energy output. We can expect that the plunged unstable part of the disc causes the black hole to be finally in a near-extreme state, with corotating accretion continued from the stable part of the disc, with the black hole spin probably being fixed at the canonical value of  $a \approx 0.998$  reflecting the effect photons captured by the black hole [36].

### 5.2.3 Near-extreme Kerr superspinars as sources of ultra-high energy particles

Banados, Silk and West recently proposed existence of ultra-high energy processes due to particle collisions in vicinity of the extreme black hole horizon [43]. They have found that assuming test-particle motion, the centre-of-mass energy can be arbitrarily high when two particles starting their motion at rest at infinity collide near the event horizon of an extreme (or a near-extreme) Kerr black hole if the angular momentum of either particle is fine-tuned to some critical value - such a process could thus represent a very efficient particle accelerator enabling testing of high energy physics in the strong gravitational field of near-extreme black holes. This phenomenon was later criticized and further discussed in a series of papers [44–47] and still represents an extremely interesting phenomenon, despite the fine tuning necessary in the processes near the black hole horizon.

We are able to show that in the field of near-extreme Kerr superspinars the centre-of-mass energy can reach ultra-high values for collisions of particles on the equatorial circular orbits at  $r = 1$  (or its very close vicinity) and any generic particle, e.g., those coming from rest at infinity, with no fine-tuning of the constants of the motion. Moreover, it can be easily demonstrated that such ultra-high energy processes can appear at  $r = 1$  and arbitrary latitude  $\theta$  due to head-on collision of particles falling from rest at infinity with zero angular momentum [48, 49] that can collide with those that inverted their motion near the ring singularity due to the gravitational repulsion of the field of Kerr superspinars [6].

The centre-of mass (CM) energy of two colliding particles having 4-momenta  $p_1^\alpha$  and  $p_2^\alpha$ , rest masses  $m_1$  and  $m_2$ , and total momentum

$$p_{(\text{tot})}^\alpha = p_1^\alpha + p_2^\alpha$$

is given by the relation

$$E_{\text{CM}}^2 = -p_{(\text{tot})}^\alpha p_{(\text{tot})\alpha} = m_1^2 + m_2^2 - 2g_{\alpha\beta} p_1^\alpha p_2^\beta = m_1^2 + m_2^2 - 2g^{\alpha\beta} p_{1\alpha} p_{2\beta}.$$

The CM energy is a scalar independent of the coordinate system so we can use the standard Boyer–Lindquist coordinates and Carter equations [23]. Assuming a Kerr superspinar with spin  $a = 1 + \delta$  and the collision at  $r = 1$ , we find that in both considered cases (particles on the corotating circular orbit colliding at the equatorial plane with particles falling from rest at infinity, and the inversely directed particles with zero angular momentum colliding at any latitude  $\theta$ ), there is the CM energy given in the first approximation by ( $m_1 = m_2 = m$ )

$$E_{\text{CM}}^2 \sim \frac{m^2}{\Delta} \sim \frac{m^2}{\delta}.$$

(See [50] for details.)

Clearly, in the field of near-extreme Kerr superspinars, close to the final stages of the conversion process, when  $\delta$  can be assumed very small, e.g.,  $\delta \sim 10^{-10}$ , or smaller, the ultra-high energy processes can occur very frequently, with no fine-tuning of the constants of motion of the colliding particles, contrary to the black-hole case where the fine-tuning is necessary. In such stages of their evolution, Kerr superspinars can work as an efficient particle accelerator being thus a laboratory of particle physics of ultra-high energies.

### 5.3 Subsequent evolution of created black holes

The characteristic time of evolution of a black hole due to self-regulated accretion is  $\sim 10^8$  yr and is comparable to the time necessary for spin-up of a non-rotating black hole due to the self-regulated accretion. For the other regimes of accretion, the characteristic times of evolution are little bit longer, but typically of the same order.

