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Extending Chubanov algorithm to generic linear program: strong polynomial time in not degenerated cases ?

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Abstract

This short paper presents an algorithm which aims to solve generic linear program ($Ax \geq b$ or $\min_{Ax \geq b} cx$) using simple projections and call to a sub solver dedicated to linear feasibility ($Ax = \mathbf{0}, x > \mathbf{0}$), recently, proven to be a strongly polynomial problem thank to Chubanov algorithm.

Then, a complexity analysis states that this algorithm is strongly polynomial in not degenerated cases.

Despite surely containing faults, this complexity analysis and this linear feasibility based algorithm call for a discussion.

1 Introduction

Interior point algorithms (e.g. [3]) are mainly the state of the art of linear program solver. But, it is not known if interior point algorithms solve linear program in strong polynomial time, and, [1] shows that major interior point family do not. Inversely, linear feasibility can now be solved in strong polynomial time thank to Chubanov algorithm [2]. But, the question about the possibility to extend this algorithm to generic linear program has not been very explored.

This short paper focus on this way, and, presents an algorithm designed to solve linear programming by solving a sequence of linear feasibility (section 2). A complexity analysis (section 3) is provided and *concludes* that the algorithm solves linear program in polynomial time on \mathbb{Q} (i.e. it solves linear program in a polynomial numbers of unitary operations read, write, test, +, -, \times , / in \mathbb{Q}).

Du to the importance of this apparent conclusion, this paper surely contains faults for which I apologize. Yet, it calls for discussion and proof consolidation.

Notation : As classically, \mathbb{N}, \mathbb{Q} are the sets of integer and rational numbers, $\forall I, J \in \mathbb{N} \setminus \{0\}$, \mathbb{Q}^I is the set of I dimensional vector of \mathbb{Q} , and, $\mathcal{M}_{J,I}(\mathbb{Q})$ is the set of matrix with J rows and I columns, with values in \mathbb{Q} . $\mathbb{U}(\mathbb{Q}^I)$ is the set of normalized vector from \mathbb{Q}^I (i.e. such that $v^T v = 1$). $\mathbb{U}(\mathcal{M}_{J,I}(\mathbb{Q}))$ will be

the set of matrix from $\mathcal{M}_{J,I}(\mathbb{Q})$ whose rows are in $\mathbb{U}(\mathbb{Q}^I)$ (only the rows, not necessarily the columns). Also, if $A \in \mathcal{M}_{J,I}(\mathbb{Q})$, then, the null vector space of A (i.e. the kernel) is written $Ker(A) = \{v \in \mathbb{Q}^I / Av = \mathbf{0}\}$.

Less classically, integer index range would be omitted when no confusion is possible, and, $\forall i$ (range omitted), x_i is i component of the vector (or i row of a matrix). A_S, b_S will be the sub matrix/vector obtained when keeping only rows/components $s \in S$. Also, if p and q are two vectors of \mathbb{Q}^I , transposition is omitted in scalar product i.e. pq would be $p^T q = \sum_i p_i \times q_i$.

2 Algorithm

The algorithm relies on a two key requirements described in section 4: any linear program is casted into a special class of linear program, and algorithm can call a sub solver used to solve linear feasibility problem. So the algorithm decrived here is mainly an iterative way to select sub solver queries.

2.1 Pseudo code

Let assume the input linear program is $\min_{Ax \geq b} cx$ with $A \in \mathbb{U}(\mathcal{M}_{M,N}(\mathbb{Q}))$, $b \in \mathbb{Q}^M$, $c \in \mathbb{U}(\mathbb{Q}^N)$, $Ac = \frac{3}{5}\mathbf{1}$, x_{start} being a trivial admissible point e.g. $(1 + \frac{5}{3} \max_m b_m)c$, and, cx being bounded by 0. The pseudo code the algorithm is:

1. compute $d = \min_m A_m x - b_m$
2. compute $D = \{m / A_m x - b_m = d\}$ the set of constraints at distance d
3. let v be the orthogonal projection of $-c$ on $Ker(A_D)$
4. if $cv < 0$, $A_D v = \mathbf{0}$ (basically if $v \neq 0$)
 - (a) compute $g = \min_{m/A_m v < 0} \frac{A_m x - b_m - d}{-A_m v}$
 - (b) let $x = x + gv$ and GO TO 1
5. call sub solver $\exists? w / \begin{pmatrix} A_D \\ -c \end{pmatrix} w > \mathbf{0}$
6. let $z = x - \frac{5d}{3}c$
7. if w exists
 - (a) $w = \frac{1}{\min_{m \in D} A_m w} w$ ($\min_{m \in D} A_m w$ is now 1) and $h = \min_{m/A_m w < 0} \frac{A_m z - b_m}{-A_m w}$
 - (b) let $x = z + \frac{h}{4}w$
 - (c) GO TO 1
8. return z and the certificate

2.2 Termination

2.2.1 Basic properties

Steps 1, 2, 5, 6, 8 are well defined.

Step 3 is well defined because D (step 2) can not be empty.

Now, by assumption cx is bounded i.e. $\forall x/Ax \geq b, cx \geq 0$. So, if there is y such that $cy < 0$ and $\forall m, A_my \geq 0$, then, one could produce an unbounded admissible point $x + \lambda y$ as $A(x + \lambda y) \geq Ax \geq b$ and $c(x + \lambda y) \xrightarrow{\lambda \rightarrow \infty} -\infty$. So, in step 4 and 7 $\{m/A_mv < 0\}$ can not be empty.

So, all steps are well defined (at this point, it can do stupid thing or looping but at least, it is well defined).

The algorithm is well defined.

In step 4, let m such that $A_mv < 0$, seeing test leading to step 4 it implies that $m \notin D$, and so that $A_mx - b_m > d$ by definition of d, D . So $g > 0$.

So, $\frac{A_mx - b_m - d}{-A_mv} \geq g > 0$ And, so $\frac{A_mx - b_m - d}{-A_mv} A_mv \leq g A_mv$ (product by a negative). So, $A_mx + g A_mv - b_m \geq A_mx + \frac{A_mx - b_m - d}{-A_mv} A_mv - b_m = d > 0$.

Also, if $A_mv \geq 0$, than $A_m(x + gv) - b_m \geq A_mx - b_m \geq d > 0$.

In step 7, let m such that $A_mw < 0$, seeing test leading to step 7 it implies that $m \notin D$, which implies that $A_mx - b_m > d$, and so that $A_mz - b_m > 0$ as $z = x - \frac{5d}{3}c$ and by assumption $A_mc = \frac{3}{5}$. So $h > 0$.

Again, $A_mz + \frac{1}{4}h A_mw - b_m \geq A_mz + \frac{1}{4} \frac{A_mz - b_m}{-A_mw} A_mw - b_m = \frac{3}{4}(A_mz - b_m) > 0$

Also, if $A_mw = 0$, $A_m(x + hw) - b_m = A_mx - b_m > 0$. And, if $A_mz - b_m = 0$, $A_mw > 0$ so $A_m(x + hw) - b_m = h A_mw > 0$.

So steps 4 and 7 make x to be in the inner space $x/Ax > b$. Then, z is admissible because x is admissible and z being built from d is admissible if x is. Let stress that by assumption $Ac = \frac{3}{5}\mathbf{1}$ - moving along c decreases equally the distance to each constraint - remember that every row and c are normalized.

The algorithm works with inner points, and, if not looping, outputs an admissible point.

Now, obviously, all moves strictly decreases cx (because by construction $vc < 0$ and $cw < 0$ and $g, h \neq 0$ - minimization excluding index from D). Let assume the algorithm returns a non optimal admissible point. Then, the step 5 just before returning should have return something. Indeed, let consider $\theta = x^* + \frac{cz}{2}c - z$ with x^* being a solution (i.e. optimal). As, x^* verifies $Ax^* \geq b$, let define $\rho = x^* + \frac{cz}{2}c$. Then, ρ verifies $A\rho > b$ but still $c\rho = \frac{cz}{2} < cz$ (because $cz > cx^*$ otherwise it would have been a solution). So, $c(\rho - z) > 0$ and $A_D(\rho - z) = A_D\rho > b_D$ (since $A_Dz = 0$ and $A_D\rho > b$). So, $\theta = x^* + \frac{cz}{2}c - z$ was a possible solution for the sub solver (precisely of $\begin{pmatrix} A_D \\ -c \end{pmatrix} w \geq \mathbf{0}$ see lemma from section 4 to extract a solution for sub solver).

