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Hamiltonian Analysis of the Higgs Mechanism for Gravity

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Abstract

In this paper we perform the canonical description of the Higgs mechanism for gravity and provide the Hamiltonian definition of the massive gravities.

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1 Introduction

Construction of the massive gravity is plagued with theoretical problems and issues that have not been resolved from the pioneered work of Fierz and Pauli [1]². One such a well known problem of massive gravity is that there is no smooth massless limit in the perturbation theory of massive gravity in the sense that the massless limit in massive gravity exists but does not agree with the results derived from General Relativity (GR) that describes massless gravitons. This pathological behavior of massive gravity is known as van Dam-Veltman-Zakharov discontinuity [6, 7].

Even if the construction of massive gravity is interesting theoretical challenge there is another more stronger reason for their formulation. In fact, generalizations of GR with a small, non-zero, graviton mass leads to large scale (or infrared) modifications of General Relativity. Since GR has been directly tested from scales of a fraction of millimeters up to Solar System scales it is possible that the structure of the theory changes on the large distances and hence infrared deviations from GR cannot be excluded. Then the massive gravities could modify GR at large cosmological scales and also can explain recent accelerated expansion of the universe without assuming the existence of mysterious dark matter and dark energy.

At present it is not clear how to formulate consistently a theory of massive gravity. Since the Einstein's theory of gravity is well tested theory it seems to be natural to add to the action of GR a term which will, in the linearized approximation, give a mass to gravitons without modifying the kinetic terms coming from GR.

Very interesting formulation of the massive theory of gravity is based on Brout-Englert-Higgs mechanism for gravity. In more details, we consider a diffeomorphism invariant action with usual Einstein-Hilbert term together with the function of the metric coupled to D scalar fields for D -dimensional gravity [8]. Then gravitons acquire a mass due to a mechanism which may be thought of as the Brout-Englert-Higgs mechanism for gravity when the vacuum expectation value of each scalar field breaks one coordinate reparameterization invariance³.

Our goal is to develop the Hamiltonian description of the systems studied in [8, 9, 10, 11, 12, 13, 14]. With the help of $(D - 1) + 1$ split formalism for the space-time metric we find corresponding Hamiltonian and show that it is linear combination of the Hamiltonian and diffeomorphism constraints. The consistency of the theory demands that these constraints are preserved during the time evolution of the system. In order to check whether they are preserved or not we calculate the Poisson brackets of these constraints. However it turns out that it is non-trivial task to calculate the Poisson bracket of the Hamiltonian constraint for the scalar field whose dynamics is governed by general action. Despite of this fact we show that the Poisson brackets of the scalar fields Hamiltonian constraints are proportional to the diffeomorphism constraints. In other words we show that the Poisson bracket of the scalar field Hamiltonian constraints takes exactly the same form as the Poisson

²For extensive reviews of various aspects of IR modification of gravities, see [2, 3, 4, 5]

³This interesting proposal of formulation of the massive gravity was also studied in [9, 10, 11, 12, 13, 14].

bracket of General Relativity Hamiltonian constraints. Collecting all these results we obtain the Poisson algebra of constraints is closed and hence these constraints are consistent with the time-evolution of the system. This result is crucial for the possibility of the fixing the gauge in the framework of the Hamiltonian formalism. We fix the gauge by introducing D gauge fixed functions that correspond to the gauge fixing conditions introduced in [8, 9, 10, 11, 12, 13, 14]. Then the system of these gauge fixing functions and original constraints form the set of the second class constraints that can be explicitly solved. Further, since the original Hamiltonian is given as the linear combination of constraints we find that in the process of the gauge fixing it strongly vanishes. On the other hand the gauge fixing implies that the reduced phase space is spanned by the spatial components of the metric h_{ij} and their conjugate momenta p^{ij} . Then we argue that the gauge fixing condition $t = \phi^0$ naturally introduces the Hamiltonian on the reduced phase-space equal to $-p_0$ where p_0 is the momentum conjugate to ϕ^0 . Note that p_0 is the function of reduced phase space variables h_{ij}, p^{ij} as a result of the solving of the Hamiltonian constraint. Say differently, we claim that the definition of the massive gravity is given by the reduced phase space variables h_{ij}, p^{ij} together with the gauge fixed Hamiltonian $H_{fix} = -p_0(h_{ij}, p^{ij})$, and where all non-dynamical modes are absent.

Experiences from many areas of theoretical physics teach us that in some cases the Lagrangian formulation of given theory is much more efficient than the Hamiltonian ones. For that reason we feel that it is useful to determine the Lagrangian formulation of the gauge fixed theory as well. It turns out that in order to find this Lagrangian it is convenient to introduce new non-dynamical modes so that the Legendre transformation from the Hamiltonian to Lagrangian formulation can be easily performed. Interestingly this Lagrangian can be written in such a form that resembles the standard Einstein-Hilbert action together with the potential term that explicitly breaks the diffeomorphism invariance of given theory.

The structure of this paper is as follows. In the next section (2) we perform the Hamiltonian analysis of the system introduced in [8, 9, 10, 11, 12, 13, 14] and we perform the gauge fixing of this system that leads to the new Hamiltonian formulation of the massive gravity. Then in section (3) we give two examples of scalar potentials that allow us to find explicit form of the gauge fixed Hamiltonian. Finally in conclusion (4) we outline our results and suggest possible extension of this work. The explicit calculation of the Poisson bracket of the constraints is given in Appendix (A).