Concerning evolution of black holes and their spin due to accretion, we can find that – according to Bardeen’s work [22] – the change of the mass caused by accretion from counterrotating disc, considering transition from initial near-extreme black hole ( $a \simeq 1$ ) to final Schwarzschild black hole ( $a = 0$ ), is

$$\frac{M_{1 \rightarrow 0}}{M_i} = \sqrt{\frac{3}{2}} \approx 1.225,$$

and accreted rest mass necessary for the black hole spin-down

$$\frac{\Delta m_{1 \rightarrow 0}}{M_i} = 3\sqrt{\frac{3}{2}} \arcsin\left(\frac{1}{9\sqrt{3}}\right) \approx 0.2359,$$

while considering spinning up from initial Schwarzschild black hole ( $a = 0$ ) to final near-extreme black hole ( $a \simeq 1$ ) due to accretion from a Keplerian corotating disc there is

$$\frac{M_{0 \rightarrow 1}}{M_i} = \sqrt{6} \approx 2.449,$$

and accreted rest mass necessary for spin-up

$$\frac{\Delta m_{0 \rightarrow 1}}{M_i} = 3 \arcsin\left(\frac{1}{\sqrt{3}}\right) \approx 1.846.$$

For the counterrotating discs the accreted rest mass necessary for a black hole spin-down ( $a \simeq 1 \rightarrow a = 0$ ) is comparable with the conversion rest mass of a Kerr superspinar with  $a_i \sim 2.9$ , while for the corotating discs the accreted rest mass necessary for a black hole spin-up ( $a = 0 \rightarrow a \simeq 1$ ) is comparable with the conversion rest mass of superspinars with  $a_i \sim 1.8$ .

Finally, inversion of the spin of a near-extreme black hole due to accretion implies for the black-hole mass change

$$\frac{M_{\text{inv}}}{M_i} = 3,$$

and for the accreted rest mass necessary for the inversion

$$\frac{\Delta m_{\text{inv}}}{M_i} = 3\sqrt{\frac{3}{2}} \left[ \arcsin\left(\frac{1}{9\sqrt{3}}\right) + \arcsin\left(\frac{1}{\sqrt{3}}\right) \right] \approx 2.497.$$

We can see that accreted mass necessary for the inversion of the black hole spin is comparable to the accreted mass necessary for conversion of a superspinar with  $a_i \sim 2.8$  due to the corotating disc, while for conversion of the same superspinar by the counterrotating disc the accreted mass is by more than one order smaller.

#### 5.4 Energy radiated during the conversion process

Energy radiated by a corotating disc during the conversion of a Kerr superspinar with given  $M_i$  and  $a_i < 3$  is by more than two orders ( $\sim 500$  times) larger in comparison with energy radiated by a counterrotating disc. The counterrotating



accretion is thus a relatively “silent” process. When both co- and counterrotating phases of accretion are combined, the total energy output essentially equals to the output of the corotating periods. If we assume, for simplicity, all near-extreme black holes observed in the era of high-redshift quasars to be created by accreting superspinars (with  $a_i < 2$ ), in half by corotating and counterrotating discs having fuel enough for generating corotating near-extreme black holes, we have to include into the energy balance the radiation generated during the spin inversion of the black holes created by the counterrotating accretion. (Recall that time necessary to obtain final corotating near-extreme black hole states is  $\sim 10^9$  yr in both cases.) Therefore, we have to find the energy output during the process of the black hole spin inversion.

For a black hole spin-down from  $a \simeq 1$  to  $a = 0$  the radiated energy

$$\frac{E_{\text{rad}}^{1 \rightarrow 0}}{M_i} = 1 + 3\sqrt{\frac{3}{2}} \arcsin\left(\frac{1}{9\sqrt{3}}\right) - \sqrt{\frac{3}{2}} \approx 0.01112.$$

For a black hole spin-up from non-rotating state to a near extreme state the radiated energy

$$\frac{E_{\text{rad}}^{0 \rightarrow 1}}{M_i} = 1 + 3 \arcsin\left(\frac{1}{\sqrt{3}}\right) - \sqrt{6} \approx 0.3969,$$

while for the spin inversion

$$\frac{E_{\text{rad}}^{\text{inv}}}{M_i} = 3\sqrt{\frac{3}{2}} \left[ \arcsin\left(\frac{1}{\sqrt{3}}\right) + \arcsin\left(\frac{1}{9\sqrt{3}}\right) \right] - 2 \approx 0.4973.$$

For counterrotating discs, energy radiated during conversion of a superspinner is slightly smaller in comparison with energy radiated during a black hole spin-down assuming comparable initial mass of the superspinner and the black hole. However, for corotating discs, energy radiated during the conversion process strongly exceeds energy radiated during a black hole spin-up.