The algorithm, if not looping, returns a solution.

2.2.2 Breaking the loop

First, each step 4 leads to strictly increase D (from ensemble point of view) because g is defined such that a new m will enter to D . And, D is bounded by $\{1, \dots, M\}$. So algorithm can not loop forever in steps 1, 2, 3, 4.

It can not happen more than M consecutive loops with steps 1, 2, 3, 4 only.

Now imagine algorithm reaches two time the step 5 with twice the same value for D (in point x_1 and x_2). Then, let consider the point z_1 and z_2 from step 4. Then, it holds both $cz_2 < cz_1$ and $A_D z_2 = A_D z_1 = \mathbf{0}$. So, step 3 should at least have returned $z_2 - z_1$, and, the algorithm should not have pass the step 4 when meeting x_1 .

Underlying idea is that algorithm explores the $\ker(A_D)$ to increase D , then when D is big enough, the algorithm moves bellow a corner of A_D . As D was big enough at this step, it means that projection on $\ker(A_D)$ is null ($\ker(A_D)$ may be not null) and that D will never be reached again.

D can not be twice the same when entering step 5.

Yet, number of D is bounded by 2^M . So it can not have more than 2^M step 5 and no more than M consecutive loops 1, 2, 3, 4 + GO TO 1. So, it can not happen more than $M2^M$ iterations.

The algorithm terminates.

3 Complexity analysis

3.1 Forgotten constraints

Let consider $p(y, S)$ the result of

1. compute $d = \min_{m \in S} A_m y - b_m$
2. compute $D = \{m \in S / A_m y - b_m = d\}$
3. let v be the orthogonal projection of $-c$ on $\ker(A_D)$
4. if $v = 0$ returns "null", y, D
5. if $\{m \in S / A_m v < 0\} = \emptyset$ return "unbounded", y, D, v
6. compute $g = \min_{m \in S / A_m v < 0} \frac{A_m y - b_m - d}{-A_m v}$
7. let $y = y + gv$
8. GO TO 1

$p(y, S)$ represents the first part of the algorithm applied on the sub matrix A_S from y .

Let assume the main algorithm meets x, D in step 7.b and let write $y = z + \frac{h}{4}w$, then let consider $p(y, D)$. This is what would happen by continuing the main algorithm from y only considering A_D .

$p(y, D)$ can not return in step 4 (of p) because otherwise q the returned point verifies $q - \frac{5d}{3}c \in \ker(A_D)$ which is impossible otherwise x should not have passed the step 4 of the algorithm. This is the basic argument of the termination: $\ker(A_D)$ has been totally explored before entering in step 5. Let stress that $q - \frac{5d}{3}c$ may not be an admissible point regarding $Ax \geq b$ but anyway, this proves that one can build a point in $\ker(A_D)$ so x should not have passed the step 4 of the algorithm.

So p will finish by returning in step 5 (of p) - looping is impossible as D increases strictly.

Let Δ be the resulting D when leaving p algorithm. It means that projection of $-c$ on $\ker(A_\Delta)$ is not null. So, $\Delta \neq D$ (because if D is met in step 7 it means again that projection of $-c$ on $\ker(A_D)$ is null).

For all x, D met in the main algorithm in step 7.b, let $y = z + \frac{h}{4}w$ and Δ built by $p(y, D)$, then Δ is a strict subset of D .

3.2 Breaking the loop earlier

In this subsection, I consider a (quite strong) *not degenerated hypothesis*:

Not degenerated hypothesis: For all x, D met in the main algorithm in step 7.b, let $y = z + \frac{h}{4}w$ and Δ built by $p(y, D)$, then $\forall k \in D \setminus \Delta$, there do not exist θ such that $A_k \theta = b_k$, $A \theta \geq b$, $c \theta < cz$.

This hypothesis is that z (step 6 of the main algorithm) is corner from which it is impossible to be closer to the constraint $k \in D \setminus \Delta$ than from the constraint kept in Δ . In other words, the idea is that if

- $Ac = \frac{3}{5}\mathbf{1}$
- $\exists w/Aw > \mathbf{0}, cw < 0$
- $\nexists v/Av = \mathbf{0}, cw < 0$

there is at least k such that $\nexists v_k/Av_k \geq \mathbf{0}, cv_k < 0$ with in addition $A_k v_k = 0$.