2 Hamiltonian Analysis of General Relativity with Massless Scalars

In this section we develop the Hamiltonian formalism for the following action

$$S = \frac{1}{16\pi G} \int d^D x \sqrt{-g} [R - L(H^{AB})] , \quad (1)$$

where G is D -dimensional Newton's constant and the induced internal metric is defined as

$$H^{AB} = g^{\mu\nu} \nabla_\mu \phi^A \nabla_\nu \phi^B , \quad (2)$$

where ϕ^A are real D scalar fields with $A = 0, \dots, D-1$ and where the indices A, B, \dots are raised and lowered using the metric $\eta_{AB} = \text{diag}(-1, 1, \dots, 1)$. Finally, L is a generic function of H^{AB} . The detailed analysis of properties of function L was performed in [10, 11, 13, 14]. It is important to stress that the form of the potential L is not arbitrary. In fact, it was argued in [10, 11, 13, 14] that this potential has to lead to the equations of motion that possess following vacuum solution

$$\phi^A = x^\mu \delta_\mu^A , \quad g_{\mu\nu} = \eta_{\mu\nu} . \quad (3)$$

This requirement leads to the constraint on the potential L

$$\frac{\delta L(H_*)}{\delta H^{AB}} = \frac{1}{2} \eta_{AB} L(H_*) , \quad (4)$$

where $H_*^{AB} = \eta^{AB}$.

Our goal is to develop the Hamiltonian formalism for system defined by the action (1). As usual in the study of the Hamiltonian formalism for gravity we introduce $(D-1) + 1$ formalism. Explicitly, let us consider D dimensional manifold \mathcal{M} with the coordinates x^μ , $\mu = 0, \dots, D-1$ and where $x^\mu = (t, \mathbf{x})$, $\mathbf{x} = (x^1, \dots, x^{D-1})$. We presume that this space-time is endowed with the metric $g_{\mu\nu}(x^\rho)$ with signature $(-, +, \dots, +)$. Suppose that \mathcal{M} can be foliated by a family of space-like surfaces Σ_t defined by $t = x^0$. Let h_{ij} , $i, j = 1, \dots, D$ denotes the metric on Σ_t with inverse h^{ij} so that $h_{ij} h^{jk} = \delta_i^k$. We further introduce the operator ∇_i that is covariant derivative defined with the metric h_{ij} . We define the lapse function $N = 1/\sqrt{-g^{00}}$ and the shift function $N^i = -g^{0i}/g^{00}$. In terms of these variables we write the components of the metric $g_{\mu\nu}$ as

$$\begin{aligned} g_{00} &= -N^2 + N_i h^{ij} N_j , & g_{0i} &= N_i , & g_{ij} &= h_{ij} , \\ g^{00} &= -\frac{1}{N^2} , & g^{0i} &= \frac{N^i}{N^2} , & g^{ij} &= h^{ij} - \frac{N^i N^j}{N^2} . \end{aligned} \quad (5)$$

Then it is easy to see that

$$\sqrt{-g} = N \sqrt{\det h} . \quad (6)$$

In the $(D-1) + 1$ formalism H^{AB} takes the form

$$H^{AB} = -\nabla_n \phi^A \nabla_n \phi^B + h^{ij} \partial_i \phi^A \partial_j \phi^B , \quad (7)$$

where

$$\nabla_n \phi^A = \frac{1}{N} (\partial_t \phi^A - N^i \partial_i \phi^A) . \quad (8)$$

Then from (1) we easily find the momenta conjugate to ϕ^A

$$p_A = \frac{\sqrt{\det h}}{8\pi G} \frac{\delta L}{\delta H^{AB}} \nabla_n \phi^B . \quad (9)$$

Using this result we find following matrix equation

$$K_{AB} = D_{AC}(H^{AB})(V^{CD} - H^{CD})D_{DB}(H^{AB}) , \quad (10)$$

where we introduced following matrices

$$K_{AB} = \left(\frac{16\pi G}{2\sqrt{\det h}} \right)^2 p_A p_B , \quad V^{AB} = h^{ij} \partial_i \phi^A \partial_j \phi^B , \quad D_{AB}(H^{AB}) = \frac{\delta L}{\delta H_{AB}} . \quad (11)$$

For further purposes we also introduce the matrix \tilde{D}^{AB} inverse to D_{AB} so that $D_{AB}\tilde{D}^{BC} = \delta_A^C$.

Now we presume that the equation (10) can be solved for H_{AB} . Let us denote this solution as $J^{AB} = J^{AB}(K, V)$. Then (10) implies following relation

$$(V^{AB} - J^{AB}) = \tilde{D}^{AC}(J)K_{CD}\tilde{D}^{DB}(J) \quad (12)$$

that will be useful below. Collecting all these results we find the Hamiltonian of the scalar fields in the form

$$H^\phi = \int d^d \mathbf{x} (\partial_t \phi^A p_A - L) = \int d^d \mathbf{x} (N(\mathbf{x})\mathcal{H}_T^\phi(\mathbf{x}) + N^i(\mathbf{x})\mathcal{H}_i^\phi(\mathbf{x})) , \quad (13)$$

where

$$\begin{aligned} \mathcal{H}_T^\phi &= \frac{1}{16\pi G} \left(2\sqrt{\det h} K_{AB} \tilde{D}^{BA} + V(J) \right) , \\ \mathcal{H}_i^\phi &= p_A \partial_i \phi^A , \end{aligned} \quad (14)$$

and where $d = D - 1$.

In the same way we should proceed with the Hamiltonian analysis of the General Relativity action. Since the procedure is well known we immediately write the final result

$$H^{GR} = \int d^d \mathbf{x} (N(\mathbf{x})\mathcal{H}_T^{GR}(\mathbf{x}) + N^i(\mathbf{x})\mathcal{H}_i^{GR}(\mathbf{x})) , \quad (15)$$

where

$$\mathcal{H}_T^{GR} = \frac{16\pi G}{\sqrt{\det h}} \pi^{ij} \mathcal{G}_{ijkl} \pi^{kl} - \frac{\sqrt{\det h}}{16\pi G} R^{(D-1)} , \quad \mathcal{H}_i = -2h_{ik} \nabla_l \pi^{lk} . \quad (16)$$

Let us explain notation used here. π^{ij} are momenta conjugate to h_{ij} . The generalized metric \mathcal{G}^{ijkl} is defined as

$$\mathcal{G}_{ijkl} = \frac{1}{2} (h_{ik} h_{jl} + h_{il} h_{jk}) - h_{ij} h_{kl} . \quad (17)$$

The inverse metric \mathcal{G}^{ijkl} is equal to

$$\mathcal{G}^{ijkl} = \frac{1}{2}(h^{ik}h^{jl} + h^{il}h^{jk}) - h^{ij}h^{kl} . \quad (18)$$