It is interesting to compare energy radiated by conversion of Kerr superspinars with  $a_i \sim 2-3$  (and  $M_i$ ) leading to a near-extreme black hole due to the corotating accretion regime only (assuming its preference) when  $E_{\text{rad}}(a_i \sim 3)/M_i \sim 2.4$  to the energy generated by the black hole spin up when  $E_{\text{rad}}^{0 \rightarrow 1}/M_i \sim 0.4$ , finding the ratio  $\epsilon \sim 6$ . In the more complex combined conversion process we have to compare  $[E_{\text{rad}}(a_i \sim 3)/M_i + E_{\text{rad}}^{\text{inv}}/M_i]/2 \sim 1.45$  with  $E_{\text{rad}}^{0 \rightarrow 1}/M_i \sim 0.4$  and we find  $\epsilon_{\text{comb}} \sim 3.8$ . Therefore, the superspinner conversion energy output can be by factor  $\epsilon \sim 4$  higher when compared with energy output due to accreting black holes, if we compare them at the era of the high-redshift quasars.

This could be of high importance in estimating the feed-back effect of the superspinar conversion process and we expect its relevant role in the evolution of the early stages of the evolution of the universe.

### 5.5 Observational signatures of Kerr superspinars

It follows from our previous discussion that the Kerr superspinars could appear in (high-redshift) quasars and AGN where supermassive black holes are usually assumed, or in Galaxy Black Hole Candidates (GBHC) observed in some X-ray binary systems and having relatively short lifetime [51]. In both classes of the black hole candidates some objects are reported with the spin estimates extremely close to the limit value  $a = 1$  and these objects could in principle serve as superspinar candidates. The spin estimates with  $a \sim 1$  are implied by the X-ray observations of AGN as MCG-6-30-15 [52] or the GBHC GRS 1915+105 [53] and are related to a variety of optical phenomena that deserve attention. Among them the most important seem to be the spectral continuum [51–56], spectral line profiles [57–70], and quasiperiodic oscillations explained by the orbital resonant models applied to near-extreme black holes [12, 71–77]. All such objects seem to be possible candidates for the Kerr superspinars and deserve attention and detailed study of data. While the spectral continuum of corotating thin discs is identical, or nearly identical, for Kerr black holes and Kerr superspinars with spin in the interval  $5/3 < a < 8\sqrt{6}/3 \approx 6.532$  [78], because of the same accretion efficiency (see Fig. 1), the other optical phenomena give clear signatures of the presence of a Kerr superspinar [16]. Similarly, some characteristic predictions appear for epicyclic frequencies of oscillations of thin discs in the field of Kerr superspinars that can give relevant signatures in the structure of high-frequency quasiperiodic oscillations [12].

## 6 Conclusions

Due to the accretion process, the spin of a Kerr superspinar can be reduced and conversion of the superspinar into a near-extreme black hole is possible. We have demonstrated that for counterrotating thin discs the conversion time can be significantly (by almost two orders) reduced in comparison with accretion from corotating thin discs. For counterrotating (retrograde) discs, the characteristic conversion time  $t_{c(r)} \sim 10^7$  yr, and the time necessary for inversion of the spin of the created extreme black hole goes up to  $t_{inv} \sim 10^9$  yr. On the other hand, the characteristic conversion time of corotating accretion  $t_{c(c)} \sim 10^9$  yr, and for initial spin  $a_i \sim 2-3$  it reaches  $t_c \sim 1.5 \times 10^9$  yr corresponding to high-redshift quasars and AGN. We can conclude that in this era most of the primordial Kerr