In other words, if w goes away from all, then it goes away from one even faster than from the other. This hypothesis is somehow related to the fact that z is a maximal *dimensionality* vertex.

Now, let write x_t, D_t be x, D the t times that algorithm enters in step 7.b, then let write y_t and Δ_t the related y and Δ .

If $k \in (D_t \setminus \Delta_t) \cap D_{t+\tau}$, then let consider the projection of $x_{t+\tau}$ on the constraint k i.e. $x_{t+\tau} - d_{t+\tau}A_k$. By definition of $d_{t+\tau}$, $A_m(x_{t+\tau} - d_{t+\tau}A_k) \geq$

$d_{t+\tau} - d_{t+\tau} A_m A_k > 0$ except for $k = m$ for which $A_k(x_{t+\tau} - d_{t+\tau} A_k) = 0$. So, one can build a direct contradiction to the *not degenerated hypothesis*

So, under the *not degenerated hypothesis*, $\forall t, k \in D_t \setminus \Delta_t, \forall \tau, k \notin D_{t+\tau}$.

Under not degenerated hypothesis $\forall t, \tau, (D_t \setminus \Delta_t) \cap D_{t+\tau} = \emptyset$.

So Δ_t can not be empty, $\Delta_i \cap \Delta_j = \emptyset$ (because $\Delta_t \subset D_t$ for all t and $\Delta_t \cap D_{t+\tau} = \emptyset$) and, finally, $\bigcup_t \Delta_t$ is bounded by $\{1, \dots, M\}$.

So the sequence of D_t has at most M elements. Worse case is when each Δ_t are singleton.

Under not degenerated hypothesis, the algorithm can not do more than M call to the subsolver.

As the algorithm can not do more than M loop without calling the subsolver, and, as loops are either simple projection or call to a strongly polynomial time algorithm, then the algorithm is strongly polynomial itself. More precisely, the algorithm is polynomial on \mathbb{Q} . Still, the size of the integer observed during the algorithm may become exponential. But by considering $+, -, \times, /$ on \mathbb{Q} to be unitary operation (false assumption on real computer but acceptable in theory), the algorithm is polynomial.

Under not degenerated hypothesis, the algorithm is strongly polynomial.

4 Lemmas

This section provide lemmas about the required assumption on linear program and the link between sub solver and Chubanov algorithm.

4.1 Linear feasibility and linear separability

[2] presents an algorithm to solve in strongly polynomial time the following problem: $\exists? x \in \mathbb{Q}^N / Ax = \mathbf{0}, x > \mathbf{0}$ under the assumption that $A \in \mathcal{M}_{M,N}(\mathbb{Q})$ has a rank of M .

Let \mathcal{A} a matrix without any assumption. Let consider the matrix $A = \begin{pmatrix} \mathcal{A} & -\mathcal{A} & -I \end{pmatrix}$ formed with \mathcal{A} concat with $-\mathcal{A}$ concat with $-I$ the opposite of identity matrix.

First, this matrix has full rank M due to the identity block.

Then, applying the Chubanov algorithm (or any linear feasibility solver) to this matrix A will lead (if a solution exists) to x_1, x_2, x_3 such that

$$\begin{pmatrix} \mathcal{A} & -\mathcal{A} & -I \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \mathbf{0}$$

and $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} > \mathbf{0}$ So, let $x = x_1 - x_2$, it holds that $\mathcal{A}x = Ix_3 = x_3 > \mathbf{0}$. And, if no solution exists, then Chubanov algorithm will provide a certificate.

Chubanov algorithm can be applied on a derived problem to solve linear separability $\exists?x \in \mathbb{Q}^N / \mathcal{A}x > \mathbf{0}$ with $\mathcal{A} \in \mathcal{M}_{M,N}(\mathbb{Q})$ (under no assumption on \mathcal{A} , especially on the rank).

4.2 Normalizing linear program

If the linear program given as input is $\min_{Ax \geq b} cx$ and verifies $A \in \mathcal{U}(\mathcal{M}_{J,I}(\mathbb{Q}))$, $b \in \mathbb{Q}^M$ with $i \neq j \Rightarrow (A_i, b_i) \neq (A_j, b_j)$, $c \in \mathcal{U}(\mathbb{Q}^N)$ and $Ac = \gamma \mathbf{1}$ with $\gamma > 0$, and, cx being bounded by 0, then the offered algorithm of section 2 can be directly used.