Finally, $R^{(D-1)}$, ∇_i are Ricci curvature and covariant derivative calculated using the metric h_{ij} . Finally due to the fact that the action (1) does not contain time derivative of N, N^i we find that corresponding conjugate momenta π_N, π_i are primary constraints of the theory

$$\pi_i \approx 0 , \quad \pi_N \approx 0 . \quad (19)$$

The condition of preservation of these constraints implies the existence of the secondary ones

$$\mathcal{H}_T = \mathcal{H}_T^{GR} + \mathcal{H}_T^\phi \approx 0 , \quad \mathcal{H}_i = \mathcal{H}_i^{GR} + \mathcal{H}_i^\phi \approx 0 \quad (20)$$

or their smeared form

$$H_T(M) = \int d^d \mathbf{x} M(\mathbf{x}) \mathcal{H}_T(\mathbf{x}) , \quad H_S(M^i) = \int d^d \mathbf{x} M^i(\mathbf{x}) \mathcal{H}_i(\mathbf{x}) , \quad (21)$$

where $M(\mathbf{x}), M^i(\mathbf{x})$ are arbitrary functions.

As the next step we have to show that these constraints are preserved during the time evolution of the systems. Since these calculations are very long and technical we included them into Appendix where an interested reader could find more details. The upshot of this analysis is that these constraints are really preserved during the time evolutions and no additional constraints are generated. Further, these constraints are the first class constraints and obey the algebra of constraints (50) presented in Appendix.

Since we showed that this system possesses D first class constraints $\mathcal{H}_T(\mathbf{x}) \approx 0, \mathcal{H}_i(\mathbf{x}) \approx 0$ we can proceed to their gauge fixing. This procedure is an analogue of the Higgs mechanism used in the construction of the massive gravity [8, 10, 11, 12, 13]. In these models the vacuum expectation values of the scalar fields coincide with D space-time coordinates. The result of this fixing is the complete breaking of the space-time diffeomorphism and emergence of the mass term for the graviton in the action when we study the small fluctuations of gravity above the flat space-time.

The standard way how to fix the gauge freedom in Hamiltonian framework is to introduce D gauge fixed functions that can be interpreted as additional constraints imposed on the system and that have non-zero Poisson brackets with the original first class constraints. As a result the extended system of original constraints together with gauge fixed functions form the collection of the second class constraints with no gauge freedom left (For review of this formalism, see [19, 20, 21].).

With analogy with the fixing the gauge given in [8, 10, 11, 12, 13] we introduce following D gauge fixing functions

$$\mathcal{G}^A(\mathbf{x}) = \phi^A(\mathbf{x}) - x^A . \quad (22)$$

Clearly

$$\{\mathcal{G}^A(\mathbf{x}), \mathcal{H}_i(\mathbf{y})\} = \delta_i^A \delta(\mathbf{x} - \mathbf{y}) \quad (23)$$

and

$$\{\mathcal{G}^A(\mathbf{x}), \mathcal{H}_T(\mathbf{y})\} = \frac{(16\pi G)^2}{2 \det h} \frac{\delta \mathcal{H}_T^\phi(\mathbf{x})}{\delta K_{AN}(\mathbf{x})} p_N(\mathbf{x}) \delta(\mathbf{x} - \mathbf{y}) . \quad (24)$$

Due to the fact that the Poisson brackets (23) and (24) are non-zero on the constraint surface we see that the collection of constraints $(\mathcal{G}^A, \mathcal{H}_T, \mathcal{H}_i)$ is the system of the second class constraints. Alternatively, the requirement of the time preservation of the constraints $\mathcal{G}^A(\mathbf{x}) \approx 0$ during the time evolution of the system implies following consistency equation

$$\begin{aligned} \frac{d\mathcal{G}^i(\mathbf{x})}{dt} &= \{\mathcal{G}^i(\mathbf{x}), H\} = \\ &= N^i(\mathbf{x}) + N(\mathbf{x}) \frac{(16\pi G)^2}{2 \det h} \frac{\delta H_T^\phi(\mathbf{x})}{\delta K_{iN}(\mathbf{x})} p_P(\mathbf{x}) = 0 , \\ \frac{d\mathcal{G}^0(\mathbf{x})}{dt} &= \partial_t \mathcal{G}^0(\mathbf{x}) + \{\mathcal{G}^0(\mathbf{x}), H\} = \\ &= -1 + N(\mathbf{x}) \frac{(16\pi G)^2}{2 \det h} \frac{\delta H_T^\phi(\mathbf{x})}{\delta K_{0N}(\mathbf{y})} \tilde{D}^{NP} p_P(\mathbf{x}) = 0 , \end{aligned} \quad (25)$$

where we used (23) and (24). We see that these equations determine N and N^i as functions of the canonical variables h_{ij}, p^{ij} with no gauge freedom left.

The fact that $\mathcal{G}^A, \mathcal{H}_T, \mathcal{H}_i$ are the second class constrains implies that they vanish strongly and can be explicit solved for p_A . As a result the reduced phase space is spanned by h_{ij} and p^{ij} . Further, since the Hamiltonian of the original system was given as a linear combination of the constraints $\mathcal{H}_T, \mathcal{H}_i$ we now see that it vanishes strongly.

On the other hand let us write the original action (1) in the form

$$S = \int dt d^d \mathbf{x} (\partial_t h_{ij} \pi^{ji} + \partial_t \phi^A p_A - H) = \int dt \left(\int d^d \mathbf{x} (\partial_t h_{ij} \pi^{ji} + p_0 (p^{ij}, g_{ji})) \right) , \quad (26)$$

where we used the fact that $H = 0$ and imposed the gauge fixing functions (22). We see from (26) that it is natural to interpret $-p_0$ as the Hamiltonian density of the reduced theory

$$\mathcal{H}_{fix} = -p_0(h_{ij}(\mathbf{x}), \mathcal{H}_T^{GR}(\mathbf{x}), \mathcal{H}_i^{GR}(\mathbf{x})) \equiv -\frac{16\pi G}{2\sqrt{\det h}} \tilde{p}_0 , \quad (27)$$

where we also used the fact that from the constraints $\mathcal{H}_i = 0$ we can express p_i as

$$p_i(\mathbf{x}) = -\mathcal{H}_i^{GR}(\mathbf{x}) . \quad (28)$$

Finally note that p_0 can be derived from the Hamiltonian constraint $\mathcal{H}_T = \mathcal{H}_T^{GR} + \mathcal{H}_T^\phi(\mathbf{x}) = 0$ at least in principle.