superspinars could be transformed into near-extreme black holes, but some of them could survive. However, it is very unlikely to observe primordial superspinars at the present era with  $t_c \sim 10^{10}$  yr. That is the reason why we should expect observations of the primordial Kerr superspinars in the relatively early universe, corresponding to the era of high-redshift quasars and AGN, rather than in the recent era, although we cannot exclude possibility that some Kerr superspinars could survive in regions with strongly reduced amount of matter to accrete. It is important that amount of rest mass  $\Delta m_c$  necessary for conversion of a Kerr superspinar with  $a_i \sim 3$  in the corotating regime is comparable to the rest mass  $\Delta m_c + \Delta m_{\text{inv}}$  necessary for conversion of superspinar in the counterrotating regime and subsequent inversion of the spin of the created black hole; moreover, times necessary for both these processes are comparable. Therefore, near-extreme black holes can appear in the era of high redshift quasars due to both processes discussed above.

The accreted rest mass necessary for conversion of a Kerr superspinar with initial spin  $a_i \sim 3$  is  $\Delta m_c \sim 0.25M_i$  for accretion counterrotating discs, but it is  $\Delta m_c \sim 3M_i$  for corotating discs, and their difference is by one order for all  $a_i < 3$ . On the other hand, the energy radiated by the corotating discs is  $\sim 500$  times larger than in the case of counterrotating discs. We can conclude that the conversion due to the counterrotation regime of accretion is relative silent, fast, and smooth process, while it is much slower and much more energetic and dramatic process in the corotating regime of accretion when an instability of a relevant part of the accretion disc occurs after the transition into a near-extreme black hole.

In the final stages of the conversion process, near-extreme Kerr superspinars can work as effective particle accelerators, since collisions of particles with extremely, ultra-high centre-of-mass energy can occur at the surface  $r = 1$ , with no fine-tuning of the constants of motion of the colliding particles. This could be considered to be an observationally very important effect giving signature of near-extreme Kerr superspinars, since in the field of near-extreme Kerr black holes such an effect can occur only for very special, fine-tuning of the motion constants of the colliding particles.

We can conclude that accretion of surrounding matter by hypothetical Kerr superspinars remaining as remnants of string phenomena occurring in the early phases of the Universe expansion could convert them relatively fast into near-extreme black holes. Converted superspinars represent a new hypothetical possibility for starting point of the evolution of supermassive black holes observed in high-redshift quasars and active galactic nuclei, along with zero-metallicity Population III stars (with  $M$  in the range  $10^2$ – $10^3 M_\odot$ ) [79–82] or hypothetical supermassive stars (with  $M \gg 10^3 M_\odot$ ) that can form when gaseous structures

generate radiation pressure sufficient to inhibit fragmentation and prevent formation of normal stars [83–85]. The string theory, or other higher-dimensional theories, recently put no restrictions on the superspinar mass, but we know that after its conversion the resulting black hole must have near-extreme spin. This could potentially alter the spin and mass evolution scenario of early supermassive black holes discussed in [20, 21]. Simultaneously, we could then expect some impact on the structure formation of the large scale universe because of radiation efficiency of Kerr superspinars and resulting near-extreme black holes. The feedback effect can be significantly modified if most of the observed supermassive black holes are created due to the evolution of primordial Kerr superspinars since the energy output due to the conversion process overcomes the standard output related to the black-hole spin-up by factor  $\sim 4$ .

Geometrically thick, toroidal accretion discs orbiting a Kerr superspinar are relevant in situations when the pressure gradients are important for the accretion discs structure.<sup>2</sup> The inner edge of the accretion disc is then closer to the superspinar surface as compared with the edge of the Keplerian disc, nevertheless it is still well above the surface assumed at  $R = 0.1M$ . Here we restricted attention to the accretion phenomena related to the Keplerian discs only, but extension of the modelling to thick discs of different types is surely relevant. Further, the role of magnetic fields related to the accretion discs, reflected by magnetohydrodynamic models, has to slightly alter the presented picture of the Kerr superspinar evolution. Let us stress that numerical models are necessary for investigation of the superspinar evolution in all of the mentioned situations. We plan to make the simulations in future work.

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2. Such structures were discussed (even with the cosmological constant included) in the case of rotating Kerr naked singularity spacetimes [11, 86]. Unexpected features were found that do not appear in the black hole spacetimes.

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