Otherwise, the linear program has to be normalized with the following scheme:

1. If the linear program is as an optimisation problem (e.g. $\max_{Ax \leq b, x \geq \mathbf{0}} cx$), it should first be converted into a inequality system $A'x \geq b'$. This could be done by combining primal and dual.
2. After that (or directly if input was an inequality system), an other normalisation is performed to reach required property (here with $\gamma = \frac{3}{5}$)
3. the important point is that from any linear program, pre processing can form an equivalent linear program meeting these requirements
4. In addition, if needed redundant rows are removed (trivial pre processing). Currently, algorithm can just be updated to directly handle identical rows. But this pre processing is that trivial that it is better to perform it: if $A_i = A_j$ after all normalisation, either $b_i = b_j$ and row j can be removed either the row with smaller b can be removed, because $A_i x \geq b_i > b_j \Rightarrow A_i x \geq b_j \dots$

4.2.1 Primal dual

The conversion of an optimisation linear program into a linear inequality system is quite classical. A brief recall is provided bellow.

Let assume original goal is to solve $\max_{A_{raw}x \leq b_{raw}, x \geq \mathbf{0}} c_{raw}x$. It is well known that the dual problem is $\min_{A_{raw}^T y \geq c_{raw}, y \geq \mathbf{0}} b_{raw}y$. Now, the primal dual is formed by combining all constraints: $A_{raw}x \leq b_{raw}$, and, $x \geq \mathbf{0}$, and, $A_{raw}^T y \geq c_{raw}$, and $c_{raw}x = b_{raw}y$, and finally, $y \geq \mathbf{0}$.

So, the problem $\max_{A_{raw}x \leq b_{raw}, x \geq 0} c_{raw}x$ can be folded into $A_{big}x_{big} \geq b_{big}$ with

$$A_{big} = \begin{pmatrix} -A_{raw} & 0 \\ I & 0 \\ 0 & A_{raw}^T \\ 0 & I \\ c_{raw} & -b_{raw} \\ -c_{raw} & b_{raw} \end{pmatrix} \text{ and } b_{big} = \begin{pmatrix} -b_{raw} \\ 0 \\ c_{raw} \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

4.2.2 Normalized primal dual error minimization

This normalisation step takes a linear program $\Gamma\chi \geq \beta$ as input, and, produces an equivalent linear program $\min_{Ax \geq b} cx$ with $A \in \mathbb{U}(\mathcal{M}_{J,I}(\mathbb{Q}))$, $b \in \mathbb{Q}^M$, $c \in \mathbb{U}(\mathbb{Q}^N)$, and, $Ac = \frac{3}{5}\mathbf{1}$, and, cx being bounded by 0.

It is sufficient to consider:

$$A = \begin{pmatrix} \frac{4}{5(\frac{\Gamma_1\Gamma_1}{2}+1)}\Gamma_1 & \frac{4}{5(\frac{\Gamma_1\Gamma_1}{2}+1)}\frac{\Gamma_1\Gamma_1}{2} & \frac{4}{5(\frac{\Gamma_1\Gamma_1}{2}+1)} & \frac{3}{5} \\ \dots & \dots & \dots & \dots \\ \frac{4}{5(\frac{\Gamma_M\Gamma_M}{2}+1)}\Gamma_M & \frac{4}{5(\frac{\Gamma_M\Gamma_M}{2}+1)}\frac{\Gamma_M\Gamma_M}{2} & \frac{4}{5(\frac{\Gamma_M\Gamma_M}{2}+1)} & \frac{3}{5} \\ \mathbf{0} & \frac{4}{5} & 0 & \frac{3}{5} \\ \mathbf{0} & -\frac{4}{5} & 0 & \frac{3}{5} \\ \mathbf{0} & 0 & \frac{4}{5} & \frac{3}{5} \\ \mathbf{0} & 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

and

$$b = \begin{pmatrix} \frac{4}{5(\frac{\Gamma_1\Gamma_1}{2}+1)}\beta_1 \\ \dots \\ \frac{4}{5(\frac{\Gamma_M\Gamma_M}{2}+1)}\beta_M \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad c = \begin{pmatrix} 0 \\ \dots \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

First, the produced linear program is in the desired form: $\min_{Ax \geq b} cx$ with $A \in \mathbb{U}(\mathcal{M}_{J,I}(\mathbb{Q}))$, $b \in \mathbb{Q}^M$, $c \in \mathbb{U}(\mathbb{Q}^N)$, and, $Ac = \frac{3}{5}\mathbf{1}$.