In summary, we found the Hamiltonian formulation of massive gravity where the physical degrees of freedom are h_{ij}, p^{ij} and where the Hamiltonian is given in (27).

Note also that the explicit form of this Hamiltonian depends on the form of the function $L(H^{AB})$. We give simple examples of two solvable potentials in the next section. Generally however it is very difficult to find explicit form of the gauge fixed Hamiltonian due to the complicated structure of the function $L(H^{AB})$.

Despite of this fact we now show that it is possible to find the Lagrangian density for given gauge fixed theory. To this we introduce four modes A, B, C_i, D^i and corresponding conjugate momenta (p_A, p_B, p^i, p_i) with non-zero Poisson brackets

$$\begin{aligned} \{A(\mathbf{x}), p_A(\mathbf{y})\} &= \delta(\mathbf{x} - \mathbf{y}) , & \{B(\mathbf{x}), p_B(\mathbf{y})\} &= \delta(\mathbf{x} - \mathbf{y}) , \\ \{C_i(\mathbf{x}), p^j(\mathbf{y})\} &= \delta_i^j \delta(\mathbf{x} - \mathbf{y}) , & \{D^i(\mathbf{x}), p_j(\mathbf{y})\} &= \delta_j^i \delta(\mathbf{x} - \mathbf{y}) . \end{aligned} \quad (29)$$

With the help of these additional modes we rewrite the Hamiltonian for gauge fixed theory as

$$\mathcal{H}_{fix} = -p_0(h_{ij}, A, C_i) + B(\mathcal{H}_T^{GR} - A) + D^i(\mathcal{H}_i^{GR} - C_i) + v_A p_A + v_B p_B + v^i p_i + v_i p^i , \quad (30)$$

where the Lagrange multipliers v_A, v_B, v^i, v_i ensure that p_A, p_B, p_i and p^i are primary constraints of the theory

$$p_A \approx 0 , \quad p_B \approx 0 , \quad p_i \approx 0 , \quad p^i \approx 0 . \quad (31)$$

Then the fact that these constraints have to be preserved during the time evolution of the system implies the secondary constraints

$$\begin{aligned} \partial_t p_A &= \{p_A, H_{fix}\} = \frac{\delta p_0}{\delta A} - B \equiv \Phi_A \approx 0 , \\ \partial_t p_B &= \{p_B, H_{fix}\} = -A + \mathcal{H}_T^{GR} \equiv \Phi_B \approx 0 , \\ \partial_t p_i &= \{p_i, H_{fix}\} = -C_i + \mathcal{H}_i^{GR} \equiv \Phi_i \approx 0 , \\ \partial_t p^i &= \{p^i, H_{fix}\} = \frac{\delta p_0}{\delta C_i} - D^i \equiv \Phi^i \approx 0 . \end{aligned} \quad (32)$$

It can be shown that the collections of the constraints $(p_A, p_B, p_i, p^i, \Phi_A, \Phi_B, \Phi^i, \Phi_i)$ are the second class constraints. Solution of these constraints reduces (30) into the original form of the gauge fixed Hamiltonian (27).

The main advantage of the extended form of the Hamiltonian density (30) is that it allows us to find corresponding Lagrangian in relatively straightforward way. In fact, from (30) we easily obtain the time derivatives of the canonical variables h_{ij}, A, B, D^i, C_i

$$\begin{aligned} \partial_t h_{ij} &= \{h_{ij}, H_{fix}\} = B \frac{32\pi G}{\sqrt{\det h}} \mathcal{G}_{ijkl} \pi^{kl} + 2\nabla_i D_j , \\ \partial_t A &= \{A, H_{fix}\} = v_A , & \partial_t B &= \{B, H_{fix}\} = v_B , \\ \partial_t D^i &= \{D^i, H_{fix}\} = v^i , & \partial_t C_i &= \{C_i, H_{fix}\} = v_i . \end{aligned} \quad (33)$$

It turns out that it is useful to introduce following object

$$\hat{K}_{ij} = \frac{1}{2B}(\partial_t h_{ij} - \nabla_i D_j - \nabla_j D_i) \quad (34)$$

that due to the first equation in (33) is related to π^{ij} as

$$\hat{K}_{ij} = \frac{16\pi G}{\sqrt{\det h}} \mathcal{G}_{ijkl} \pi^{kl} . \quad (35)$$

Then it is easy to find corresponding Lagrangian

$$\begin{aligned} L_{fix} &= \int d^d \mathbf{x} \left(\frac{\sqrt{\det h}}{16\pi G} B(\hat{K}_{ij} \mathcal{G}^{ijkl} \hat{K}_{kl} + R^{(D-1)}) + \frac{\sqrt{\det h}}{8\pi G} \tilde{p}_0(h_{ij}, A, C_i) + BA + D^i C_i \right) = \\ &= \int d^d \mathbf{x} \left(\frac{1}{16\pi G} \sqrt{-\hat{g}} \hat{R} + \frac{\sqrt{\det h}}{8\pi G} \tilde{p}_0(h_{ij}, A, C_i) + BA + D^i C_i \right) , \end{aligned} \quad (36)$$

where \hat{g} is D -dimensional metric with components

$$\hat{g}_{00} = -B^2 + D_i h^{ij} D_j , \quad \hat{g}_{0i} = D_i , \quad \hat{g}_{ij} = h_{ij} . \quad (37)$$

and where \hat{R} is $D+1$ Ricci scalar built from this metric. We see that the last form of the Lagrangian (36) can be interpreted as the sum of the General Relativity action with additional potential terms that breaks the full diffeomorphism invariance of the theory. It is important to stress that this potential term depends on the auxiliary fields A and C_i . In principle these terms could be integrated out however we expect that the resulting Lagrangian would be very complicated.

3 Examples of Potentials $L(H^{AB})$

In this section we give two solvable examples of the scalar function $L(H^{AB})$ that allow to find the explicit form of Lagrangian for massive gravity.

In the first case we follow [8] and consider function $L(H^{AB})$ in the form ⁴

$$L = \Lambda + H_{AB} \eta^{BA} . \quad (38)$$

Then it is easy to see that

$$\frac{\delta L}{\delta H_{AB}} = \eta_{BA} \quad (39)$$

and

$$J^{AB} \eta_{BA} = V^{AB} \eta_{BA} - K^{AB} \eta_{BA} . \quad (40)$$

⁴We would like to stress that the similar model with $L = \Lambda + H_{AB} \delta^{AB}$ has been analyzed previously in [22] with results that overlap us.

Using these results it is straightforward exercise to find scalar field Hamiltonian density

$$\mathcal{H}_T^\phi = \frac{\sqrt{\det h}}{16\pi G} (K_{AB}\eta^{BA} + V^{AB}\eta_{BA} + \Lambda) . \quad (41)$$

Then following the general procedure outlined in previous section we find

$$\mathcal{H}_{fix} = \frac{\sqrt{\det h}}{8\pi G} \sqrt{\frac{16\pi G}{\sqrt{\det h}} \mathcal{H}_T^{GR} + \left(\frac{8\pi G}{\sqrt{\det h}}\right)^2 \mathcal{H}_i^{GR} \delta^{ij} \mathcal{H}_j^{GR} + h^{ij} \delta_{ji}} + \Lambda . \quad (42)$$

Finally we find the Lagrangian density of the gauge fixed theory in the form

$$\begin{aligned} \mathcal{L}_{fix} &= \frac{\sqrt{\det h}}{16\pi G} B(\hat{K}_{ij} \mathcal{G}^{ijkl} \hat{K}_{kl} + R^{(D-1)} + \Lambda) + D_i C^i + AB - \\ &- \frac{\sqrt{\det h}}{8\pi G} \sqrt{A + \left(\frac{8\pi G}{\sqrt{\det h}}\right)^2 C_i \delta^{ij} C_j + h^{ij} \delta_{ji}} . \end{aligned} \quad (43)$$

We would like to stress that we can integrate out auxiliary fields A, B, C^i, D_i from the Lagrangian (43) and then to derive the Lagrangian density for dynamical modes h_{ij} only. However the resulting Lagrangian would be very complicated and hence we prefer to work with the extended Lagrangian (43).

As the second example of exactly solvable theory we consider the Lagrangian function $L(H^{AB})$ in the form

$$L(H_{AB}) = \Lambda + \sqrt{\Omega + H^{AB}\eta_{BA}} , \quad (44)$$

where Λ and Ω are constants. For (44) we easily find

$$D_{AB} = \frac{1}{2} \frac{\eta_{BA}}{\sqrt{\Omega + H^{AB}\eta_{BA}}} , \quad \tilde{D}^{AB} = 2\eta^{AB} \sqrt{\Omega + H^{AB}\eta_{BA}} \quad (45)$$

and hence

$$J^{AB}\eta_{BA} = \frac{V^{AB}\eta_{BA} - 4\Omega K_{AB}\eta^{BA}}{1 + 4K_{AB}\eta^{BA}} . \quad (46)$$

After some calculation we obtain the Hamiltonian density for the scalar field in the form

$$\mathcal{H}_T^\phi = \frac{\sqrt{\det h}}{16\pi G} \sqrt{\Omega + V^{AB}\eta_{BA}} \sqrt{1 + 4K_{AB}\eta^{BA}} - \frac{\sqrt{\det h}}{16\pi G} \Lambda . \quad (47)$$

Following the analysis presented in previous section (2) we find the gauge fixed Hamiltonian density in the form

$$\mathcal{H}_{fix} = \frac{\sqrt{\det h}}{16\pi G} \sqrt{1 + \left(\frac{16\pi G}{\sqrt{\det h}}\right)^2 \mathcal{H}_i^{GR} \delta^{ij} \mathcal{H}_j^{GR} - \left(\frac{16\pi G}{\sqrt{\det h}} \mathcal{H}_T^{GR} - \Lambda\right)^2 \frac{1}{\Omega + h^{ij} \delta_{ji}}} \quad (48)$$

and corresponding Lagrangian

$$\begin{aligned}
L_{fix} &= \int d^D \mathbf{x} \left(\frac{1}{16\pi G} \sqrt{-\det \hat{g}} (\hat{R} + \Lambda) + BA + D^i C_i - \right. \\
&\quad \left. - \frac{\sqrt{\det h}}{16\pi G} \sqrt{1 + \left(\frac{16\pi G}{\sqrt{\det h}} \right)^2 C_i \delta^{ij} C_j - \left(\frac{16\pi G}{\sqrt{\det h}} A \right)^2 \frac{1}{\Omega + h^{ij} \delta_{ji}}} \right).
\end{aligned} \tag{49}$$

In this section we gave two explicit examples of Hamiltonians and Lagrangians for massive gravity. Clearly it would be desirable to understand properties of these models further.

4 Conclusion

This paper was devoted to the study of the Higgs mechanism for gravity from the point of view of the Hamiltonian formalism. We performed the fixing of the space-time diffeomorphism and we argued that the resulting Hamiltonian corresponds to the Hamiltonian of the massive gravity. The main advantage of our approach is that this theory is defined on the reduced phase space spanned by the physical degrees of freedom h_{ij}, p^{ij} only. On the other hand the price we pay for this property is that it is difficult to find the form of the gauge fixed Hamiltonian for general potential $L(H^{AB})$. In fact, we are not able to find explicit form of the gauge fixed Hamiltonian for the specific form of the scalar actions introduced in [9, 10, 11, 12, 13, 14]. On the other hand introducing additional auxiliary fields we can determine Lagrangian for massive gravity that has the form of the ordinary General Relativity action with specific potential terms that break diffeomorphism invariance. Then we can ask the question how this Lagrangian is related to the original Lagrangian where we fix the gauge as in [9, 10, 11, 12, 13, 14]. One can hope that these actions could be related by some fields redefinitions. However finding this redefinition seems to be very complicated due to the presence of the auxiliary fields A, C_i in the Lagrangian (36) whose explicit integration out would lead to very obscure form of the Lagrangian.