Trivially, $Ac = \frac{3}{5}\mathbf{1}$ by construction, and, all rows of A are normalized either directly because $(\frac{4}{5})^2 + (\frac{3}{5})^2 = 1$, or, because of that, and the fact that, $(\frac{1}{\frac{\Gamma_m\Gamma_m}{2}+1})^2\Gamma_m\Gamma_m + (\frac{1}{\frac{\Gamma_m\Gamma_m}{2}+1})^2\frac{(\Gamma_m\Gamma_m)^2}{4} + (\frac{1}{\frac{\Gamma_m\Gamma_m}{2}+1})^2$ is $(\frac{1}{\frac{\Gamma_m\Gamma_m}{2}+1})^2 \times (\Gamma_m\Gamma_m + \frac{(\Gamma_m\Gamma_m)^2}{4} + 1)$ which is $(\frac{1}{\frac{\Gamma_m\Gamma_m}{2}+1})^2 \times (\frac{\Gamma_m\Gamma_m}{2} + 1)^2$ which is 1!

Then, the 4 last constraint prevent x_{N+3} to be negative so cx is well bounded by 0. Indeed, if $x_{N+2} + x_{N+3} \geq 0$ and $-x_{N+2} + x_{N+3} \geq 0$, then $x_{N+3} \geq 0$, and, when $x_{N+3} = 0$ these constraints force $x_{N+1} = x_{N+2} = 0$ because the three constraints $x_{N+2} + x_{N+3} \geq 0$, $-x_{N+2} + x_{N+3} \geq 0$, and $x_{N+3} = 0$ can be reduced to $x_{N+2} \geq 0$ and $-x_{N+2} \geq 0$ which force $x_{N+2} = 0$.

Now, the goal is to minimize $cx = x_{N+3}$. So, either the minimum is $x_{N+3} = 0$ or either there is no such solution. In the case $Ax \geq b, x_{N+3} = 0$, it holds $x_{N+1} = x_{N+2} = x_{N+3} = 0$, and, $x_1, \dots, x_N = \chi$ with $\frac{4}{5(\frac{\Gamma_m \Gamma_m}{2} + 1)} \Gamma_m \chi \geq \frac{4}{5(\frac{\Gamma_m \Gamma_m}{2} + 1)} \beta_m$. But this last inequality can be reduced to $\Gamma_m \chi \geq \beta_m$. So, if the solution of the derived linear program is x with $Ax \geq b, x_{N+3} = 0$, then $x_1, \dots, x_N = \chi$ is a solution of the original set of inequality.

And, inversely, if there is a solution χ , then, $x = \chi, 0, 0, 0$ is a solution of the optimisation problem (because x_{N+3} is bounded by 0).

So, this derived linear program is equivalent to the inequality set.

For any linear program, it is possible to create a derived form meeting the requirement of the offered algorithm.

All this normalization is entirely done in \mathbb{Q} i.e. no square root are needed.

Discussion

Seeing the very important result, this short paper claims to prove, it surely contains some wrong statements.

But, this short paper calls for a discussion. First, because, the offered algorithm behaves like expected in toy numerical experiments (including Klee-Minty cube). And, because, this way of solving linear program may be updated to reach strong polynomial time algorithm for linear programming.

References

- [1] Xavier Allamigeon, Pascal Benchimol, Stéphane Gaubert, and Michael Joswig. Log-barrier interior point methods are not strongly polynomial. *SIAM Journal on Applied Algebra and Geometry*, 2(1):140–178, 2018.
- [2] Sergei Chubanov. A polynomial projection algorithm for linear feasibility problems. *Mathematical Programming*, 153(2):687–713, 2015.
- [3] Yurii Nesterov and Arkadii Nemirovskii. *Interior-point polynomial algorithms in convex programming*, volume 13. Siam, 1994.