The next important step in our investigation would be to analyze the spectrum of fluctuations around the flat space-time background. It would be also very interesting to study the classical solutions corresponding to these form of massive gravities. We hope to return to the analysis of these problems in future.

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A Appendix: Algebra of Hamiltonian Constraints

In this appendix we explicitly prove the stability of the constraints (20), or equivalently, we show that these constraints are preserved during the time evolution of the system. In fact, the careful analysis of these constraints was performed in 70's in the geometrodynamics program [15, 16, 17, 18]. It was argued there that for the consistency of theory the Poisson brackets of constraints should have the form

$$\begin{aligned} \{H_S(N^i), H_S(M^i)\} &= H_S(N^i \partial_i M^j - M^i \partial_i N^j) , \\ \{H_S(N^i), H_T(M)\} &= H_T(N^i \partial_i M) , \\ \{H_T(N), H_T(M)\} &= H_S((M \partial_j N - N \partial_j M) h^{ji}) . \end{aligned} \quad (50)$$

The aim of this appendix is to show that the Poisson brackets of constraints (20) obey the algebra (50). In fact, it is well known that the General Relativity constraints

$$H_T^{GR}(N) = \int d^D \mathbf{x} N(\mathbf{x}) \mathcal{H}_T^{GR}(\mathbf{x}) , \quad H_S^{GR}(N^i) = \int d^D \mathbf{x} N^i(\mathbf{x}) \mathcal{H}_i^{GR}(\mathbf{x}) \quad (51)$$

obey the Poisson brackets relations (50). Further in case of scalar field diffeomorphism constraint we easily find

$$\left\{ H_S^\phi(N^i), H_S^\phi(M^i) \right\} = H_S^\phi(N^i \partial_i M^j - M^i \partial_i N^j) . \quad (52)$$

Then using the fact that the mixed Poisson brackets $\left\{ H_S^\phi(N^i), H_S^{GR}(M^i) \right\}$ are trivially zero we find that the Poisson brackets of diffeomorphism constraints obey the first equation in (50).

As the next step we calculate the Poisson bracket between diffeomorphism constraint $H_S(N^i)$ and $H_S^\phi(M)$. Using following Poisson brackets

$$\begin{aligned} \{H_S(N^i), \phi^A\} &= -N^i \partial_i \phi^A , \\ \{H_S(N^i), p_A\} &= -\partial_i (N^i p_A) , \\ \{H_S(N^i), h_{ij}\} &= -N^k \partial_k h_{ij} - \partial_i N^k h_{kj} - h_{ik} \partial_j N^k , \\ \left\{ H_S(N^i), \sqrt{\det h} \right\} &= -N^k \partial_k \sqrt{\det h} - \partial_k N^k \sqrt{\det h} \end{aligned} \quad (53)$$

we easily find

$$\left\{ H_S(N^i), K_{AB} \right\} = -N^k \partial_k K_{AB} , \quad \left\{ H_S(N^i), V^{AB} \right\} = -N^k \partial_k V^{AB} . \quad (54)$$

Collecting all these results we obtain

$$\left\{ H_S(N^i), \mathcal{H}_T^\phi \right\} = -N^k \partial_k \mathcal{H}_T^\phi - \partial_k N^k \mathcal{H}_T^\phi . \quad (55)$$

Finally using the fact that $\{H_S^{GR}(N^i), H_T^{GR}(M)\} = H_T^{GR}(N^i \partial_i M)$ and the equation (55) we find that

$$\{H_S(N^i), H_T(M)\} = H_T(N^i \partial_i M) . \quad (56)$$

As the last step we calculate the Poisson brackets of the Hamiltonian constraints. While it is well known that the Poisson bracket of the GR Hamiltonian constraints takes precisely the form given in (50) it is far from obvious that for the system defined by the action (1) it holds as well. In case of simple scalar action $\sim g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ the result is well known, see for example [15, 16, 17, 18]. Further, it was shown very elegantly in [15] that the scalar field action in the form $L(-g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi)$ leads to the Hamiltonian formulation where the constraints obey the Poisson brackets (50). On the other hand the scalar field Hamiltonian constraint (14) is more general than the form of the Hamiltonian constraints studied in [15] so that we perform the explicit calculation of the Poisson brackets of the smeared form of the Hamiltonian constraints (14) below.

Before we proceed to this calculation we discuss the calculation of the mixed Poisson brackets between the GR Hamiltonian constraint and scalar fields Hamiltonian constraint. The crucial point is that the constraint H_T^ϕ depends on g_{ij} only. Then it is easy to see that

$$\begin{aligned} & \{H_T^{GR}(N), H_T^\phi(M)\} + \{H_T^\phi(N), H_T^{GR}(M)\} = \\ & = \int d^D \mathbf{x} d^D \mathbf{y} N(\mathbf{x}) M(\mathbf{y}) \left(\{ \mathcal{H}_T^{GR}(\mathbf{x}), \mathcal{H}_T^\phi(\mathbf{y}) \} + \{ \mathcal{H}_T^\phi(\mathbf{x}), \mathcal{H}_T^{GR}(\mathbf{y}) \} \right) = 0 \end{aligned} \quad (57)$$

using the fact that $\{ \mathcal{H}_T^\phi(\mathbf{x}), \mathcal{H}_T^{GR}(\mathbf{y}) \} \sim \delta(\mathbf{x} - \mathbf{y})$.

A.1 Poisson Brackets of Hamiltonian Constraints for Scalar Field

In this section we determine the Poisson bracket between $H_T^\phi(N), H_T^\phi(M)$. As a warm example we begin with the single scalar field action

$$S = \int d^D x \sqrt{-g} L(-I/2) , \quad (58)$$

where $I = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$. In $(D - 1) + 1$ formalism this action takes the form

$$S = \int d^D x N \sqrt{\det h} L\left(\frac{1}{2}(\nabla_n^2 \phi - h^{ij} \partial_i \phi \partial_j \phi)\right) . \quad (59)$$

Then it is easy to see that the momentum conjugate to ϕ is equal to

$$p = \sqrt{\det h} L'(I/2) \nabla_n \phi \quad (60)$$

and consequently

$$K = \left[\frac{p}{\sqrt{\det h}} \right]^2 = L'^2(I/2)(I + V) , \quad (61)$$

where $V = h^{ij} \partial_i \phi \partial_j \phi$ and where $L'(x) = \frac{dL(x)}{dx}$. Let us presume that the equation (61) can be solved for I and we denote this solution as $I = J(K, V)$. Then (60) implies

$$\nabla_n \phi = \frac{1}{\sqrt{h} L'(J/2)} \pi \quad (62)$$

and hence the Hamiltonian takes the form

$$H = \int d^d \mathbf{x} (N(\mathbf{x}) \mathcal{H}_T(\mathbf{x}) + N^i(\mathbf{x}) \mathcal{H}_i(\mathbf{x})) , \quad (63)$$

where

$$\mathcal{H}_T = \frac{p^2}{\sqrt{\det h} L'(J/2)} - \sqrt{\det h} L(J/2) . \quad (64)$$

Our goal is to calculate the Poisson bracket $\{H_T^\phi(N), H_T^\phi(M)\}$. To do this we start with the calculation of the following Poisson bracket

$$\begin{aligned} \{\phi, H_T\} &= \int d^d \mathbf{x} \{\phi, N(\mathbf{x}) \mathcal{H}_T(\mathbf{x})\} = N \frac{2p}{\sqrt{\det h} L^2(J/2)} - \\ &- N \frac{p^3}{(\sqrt{\det h})^3} \frac{L''(J/2)}{L^2(J/2)} \frac{\delta J}{\delta K} - N \frac{p}{\sqrt{\det h}} L'(J/2) \frac{\delta J}{\delta K} , \end{aligned} \quad (65)$$

where we used

$$\{\phi(\mathbf{x}), J(\mathbf{y})\} = 2 \frac{p}{\det h} \frac{\delta J}{\delta K}(\mathbf{x}) \delta(\mathbf{x} - \mathbf{y}) . \quad (66)$$

To proceed further we note that J obeys the equation

$$K = L^2(J/2)(J + V) . \quad (67)$$

When we difference this equation with respect to K and use the fact that $J = J(K, V)$ we find

$$K L''(J/2) \frac{\delta J}{\delta K} = L'(J/2) \left(1 - L^2(J/2) \frac{\delta J}{\delta K} \right) . \quad (68)$$

Inserting this result into (65) and performing the appropriate manipulation we find that the Poisson bracket (65) takes the form

$$\{\phi, H_T(N)\} = N \frac{\pi}{\sqrt{\det h} L'(J/2)} = N \nabla_n \phi \quad (69)$$

that agrees with the result derived in [15].

As the next step we determine the Poisson bracket between p and $H_T(N)$

$$\begin{aligned} \{p, H_T(N)\} &= \int d^d \mathbf{x} N(\mathbf{x}) \times \\ &\times \left(-\frac{p^2}{2\sqrt{\det h} L'^2(J/2)} L''(J/2) \frac{dJ}{dV}(\mathbf{x}) \{p, V(\mathbf{x})\} - \frac{1}{2} \sqrt{\det h} L'(J/2) \frac{\delta J}{dV}(\mathbf{x}) \{p, V(\mathbf{x})\} \right). \end{aligned} \quad (70)$$

With the help of the relation (67) we find

$$K L''(J/2) \frac{dJ}{dV} = -L^3(J/2) \left(1 + \frac{dJ}{dV} \right), \quad \frac{dJ}{dV} = -L' \frac{dJ}{dK}. \quad (71)$$

Inserting these expressions into (70) we find

$$\{p, H_T(N)\} = \frac{1}{2} \int d^d \mathbf{x} N \sqrt{\det h} L'(\mathbf{x}) \{p, V(\mathbf{x})\} = \partial_i [N \sqrt{\det h} h^{ij} L' \partial_j \phi], \quad (72)$$

where we also used

$$\{p(\mathbf{x}), V(\mathbf{y})\} = -2h^{ij}(\mathbf{y}) \partial_{y^j} \phi(\mathbf{y}) \partial_{y^i} \delta(\mathbf{x} - \mathbf{y}). \quad (73)$$

Now we are ready to determine the Poisson bracket $\{H_T(N), H_T(M)\}$. To do this we consider following expression

$$\{\{\phi, H_T(N)\}, H_T(M)\} - \{\{\phi, H_T(M)\}, H_T(N)\}. \quad (74)$$

In the first step we use (69) and we find

$$\left\{ N \frac{\pi}{\sqrt{\det h} L'(J/2)}, H_T(M) \right\} - \left\{ M \frac{\pi}{\sqrt{\det h} L'(J/2)}, H_T(N) \right\}. \quad (75)$$

Clearly all terms in (75) that are not proportional to the partial derivatives of N, M cancel. To proceed further in the calculation of (75) we have to calculate following Poisson bracket

$$\begin{aligned} \left\{ \frac{1}{L'}, H_T(N) \right\} &= -2N \frac{L''(J/2)}{L'^2(J/2)} \frac{dJ}{dK} \times \\ &\times \left(-L'(J/2) h^{ij} \partial_i \phi \partial_j \left[\frac{p}{\sqrt{\det h} L'(j/2)} \right] + \frac{p}{\det h} \partial_i [\sqrt{\det h} h^{ij} \partial_j \phi] \right). \end{aligned} \quad (76)$$

Then it is easy to see that $\frac{\pi}{\sqrt{\det h}} (M \{\frac{1}{L'}, H_T(N)\} - N \{\frac{1}{L'}, H_T(M)\}) = 0$. Finally with the help of (72) we find that (74) is equal to

$$\{\{\phi, H_T(N)\}, H_T(M)\} - \{\{\phi, H_T(M)\}, H_T(N)\} = (N \partial_i M - M \partial_i N) h^{ij} \partial_j \phi. \quad (77)$$

With the help of the Jacobi identity we can rewrite this expression into the form

$$\begin{aligned} \{\phi, \{H_T(N), H_T(M)\}\} &= (N\partial_i M - M\partial_i N)h^{ij}\partial_j\phi = \\ &= \{\phi, H_S(N\partial_i M - M\partial_i N) + f(\phi)\} \end{aligned} \quad (78)$$

for arbitrary function $f(\phi)$. On the other hand performing the same step with π we find that f does not depend on ϕ as well and hence could be taken to vanish at least at classical level. In summary, we proved that the Poisson bracket of the Hamiltonian constraints of the scalar field with general action (58) takes the desired form

$$\{H_T(N), H_T(M)\} = H_S((N\partial_i M - M\partial_i N)h^{ij}) . \quad (79)$$

Now we are ready to proceed to the calculation of the Poisson bracket of the Hamiltonian constraint when the action for the scalar fields is given in (1). We again start with the calculation of the Poisson bracket between ϕ^X and H_T^ϕ

$$\begin{aligned} \{\phi^X, H_T^\phi\} &= N\sqrt{\det h} \left(2\{\phi^X, K_{AB}\} \tilde{D}^{BA} - \right. \\ &\quad \left. - 2K_{AB}\tilde{D}^{BC} \frac{\delta D_{CD}}{\delta K_{MN}} \tilde{D}^{DA} \{\phi^X, K_{MN}\} + D_{AB} \frac{\delta J^{BA}}{\delta K_{MN}} \{\phi^X, K_{MN}\} \right) . \end{aligned} \quad (80)$$

To proceed we note that the equation (12) implies following relation

$$K_{AB} = D_{AC}(V^{CD} - J^{CD})D_{DB} , \quad (81)$$

where D_{AB} depends on J . Then we calculate the Poisson bracket between ϕ^X and the relation given above, multiply the result with \tilde{D}^{BA} and finally take the traces over capital indices. As a result we find

$$\tilde{D}^{BA} \{\phi^X, K_{AB}\} = 2\tilde{D}^{BA} \frac{\delta D_{AC}}{\delta K_{MN}} \{\phi^X, K_{MN}\} \tilde{D}^{CD} K_{DB} + \frac{\delta J^{BD}}{\delta K_{MN}} D_{DB} \{\phi^X, K_{MN}\} . \quad (82)$$

Using this result in (80) we find

$$\{\phi^X, H_T^\phi(N)\} = N \frac{1}{2\sqrt{\det h}} \tilde{D}^{XB} p_B = N \nabla_n \phi^X . \quad (83)$$

In the similar way we proceed with the calculation of the Poisson bracket between p_X and H_T^ϕ

$$\{p_X, H_T^\phi(N)\} = 2\partial_i [\sqrt{\det h} h^{ij} N D_{XA} \partial_j \phi^A] , \quad (84)$$

where we used the Poisson bracket between p_X and (81) that implies

$$2K_{PB}\tilde{D}^{BA}\frac{\delta D_{AC}}{\delta J^{MN}}\frac{\delta J^{MN}}{\delta V^{PQ}}\{p_X, V^{PQ}\} = \frac{\delta J^{CD}}{\delta V^{PQ}}D_{DC}\{p_X, V^{PQ}\} - \{p_X, V^{CD}\}D_{DC} . \quad (85)$$

As in case of single scalar field we consider following expression

$$\left\{ \left\{ \phi^A, H_T^\phi(N) \right\} H_M^\phi(M) \right\} - \left\{ \left\{ \phi^A, H_T^\phi(M) \right\} H_T^\phi(N) \right\} , \quad (86)$$

where the first double Poisson bracket is equal to

$$\begin{aligned} \left\{ \left\{ \phi^A, H_T^\phi(N) \right\} H_M^\phi(M) \right\} &= \frac{N}{\sqrt{\det h}}\tilde{D}^{AB}\partial_i[M\sqrt{\det h}h^{ij}D_{BC}\partial_j\phi^C] - \\ - \frac{N}{\sqrt{\det h}}\tilde{D}^{AC} &\left(\frac{\delta D_{CD}}{\delta J^{MN}}\frac{\delta J^{MN}}{\delta K_{PQ}}\left\{ K_{PQ}, H_T^\phi(M) \right\} + \frac{\delta D_{CD}}{\delta J^{MN}}\frac{\delta J^{MN}}{\delta V^{PQ}}\left\{ V^{PQ}, H_T^\phi(M) \right\} \right) \tilde{D}^{DB}p_B . \end{aligned} \quad (87)$$

To proceed further we have to determine the Poisson bracket between D_{AB} and $H_T^\phi(N)$. Note that when we compare the variation of (81) with respect to K_{MN} with the variation of (81) with respect to V_{MN} we find following relation

$$\frac{\delta J^{MN}}{\delta V^{PQ}} = -\frac{\delta J^{MN}}{\delta K_{RS}}D_{RP}D_{SQ} . \quad (88)$$

Then using this relation in (87) and after some manipulation we find

$$\left\{ \left\{ \phi^A, H_T^\phi(N) \right\} H_T^\phi(M) \right\} - \left\{ \left\{ \phi^A, H_T^\phi(M) \right\} H_T^\phi(N) \right\} = (N\partial_i M - M\partial_i N)h^{ij}\partial_j\phi^A . \quad (89)$$

Performing the same analysis with conjugate momenta p_A we find the desired result

$$\left\{ H_T^\phi(N), H_T^\phi(M) \right\} = H_S^\phi((N\partial_i M - M\partial_i N)h^{ij}) . \quad (90)$$

The upshot of this long analysis is the proof that the Poisson brackets of Hamiltonian constraints $H_T(N) = H_T^{GR}(N) + H_T^\phi(N)$ take exactly the same form as in (50). Using this fact we immediately obtain that the Hamiltonian and diffeomorphism constraints are preserved during the time evolution of the system. This is very important result since only after determining the complete constraint structure of given theory it is possible to perform the Hamiltonian gauge fixing.

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